Instituto Complutense de Análisis Económico

Inequalities in the longevity of females and males aged 65+ in Spain, 2008-2021. The role of pension income.

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Abstract

Studies using pensions and/or pensionable income as proxies for life expectancy are few and far between. This paper looks at Spain, a country for which until recently very little was known about the life expectancy of pensioners by pension income (PI) level. We use a large administrative data set to estimate inequalities in longevity among pensioners grouped according to their PI levels. We present the results for mortality trends among retirement pensioners aged 65 and over for nine rolling windows covering six years each for the period 2008-2021. We find that life expectancy by PI level at ages 65 (LE_{65}) has a positive link with the PI level for both males and females, and this is true for all the periods analysed. The absolute differences in LE_{65} between pensioners in the highest and the lowest PI groups fluctuate across the nine rolling windows examined. For males, the differences increase from the beginning (2.67 years) until 4.06 years and then tend to decrease (2.83 years). For females there are also fluctuations (between 1.84 and 2.67 years), but the absolute differences are always smaller than those observed for males. Another finding is that socioeconomic inequality in longevity by PI group is lower when measured with M_{65} than LE_{65}. We also find that the pensioner population seems to have been more affected by the COVID-19 pandemic than the general population and that two groups of pensioners – the lowest PI group of males and the highest PI group of females – appear to have improved their longevity during the pandemic.

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KEYWORDS: Continuous Sample of Working Lives; Mortality; Public Pension System; Retirement; Spain.

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Inequalities in the longevity of females and males aged 65+ in Spain, 2008-2021. The role of pension income.

Desigualdades en la longevidad de mujeres y hombres mayores de 65 años en España, 2008-2021. El papel de los ingresos por pensiones.

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Abstract

Studies using pensions and/or pensionable income as proxies for life expectancy are few and far between. This paper looks at Spain, a country for which until recently very little was known about the life expectancy of pensioners by pension income (PI) level. We use a large administrative data set to estimate inequalities in longevity among pensioners grouped according to their PI levels. We present the results for mortality trends among retirement pensioners aged 65 and over for nine rolling windows covering six years each for the period 2008-2021. We find that life expectancy by PI level at ages 65 (LE_{65}) has a positive link with the PI level for both males and females, and this is true for all the periods analysed. The absolute differences in LE_{65} between pensioners in the highest and the lowest PI groups fluctuate across the nine rolling windows examined. For males, the differences increase from the beginning (2.67 years) until 4.06 years and then tend to decrease (2.83 years). For females there are also fluctuations (between 1.84 and 2.67 years), but the absolute differences are always smaller than those observed for males. Another finding is that socioeconomic inequality in longevity by PI group is lower when measured with M_{65} than LE_{65}. We also find that the pensioner population seems to have been more affected by the COVID-19 pandemic than the general population and that two groups of pensioners – the lowest PI group of males and the highest PI group of females – appear to have improved their longevity during the pandemic.

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Resumen

Los estudios que utilizan la cuantía de la pensión y/o la base reguladora como indicadores de la esperanza de vida son escasos. Este trabajo se centra en España, un país del que hasta hace poco se sabía muy poco sobre la esperanza de vida de los pensionistas según la cuantía de la pensión de jubilación (IP). Se utiliza un amplio conjunto de datos administrativos para estimar las desigualdades en longevidad entre pensionistas agrupados según cuantía de pensión (IP). Se presentan resultados de las tendencias de la mortalidad entre los pensionistas de 65 años o más para nueve ventanas móviles de seis años cada una para el periodo 2008-2021. Se encuentra que la esperanza de vida por cuantía de pensión (IP) a los 65 años (LE_{65}) tiene una relación positiva tanto para hombres como para mujeres, y esto es así para todos los periodos analizados. Las diferencias absolutas en la LE_{65} entre los pensionistas con mayor y menor cuantía fluctúan a lo largo de las nueve ventanas móviles examinadas. En el caso de los hombres, las diferencias aumentan desde el principio (2,67 años) hasta los 4,06 años y luego tienden a disminuir (2,83 años). En el caso de las mujeres también hay fluctuaciones (entre 1,84 y 2,67 años), pero las diferencias absolutas son siempre menores que las observadas en los hombres. Otra conclusión es que la desigualdad socioeconómica en la longevidad por cuantía (IP) es menor cuando se mide con la moda (M_{65}) que con LE_{65}. También observamos que la población de pensionistas parece haberse visto más afectada por la pandemia de COVID-19 que la población general y que dos grupos de pensionistas -el grupo de varones con menor cuantía y el grupo de mujeres con mayor - parecen haber mejorado su longevidad durante la pandemia.

PALABRAS CLAVE: Muestra Continua de Vidas Laborales; Mortalidad; Sistema Público de Pensiones; Jubilación; España.

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Availability of data and material: Ethics approval is not required to use CSWL; its use for scientific purposes is regulated since inception. Researchers can request versions of the CSWL by post. A separate request must be made for each version. Requests consist of a user profile de-scribing the project being carried out and a document accepting the CSWL's conditions of use. These are available at the following address: http://www.seg-social.es/wps/portal/wss/internet/EstadisticasPresupuestosEstudios/Estadisticas/EST211

DECLARATIONS

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Inequalities in the longevity of females and males aged 65+ in Spain, 2008-2021. The role of pension income.

1.-Background

The issue of inequalities in longevity by socioeconomic status is currently attracting considerable interest as it could have important implications for public pension reform and system design, the actuarial fairness and progressivity of public pension systems, the statutory retirement age, the economic status of future beneficiaries and the actuarial valuation of pension liabilities to pensioners (Duggan et al., 2008; Ayuso et al., 2016; Holzmann et al., 2019; Reznik et al., 2019; Sánchez-Romero et al., 2020; Hann et al., 2020; Diakite and Devolder, 2021; Sheshinski and Caliendo, 2021; Alvarez et al., 2021; Jijiie et al., 2021; Reznik et al., 2021; Garvey et al., 2022; Simonovits and Lacko, 2023)

Life expectancy is the most common metric of survival. It is the hypothetical average age at death given age-specific death rates in a given year. We are interested in examine the inequalities in life expectancy of retirement pensioners in Spain at ages 65 (LE_{65}), 75 (LE_{75}) and 85 (LE_{85}) by pension income (PI) level. There are very few studies that focus on income inequality in connection with old-age life expectancy, yet there is evidence that high-income countries experience substantial and potentially increasing inequality in late-life longevity (Fors et al., 2021). Studies using the amount of pension and/or pensionable earnings as a proxy for life expectancy are even scarcer, although some have been conducted in recent years for countries including Germany (Wenau et al., 2019; Tetzlaff et al., 2020), the USA (Waldron, 2007; Goldman and Orszag, 2014; Bosley et al., 2018), Italy (Ardito et al., 2020), Canada (Adam, 2012; Osfic, 222; Wen et al., 2020), Chile (Edwards et al., 2023), Argentina (Bramajo and Grushka, 2019) and Spain (Pérez-Salamero González et al., 2021 and 2022).

This paper focuses on Spain, a country for which little information about pensioners' life expectancy by PI level was known until recently. To the best of our knowledge, only two studies have examined this issue, and only for male pensioners (Pérez-Salamero González et al., 2021 and 2022). These authors found that individuals' PI levels contained information about their mortality experience and that this variable could be used as a proxy for socioeconomic status.

Pérez-Salamero González et al. (2021) examined the differences in life expectancy between self-employed (SEP) and paid employee (PEP) workers when they become retirement pensioners. They did this by looking at levels of PI using administrative data from Spanish social security records. They found that LE_{65} was slightly higher for SEP than for PEP retirement pensioners, despite the fact that the average retirement benefit is (much) lower for SEP than for PEP workers. Their following paper (Pérez-Salamero González et al., 2022) showed that disparities in LE_{65} and LE_{75} between pensioners in the lowest and highest income groups were relatively small, although slightly higher than previously reported for Spain in Pérez-Salamero González et al. (2021). This gap in LE_{65} widens over time, from 1.49 (2005-2009) to 2.54 years (2015-2018). These differences are statistically significant. The authors point out that they can mainly be explained by the exclusion of SEP retirement pensioners, by the improvements made to the procedure for obtaining life expectancies within groups, and by certain additional adjustments made to the data set used. These conclusions are in line with previous findings for Spain involving older adults and using very different methodologies and/or databases (Regidor et al., 2012; Kulhánová, et al., 2014; Permanyer et al., 2018; Solé-Auro et al., 2020; González and Rodríguez-González, 2021).

While we recognize the importance of the findings for the Spanish case, the two papers mentioned above have several limitations. They both assign pensioners to groups on the basis of income cut-off points deriving from the PI distribution of the total retirement pensioner population. This means that the number of individuals assigned to each group is very uneven and changes substantially over the periods studied (in that particular case three periods, one covering five years of deaths and two covering four years each).

Another important limitation was that they did not study the case of females. They argued that this was because of lower labour force participation rates for female cohorts in Spain and the fact that women sometimes have shorter careers (in terms of years of employment) and may work less intensively than men due to family roles and commitments.

The matter of (three fixed) observation windows was also limiting when explaining the variation in results from one period to another. In the end longevity was only measured in terms of life expectancy, with no other indicators being examined. The variation in ages at death, captured by a metric of life span variation, should be used to supplement measures of average longevity when comparing or monitoring population subgroups (van Raalte et al., 2018).

What we want to know now is how the way the PI level is defined impacts the estimation of life expectancy, as this seems to be relevant. (Shi et al., 2021). We would also like to assess the impact of the COVID-19 pandemic on the life expectancy of different groups of pensioners according to their PI levels. To do this we use a large administrative data set (CSWL, the Continuous Sample of Working Lives or la Muestra Continua de Vidas Laborales in Spanish) to estimate inequalities in longevity among male and female pensioners grouped according to their PI levels. We work with the longest possible period covered by this data source – 2004 to 2021 – but only present results for mortality trends among retirement pensioners aged 65 and over for the period 2008-2021. Due to various data coherence problems (especially for females), we have ruled out the years from 2004 to 2007. We therefore present results for nine rolling windows of six years each.

The PI-longevity gradient is quantified in two ways. The first is by estimating the changes in total life expectancy by PI level at ages 65 (LE_{65}), 75 (LE_{75}) and 85 (LE_{85}) over time. The second is by introducing some additional longevity and life span variation indicators by PI level at age 65, these being the median age at death, the interquartile range and the modal age at death (Wilmoth and Horiuchi,1999; Cheung et al., 2005). As a supplementary approach, we also provide a comparison with the LE_{65}, LE_{75} and LE_{85} for the Spanish population as a whole.

Our paper is guided by the following research questions: Are there any differences in longevity between PI groups for male/female pensioners? If so, are they statistically significant? Are there differing trends in life expectancy between PI groups that lead to a widening or narrowing of inequalities over time? Is the evolution of longevity by PI group similar for both males and females? Are the additional longevity indicators in line with life expectancy (determined for pensioners grouped according to their PI levels)? How has the longevity of the general population and of the different groups of pensioners classified by their PI levels evolved? How has the COVID-19 pandemic impacted the life expectancy of pensioners classified according to their PI levels?

2.-Methods

We use the Continuous Sample of Working Lives (CSWL), a large Spanish administrative data set that has a number of advantages over survey data, such as larger sample sizes, lower costs and a lighter respondent burden. An edition of the CSWL dataset has been released every year since 2004. Each contains the social security records for a 4% non-stratified random sample of the population who during that year had some relationship with Spain's social security system, i.e. anyone who was working, receiving unemployment benefit or receiving a pension. Individuals who for any reason had no connection with social security in a particular year do not appear. Civil servants are also excluded.

The CSWL contains administrative data on the working lives that form the basis of the sample, which is taken from social security records and comprises anonymized microdata with detailed information on individuals (Pérez-Salamero et al., 2017; Nuñez-Antón et al., 2020).

The first wave covers people who had some kind of financial link with the social security system in 2004 and provides the entire working history of the sample population. The sample is updated every year using information for the variables provided by the social security system dating back to when computerized records began, along with data from other administrative sources that record additional information on individuals. At the time of writing this paper, the data available to researchers run from 2004 to 2021.

In our study the initial population is made up of (true) retirement pensioners whose retirement age was 65 (the ordinary retirement age) or over and who were categorized under the general scheme (the main part of the Spanish social insurance system). Until 31 December 2012, the statutory retirement age in Spain was 65. From 2027 onwards there will be two standard retirement ages: 65 with 38.5 years' contributions and 67 with 37 years' contributions. The shift from 65 to 67 is being made gradually between 2013 and 2027.

In line with the two previous papers that have used the CSWL to analyse the mortality and longevity of pensioners by PI level (Pérez-Salamero González et al., 2021 and 2022), we also exclude retirement pensioners from the special system for the self-employed (SEP), disabled people whose benefits were reclassified as retirement benefits and early retirees who were able to access benefits, before the statutory retirement age.

2.1.-Variables and socioeconomic groups

We have divided the pensioners into specific income quartiles, in this case four equal-sized segments that each contain approximately a quarter (25%) of the individuals. These segments are denoted G_1 – the PI group for pensioners with the lowest 25% of retirement benefits ("lowest"); G_2 – for pensioners with retirement benefits between 25% and 50% (up to median, "second"); G_3 – in which the benefit amount is between 50% to 75% (above the median, "third"); and G_4 – which covers those pensioners with the highest 25% of retirement benefits ("highest"). The people within each group do not represent exactly 25%, since the number of deaths within each group varies over the six years of each window. The individuals (alive or dead) assigned to each group according to their

PI level represent exactly 25% of the sample. Quartiles were calculated separately by sex and by rolling window.

We are mainly interested in the first (lowest) and fourth (highest) groups. Table 1 shows the exposures in person-years and number of deaths (percentages) for three of the periods studied (three of the nine rolling windows).

Between the first rolling window and the last, the number of exposures in person-years increases by almost 50% (from 243,950 to 364,119), with a 66% increase for women and a 37% increase for men. The share of women in the total number of exposed pensioners increases from 41.23% to 46.10%.

Table 1: Pensioners by PI level: exposures (Exp) in person-years and number of deaths (De) for selected periods (W_1 2008-2013, W_5 2012-2017 and W_9 2016-2021)

Periods	**Items**	**Pension income quartile (females)**				**Pension income quartile (males)**			
		G_1	**G_2+G_3**	**G_4**	**Total**	**G_1**	**G_2+G_3**	**G_4**	**Total**
W_1	**Exp**	25,758	48,971	26,009	100,737	31,763	75,601	35,850	143,213
	%	25.57	48.61	25.82	100	22.18	52.79	25.03	100
	De	1,144	1,000	522	2,666	1,692	2,611	699	5,002
	%	42.91	37.51	19.58	100	33.83	52.20	13.97	100
W_5	**Exp**	36,620	72,485	34,419	143,523	44,287	92,771	46,590	183,648
	%	25.52	50.50	23.98	100	24.12	50.52	25.37	100
	De	1,788	1,345	555	3,688	2,382	2,993	830	6,205
	%	48.48	36.47	15.05	100	38.39	48.24	13.38	100
W_9	**Exp**	41,006	85,660	41,212	167,877	43,765	100,655	51,823	196,242
	%	24.43	51.03	24.55	100	22.30	51.29	26.41	100
	De	2,148	1,519	599	4,266	2,328	3,684	1,064	7,076
	%	50.35	35.61	14.04	100	32.90	52.06	15.04	100

Source: Own work based on CSWL (2008-2021)

Table 2 displays the maximum life span observed in each rolling window, which literally corresponds to the current age of the oldest living member.

The age of the oldest living pensioner in G_1 has decreased by four years for females and three years for males, and the differences in the maximum life span observed between groups (G_1-G_4) for both sexes have also decreased over the period.

Table 2: Maximum life span						
Items	**Females**			**Males**		
	G_1	**G_2+G_3**	**G_4**	**G_1**	**G_2+G_3**	**G_4**
2008-2013	113	112	103	111	112	101
2009-2014	113	112	103	111	113	102
2010-2015	113	112	104	111	114	102
2011-2016	113	112	104	111	114	102
2012-2017	113	112	104	111	114	102
2013-2018	113	108	104	107	114	102
2014-2019	109	109	104	107	114	102
2015-2020	109	110	105	107	114	100
2016-2021	109	108	106	108	106	101
Source: Own work based on CSWL (2008-2021)						

Note that, through all the rolling windows and for both females and males, the age of the oldest living individual in the group of the most advantaged pensioners (G_4) is the lowest of all the groups considered. Nevertheless, as we will see later, those in group G_4 have the longest life expectancy.

2.2.-Methodology

The PI-longevity gradient is quantified in two ways. The first is by considering changes in total life expectancy by PI level at ages 65 (LE_{65}), 75 (LE_{75}) and 85 (LE_{85}) over time. We use the Mort1Dsmooth function in the MortalitySmooth R package (Camarda, 2012) – which is tailored to smooth mortality rates across different ages with P-splines – to construct complete-period life tables from age 65 to age 105 and to calculate LE_{65}, LE_{75} and LE_{85} for each rolling window of six years. Data above age 105 are unreliable because the exposures are small and, in addition, the reporting of deaths is questionable for very old ages.

In order to check the robustness of the estimated changes in total life expectancy by PI, we have also used the R Package DBKGrad for Mortality Rates Graduation by Discrete Beta Kernel Techniques (Mazza and Punzo, 2014) and the generalized additive modelling functions provided in the R package MGCV developed by Wood (2017).

One important reason for measuring mortality/longevity is to detect differences between populations/groups (Li, 2005). Deterministic life tables might therefore specify whether a life-table variable for one population is bigger than the same variable for another, while a probabilistic life table can further test whether such differences are statistically significant or appear merely by chance. The technical details for testing whether there is a significant positive difference in life expectancy can be found in the technical appendix.

The second way of quantifying the PI-longevity gradient is by estimating some additional indicators of longevity and life span variation by PI level at age 65. Median and modal ages at death are seldom proposed as measures for studying longevity. The mean age at death, i.e. life expectancy, is generally preferred. However, all three measures show central tendencies and are therefore important. They complement one another with information about the “centre” of the distribution of deaths. (Canudas-Romo, 2010).

The median age at death, Md, is the age when half of the hypothetical cohort members have died, i.e. when the number of people surviving to the exact age of x (l_x) is equal to half the initial cohort aged x_e, $l_{Md} = \frac{l_{x_e}}{2}$. In our case the cohort is aged 65 and over.

The adult modal age at death, *M*, is the age (beyond infancy) at which the largest single number of deaths occur and is used as an indicator to analyse mortality disparities at older ages. It is both a natural measure of the length of life and a good basis for measuring its dispersion (Kannisto, 2001). Under a given mortality regime, *M* represents the most common or "typical" length of life among adults (Diaconu et al., 2022). Compared with conventional measures of old-age mortality such as life expectancy, *M* has two valuable and desirable properties: first, it does not depend on an arbitrary choice of "old" age threshold, and second, it is determined solely by mortality at older ages. Despite its intuitive meaning and desirable properties, *M* has not been commonly used for examining socioeconomic differences in mortality.

There are several measures for calculating life span variation and each has different underlying properties (Van Raalte and Caswell, 2013). They all aim to calculate the amount of heterogeneity in age at death across all individuals in a population (Hiam et al., 2021; Seaman et al., 2019).

The interquartile range (IQR), also known as the middle 50% (Wilmoth and Horiuchi, 1999), is a measure that equals the distance between the lower and upper quartiles of the age distribution of death in a life table. It decreases as the age at death becomes less variable and has a twofold appeal as a single measure of variability in the life table. First, it is very simple to calculate because it equals the difference between the ages where the survivorship curve, S(x), crosses 0.25 and 0.75, and second, it is easy to interpret because it is the span of ages containing the middle 50% of deaths.

Life span variation indicators have two important implications (Van Raalte et al., 2018), one at the individual (micro) level and the other at the population (macro) level. Life span variation reflects both individual uncertainty in the timing of death and heterogeneity in the underlying population health. At the micro level, it measures uncertainty regarding the timing of death. From our perspective, a diverging life span variation between PI groups means that an often overlooked dimension of social inequality in longevity is increasing, i.e. the fact that pensioners from the highest PI level can more effectively plan their remaining life course, whereas less advantaged groups face increasing uncertainty about their survival. At the macro level, life span variation is an indicator of heterogeneity in underlying population longevity. In our context, an increasing life span variation among disadvantaged groups implies that the pensioners belonging to such groups are living increasingly diverse lives.

3.-Results

We will show the results mainly through figures and tables.

Table 3 (females) and Table 4 (males) show the 95% confidence intervals (CIs) of LE_{65} for all PI groups of pensioners and all nine periods analysed.

Table 3: LE_{65}. 95% confidence intervals by PI group. Females.									
Items	**G_1**			**Whole**			**G_4**		
Periods	**2.5th**	**50th**	**97.5th**	**2.5th**	**50th**	**97.5th**	**2.5th**	**50th**	**97.5th**
2008-2013 (W_1)	23.13	**23.67**	24.20	24.94	**25.20**	25.46	24.98	**25.59**	26.19
2009-2014 (W_2)	22.74	**23.29**	23.85	25.15	**25.40**	25.64	25.08	**25.65**	26.21
2010-2015 (W_3)	22.98	**23.58**	24.18	25.24	**25.47**	25.71	25.12	**25.66**	26.20
2011-2016 (W_4)	23.43	**24.02**	24.61	25.41	**25.64**	25.87	25.48	**26.02**	26.56
2012-2017 (W_5)	23.35	**23.94**	24.52	25.52	**25.74**	25.95	25.60	**26.13**	26.66
2013-2018 (W_6)	23.84	**24.37**	24.89	25.66	**25.87**	26.08	25.77	**26.32**	26.86
2014-2019 (W_7)	24.24	**24.77**	25.29	25.82	**26.03**	26.24	26.07	**26.60**	27.13
2015-2020 (W_8)	23.87	**24.37**	24.87	25.63	**25.84**	26.04	26.45	**27.03**	27.61
2016-2021 (W_9)	23.94	**24.36**	24.78	25.70	**25.91**	26.11	26.46	**27.03**	27.61
Source: Own work based on CSWL (2008-2021)									

Table 4: LE_{65}. 95% confidence intervals by PI group. Males.									
Items	**G_1**			**Whole**			**G_4**		
Periods	**2.5th**	**50th**	**97.5th**	**2.5th**	**50th**	**97.5th**	**2.5th**	**50th**	**97.5th**
W_1	19.75	**20.19**	20.64	21.50	**21.71**	21.92	22.20	**22.86**	23.53
W_2	19.65	**20.08**	20.50	21.68	**21.89**	22.09	22.74	**23.47**	24.20
W_3	19.26	**19.67**	20.09	21.63	**21.82**	22.02	22.49	**23.09**	23.70
W_4	19.11	**19.49**	19.88	21.70	**21.90**	22.09	22.92	**23.56**	24.19
W_5	19.35	**19.75**	20.15	21.92	**22.11**	22.30	22.99	**23.54**	24.10
W_6	19.33	**19.73**	20.14	21.95	**22.14**	22.32	23.21	**23.74**	24.28
W_7	19.45	**19.86**	20.28	21.92	**22.11**	22.30	22.98	**23.48**	23.97
W_8	19.65	**20.07**	20.49	21.79	**21.97**	22.16	22.73	**23.17**	23.62
W_9	19.83	**20.25**	20.66	21.71	**21.89**	22.08	22.65	**23.08**	23.51
Source: Own work based on CSWL (2008-2021)									

It can be seen that LE_{65} has a positive link with PI levels for both males and females, and this is true for all periods analysed. It can also be seen that the LE_{65} for females exceeds that of males in all periods.

The minimum values are circled in red and the maximum values in green. Note that the maximum and minimum values are different for the two groups of pensioners analysed.

The evolution of LE_{65} for males and females by PI level is clearly different. The least advantaged group of female retirement pensioners (G_1) reaches the highest value for LE_{65} in window W_7 (2014-2019) with 24.77 years, then apparently because of the COVID-19 pandemic effect it drops to 24.36 years in window W_9. For the most advantaged group of females (G_4), the highest value for LE_{65} (27.03 years) is reached in the final window, indicating that the COVID-19 pandemic seems to have affected these retirement pensioners very modestly.

The evolution of LE_{65} for males by PI level seems to be the opposite of that for females. The most advantaged group reaches its highest value for LE_{65} in the period 2014-2018 (W_6), with 23.74 years, and it then gradually decreases to 23.08 years. However, the LE_{65} for pensioners in the most disadvantaged group increases from its lowest value of 19.49 years in the period 2011-2016 (W_4) to reach its maximum in the period 2016-2021 (20.25 years), showing that the COVID-19 pandemic seems to have had little effect on this group. This means that the evolution of LE_{65} for all the female pensioners as a whole would be more like that of the G_1 group, and that for the males more like that of the G_4 group.

Figure 1 shows the absolute differences in LE_{65} by PI level and sex for the periods studied. The figure is broken down into 2 graphs: the first shows the difference in LE_{65} between the G_4 and G_1 groups for each sex, while the second shows the absolute difference in years between females and males for a given PI group.

The absolute differences in LE_{65} between pensioners in the highest PI groups compared to the lowest fluctuate over the nine rolling windows examined. For males, the minimum is reached in the first window (2.67 years) and the maximum in the fourth (4.06 years). To begin with (see the first four rolling windows) the differences widen, but after that the trend is downwards. For females there have also been fluctuations (between 1.84 and 2.67 years), but the absolute differences are always smaller than those observed for males.

The maximum (2.67 years, W_9) and minimum (1.84 years, W_7) values are very close in time. The maximum value reached by females (2.67 years, W_9) coincides with the minimum value reached by males (2.67 years, W_1). As far as the absolute difference in years between females and males for a given PI group is concerned, a clear upward trend can be observed.

The minimum values are found in the first two windows and the maximum values for G_4 and the whole group appear in the last window. For G_1, the maximum value is reached in W_7. The differences between females and males are greater in the lowest PI group (from 4.90 to 3.22 years) than in the highest (from 3.95 to 2.18 years). The absolute difference in LE_{65} between females and males for the pensioner group as a whole continues to increase over time from 3.49 to 4.01 years.

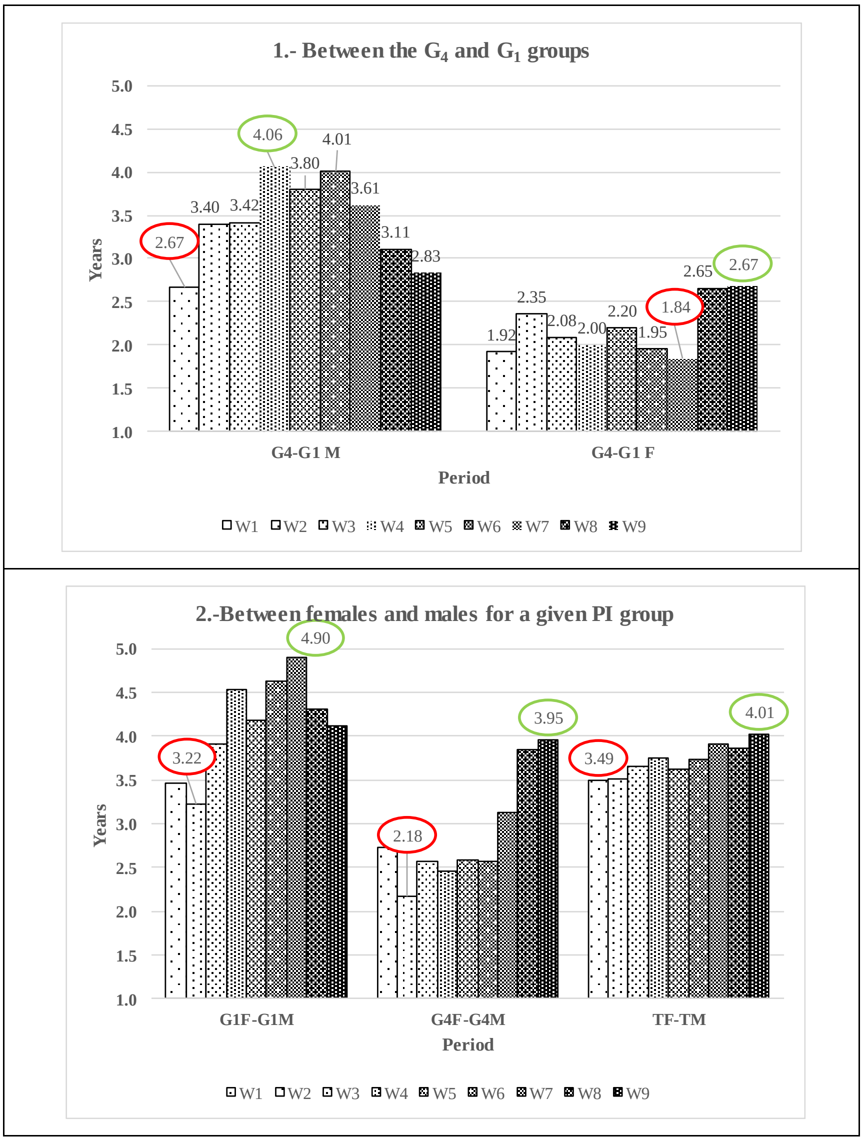


Figure 1: Absolute differences in LE_{65} (PI groups and sexes).

The results in Tables 3 and 4 and here in Figure 1 raise the question as to whether or not the differences in LE_{65} between PI levels are statistically significant. Tables 5-7 can provide us with more detailed information.

For all nine periods analysed, Table 5 (females) and Table 6 (males) show the differences in LE_{65} (written in the tables as DLE_{65}) between the total group and the G_1 group (the most disadvantaged) and between the G_4 group (the most advantaged) and the total group, together with the standard error for those differences and the z-score value of the test statistic to test the null hypothesis that the difference in life expectancy is zero against the alternative of its being positive.

Table 5: Absolute differences in LE_{65} between PI groups by period (females)									
Items	**W_1**	**W_2**	**W_3**	**W_4**	**W_5**	**W_6**	**W_7**	**W_8**	**W_9**
Differences between the whole group and the G_1 group									
DLE_{65}	1.53	2.10	1.89	1.62	1.80	1.51	1.26	1.46	1.55
$Se(DLE_{65})$	0.30	0.31	0.33	0.32	0.32	0.29	0.29	0.27	0.24
z score	5.06***	6.83***	5.74***	5.03***	5.68***	5.18***	4.39***	5.33***	6.53***
Differences between the G_4 group and the whole group									
DLE_{65}	0.39	0.25	0.19	0.38	0.39	0.45	0.57	1.19	1.12
$Se(DLE_{65})$	0.34	0.31	0.30	0.30	0.29	0.30	0.29	0.31	0.31
z score	1.15	0.80	0.64	1.27	1.35*	1.50*	1.96**	3.79***	3.62***
*** significant at 1% one-tailed test. ** significant at 5% one-tailed test. * significant at 10% one-tailed test.									

The results for females (Table 5) show that the differences in LE_{65} between the total group and the G_1 group are all statistically significant at 1%. Those between the G_4 group and the total group for the four first windows (W_1-W_4) are not significant at 10%, whereas for the next two windows (W_5-W_6) they are significant at 10%. W_7 is significant at 5%, and the last two windows (W_8-W_9) are significant at 1%.

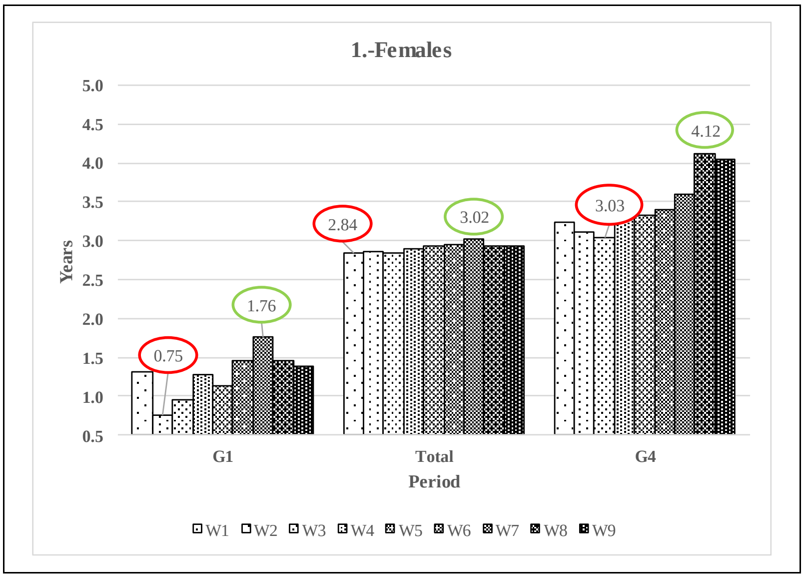
The results for males (Table 6) show that all differences in LE_{65} are statistically significant at 1%.

Table 6: Absolute differences in LE_{65} between PI groups by period (males)									
Items	**W_1**	**W_2**	**W_3**	**W_4**	**W_5**	**W_6**	**W_7**	**W_8**	**W_9**
Differences between the whole group and the G_1 group									
DLE_{65}	1.52	1.81	2.15	2.40	2.36	2.40	2.25	1.90	1.65
$Se(DLE_{65})$	0.25	0.24	0.23	0.22	0.23	0.23	0.23	0.23	0.23
z score	6.06***	7.56***	9.2***	10.9***	10.4***	10.6***	9.6***	8.1***	7.1***
Differences between the G_4 group and the whole group									
DLE_{65}	1.15	1.59	1.27	1.66	1.44	1.61	1.36	1.20	1.19
$Se(DLE_{65})$	0.35	0.39	0.33	0.34	0.30	0.29	0.27	0.25	0.24
z score	3.25***	4.11***	3.90***	4.92***	4.81***	5.52***	5.05***	4.90***	4.98***
*** significant at 1% one-tailed test. ** significant at 5% one-tailed test. * significant at 10% one-tailed test.									

Finally, Table 7 shows the results for the differences in LE_{65} between PI groups by period and sex (females-males). For all three cases, the differences between G_1 groups, G_4 groups and total groups are statistically significant at 1%.

These results support the hypothesis that there is highly significant evidence of a positive relationship between LE_{65} and PI groups, and that this is true for both males and females.

Table 7: Absolute differences in LE_{65} between PI groups by period and sex (females-males)									
Items	**W_1**	**W_2**	**W_3**	**W_4**	**W_5**	**W_6**	**W_7**	**W_8**	**W_9**
Differences between G_1 groups									
DLE_{65}	3.47	3.22	3.91	4.53	4.19	4.63	4.90	4.30	4.11
Se(DLE_{65})	0.36	0.36	0.37	0.36	0.36	0.34	0.34	0.33	0.30
z score	9.77***	9.06***	10.5***	12.6***	11.6***	13.7***	14.4***	12.9***	13.7***
Differences between G_4 groups									
DLE_{65}	1.53	0.67	0.96	0.83	1.03	1.19	1.50	2.51	2.55
Se(DLE_{65})	0.52	0.55	0.46	0.47	0.41	0.40	0.37	0.37	0.36
z score	2.92***	1.21***	2.10***	1.74***	2.51***	2.95***	4.04***	6.89***	7.14***
Differences between whole groups									
DLE_{65}	3.49	3.51	3.65	3.74	3.63	3.74	3.92	3.86	4.01
Se(DLE_{65})	0.17	0.16	0.16	0.15	0.15	0.15	0.14	0.14	0.14
z score	20.7***	21.8***	23.3***	24.8***	24.6***	25.7***	27.1***	27.5***	28.6***
*** significant at 1% one-tailed test. ** significant at 5% one-tailed test. * significant at 10% one-tailed test.									



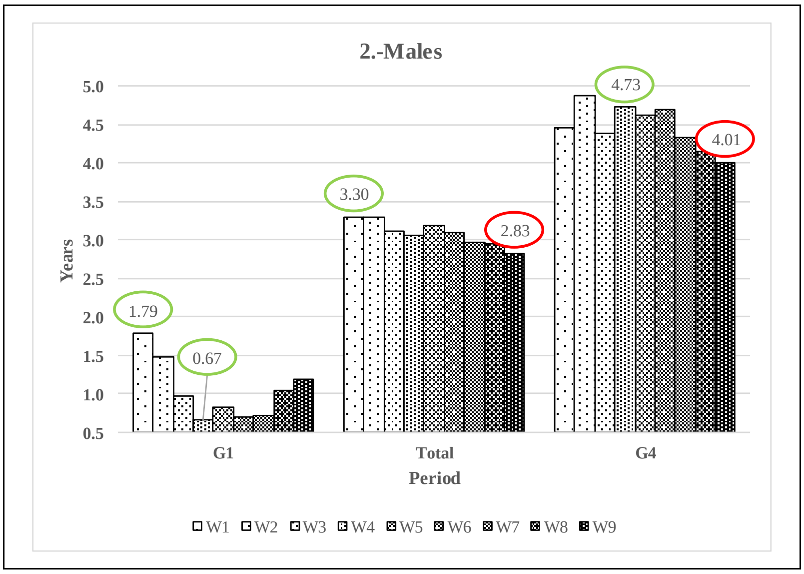


Figure 2: $ADLE_{65}$. Pensioners versus general population

Figure 2 presents a comparison of our results with the LE_{65} for the Spanish population as a whole. The top part shows the absolute differences in LE_{65} (hereafter $ADLE_{65}$) for females (pensioners minus general population), while the bottom part shows them for males.

We can see that the $ADLE_{65}$ are positive for all PI groups and periods. The differences fluctuate over time, but it could be argued that for the female group as a whole the $ADLE_{65}$ show a slightly upward trend. For the male group, however, they show a downward trend.

The trends are less clear when we look at the differences between the highest and lowest PI groups. The females and males in group G_1 show great variability in $ADLE_{65}$ (compared to the general population), but there is a stable trend for females and a slightly downward trend for males. There seems to be less variability among the females and males in group G_4 than in group G_1, but in this case the females show an upward trend in the evolution of $ADLE_{65}$ and males a downward trend. At the end of the period examined, the $ADLE_{65}$ for both sexes show a certain convergence for G_4 (of around 4 years) and for the whole group (of around 3 years).

It is not surprising that those who receive retirement pensions live longer than the general population, since one of the conditions for receiving a contributory pension is to have paid contributions for at least 15 years, including at least 2 of the last 15. This requirement is likely to exclude some of the most vulnerable members of the Spanish population. It should be remembered that disabled pensioners and early retirees, both groups with lower life expectancy than the general population, were also excluded from our sample.

But what about life expectancy at older ages? In Section 7.2. of the Appendix (Further results) we provide the information for LE_{75} and LE_{85}. We detect highly significant evidence of a positive relationship between life expectancy at older ages and PI groups. This is also true for males and females, but the statistical significance diminishes slightly.

Tables 8-11 present further indicators of longevity in years. Tables 8 (females) and 9 (males) show the values for the G_1 group (the most disadvantaged), while Tables 10 (females) and 11 (males) show the values for the G_4 group (the most advantaged).

The four tables contain the following items:

$\boldsymbol{Q^3_{65}}$ is the age at which only 25% of the pensioners who were originally aged 65 are still alive (the third quartile).

$\boldsymbol{Q^1_{65}}$ is the age at which 75% of the pensioners who were originally aged 65 are still alive (the first quartile).

$\boldsymbol{IQR_{65}}$ indicates the table-specific difference between the 25th and 75th percentiles in survivorship, i.e. $\boldsymbol{Q^3_{65} - Q^1_{65}}$.

$\boldsymbol{Md_{65}}$ is the age at which half of the pensioners aged 65 have died (or are still alive).

$\boldsymbol{Ad_{65}}$ is the average age of death for pensioners aged 65.

$\boldsymbol{M_{65}}$ is the age (beyond 65) at which the largest number of deaths occur. The percentage of survivors corresponding to the modal age is shown in brackets.

In Tables 8-11 the minimum values are circled in red and the maximum values in green.

Table 8: Further longevity indicators (in years) for females in G1 groups by period

Items	Q^3_{65}	Q^1_{65}	IQR_{65}	Md_{65}	Ad_{65}	M_{65}
2008-2013	94.31	83.02	11.29	89.33	88.67	91.98 (36.68%)
2009-2014	94.02	82.52	11.50	88.97	88.29	91.46 (37.66%)
2010-2015	94.62	82.62	11.99	89.36	88.58	92.28 (36.14%)
2011-2016	94.90	83.33	11.57	89.68	89.02	92.53 (36.33%)
2012-2017	95.05	82.92	12.13	89.39	88.94	92.98 (34.25%)
2013-2018	94.98	83.67	11.31	89.96	89.37	92.54 (37.14%)
2014-2019	95.23	84.27	10.96	90.37	89.77	92.83 (37.37%)
2015-2020	94.67	83.93	10.74	89.80	89.37	92.40 (36.67%)
2016-2021	94.67	83.89	10.79	89.82	89.36	92.47 (36.38%)

Source: Own work based on CSWL (2008-2021)

Table 9: Further longevity indicators (in years) for males in G1 groups by period

Items	Q^3_{65}	Q^1_{65}	IQR_{65}	Md_{65}	Ad_{65}	M_{65}
2008-2013	91.01	78.34	12.67	85.49	85.19	88.29 (37.12%)
2009-2014	90.79	78.24	12.56	85.45	85.08	88.12 (37.26%)
2010-2015	90.46	78.07	12.38	85.09	84.67	88.15 (35.67%)
2011-2016	90.30	77.87	12.44	84.99	84.49	88.14 (35.20%)
2012-2017	90.89	77.99	12.90	85.26	84.75	88.79 (34.44%)
2013-2018	90.88	78.03	12.86	85.18	84.73	88.37 (36.07%)
2014-2019	91.09	78.02	13.07	85.24	84.86	88.26 (37.08%)
2015-2020	91.24	78.24	13.00	85.34	85.07	88.29 (37.50%)
2016-2021	91.44	78.17	13.27	85.39	85.25	85.98 (47.31%)

Source: Own work based on CSWL (2008-2021)

It can be seen from Tables 8 and 9 that the average age of death for pensioners aged 65 ($\boldsymbol{Ad_{65}}$) and the age at which half of the pensioners aged 65 have died ($\boldsymbol{Md_{65}}$) show very close values. However, the age (beyond 65) at which the largest number of deaths occur ($\boldsymbol{M_{65}}$) is between 1.5 and 3 years higher than the average age of death. This is consistent with the result reported by Canudas-Romo (2010), who showed the similarity between the median and mean age at death in both time trends and record-holding countries for a series of record measures of longevity for the period 1840-2005. He also found that the record modal age at death in both the time trends and the record-holding countries was very different from the other measures.

The proportion of pensioners surviving to $\boldsymbol{M_{65}}$ is relatively constant over time for G_1 (at around 36-37%) and for G_4 (40-41%), the exceptions being G_1 over the period 2012-2017 (34.25%) and G_4 over the period 2008-2013 (37.08%).

But what can life span variation reveal that life expectancy does not? If we look at female pensioners in the 25th to 75th percentile (the interquartile range) we can see that, although the value was higher for G_1 in the early periods and reached a maximum of 12.13 in the period 2012-2017, there is a trend towards convergence between the two groups (around 11 years). The values calculated for the interquartile range are more stable for G_4.

Table 10: Further longevity indicators (in years) for females in G_4 groups by period

Items	$\boldsymbol{Q^3_{65}}$	$\boldsymbol{Q^1_{65}}$	$\boldsymbol{IQR_{65}}$	$\boldsymbol{Md_{65}}$	$\boldsymbol{Ad_{65}}$	$\boldsymbol{M_{65}}$
2008-2013	96.36	84.89	11.47	91.27	90.59	93.90 (37.08%)
2009-2014	96.02	85.24	10.78	91.14	90.65	92.89 (40.90%)
2010-2015	95.96	85.36	10.60	91.17	90.66	92.95 (40.58%)
2011-2016	96.26	85.79	10.48	91.51	91.02	93.20 (40.87%)
2012-2017	96.25	86.05	10.20	91.59	91.13	93.09 (41.82%)
2013-2018	96.52	86.12	10.40	91.79	91.32	93.47 (41.02%)
2014-2019	96.62	86.79	9.83	92.13	91.60	93.57 (41.82%)
2015-2020	97.51	86.64	10.87	92.62	92.03	94.55 (40.05%)
2016-2021	97.60	86.59	11.00	92.68	92.03	94.68 (39.72%)

Source: Own work based on CSWL (2008-2021)

According to the data presented for males in Tables 9 and 11, the evolutionary trend of the values for modal age at death is very stable for both groups (around 88 years for G_1 and 90 for G_4) except for the last period for G_1 where the estimated value might be considered anomalous (85.98 years).

As in the case of the females, the proportion of male pensioners surviving to $\boldsymbol{M_{65}}$ is relatively constant over time for G_1 (at around 35-37%) and G_4 (40-41%), except for G_1 in the last window (2016-2021, 47.31%) and G_4 in the first window (2008-2013, 37.39%).

Contrary to what we found for females, however, the $\boldsymbol{IQR_{65}}$ value for G_1 and G_4 shows a divergent trend. This was already higher at the beginning of the period studied – 12.67 years for G_1 compared to 11.44 years for G_4 – and this difference increased (13.27 years for G_1 compared to 10.85 years for G_4).

In the last period studied (2016-2021), the LE_{65} for pensioners in the G_4 group is 23.08 years compared to 20.25 years for those in the G_1 group. This indicates a remarkable socioeconomic disparity in longevity. Pensioners in the 25th to 75th percentile (the interquartile range) are expected to die between 78.17 and 91.44 years in the G_1 PI group, while those in the G_4 group are expected to die between 82.60 and 93.45 years. Although LE_{65} for G_1 pensioners is (only) 12.27% lower than for G_4 pensioners, the age window in which these deaths are expected to occur is 22.27% wider for G_1 pensioners.

In general, male pensioners in the most disadvantaged group not only die earlier on average than their more affluent counterparts, they also face greater variation in the time of death - a double burden of inequality that has increased over time.

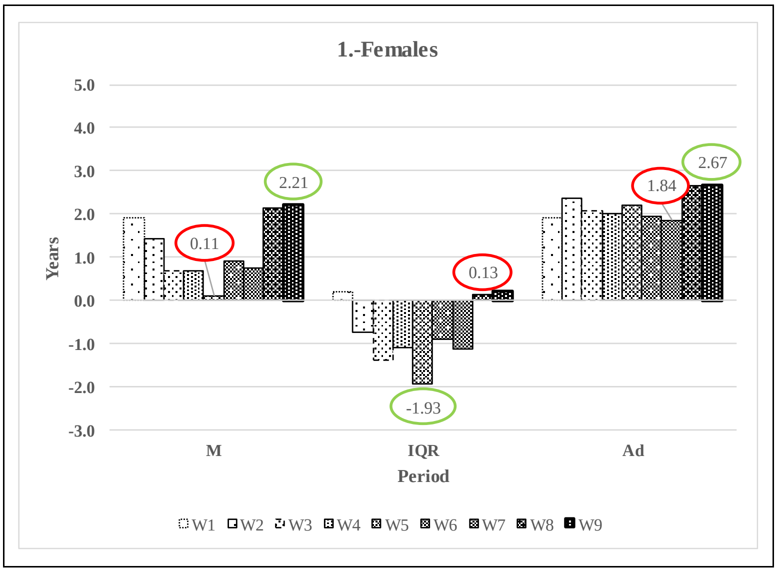
Table 11: Further longevity indicators (in years) for males in G_4 groups by period

Items	Q^3_{65}	Q^1_{65}	IQR_{65}	Md_{65}	Ad_{65}	M_{65}
2008-2013	93.34	81.91	11.44	88.16	87.86	90.34 (37.39%)
2009-2014	94.22	82.26	11.95	88.74	88.47	90.59 (41.27%)
2010-2015	93.80	82.03	11.77	88.47	88.09	90.56 (40.07%)
2011-2016	94.28	82.47	11.81	88.93	88.56	90.73 (41.40%)
2012-2017	94.17	82.75	11.42	89.06	88.54	90.79 (41.43%)
2013-2018	94.42	83.12	11.30	89.41	88.74	91.22 (40.90%)
2014-2019	93.92	83.00	10.92	89.09	88.48	90.73 (41.37%)
2015-2020	93.41	82.85	10.56	88.78	88.17	90.56 (40.22%)
2016-2021	93.45	82.60	10.85	88.70	88.08	90.59 (39.94%)

Source: Own work based on CSWL (2008-2021)

Figure 3 shows the absolute differences between the G_4 and G_1 groups in IQR_{65}, Md_{65} and Ad_{65} by PI level for the periods studied. The figure is broken down into two parts: females and males.

The absolute differences in Ad_{65} are identical to the absolute differences in LE_{65} (see Figure 1, top) and are included so that they can be compared with the absolute differences of the mode value (ADM_{65}). Similarly, the ADM_{65} between pensioners in the highest compared to the lowest PI groups also fluctuate over the nine rolling windows examined. For both males and females the minimum is reached in the fifth window (0.11 years females, 2 years males) and the maximum in the last window (2.21 years females, 4.61 years males).



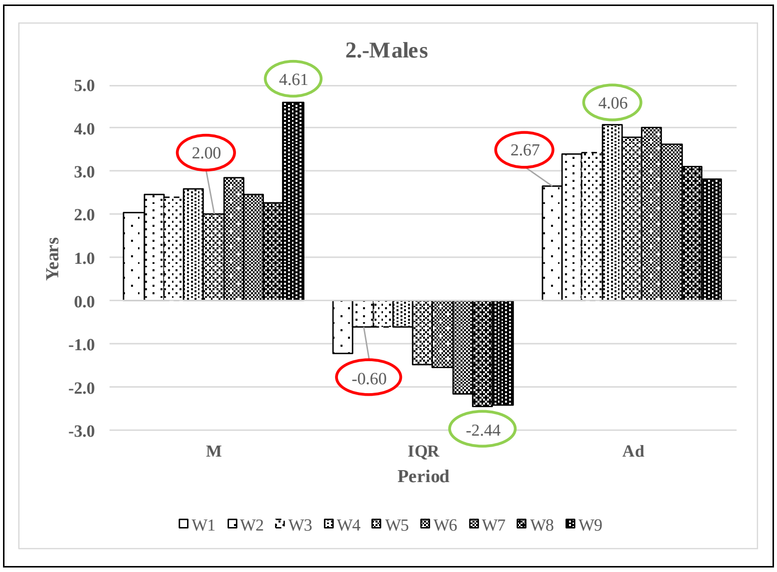


Figure 3: Absolute differences between the G_4 and G_1 groups in IQR_{65}, M_{65} and Ad_{65}

For males, the ADM_{65} between pensioners in the highest and lowest PI groups were always smaller than the $ADLE_{65}$, except in the last period. It could be said that socioeconomic inequality in longevity is lower measured with M_{65} than with LE_{65}. Over the whole period, the average inequality in longevity measured with LE_{65} is 30.37% higher than with M_{65}.

For females, too, socioeconomic inequality in longevity as measured by the M_{65} is far lower than when measured with the LE_{65}. Except for the first rolling window, when ADM_{65} coincides with $ADLE_{65}$, the ADM_{65} between pensioners in the highest and lowest PI groups is always smaller than the $ADLE_{65}$. For the entire period, on average, the inequality in longevity measured with the LE_{65} is 81.51% higher than when measured with the M_{65}.

It can be said for both males and females that the evolution of the ADM_{65} and $ADLE_{65}$ between pensioners in the highest and lowest PI groups shows a moderate positive correlation.

But what about the trends in life span variation? What we observe for males and what we observe for females are clearly different and depend on PI groups. In general there is an inverse correlation between life span variation and LE_{65} for females, whereas increases in life span variation for males are associated with increases in LE_{65}, thereby changing the historically observed correlation (in most developed countries) from negative to positive.

More specifically, for females the inverse correlation between life span variation and LE_{65} is moderate for the G1 group and weak for the G4 group both for females (negative or inverse) and males (positive)

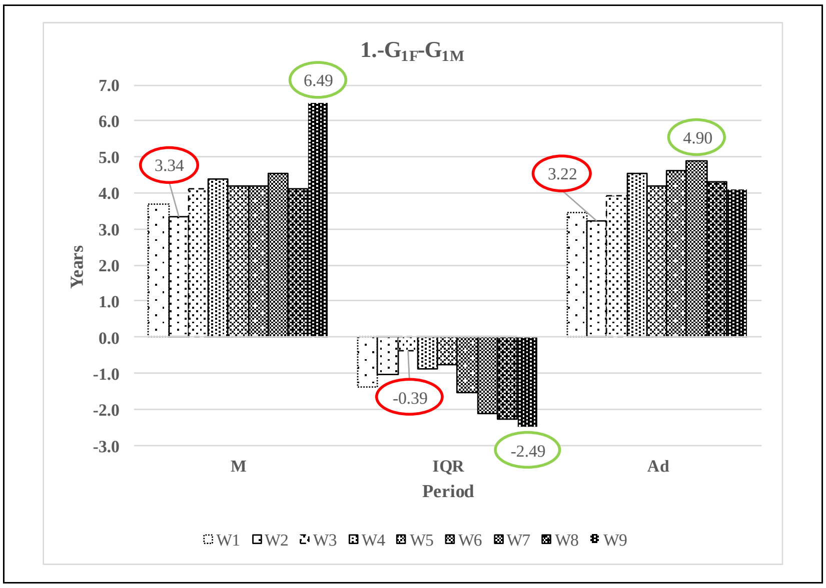
It should be remembered that the last two rolling windows were affected by the COVID-19 pandemic, which modified the association between life span variation and LE_{65}. If the last two windows had not been taken into account, we would have said that for females the inverse correlation between life span variation and LE_{65} is moderate for the G_1 group and strong for the G_4 group, while for males the positive correlation is weak for the G_1 group and inverse and weak for the G_4 group.

In short, the absolute differences between the G_4 and G_1 groups in IQR_{65} ($ADIQR_{65}$) have evolved very differently over time for males and females. For males the differences have increased despite some fluctuations, while for females, although there have also been large fluctuations, the differences have decreased and even changed sign in the last two rolling windows (W_8 and W_9), which, as already mentioned, have been influenced by the effect of the COVID-19 pandemic.

To end this section, Figure 4 (below) shows the absolute differences between females and males for a given PI group in IQR_{65}, Md_{65} and Ad_{65} for the periods studied. Again we have split the figure in two: top (differences for the G_1 groups) and bottom (differences for the G_4 groups).

Like in Figure 3, the absolute differences in Ad_{65} are identical to $ADLE_{65}$ (see Figure 1, Graph 2) and are included in the graphs so that they can be compared with the absolute differences of the mode value (ADM_{65}).

Contrary to what we saw in Figure 3, the socioeconomic inequality in longevity between the sexes for a given PI group is slightly higher for M_{65} than for LE_{65}. Over the whole period, the average inequality in longevity as measured with the M_{65} is respectively 4.85% and 0.6% higher for individuals belonging to the G_1 and G_4 groups than it is when measured with the LE_{65}. Finally, as regards the absolute differences between women and men for a given PI group in IQR_{65} ($ADIQR_{65}$), these have evolved very differently over time for G_1 and G_4. For individuals in the G_1 group the differences have increased despite fluctuations, but for individuals in the G_4 group, although there have also been some small fluctuations, the differences have decreased and even changed sign in the last two rolling windows.



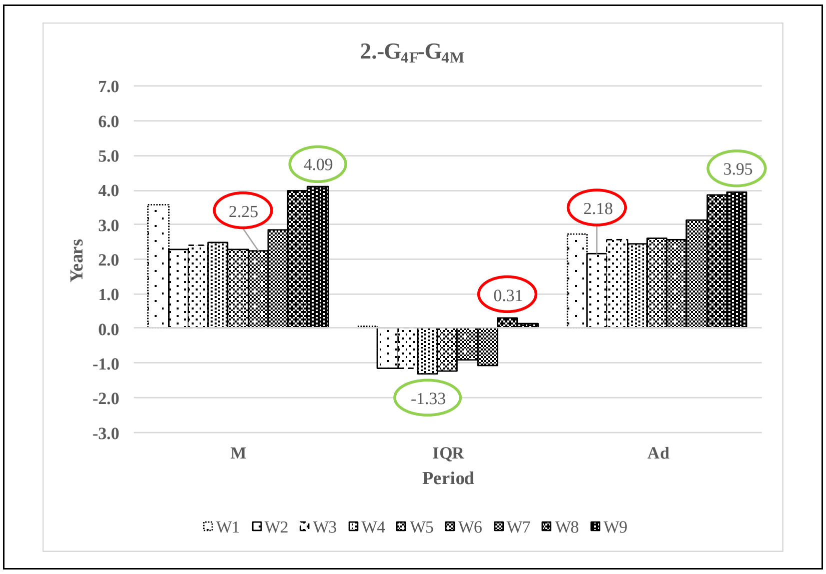


Figure 4: Absolute differences between females and males for a given PI group in IQR_{65}, Md_{65} and Ad_{65}

4.-Discussion

One question that immediately comes to mind is: are our results in line with those obtained in other countries using pension amounts and/or pensionable income?

Tables 12 (females) and 13 (males) show LE_{65} (in years) by PI group (lowest and highest) and the absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$) in LE_{65}. We do not want to distort comparisons with other countries by including the years affected by the COVID-19 pandemic, so the last data shown for Spain are for the 2013-2019 window.

Table 12: LE_{65} (years) by PI group (lowest and highest). Absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$). Females

Countries	Periods/Cohorts	Lowest	Highest	$ADLE_{65}$	$RDLE_{65}$	Trend
Spain	**2008-2013**	23.67	25.59	1.92	8.11%	
	2011-2016	24.02	26.02	2.00	8.31%	
	2014-2019	24.77	26.60	1.84	7.41%	
Argentina	**2015-2016**	18.80	21.20	2.40	12.77%	n.a.
Germany (a)	**2005-2008**	20.00	21.40	1.40	7.00%	
	2009-2012	20.63	21.93	1.30	6.30%	
	2013-2016	20.87	22.07	1.20	5.75%	
Germany (b)	**1996-1999**	15.09	17.34	2.25	14.90%	
	2004-2007	15.26	19.10	3.84	25.14%	
	2012-2015	14.64	21.16	6.52	44.57%	
Canada	**1999**	18.80	20.80	2.00	10.64%	
	2009	20.10	22.20	2.10	10.45%	
	2019	20.70	23.20	2.50	12.08%	
Netherlands	**1996-2008 (s)**	17.38	19.92	2.54	14.61%	n.a.
	1996-2008 (c1)	18.68	21.25	2.57	13.76%	

Table 12: LE_{65} (years) by PI group (lowest and highest). Absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$). Females						
Countries	**Periods/Cohorts**	**Lowest**	**Highest**	**$ADLE_{65}$**	**$RDLE_{65}$**	**Trend**
	1996-2008 (c2)	19.5	21.74	2.24	11.49%	
Italy	**1990-1994**	19.57	19.49	-0.08	-0.41%	→
	1995-1999	19.61	19.74	0.13	0.66%	
	2000-2004	19.61	19.95	0.34	1.73%	
Sweden	**2007**	19.5	22.1	2.60	13.33%	→
	2011	20.00	22.90	2.90	14.50%	
	2015	19.80	23.20	3.40	17.17%	
US	**1950**	21.20	25.40	4.20	19.81%	→
	1960	20.00	25.50	5.50	27.50%	
	1970	19.90	26.00	6.10	30.65%	
Source: Own based on Bramajo and Grushka (2019) (Argentina); Teezlaff et al. (2020) (Germany (a)); Lampert et al. (2019) (Germany (b)); Edwards et al. (2023) (Chile); Osfic (2022) (Canada); Kalwij et al. (2013) (Netherlands); Ardito et al. (2022) (Italy); Fors et al. (2021) (Sweden); and Reznik et al. (2021) (US)						

A certain amount of caution is needed when making comparisons, given that the individuals (pensioners) included in the so-called "lowest" and "highest" groups are defined differently in each country.

For Argentina, Bramajo and Grushka (2019) considered four income category scenarios corresponding to 1BM (minimum benefit), 2BM, 4BM and 8BM (this being an extremely high value compared to the average). The social security records held by the National Social Security Administration of Argentina (ANSES) on a single database were the main source of data for this study. In Tables 12 and 13 the "lowest" group includes those pensioners with 1 unit of minimum benefit (1BM), while the "highest" includes those with 8 units (8BM).

Table 13: LE_{65} (years) by PI group (lowest and highest). Absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$). Males						
Countries	**Periods/Cohorts**	**Lowest**	**Highest**	**$ADLE_{65}$**	**$RDLE_{65}$**	**Trend**
Spain	**2008-2013**	20.19	22.86	2.67	13.21%	→
	2011-2016	19.49	23.56	4.06	20.85%	
	2014-2019	19.86	23.48	3.61	18.19%	
Argentina	**2015-2016**	15.20	17.70	2.50	16.45%	n.a.
Germany (a)	**2005-2008**	14.76	17.86	3.10	21.00%	→
	2009-2012	15.22	18.72	3.50	23.00%	
	2013-2016	15.19	19.29	4.10	27.00%	
Germany (b)	**1996-1999**	8.71	16.55	7.84	90.07%	→
	2004-2007	9.11	16.85	7.74	84.87%	
	2012-2015	9.87	20.54	10.67	108.15 %	
Chile	**2019**	19.50	22.50	3.00	15.38%	n.a.
Canada	**1999**	14.50	16.90	2.40	16.55%	

→

Table 13: LE_{65} (years) by PI group (lowest and highest). Absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$). Males						
Countries	**Periods/Cohorts**	**Lowest**	**Highest**	**$ADLE_{65}$**	**$RDLE_{65}$**	**Trend**
	2009	16.50	19.00	2.50	15.15%	
	2019	17.40	20.30	2.90	16.67%	
Netherlands	**1996-2008 (s)**	10.85	12.64	1.79	16.50%	n.a.
	1996-2008 (c1)	15.21	17.98	2.77	18.21%	
	1996-2008 (c2)	15.5	18.07	2.57	16.58%	
Italy	**1990-1994 C**	16.63	17.72	1.09	6.55%	↗
	1995-1999 C	16.72	18.23	1.51	9.03%	
	2000-2004 C	16.77	18.55	1.78	10.61%	
Sweden	**2007**	16.1	19.7	3.60	22.36%	↗
	2011	16.40	20.30	3.90	23.78%	
	2015	16.60	21.10	4.50	27.11%	
US	**1950C**	18.70	21.60	2.90	15.51%	↗
	1960C	18.80	22.70	3.90	20.74%	
	1970C	18.40	22.50	4.10	22.28%	
Source: Own based on Bramajo and Grushka (2019) (Argentina); Teezlaff et al. (2020) (Germany (a)); Lampert et al. (2019) (Germany (b)); Edwards et al. (2023) (Chile); Osfic (2022) (Canada); Kalwij et al. (2013) (Netherlands); Ardito et al. (2022) (Italy); Fors et al. (2021) (Sweden); and Reznik et al. (2021) (US).						

In the paper for Germany (a), Teezlaff et al. (2020) divided individuals into three groups. The low-income group included those with a pre-tax annual income of <60% of the average income, the high-income group was for those with ≥80% of the average income, and the middle-income group was defined as falling between these categories. We use their 'low-income group' and 'high-income group' as our ‘lowest' and 'highest' respectively. For their study the authors used claims data from a large statutory health insurer in Lower Saxony (AOK Niedersachsen (AOKN)).

For the case of Chile, Edwards et al. (2023) use three income brackets based on monthly retirement benefits (low, medium and high). We use their 'low group' and 'high group' as our 'lowest' and 'highest' respectively. Their outcomes rely on the annuities issued since the inception of the new social security system in Chile (1983). The authors do not show LE_{65} for females.

As regards Canada (OSFIC, 2022), low-income pensioners (those who receive GIS, a guaranteed income-related supplement available to low-income Old Age Security (OAS) pensioners) are included in the "lowest" group, while higher-income pensioners (who do not receive GIS) are included in the "highest" group. The study, which was conducted by the Office of the Chief Actuary of the Canada Pension Plan (CPP), is based on data from OAS recipients over the experience periods from 1 January 1999 to 31 December 2019.

In the case of the Netherlands, Kalwij et al. (2013) split individuals into three groups: low income (those receiving an amount equal to the public pension benefit), medium income, and high income. They also distinguished between three household situations: single household (s), couple (c1) (if before the age of 65 the man worked full-time and the woman did not work), and couple (c2) (before the age of 65 the man worked full-time and the woman worked part-time). They used data from the Dutch Income Panel Study 1996-2007 (IPO, Inkomens Panel Onderzoek) and the Causes of Death Register 1997-2008 (DO, Doodsoorzaken), both collected by Statistics Netherlands.

For Italy, Ardito et al. (2022) used data from the administrative archives of the Italian National Social Security Institute (INPS). They used average weekly earnings as an indicator of income. Individuals were classified into quartiles according to average weekly wage, calculated using cut-off points deriving from the final data set of the analysis, and separately for men and women and for each time period. Tables 12 and 13 use their results for the lowest and highest quartiles.

As for Sweden, Fors et al. (2021) ranked older adults according to their disposable income relative to all other people of the same sex and age for each year and divided them into quartiles. They characterized each person's income level using data from the Swedish Income and Taxation Register, which covers all registered persons in that country. Tables 12 and 13 show their results for the lowest and highest quartiles.

Finally for the US, Reznik et al. (2021) classified individuals into lifetime earnings quartiles based on average indexed monthly earnings (AIME) at age 65. AIME reflects the average of an individual's highest 35 years of wage-indexed earnings and is used to calculate social security benefits. Quartiles are calculated separately by sex and cohort. The results of this paper differ significantly in terms of methodology from the other results presented in this discussion. They are based on projections. It should be noted that these projections of mortality and the way it might be distributed relative to mid-career earnings and by sex are subject to uncertainty from a number of sources (NASEM, 2015).

Tables 12 and 13 show the results for the lowest and highest quartiles, like in the previous cases.

Table 12 indicates that $RDLE_{65}$ for females in the case of Spain are relatively small and that the trend is slightly decreasing. Only Germany (a) and Italy show smaller differences than Spain. The studies of just two countries (Spain and Germany (a)) show narrowing trends (green arrows pointing downwards in the last column of Table 12), while the trend in four countries (Canada, Sweden, the US and Italy) is towards widening inequalities (red arrows pointing upwards in the last column of Table 12).

As mentioned earlier, comparisons should be treated with caution. As regards Germany (b), for example, the study by Lampert et al. (2019) reports much higher absolute and relative differences than the study by Tezlaff et al. (2020). It is not just the size of the $ADLE_{65}$ that is striking, but also the fact that in some cases the trends are moving in opposite directions. There could be several reasons for such large discrepancies in the results: the database, the methodology, sample size, sample period and, most importantly, the range of income values used to define the highest and lowest income groups.

As already noted, Teezlaff et al. (2020) divided individuals into three groups. Lampert et al. (2019), on the other hand, used the equivalized net income of the population in Germany to define five income categories: less than 60% of the median equivalized net income, between 60% and less than 80%, between 80% and less than 100%, between 100% and less than 150%, and 150% or more. In line with sociopolitical definitions, households with an income of less than 60% of the median can be described as living in or at risk of poverty.

We can see from Table 13 that the $ADLE_{65}$ and $RDLE_{65}$ are higher for men than for women. In all the countries analysed there is an increasing trend in inequality between the highest and lowest groups (upward-pointing red arrows in the last column of Table 13).

In Spain the inequalities in $RDLE_{65}$ are between 1.63 and 2.45 times greater for men than for women. These differences would be lower in Canada and the US but much higher in the cases of Italy and Germany.

Tables 14 (lowest groups) and 15 (highest groups) show the LE_{65} (in years) by sex along with the absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$) in LE_{65} within each group.

Spain is the only country in which the $ADLE_{65}$ and $RDLE_{65}$ have increased between the sexes for both groups of pensioners (lowest and highest). Most of the countries in the studies analysed – apart from Germany (a) for the lowest group of pensioners – show a reduction in inequality between the sexes in LE_{65} for both groups of pensioners.

The increase in inequality in LE_{65} between females and males in the lowest-income group of pensioners in Spain means that it is now only higher than in Germany (a) (b) and the Netherlands.

However, despite the increase in inequality between females and males at the highest pension level in Spain, it is still one of the countries with the lowest rates of inequality at this level (LE_{65}), together with Sweden, Germany (b) and Italy.

Table 14: LE_{65} values by sex for certain PI groups for selected countries. Absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$). Lowest income groups.

Countries	Periods/Cohorts	Males	Females	$ADLE_{65}$	$RDLE_{65}$	Trend
Spain	**2008-2013**	20.19	23.67	3.47	17.19%	↗
	2011-2016	19.49	24.02	4.53	23.23%	
	2014-2019	19.86	24.77	4.90	24.68%	
Argentina	**2015-2016**	15.20	18.80	3.60	23.68%	n.a.
Germany (a)	**2005-2008**	14.76	20.00	5.24	35.48%	↗
	2009-2012	15.22	20.63	5.42	35.60%	
	2012-2016	15.19	20.87	5.68	37.43%	
Germany (b)	**1996-1999**	8.71	15.09	6.39	73.34%	↘
	2004-2007	9.11	15.26	6.15	67.46%	
	2012-2015	9.87	14.64	4.77	48.33%	
Canada	**1999**	14.50	18.80	4.30	29.66%	↘
	2009	16.50	20.10	3.60	21.82%	
	2019	17.40	20.70	3.30	18.97%	
Netherlands	**1996-2008 (s)**	10.85	17.38	6.53	60.18%	n.a.
	1996-2008 (c1)	15.21	18.68	3.47	22.81%	
	1996-2008 (c2)	15.50	19.50	4.00	25.81%	
Italy	**1990-1994 C**	16.63	19.57	2.94	17.68%	→

Table 14: LE_{65} values by sex for certain PI groups for selected countries. Absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$). Lowest income groups.						
Countries	**Periods/Cohorts**	**Males**	**Females**	**$ADLE_{65}$**	**$RDLE_{65}$**	**Trend**
	1995-1999 C	16.72	19.61	2.89	17.28%	
	2000-2004 C	16.77	19.61	2.84	16.94%	
Sweden	**2007**	16.10	19.50	3.40	21.12%	→
	2011	16.40	20.00	3.60	21.95%	
	2015	16.60	19.80	3.20	19.28%	
US	**1950C**	18.70	21.20	2.50	13.37%	→
	1960C	18.80	20.00	1.20	6.38%	
	1970C	18.40	19.90	1.50	8.15%	
Source: Own based on Bramajo and Grushka (2019) (Argentina); Teezlaff et al. (2020) (Germany); Lampert et al. (2019) (Germany*); Edwards et al. (2023) (Chile); Osfic (2022) (Canada); Kalwij et al. (2013) (Netherlands); Ardito et al. (2022) (Italy); Fors et al. (2021) (Sweden); and Reznik et al. (2021) (US).						

In Spain, the inequalities as regards LE_{65} are between 1.44 and 2.22 times higher for the most disadvantaged group of pensioners than for the most well-off group. These differences are similar to those found in Germany and Italy, but much higher those found in Sweden, the Netherlands, Canada, Argentina and the US.

Table 15: LE_{65} values by sex for certain PI groups for selected countries. Absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$). Highest income groups.						
Countries	**Periods/Cohorts**	**Males**	**Females**	**$ADLE_{65}$**	**$RDLE_{65}$**	**Trend**
Spain	**2008-2013**	22.86	25.59	2.72	11.91%	→
	2011-2016	23.56	26.02	2.46	10.44%	
	2014-2019	23.48	26.60	3.13	13.32%	
Argentina	**2015-2016**	17.70	21.20	3.50	19.77%	n.a.
Germany (a)	**2005-2008**	17.86	21.40	3.54	19.81%	→
	2009-2012	18.72	21.93	3.22	17.19%	
	2012-2016	19.29	22.07	2.78	14.44%	
Germany (b)	**1996-1999**	16.55	17.34	0.79	4.79%	→
	2004-2007	16.85	19.10	2.25	13.36%	
	2012-2015	20.54	21.16	0.62	3.02%	
Canadá	**1999**	16.90	20.80	3.90	23.08%	→
	2009	19.00	22.20	3.20	16.84%	
	2019	20.30	23.20	2.90	14.29%	
Netherlands	**1996-2008 (s)**	12.64	19.92	7.28	57.59%	n.a.
	1996-2008 (c1)	17.98	21.25	3.27	18.19%	
	1996-2008 (c2)	18.07	21.74	3.67	20.31%	
Italy	**1990-1994 C**	17.72	19.49	1.77	9.99%	→
	1995-1999 C	18.23	19.74	1.51	8.28%	
	2000-2004 C	18.55	19.95	1.40	7.55%	
Sweden	**2007**	19.70	22.10	2.40	12.18%	→
	2011	20.30	22.90	2.60	12.81%	

Table 15: LE_{65} values by sex for certain PI groups for selected countries. Absolute ($ADLE_{65}$) and relative differences ($RDLE_{65}$). Highest income groups.						
Countries	**Periods/Cohorts**	**Males**	**Females**	**$ADLE_{65}$**	**$RDLE_{65}$**	**Trend**
	2015	21.10	23.20	2.10	9.95%	
US	**1950C**	21.60	25.40	3.80	17.59%	→
	1960C	22.70	25.50	2.80	12.33%	
	1970C	22.50	26.00	3.50	15.56%	
Source: Own based on Bramajo and Grushka (2019) (Argentina); Teezlaff et al. (2020) (Germany); Lampert et al. (2019) (Germany*); Edwards et al. (2023) (Chile); Osfic (2022) (Canada); Kalwij et al. (2013) (Netherlands); Ardito et al. (2022) (Italy); Fors et al. (2021) (Sweden); and Reznik et al. (2021) (US).						

None of the papers cited above included any additional indicators of longevity, so valuable information that was not revealed by life expectancy, as we have just seen, may have been lost. Apart from the case of Sweden (Fors et al., 2021), none of the papers cited included indices of life span variation.

Although it is not the main focus of this paper, we would like to briefly discuss the effect of the COVID-19 pandemic on LE_{65} and compare it with the equivalent LE_{65} of the general population as determined by the rolling windows. Table 16 shows the relative differences in LE_{65} between the selected periods by sex and PI group.

Table 16: Relative differences in LE_{65} between the selected periods by sex and PI group.								
Items	**General population**		**Males**			**Females**		
	Males	**Females**	**G_1**	**Average**	**G_4**	**G_1**	**Average**	**G_4**
W_8-W_7	-0.62%	-0.44%	1.03%	-0.63%	-1.28%	-1.59%	-0.73%	1.60%
W_9-W_8	0.21%	0.31%	0.89%	-0.37%	-0.42%	-0.05%	0.27%	0.02%
W_9-W_7	-0.41%	-0.14%	1.93%	-0.99%	-1.69%	-1.64%	-0.46%	1.62%
Source: Own								

The last two rolling windows are (partially) affected by the COVID-19 pandemic. The penultimate window (W_8: 2015-2020) is affected in its final year and the last window (W_9: 2016-2021) in its final two years. The antepenultimate window (W_8: 2015-2020), which is assumed to be unaffected by the COVID-19 pandemic, is used as a benchmark period for comparison. It is noticeable that the pensioner population seems to be more affected than the general population, and that males seem to be more affected by COVID-19 than females, in line with the general population. But the most striking aspect of all is the unequal impact within the different groups of pensioners by PI level.

There are two groups of pensioners – the least advantaged group of males and the most advantaged group of females – that appear to be "beneficiaries" of the COVID-19 pandemic. This clearly seems to be an anomaly. It is very difficult to know the causes of such an anomaly within our database, but as Aburto et al. (2022) point out, annual changes in life expectancy are subject to random fluctuations that are greater for smaller populations/groups, and that could be the case here. To shed some light on the issue it might be a good idea to apply Navarro and Requena's (2023) methodology based on an analysis of mortality rates, but doing so would clearly go beyond the scope of this paper.

To conclude this section we should consider the usefulness of our results, i.e. how they could be used to improve the information that the (Spanish) social security system ought to provide to stakeholders.

The use of life tables arranged by PI level could be of benefit to several studies that have been carried out on subjects such as the actuarial fairness of the contributory pension system, the implicit redistribution embedded in the pension calculation formula and the valuation of liabilities to pensioners, to name just a few.

It has long been recognized that the Social Security Administration in Spain does not produce enough actuarial studies on the contributory pension system, possibly because there is no Office of the Chief Actuary or any similar research department. In countries such as the United States[1], the United Kingdom[2], Canada[3], Sweden[4] and Japan[5], these offices play an important role. In Spain, the function most similar to that carried out by the Office of the Chief Actuary would be the work done by the Independent Authority for Fiscal Responsibility (AIReF)[6] and the Analysis and Research Department of the Bank of Spain[7]

Spain's state contributory pension system is essentially funded on a pay-as-you-go (PAYGO) basis. In a PAYGO system the benefits do not depend on the accumulation of individual contributions like in a defined contribution plan, and neither do annual contributions depend on the planned future benefits of current workers and pensioners like in a pre-funded defined benefit plan. Instead, the total benefits paid in a year determine the combined amount that workers and employers must contribute to fund the scheme for that year.

In the context of the state contributory pension system in Spain, it would be very useful if analyses of theoretical money's worth ratios for hypothetical workers/pensioners were regularly presented, including different earnings patterns and levels. For an individual or a cohort of workers, the money's worth ratio is the ratio of the present value of expected benefits to the present value of expected contributions. A ratio greater than one would indicate that, on a present value basis, more money is expected to be received in benefits

[1] https://www.ssa.gov/OACT/
[2] https://www.gov.uk/government/organisations/government-actuarys-department
[3] https://www.osfi-bsif.gc.ca/Eng/oca-bac/Pages/mnd.aspx
[4] https://www.pensionsmyndigheten.se/other-languages/english-engelska/english-engelska/publications
[5] https://www.mhlw.go.jp/english/org/policy/p36-37a.html
[6] The AIReF is an independent administrative authority (Spanish acronym: AAI). It has legal personality and full public and private capacity. It acts objectively, transparently and impartially and carries out its functions autonomously and independently of the general government. Its official name is "Autoridad Independiente de Responsabilidad Fiscal, AAI". However, in its public activity it also uses the name "Independent Authority for Fiscal Responsibility" or AIReF. https://www.airef.es/en/about-us/
[7] Economic analysis and research are among the Bank of Spain's main tasks. These include studying and monitoring the Spanish economy and its environment, economic policy (with particular reference to the monetary policy of the Eurosystem), and the banking and financial system.

than the amount expected to be paid in contributions over the lifetime of the individual or cohort.

Such a practice is regularly undertaken by the US Social Security Administration, specifically the Office of the Chief Actuary (Clingman et al., 2021). However, these authors acknowledge that the money's worth ratios do not reflect differences in mortality by income level. This means that the results are partially biased.

Nevertheless, there are various measures of intergenerational distribution that can be used to assess the distributional characteristics of existing pension systems and the impact of pension reform proposals (Knell, 2010). A widely used measure is the internal rate of return (IRR), which has an added advantage in that it does not need a suitable discount rate to be decided on. A pension scheme's IRR is the discount rate that equals the present value of contributions and the discounted value of pension entitlements accrued through those contributions (Vidal-Meliá et al., 2016). To a large extent, the contributors' IRR depends on several assumptions, such as the age they started work, the life tables used and the time the calculation was carried out (when starting work, when retiring, a few years after entering the labour market, etc.). This IRR could be interpreted as the return a new retiree might expect to obtain from their participation in the contributory (retirement) pension system.

This measure is regularly reported by the US Social Security Administration (Leimer, 1996; 2007) and the Office of the Chief Actuary in Canada (Osfic, 2016).

The Social Security Administration in Spain, on the other hand, does not publish values for the IRR, although the Bank of Spain did estimate it for a sample of new pensioners in 2017 (Moraga and Ramos, 2020). Although the analysis is interesting, it could certainly be improved by using survival probabilities by PI level instead of those taken from the male and female mortality tables produced by the National Statistics Institute (INE). It seems clear that, if they were used, there would be an increase in the heterogeneity of the returns provided by the system and that the IRR would be higher in all cases. This is because, as we saw earlier, the average LE_{65} of the pensioner population is higher (between 2 and 3 years) than that of the general population.

Finally, our life tables by PI level could be used to better estimate liabilities to pensioners. Since 2017 EU regulations have required all Member States to disclose their accrued-to-date pension liability (ADL) using a standard actuarial cost method and some common assumptions. These pension liabilities[8] have to be disclosed in a supplementary document known as Table 29 (Garvey et al., 2022).

In the case of social security pension schemes that cover a large part of the population, demographic data provided by Eurostat ensure a comparable dataset across EU countries. Eurostat provides age- and sex-specific mortality tables for all EU countries. However, it is only if specific data on the mortality of social security members are available that such data should be used. Since mortality rates differ widely between men and women, a sex-specific differentiation of mortality data is necessary (Eurostat, 2020).

[8] Liabilities are expressed in terms of "actuarial present value" (APV). The APV is the sum of money needed now which, invested over the duration of the scheme's pension commitments, is expected to be sufficient to pay out all the pensions promised.

In order to compile Table 29 showing Spain's pension liabilities at the end of 2018, the INE (2020) used the mortality tables projected for Spain in the Eurostat Population Projections 2019-2100. Given that the 2019 life tables of the general population in Spain were used to value the pensions in payment, i.e. the liability to pensioners, it seems clear that this liability would have been undervalued. If it had been valued using life tables arranged by pension amount, the result would have been higher.

4.1.-Limitations

We need to consider the study's limitations before we conclude this section.

First, we are aware that PI is not a perfect indicator of a pensioner's total income. Other forms of income may arise from part-time work after retirement, income from investments and savings, government and private transfers and so on. However, for 70% of Spanish pensioners the state pension is their only source of income, which means that only 30% of retired people have additional income from private pensions and savings, insurance plans, financial products and rental income (Pérez-Salamero et al., 2022). It could be said that for retired people with lower pensions (our G_1 group) the amount of their pension is a good indicator of their total income, whereas for retired people with higher pensions (our G_4 group) this amount is not such a good proxy.

Second, we excluded certain groups of pensioners for whom applying our longevity indicator may not have been appropriate for various reasons (disabled pensioners, early retirees, those covered by special schemes such as the self-employed).

Third, we could not include the collective of pensioners belonging to the Régimen de Clases Pasivas (civil servants) because they are not included in the database used.

Fourth, the sample we used does not allow us to draw conclusions for the entire population, or even for all retirement pensioners. However, it does work for a large part of this collective, since those included in the general scheme (4,448,130) represent 91.72% of the total number of pensioners excluding self-employed retirement pensioners (6,165,400 – 1,315,700) in 2021 (BEL, 2023).

A fifth limitation could arise from our classification of PI levels into quartiles. Dividing the PI distribution into only four groups could lead to the omission of some heterogeneity by PI level. If we had access to all the records held by the Spanish Social Security Institute along with details of any additional sources of income that the pensioners may have, we could perhaps have used quintiles or deciles instead of quartiles. This would have made the estimation of inequalities in mortality much more accurate.

5.-Conclusions

It can be said that the paper's objective has been achieved, given that the basic research questions have been answered. We have found that LE_{65} has a positive link with the PI level for both males and females and that this is true for all the periods analysed. It can also be seen that the LE_{65} for females exceeds that for males in all periods and that its evolution for males and females by PI level is clearly different. We can say that for the female pensioners as a whole, the evolution of the LE_{65} would be closest to that of the G_1 group (the most disadvantaged group), whereas for males it would be closest to that of the G_4 group (the most affluent). The absolute differences in LE_{65} between pensioners in the highest and the lowest PI groups fluctuate over the nine rolling windows examined.

For males, the differences increase at the beginning and then tend to decrease. For females there are also fluctuations, but the absolute differences are always smaller than those observed for males.

We have also found that the absolute differences in LE_{65} between females and males are greater in the most disadvantaged group than in the most advantaged. The absolute differences in LE_{65} (DLE_{65}) between females and males for the whole group of pensioners kept growing, increasing over time from 3.49 to 4.01 years. For most of the cases analysed, the DLE_{65} between G_1 groups, G_4 groups and total groups are statistically significant at 1%.

We have also detected highly significant evidence of a positive relationship between life expectancy at older ages ($LE_{75;}$ LE_{85}) and PI groups. This is also true for males and females separately, but the statistical significance diminishes slightly.

In addition to the classical longevity indicator, we have also presented other indicators of longevity and life expectancy variation by PI level. These indicators have revealed some results that life expectancy usually conceals. It could be said that socioeconomic inequality in longevity by PI group is lower measured with M_{65} than with $LE_{65.}$ We have found an inverse correlation between life span variation and LE_{65} for females and males in the highest PI group, whereas increases in life span variation are associated with increases in LE_{65} for males in the least advantaged group, i.e. the historically observed correlation (in most developed countries) changes from negative to positive.

Male pensioners in the most disadvantaged group not only die earlier on average than their more affluent counterparts, they also face greater variation in time of death – a double burden of inequality that has increased over time. Unlike what was observed when a PI group for a given sex is analysed, the socioeconomic inequality in longevity between the sexes for a given PI group is slightly higher for M_{65} than for LE_{65}.

It therefore seems clear that it is important to complement the information provided by life expectancy with additional indicators of longevity and life span.

We have also found that the pensioner population seems to have been more affected by the COVID-19 pandemic than the general population, and that males seem to have been more affected than females, which is in line with the general population. The most striking aspect of all the results is the unequal impact within the different groups of pensioners by PI level. There are two groups of pensioners, the least advantaged males and the most advantaged females, who seem to have benefitted from the COVID-19 pandemic. This naturally looks like an anomaly and further research is needed to shed light on it.

Last but not least, in the context of the Spanish contributory public pension system, the use of the life tables by PI level that have been constructed in this paper could improve several studies that have looked at areas such as the actuarial fairness of the contributory pension system, the implicit redistribution embedded in the pension calculation formula and the valuation of liabilities to pensioners, to give just a few examples.

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7.-Appendix

7.1.-Technical details

The observed probability of death

For all pensioner groups classified by PI level m, the crude mortality rate for a given period-year interval, $\mathrm{P} = \{a, a+1, \cdots, n\}$, age x, and sex j, is defined as the observed probability that a person of age x nearest birthday will die between ages x and $x+1$ during the period-year interval P. n represents 31 December for the last calendar year within the period, and a represents 31 December for the first year.

The observed probability of death is calculated by simply dividing the relevant number of deaths ($D_{x,\mathrm{P}}^{j,m}$) by the number of life-years of exposure over the given year or period ($E_{x,\mathrm{P}}^{j,m}$).

The size of the exposure population is estimated by averaging the population sizes at the beginning and end of the year. In our case, the crude mortality rate $\hat{q}_{x,\mathrm{P}}^{j,m}$ is calculated as follows:

$$\hat{q}_{x,\mathrm{P}}^{j,m} = \frac{D_{x,\mathrm{P}}^{j,m}}{E_{x,\mathrm{P}}^{j,m}} = \frac{D_{x,a+1}^{j,m} + \cdots + D_{x,n}^{j,m}}{\frac{1}{2}\left(D_{x,a+1}^{j,m} + \ldots + D_{x,n}^{j,m} + L_{x,a}^{j,m} + L_{x,n}^{j,m}\right) + L_{x,a+1}^{j,m} \ldots + L_{x,n-1}^{j,m}} \quad (1)$$

where $D_{x,\mathrm{t}}^{j,m}$ is the observed number of deaths of individuals who have attained age x on their nearest birthday for PI level group m, gender j in calendar year $t \in \{a+1, \cdots, n\}$; and $L_{x,\mathrm{t}}^{j,m}$, with $t \in \{a, \ldots. n\}$, is the observed number of retirement pensioners aged x at their nearest birthday in PI level group m and gender j, at the end of year $t \in P$.

Given that the levels of exposure are not sufficiently high for some age groups, the initial estimates need to be revised to produce smoother estimates (graduated mortality rates) using a procedure called graduation. In our case the average crude death rates are graduated through the age, PI level, sex and period dimensions to reflect a compromise between smoothness and fit.

Testing for differences between populations/groups

Testing whether there is a significant positive difference in life expectancy $LE_{x,P}^{m}$ between two pensioner income groups for a given age and period, one with a higher PI than the other, can be done by using one-tailed statistical tests based on normal distribution (Scherbov and Ediev, 2011; Li, 2015). Under the null hypothesis, this difference will be zero and under the alternative it will be positive.

The z score statistic is defined as the ratio of $DLE_{x,P}^{m_i-m_j}$, the difference in life expectancy between the groups, to the standard error of that difference, $SDLE_{x,P}^{m_i-m_j}$, which is computed as the square root of the sum of the variances of the corresponding $LE_{x,P}^{m}$ for each group, $VLE_{x,P}^{m_i}$ and $VLE_{x,P}^{m_j}$, respectively:

$$z_{x,P}^{m_i-m_j} = \frac{DLE_{x,P}^{m_i-m_j}}{SDLE_{x,P}^{m_i-m_j}} = \frac{DLE_{x,P}^{m_i-m_j}}{\sqrt{VLE_{x,P}^{m_i} + VLE_{x,P}^{m_j}}} \quad (2)$$

Following Chiang (1984), the variance of the corresponding life expectancy at age x for a given group m and period P can be calculated as

$$VLE_{x,P}^{m} = \sum_{k=0}^{w-1-x} \left[({}_{k}p_{x,P}^{m})^2 \cdot (LE_{x+k+1,P}^{m})^2 \cdot \frac{(q_{x+k,P}^{m})^2 \cdot (1 - q_{x+k,P}^{m})}{D_{x+k,P}^{m}} \right] \quad (3)$$

where ${}_{k}p_{x,P}^{m}$ is the probability of surviving from age x to age $x+k$, $q_{x+k,P}^{m}$ is the probability that an individual aged $x+k$ will die within the year, $\mathrm{LE}_{\mathrm{x+k+1,P}}^{\mathrm{m}}$ is life expectancy at age x+k, and $D_{x+k,P}^{m}$ is the number of deaths at age $x+k$. These elements refer to period P and PI group m.

We reject the null hypothesis if the sample value of the statistic $z_{x,P}^{m_i-m_j}$ is greater than the critical value at given level of significance α in the normal distribution. If that is the case, then there is statistically significant evidence that $LE_{x,P}^{m}$ is greater for the higher PI group than for the other.

95% confidence intervals (CIs) of $\mathrm{LE}_{\mathrm{x,P}}^{\mathrm{m}}$ are determined by:

$$95\%CI\left(LE_{x,\mathrm{P}}^{m}\right) = LE_{x,\mathrm{P}}^{m} \pm 1.96 \cdot \sqrt{VLE_{x,P}^{m}} \tag{4}$$

The median age at death, Md

When the value of Md is found between two complete single ages x and $x+1$, its value needs to be interpolated as $Md = x + \gamma$, where γ is a function of the number of people surviving in the same health state between ages x and $x+1$. Assuming linearity in this interval, age Md is located as:

$$Md = x + \frac{(l_x - \frac{l_{x_e}}{2})}{(l_x - l_{x+1})} \tag{5}$$

Alternatively, the median age at death can be defined as the age at which the survival function is equal to one half (Canudas-Romo, 2008), and using the discrete distribution of deaths is

$$Md = \left\{x \text{ such that } S_{x_e}(x) = {}_{x-x_e}p_{x_e} = \frac{1}{2}\right\} \tag{6}$$

The value of Md with decimal precision points can also be estimated by linear interpolation between two complete single ages x and $x + 1$

$$Md = x + \frac{(S_{x_e}(x) - 0.5)}{(S_{x_e}(x) - S_{x_e}(x+1))} \tag{7}$$

The adult modal age at death, *M*.

In our case the cohort is aged $x_e = 65$ or over:

$$M = \{x \text{ such that } x > x_e \text{ and } d_x > d_a \;\; \forall\, a > x_e\} \tag{8}$$

where d_x is the number of deaths between the exact ages of x and $x+1$.

Alternatively, and more appropriately for actuarial purposes, the modal age at death can be defined as

$$M = \left\{x \text{ such that } Max\left[d_{x_e}(x)\right] \;\; \forall\, x > x_e\right\} \tag{9}$$

where $d_{x_e}(x)$ is the life span for the cohort aged x_e, i.e. the life table density function describing the distribution of deaths for a cohort starting from age x_e. To obtain the expression for the modal age at death with decimal precision, its value is estimated in the range $[x-1, x+1]$ by the parabola (a quadratic polynomial approximation) that has the right areas below it to produce the observed values $d_{x_e}(x-1)$, $d_{x_e}(x)$ and $d_{x_e}(x+1)$, i.e. (Canudas-Romo, 2010)

$$M = x + \frac{d_{x_e}(x) - d_{x_e}(x-1)}{\left(d_{x_e}(x) - d_{x_e}(x-1)\right) + \left(d_{x_e}(x) - d_{x_e}(x+1)\right)} \quad (10)$$

The interquartile range (IQR)

This is computed by the mathematical difference between the third and first quartiles of the data:

$$Q_3 - Q_1 \quad (11)$$

where Q_1 and Q_3 are, respectively, the first and third age quartiles, values that will have to be interpolated with a discrete distribution (van Raalte & Caswell, 2013). The IQR is not sensitive to outlier data. It is sensitive to transfers between quartiles but not to transfers within quartiles. In our context, it indicates the table-specific difference between the 25th and 75th percentiles in survivorship. The larger range in this measure indicates more variability and uncertainty, whereas a smaller range signals greater regularity in life spans.

7.2.-Further results

These results are available upon request from the authors.