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## “Mixed oligopoly and predatory public firms”

Joan-Ramon Borrell and Carlos Suarez

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## *Abstract*

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In this paper, we propose a mixed duopoly model in which the public company aims to maximize a weighted function of profits and a function of its production scale. We found that if the weight to the scale of production is high the public firms may exclude its rivals from the market (exercising predatory prices). We also find that the profit sacrifice by the public firm to get this exclusion is higher if there are marked differences between the cost efficiency of private and public firms.

*JEL classification:* L13, L94, C10.

*Keywords:* Mixed Oligopoly, Predatory prices, Public firm.

Joan-Ramon Borrell: Universitat de Barcelona, Institut d'Economia Aplicada (IREA) - Grup de Governos i Mercats (GiM); and, University of Navarra, IESE Business School, Public-Private Sector Research Center. Email: [jrborrell@ub.edu](mailto:jrborrell@ub.edu)

Carlos Suarez: Universidad Jorge Tadeo Lozano, Grupo de Investigación en Energía, Ambiente y Desarrollo Sostenible; Universitat de Barcelona, Institut d'Economia Aplicada (IREA) - Grup de Governos i Mercats (GiM). Email: [carlos.suarez@ub.edu](mailto:carlos.suarez@ub.edu)

# Mixed oligopoly and predatory public firms\*

Joan-Ramon Borrell<sup>†</sup> and Carlos Suarez<sup>‡</sup>

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## Abstract

In this paper, we propose a mixed duopoly model in which the public company aims to maximize a weighted function of profits and a function of its production scale. We found that if the weight to the scale of production is high the public firms may exclude its rivals from the market (exercising predatory prices). We also find that the profit sacrifice by the public firm to get this exclusion is higher if there are marked differences between the cost efficiency of private and public firms.

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## 1 Introduction

To date, compelling predatory pricing theories study markets in which the firms involved are profit maximizers. According to Fumagalli et. to the. (2018) to consider a predatory pricing theory as convincing, this behavior must be profitable and at the same time rational, that is, predation must be the most profitable strategy among the different alternatives available to the incumbent company.

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\*Disclaimer: The authors have been consulting experts for some complainers and plaintiffs in cases of predation and State aid.

<sup>†</sup>Universitat de Barcelona, Institut d'Economia Aplicada (IREA) - Grup de Governos i Mercats (GiM); and, University of Navarra, IESE Business School, Public-Private Sector Research Center. jrborrell@ub.edu

<sup>‡</sup>Universidad Jorge Tadeo Lozano, Grupo de Investigación en Energía, Ambiente y Desarrollo Sostenible; Universitat de Barcelona, Institut d'Economia Aplicada (IREA) - Grup de Governos i Mercats (GiM). carlos.suarez@ub.edu

In many liberalized markets and under competition the incumbent company is a public company. Likewise, although the maximization assumption is reasonable when studying the interaction of private companies, there are theoretical arguments and empirical evidence that indicates that the final objectives of public companies may be different from profit maximization.

However, despite these observations, to date no particular predatory pricing model has been devised for the case where the incumbent company is a public company. This paper aims to contribute filling this gap in the literature.

This research is of special interest in the current context, in which due to the powerful effects of the COVID 19 pandemic, state aid is being revalued as an instrument of economic policy. In many of the markets in which public companies are the incumbents, they are also in charge of the provision of universal public service obligations and receive large amounts of state aid. In this study we want to contribute to understanding whether state aid to incumbent public companies can have adverse effects on competition.

The mixed oligopoly studies have analyzed theoretically the strategic interaction between public and private firms in non-perfect competitive markets (Harris et al 1980;. Cremer et. al., 1989). This branch of the literature has proposed models in which public companies compete with private companies.

In several cases the difference between both types of company is the objective function that they must maximize. Traditional approaches to public firms have mainly viewed them as instruments of government policy and planning (Bös, 2015). Following this approach, the mixed oligopoly models assumes that the objective functions of public firms is to promote social welfare and private companies behave as profit maximizers.

In fact, many of the conclusions drawn from these models arise from this objective function difference.<sup>1</sup> In this type of models, under the assumption of equal productive efficiency of public and private firms, the impact of privatization on welfare is driven by its effect on the intensity of competition.

However, public firms may have different objectives to profit maximization and even may have multiplicity of objectives (Kay and Thompson, 1986). In the field, the objective function

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<sup>1</sup>Several authors of these types of models have concluded that full privatization is not recommendable because it can have counter-competitive effects in the market.

of this type of firms depends of several issues related with the final objective of the government and the incentives provided to their managers (Fershtman and Judd, 1987).

Hence, the literature of mixed oligopoly takes a somewhat naive view of the incentives for public enterprises.<sup>2</sup> For instance, if there are political pressure from voters to decrease prices, SOEs may try to mitigate market power, even applying predatory prices. Conversely, if a government wants to solve a problem of fiscal deficit, its SOEs may try to maximizing profits using market power marks up as covered taxes.

Likewise, governments committed with a privatization program, they will boost the profit performance of the public firm in order to increase the sale price. In addition, in the particular of mixed firms a government majority share, the board members have a fiduciary duty to the minority shareholders and therefore they cannot ignore profit-maximization incentives. According to Thompson and Kay (1986) *“Public sector managers could be expected to respond to the particular personal incentives with which they were faced. Such incentives might lead to a desire to maximise the scale of operations of the business, subject to any external financial constraint, or to seek a quiet life untroubled by changes in working practices or difficulties in labour relations, rather than to pursue a nebulous public good.”*

In this paper, we propose a model in which the public company aims to maximize its size in order to increase the bureaucratic apparatus and enhance its clientelism (in political terms) of the company. We adopt two alternatives in order to model the problem of the public company. In the first approach the objective function of the bureaucratic public firm is a linear combination of the total sales and the benefits of the firm. In the second alternative we model the objective function of the bureaucratic public firm as a linear combination of the total productio and the benefits of the firm.

We found that if the weight to the bureaucratic objective is high the public firms excludes its rivals from the market (exercising predatory prices). This result is interesting because, pose a predatory situation in which it is not necessary a recovery phase. In this context, the recovery phase is not necessary for two reasons: i) The public company receives state aid that compensates it for its losses and ii) The public company does not seek to maximize profits,

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<sup>2</sup>Note that maximizing social welfare in the long term also implies adopting technologies that minimize costs. This is contrary to the results of several empirical studies that indicate that public companies are less cost-efficient than private ones. This evidence can be interpreted as a symptom that public companies should not be maximizing social welfare.

therefore it does not care to recover the losses.

The rest of this paper is divided into four sections. The second frames our contribution in the literature of mixed oligopoly and presents evidence that supports the adoption of an objective function that weights the scale of production. Section three presents the model, characterizes the market equilibria and presents the conditions for predatory prices. The final section summarizes the results and concludes.

## 2 Literature review

This paper lies at the intersection of two branches of the theory of industrial organization: i) the literature of mixed oligopoly, interested in the study of the interaction in the same relevant market of public and private firms, and ii) the theoretical models of predation.

The mixed oligopoly literature studies the strategic interaction of public and private companies that compete in the same relevant market. The seminal papers of this group of studies has focused on the optimal level of privatization (De Fraja, et al., 1989; Fershtman, 1990; Matsumura, 1998). Other authors, as Harris et al., 1980; Beato et al., 1984; Cremer et al, 1989; Brandao et al, 2007; Bel et al, 2009 study the role of public companies as an instrument of economic policy. For instance, Bel et. al. (2009) analyses how the size of the universal service obligations (USO) and the mechanisms traditionally used to finance it have prevented privatization in the postal sector. Finally, a third group of studies uses the mixed oligopoly framework for analyzing the incentive compatibility between corporate managers and objectives in both private and public companies (Freshtman et al., 1987; Barros, 1995). All these studies assumes a public interest vision of the objective function of private firms.

Few mixed oligopoly models have explored objective functions that incorporates the elements of the point of view of the private interest and public choice theories of the economic regulation. Nett (1994) studied the technology decision by public and private firms in a mixed duopoly framework. He found that a private firm has an incentive to operate at lower variable costs but higher fixed costs than a public firm. He assumes that a public firm maximizes output while facing a zero-budget constraint. The author justify this assumption presenting the arguments posed in Rees (1984). This author argues that for several reasons the manager of a public firm has the possibility to pursue his own objective and given this alternative, the

manager typically is interested in status, prestige, number of employees etc. These objectives are highly correlated with output. Sappington and Sidak (2003) focus their theoretical analysis on the setting where the public enterprise seeks both profit and expanded scale of operations. These authors found that a reduced focus on profit may generate stronger incentives to pursue anticompetitive activities, such as prices below marginal cost and erecting entry barriers, than does a private profit-maximizing firm. White (2002) demonstrates in a mixed oligopoly framework how a government can maximize its true objective by assigning a public firm a different one, which can be used to disguise an unpopular political agenda or outside influence on the regulatory body.

Regarding the formal approach, the typical model of mixed oligopoly poses that the problem of the public company is the maximization of an objective function that is the linear combination of social welfare and the benefits of the public firm (Matsumura, 1998; Lee et.al, 2003; Claude et.al, 2005 ):

$$V = (1 - \theta) W + \theta \pi_0$$

Where  $V$  is the objective function,  $W$  is social welfare,  $\pi_0$  is the profits of the public company and  $\theta$  is a function of private shareholding in the public firm. On the contrary the studies of Nett(1994) and Sappington and Sidak (2003) discard the inclusion of the welfare in the objective function and instead they weight a function of the production scale. We adopt this type of approach. The assumption of this type of behavior by public firms is justified by theoretical and empirical studies.

From the theoretical perspective, Shleifer and Vishny (1994) studied the behavior of private and public enterprises in situations where politicians try to influence firms to pursue political objectives. They argue that public enterprises are highly inefficient, and their inefficiency is the result of political pressures from the politicians who control them. They put emphasis in the excess of employment of public firms and point out that the beneficiaries of the excess employment are often political supporters of the government, who value these jobs because they pay more than market wages. In their model, to persuade the manager of the public firm to increase the extra employment, the politician subsidize the firm. Boycko, Shleifer and Vishny (1996) claim that public firms are highly inefficient, primarily because they pursue strategies,

such as excess employment, that satisfy the political objectives of the officeholders. They also argue that privatization contributes to reducing these inefficiencies because the payments to private firms necessary to incentive them to maintain inefficient levels of labor and contracting are not politically sustainable in comparison to the alternative economic policy tool such as subsidies for public firms. Sappington and Sidak (2004) argue that often public firms may act more aggressively toward their competitors than would private firms. They suggest that potential conducts to disadvantage competitors include setting prices below cost, misstating costs and choosing inefficient technologies to circumvent restrictions on below-cost pricing, raising the operating costs of existing rivals, and erecting entry barriers to preclude the operation of new competitors. These authors also point out that the directives given to public firms to increase local employment and/or to ensure that affordable service is provided ubiquitously to low income families, provokes that the public firm act as if they value the expanded scale of production. Also, these authors argue that even though SOEs may value the profit, they also pursue anticompetitive activities that expand its own output and revenue.

There are empirical evidence that points out that public firms may have interest in considering in its objective function the scale of production. Bhaskar and Khan (1995) studied the effects of privatization upon employment and output in the Jute industry in Bangladesh. They found that the employment reduction after privatization was substantially larger among white collar employees as compared with permanent manual workers. They interpret this result as evidence that public owned firms' employment of white-collar workers was excessive. They suggest that this indicative of the clientelism incentives of public sector in Bangladesh. Colonelli, Teso and Prem (2020) study the use of public sector jobs to reward the supporters of the party in power in Brazilian local governments. They find a direct relation between being supporter of the party in power and the probability of having a public sector job. Quaresma and Foirillo (2016) investigates the influence of politician upon the managerial boards in Italian state-owned enterprises. They found evidence that the appointment of ex members of the parliament in the board of state owned firms is used by political parties as a reward for their loyalty during the legislature. Majumdar (1999) evaluated and compared the returns to scale efficiency of state-owned, private and foreign-owned enterprises in India. He found that State-owned units suffer from decreasing returns to scale.

Given the theoretical arguments and empirical evidence presented above, we interpret that a

simple alternative to model the behavior of a public firm with bureaucratic objectives is include a weight of the production of scale. The next section develops the implication of this new modeling approach.

This paper is also related with the theoretical literature of predation. According to Funagalli et. al. (2018) Predatory pricing refers to a practice whereby an incumbent firm (the predator) sets prices very aggressively, sacrificing its profits, with the objective of excluding a rival from the market and subsequently obtaining higher profits in the future through the exercise of market power. Hence the predation theory recognise two stages in this phenomenon: the sacrifice period and the recoupment period.

During a long there was a lack of a convincing theory of predatory prices. For being convincing a predation theory has to prove that this behavior is profitable and rational. According to Funmagalli et. al. (2018) for predation to be rational, it must be not only feasible but also more profitable than alternative instruments.

It is important to note that until date the theorists of the predation phenomenon only have considered as rational the profit maximization behavior, and that it is why predation must be the most profitable strategy. Under this rationality concept the theorist of predation have resorted to asymmetric information problems in order to provide a convincing theory of predation.

Kreps and Wilson (1982) propose a theory of entry deterrence based on the reputation of aggressive competitor of the incumbent in a repeated game context. This theory is built on the information constraints of the potential entrant regarding the aggressiveness of a multi-market monopolist. Given this uncertainty, the latter pretends to be a very tough competitor in order to create a reputation in early stages of the game and keep its monopolist position in several markets in subsequent stages.

Also in the context of repeated game interaction, Milgrom and Roberts (1982) propose a signalling model of rational predation in which the incumbent mimic the behavior of a low cost firm. As in the Kreps and Wilson (1982) model, the incumbent exploits the limited information of the rivals. Given the uncertainty of the prey about the cost of the incumbent, the latter pretends to be very competitive in order to discourage the permanence of the rival in the market and to prevent further entry into the market in the future.

In a different line from the reputational models, Scharfstein (1984) and Fundenberg and

Tirole (1986) propose models of rational predation, in which the objective of the predatory behavior of the incumbent is to sabotage the acquisition of information by the prey from its participation in the market. In a different way to reputational models, the predatory behavior of the incumbent aims to prevent the incoming rival from obtaining information that allows it to infer the true profitability of the market.

Bolton and Scharfstein (1990) proposed a theory of predation based on agency problems in financial contracting. This theory is built on the limited information of the external funding providers regarding the long run profitability of the prey. This theory considers the access to external funding as endogenous, and suggest that the mechanism used by the incumbent for the exclusion of the rival of the market consist in the reduction of profitability of the latter in early stages of its activity in the market. This reduction affects the perceived risk of lending money by the rival and hence constrains the external financial sources necessary for the survival of the prey.

Regarding the rational predation theories based in different aspects to asymmetric information, Fumagalli and Motta (2013) propose a model in which predation prevents the rival from reaching the scale it needs to operate efficiently.

In the predation models cited above, the rationality of the incumbent is linked with the profitability of the predatory practice, and hence depends of the existence of a recoupment phase. In this paper we explore the plausibility of predatory behavior by public firms. Different from analysis of private firms competition, the rationality of public firms do not necessarily obeys to the principle of profitability. As the literature of mixed oligopoly has proposed, the objective function of the public firm might be different of profit maximization and even it can be the combination of several objectives.

In this paper we explore if predatory pricing by public firms may arise if it weights other objectives than profit maximization. This approach entails a new perspective of the necessary conditions for a convincing rational model of predation. First, this approach relaxes the link between rationality and profitability. Second, as a consequence of this relaxation, the possibility that a recoupment phase be unnecessary arises.

### 3 The Model

In this section we present different models of duopoly in which a private company competes with a public company whose objective is to maximize bureaucracy. In relation to the duopoly competition model, Cournot competition and price competition in a framework of differentiated goods will be explored. On the other hand, we will model the bureaucracy of the public company in two ways: In the first we will assume that the bureaucracy is proportional to the company's income; in the second way, the bureaucracy will be assumed to be proportional to the quantities sold.

#### 3.1 Public company weighs its sales

Model in which a private company that maximizes profits that is identified with the subscript 1 and a public company that maximizes a weight of sales and profits compete (Freshtman and Judd, 1987). The weight for profits is  $\alpha$  and the weight for sales is  $(1 - \alpha)$ . Competition a la Cournot (Quantities). Constant marginal costs:  $c_0 > c_1$ , private enterprise is more cost efficient. This assumption implicitly reflects the idea that the public company, when trying to maximize the bureaucracy, creates a production technology that favors the hiring of certain factors of production and that therefore is below the frontier of efficient production possibilities. Nett (1994) proposed a mixed duopoly model allowing private and public firms to choose among various technologies. He argues that the corresponding cost functions can be interpreted as the consequence of different investment levels in research and development.

The Inverse linear demand function is:  $p = a - bQ$  where  $p$  is the price and  $Q$  the total quantity,  $Q = q_0 + q_1$

The objective function of the private company is:

$$\pi_1 = pq_1 - c_1q_1 = (a - b(q_0 + q_1))q_1 - c_1q_1$$

The objective function of the public company:

$$v_0 = \alpha(pq_0 - c_0q_0) + (1 - \alpha)(pq_0) = (a - b(q_0 + q_1))q_0 - \alpha c_0q_0$$

The best response function of the private company is given by:

$$q_1(q_0) = \frac{a - bq_0 - c_1}{2b}$$

The best response function of the public company is given by:

$$q_0(q_1) = \frac{a - bq_1 - \alpha c_0}{2b}$$

The equilibrium quantity of the private firm is given by:

$$q_1^* = \frac{a - 2c_1 + \alpha c_0}{3b}$$

The equilibrium quantity of the public company is given by:

$$q_0^* = \frac{a - 2\alpha c_0 + c_1}{3b}$$

The total market quantity is given by:

$$Q^* = \frac{2a - c_1 - \alpha c_0}{3b}$$

The equilibrium price is given by:

$$p^* = \frac{a + c_1 + \alpha c_0}{3}$$

*Corollary 1: In the Cournot duopoly in which the public firm weights the total sales, the higher the profit maximization weight, the more difficult it is to incur exclusive predatory pricing.*

*Proof:* Note that the public company incurs predatory exclusionary prices,  $p^* < c_1$ , if:

$$\alpha < \frac{2c_1 - a}{c_0}$$

The condition of exclusive predatory prices depends on the value of the maximum willingness to pay, the parameter  $a$ , that the relative value of the costs between the private company and the public company is not very small and that the weighting of the benefits by the public company is not very great. When the weighting of sales tends to 1, exclusionary prices are

incurred.□

*Corollary 2: In the Cournot duopoly in which the public firm weights the total sales, the higher the profit maximization weight, the greater the difference between the costs of the private company and the public company, the more the sacrifice by the public to impose exclusionary prices.*

*Proof:* In the model presented, the marginal costs of public and private firms are constant, therefore it is always possible to express  $c_1 = \beta c_0$ , where  $\beta = \frac{c_1}{c_0}$ . Given that private incoming company is more efficient  $\beta < 1$ .

The exclusionary predatory pricing condition ,  $p^* < c_1$ , is given by:

$$\beta > \frac{a + \alpha c_0}{2c_0}$$

This entails that the sacrifice by public firm for exclusionary prices to occur is higher if the inefficiency of the public company is great in relation to the private company.□

### 3.2 Public company weighs its production volumes

In this second version of the model, a private company that maximizes profits that is identified with the subscript 1 competes with a public company that maximizes a weighting of its sales volumes and its profits. In this second version of the model we also assume competition to the Cournot (Quantities), constant marginal costs, such that  $c_0 > c_1$  that is, the private company is more cost efficient. As in the previous case, we consider an inverse linear demand function:  $p = a - bQ$  where  $p$  the price y  $Q$  the total quantity,  $Q = q_0 + q_1$ .

As in model 1, the objective function of the private company is:

$$\pi_1 = pq_1 - c_1q_1 = (a - b(q_0 + q_1))q_1 - c_1q_1$$

In this case, the objective function of the public company is expressed as follows:

$$v_0 = \alpha(pq_0 - c_0q_0) + (1 - \alpha)q_0 = \alpha((a - b(q_0 + q_1))q_0 - c_0q_0) + (1 - \alpha)q_0$$

The best response function of the private company is given by:

$$q_1(q_0) = \frac{a - bq_0 - c_1}{2b}$$

The best response function of the public company is given by:

$$q_0(q_1) = \frac{\gamma + a - bq_1 - c_0}{2b}$$

- Where  $\gamma = \frac{(1-\alpha)}{\alpha}$

The equilibrium quantity of the private firm is given by:

$$q_1^* = \frac{a - 2c_1 + c_0 - \gamma}{3b}$$

The equilibrium quantity of the public company is given by:

$$q_0^* = \frac{2\gamma + a - 2\alpha c_0 + c_1}{3b}$$

The total quantity sold in the market is given by:

$$Q^* = \frac{2a - c_1 - c_0 + \gamma}{3b}$$

The equilibrium price is given by:

$$p^* = \frac{a + c_1 + c_0 - \gamma}{3}$$

In this model the condition of exclusionary predatory prices,  $p^* < c_1$ , is given by:

$$\gamma > a - 2c_1 + c_0$$

*Corollary 3: In the Cournot duopoly in which the public firm weights the production scale, the higher the profit maximization weight, the more difficult it is to incur exclusive predatory pricing.*

*Proof:* In this case, the condition of predation depends on the value of the maximum willingness to pay, the parameter  $a$ , that the relative difference in costs between the private company and the public company is not very large and that the weighting of the benefits by of

the public company is not very large (Remember that  $\gamma = 0$  if the public company maximizes profits). When the weighting of sales tends to 1, exclusionary prices are incurred.  $\square$

*Corollary 4: In the Cournot duopoly in which the public firm weights the production scale, the higher the profit maximization weight, the greater the difference between the costs of the private company and the public company, the more the sacrifice by the public to impose exclusionary prices.*

*Proof:* In the model presented, the marginal costs are constant, therefore it is always possible to express  $c_1 = \beta c_0$ , where  $\beta = \frac{c_1}{c_0}$ . Hence, private companies are more efficient  $\beta < 1$ .

The exclusionary predatory pricing condition,  $p^* < c_1$ , is given by:

$$\beta > \frac{a + c_0 - \gamma}{2c_0}$$

This condition implies that the sacrifice by public firm for exclusionary prices to occur is higher if the new competitor (private) is efficient and if the incumbent (public) is very inefficient.  $\square$

### 3.3 Differentiated goods - Public company weighs its sales

Some of the services provided by public companies are not homogeneous goods. Activities such as postal or air transport services can be conceived as differentiated goods. For this reason, this section proposes a Bertrand model with differentiated goods in which a private company that maximizes profits that is identified with the sub-index 1 and a public company that maximizes a weighting of its sales and profits. As in the previous case, constant marginal costs are assumed and that the costs of the private company are lower than those of the public company,  $c_0 > c_1$ . We model product differentiation using demand functions with a substitution parameter. Linear inverse demand functions are assumed for both products as follows:

$$p_i(q_i, q_j) = a - b(q_i + \theta q_j)$$

where  $p_i$  is the price,  $q_i$  the quantity sold by the firm  $i$  and  $\theta$  is the substitution parameter. Therefore, the demand functions of each firm can be expressed as follows:

- $q_i(p_i, p_j) = \frac{a(1-\theta)-p_i+\theta p_j}{(1-\theta^2)b}$

The objective function of the private company is:

$$\pi_1 = (p_1 - c_1)q_1 = (p_1 - c_1)\left(\frac{a(1 - \theta) - p_1 + \theta p_0}{(1 - \theta^2)b}\right)$$

The objective function of the public company:

$$V_0 = \alpha((p_0 - c_0)q_0) + (1 - \alpha)(p_0 q_0) = (p_0 - \alpha c_0)\left(\frac{a(1 - \theta) - p_0 + \theta p_1}{(1 - \theta^2)b}\right)$$

The best response function of the private company is given by:

$$p_1(p_0) = \frac{a(1 - \theta) + \theta p_0 + c_1}{2}$$

The best response function of the public company is given by:

$$p_0(p_1) = \frac{a(1 - \theta) + \theta p_1 + \alpha c_0}{2}$$

The equilibrium quantity of the private firm is given by:

$$p_1^* = \frac{a(\theta + 2)(1 - \theta) + \theta \alpha c_0 + 2c_1}{4 - \theta^2}$$

The equilibrium quantity of the public company is given by:

$$p_0^* = \frac{a(\theta + 2)(1 - \theta) + \theta c_1 + 2\alpha c_0}{4 - \theta^2}$$

*Corollary 5: In the Bertrand model with differentiated products duopoly in which the public firm weights the total sales, the higher the profit maximization weight, the more difficult it is to incur exclusive predatory pricing.*

*Proof:* In this model, the condition of exclusive predatory prices for the private company,  $p_1^* < c_1$ , it is given by:

$$\alpha < \frac{(2 - \theta^2)c_1 - a(\theta + 2)(1 - \theta)}{\theta c_0}$$

This condition depends on the value of the substitution parameter, that the inefficiency of the costs of the public company is not very great in relation to the costs of the private

company and that the weighting of the benefits by the public company is not very big. When the weighting of sales by public firm tends to 1, exclusionary prices are incurred.  $\square$

*Corollary 6: In the Bertrand model with differentiated products duopoly in which the public firm weights the total sales, the higher the profit maximization weight, the greater the difference between the costs of the private company and the public company, the more the sacrifice by the public to impose exclusionary prices.*

*Proof:* If, as in the models exposed above, it is expressed  $c_1 = \beta c_0$ , where  $\beta = \frac{c_1}{c_0}$  and  $\beta < 1$ , it is possible to express the condition of exclusionary predatory prices,  $p^* < c_1$ , as follows:

$$\beta > \frac{a(\theta + 2)(1 - \theta) + \alpha\theta c_0}{(2 - \theta^2)c_0}$$

In this condition, it is possible to observe that the sacrifice by public firm for exclusionary prices to occur is higher if the cost inefficiency of the public company in relation to the private company is great.  $\square$

### 3.4 Differentiated goods - Public company that weights its production volumes

In this version of the model, a private company that maximizes profits and a public company that maximizes a weighting of its volumes and profits compete. These companies interact in an environment of competition à la Bertrand with differentiated goods. As in the previous model, constant marginal costs are assumed and that the costs of the private company are lower than those of the public company,  $c_0 > c_1$  and product differentiation is modeled by using demand functions with a substitution parameter.

The objective function of the private company is:

$$\pi_1 = (p_1 - c_1)q_1 = (p_1 - c_1)\left(\frac{a(1 - \theta) - p_1 + \theta p_0}{(1 - \theta^2)b}\right)$$

The objective function of the public company:

$$W_0 = \alpha(p_0 - c_0)q_0 + (1 - \alpha)q_0 = (\alpha(p_0 - c_0) + (1 - \alpha))\left(\frac{a(1 - \theta) - p_0 + \theta p_1}{(1 - \theta^2)b}\right)$$

The best response function of the private company is given by:

$$p_1(p_0) = \frac{a(1-\theta) + \theta p_0 + c_1}{2}$$

The best response function of the public company is given by:

$$p_0(p_1) = \frac{a(1-\theta) + \theta p_1 + c_0}{2} - \frac{\gamma}{2}$$

- Where  $\gamma = \frac{(1-\alpha)}{\alpha}$

The equilibrium quantity of the private firm is given by:

$$p_1^* = \frac{a(\theta+2)(1-\theta) + \theta c_0 + 2c_1}{4-\theta^2} - \frac{\theta\gamma}{4-\theta^2}$$

The equilibrium quantity of the public company is given by:

$$p_0^* = \frac{a(\theta+2)(1-\theta) + \theta c_1 + 2c_0}{4-\theta^2} - \frac{2\gamma}{4-\theta^2}$$

*Corollary 7: In the Bertrand model with differentiated products duopoly in which the public firm weights the production scale, the higher the profit maximization weight, the more difficult it is to incur exclusive predatory pricing.*

*Proof:* In this model the condition of exclusive predatory prices of the private company,  $p_1^* < c_1$ , is given by:

$$\gamma > \frac{a(\theta+2)(1-\theta) + \theta c_0 + (\theta^2 - 2)c_1}{\theta}$$

It is possible to observe that this condition depends on the value of the substitution parameter, that the cost inefficiency of the public company in relation to the private company is not very large and that the weighting of the benefits by the public company is not very big. When the weighting of sales tends to 1, exclusionary prices are incurred.  $\square$

*Corollary 8: In the Bertrand model with differentiated products duopoly in which the public firm weights the production scale, the higher the profit maximization weight, the greater the difference between the costs of the private company and the public company, the more the sacrifice by the public to impose exclusionary prices.*

*Proof:* If we express the cost of the private firm as  $c_1 = \beta c_0$ , where  $\beta = \frac{c_1}{c_0}$  and  $\beta < 1$ .

The exclusionary predatory pricing condition,  $p^* < c_1$ , is given by:

$$\beta > \frac{a(\theta + 2)(1 - \theta) + \theta c_0 - \gamma \theta}{(2 - \theta^2)c_0}$$

Therefore, the sacrifice by public firm for exclusionary prices to occur is higher if the new competitor (private) and the public firm have large differences in efficiency.  $\square$

## 4 Conclusions

In this document we have explored the results on the competition of having a public company that, in addition to maximizing profits, is also interested in maximizing its scale of operation. The objective of this theoretical exercise was to try to establish whether, given said incentive scheme, the public company could engage in anti-competitive behavior.

In line with the results of Sappington and Sidek (2003), we have found that the public company can incur exclusive predatory prices when the weighting of the benefits is low and when there is no substantial difference between the efficiency of private and public companies.

Given these results, low cost efficiency of public companies can harm competition in environments in which public companies have incentives to increase the scale of production, for example, in companies that are used by politicians as instruments of clientelism or in which the administrators of public companies are under high pressure to provide quasi-free services.

An alternative to solve this dilemma may be the old recipe for privatization. However There are new lines of research that suggest that in specific sectors, such as the delivery industry, the introduction of new operators can improve the efficiency of large incumbents. Ennis (2021) proposes a new definition of natural monopoly in which the incumbent's cost function may be interior to the frontier cost function and hence the possibility of total cost reduction by the entry of new providers (more efficient) arises.

Another option is to implement an incentive mechanism in which government institutions avoid the temptation to exert pressure on the managers of public companies and offer them a contract that is compatible in incentives with profit maximization. Unfortunately, in either case, political parties are required to push for this reform, which is doubtful, knowing that it will reduce the scope of their tentacles of power.

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Institut de Recerca en Economia Aplicada Regional i Pública  
*Research Institute of Applied Economics*

**Universitat de Barcelona**

Av. Diagonal, 690 • 08034 Barcelona

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**WEBSITE:** [www.ub.edu/irea/](http://www.ub.edu/irea/) • **CONTACT:** [irea@ub.edu](mailto:irea@ub.edu)

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