Incomplete Wage Posting

Claudio Michelacci
CEMFI

Javier Suarez
CEMFI

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CEMFI, Casado del Alisal 5, 28014 Madrid, Spain
www.cemfi.es

Abstract

We consider a competitive search model where firms with vacancies choose between posting a wage ex ante and bargaining it with workers ex post. Workers apply for vacancies after observing firms' wage setting decisions, and differ in some observable but not verifiable qualifications that affect their productivity in the job. Thus posted wages prevent the hold-up problem associated with bargaining but are incomplete since they cannot be contingent on worker qualifications. In contrast, bargained wages are increasing in them and, thus, may serve to entice better workers into the vacancy. We find that when the hold-up problem is mild and workers' heterogeneity is large, firms opt for bargaining. Yet, equilibria with bargaining always fail to maximize aggregate net income and sometimes fail to be constrained Pareto optimal.

JEL classification: J30, J41
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1 Introduction

Job advertisements frequently announce that the salary will be negotiated according to the qualifications and experience of the selected candidate. This practice contrasts with the theoretical prediction that firms should post wages in order to prevent the *hold-up problem* (or problem of inadequate compensation) associated with bargaining over the wages once firms' and workers' investments in the vacancy are already sunk.[1] Ideally, firms facing heterogeneous workers should post wage schedules specifying how wages will depend on workers' qualifications. Yet, job openings are very rarely accompanied by the announcement of complex wage schedules —at the very most, wages may be a function of easily assessable variables such as age, formal education, or demonstrable years of tenure in a prior employment.[2] In this paper we argue that firms' preference for bargained wages can be explained by the impossibility of making posted wages contingent on some of workers' relevant qualifications.

The idea is that some determinants of a worker's productivity in a job can be assessed by the end of the hiring process (say, in a job interview or after a probation period) but are hard to incorporate into an enforceable, predetermined, wage schedule. Indeed, some qualifications are difficult to describe in a precise or objectively measurable manner (for example, "relevant experience," "vision," "drive," "good presence"). In other cases, announcing wages contingent on certain characteristics (such as gender, race or marital status) may constitute a (flagrant) violation of anti-discrimination laws. One way or another, posted wages become *incomplete*, that is, not fully contingent on some of workers' relevant characteristics.[3]

[1] Peters (1991) first showed that in an environment where (homogenous) buyers direct their search after observing the price posted by each seller, sellers always have the incentive to pre-commit to a given price.

[2] For example, in Holzer et al. (2000), 25% of the employers offering vacancies recognize that their salaries will not depend on the applicant's skill and experience.

[3] Of course if wages can be made contingent on a (noiseless) signal of the worker's productivity, the incompleteness vanishes. In many relevant circumstances, however, certifying each worker's contribution to the firm's final revenue can be prohibitively expensive —especially with team production

Under incomplete wage posting, workers of different productivity can access the same posted wage. As firms base their wage offers on the expected productivity of their prospective employees, workers whose productivity is above the average end up "subsidizing" those with productivity below the average. In contrast, when the wage is bargained at the end of the hiring process, rent sharing implies that wages increase with each worker's productivity. So highly productive workers are the most attracted to the vacancies with bargained wages and firms may find that offering a bargained wage allows them to entice a better pool of applicants.[4] In this sense, wage posting suffers an *adverse selection* problem akin to that typically described in imperfect information environments. All in all, firms must trade-off the advantage of wage bargaining in relation to this adverse selection problem with the advantage of wage posting in relation to the hold-up problem mentioned above.

We analyze the resolution of this trade-off in the context of a *competitive search* model.[5] Workers differ in some observable but not verifiable qualification that affects their productivity in the job.[6] Firms create vacancies and choose between posting a (non-contingent) wage or leaving it subject to bargaining. Workers direct their search towards the vacancies with their favorite wage setting mechanism. In line with the standard prediction, wage posting prevails when bargaining powers imply a very unbalanced compensation of workers' and firms' sunk investments and when worker heterogeneity is small. But when the hold-up problem is mild and workers' heterogeneity makes the adverse selection problem sufficiently severe, wage bargaining emerges. Interestingly, when both the hold-up problem and the adverse selection problem are mild, the labor market gets segmented: some firms set wages through

or when labour is combined with other factors of production.

[4] Human resources experts concede that linking remuneration to individual merits helps firms to attract the best workers; see Baron and Kreps (1999). This claim is empirically supported by Highhouse et al. (1999).

[5] Our baseline model is close to Moen (1997) and Acemoglu and Shimer (1999a, 1999b).

[6] In the parlance of the incomplete contracts literature, outside authorities (say courts) cannot enforce contracts contingent on information which is observable but *not verifiable*. For an authoritative introduction to incomplete contracts, see Hart (1995).

bargaining and attract the most productive workers, while the remaining firms post a wage and entice the least productive ones.

Over some range of the parameter space, equilibria with and without bargaining coexist, which reflects an *externality* which operates through adverse selection. Specifically, when sufficiently many firms bargain their wages, opting for bargaining allows high productivity workers to attain larger utility than low productivity ones and makes them no longer willing to apply for vacancies with a posted wage. But, as high productivity workers abandon the posting segment of the market, the average productivity of the pool of applicants for vacancies with a posted wage falls and so it does the profitability of posting a wage. Thus, the sustainability of posting depends negatively on the number of firms that opt for bargaining.

Equilibria with bargaining associate with socially *inefficient* outcomes. Bargaining copes with the underlying adverse selection problem by redistributing income from low to high productivity workers, generating no gain in aggregate income. Actually, its hold-up problem translates into either excessive vacancy creation or excessive unemployment so that net aggregate income is always lower than if firms were posting a wage.[7] Moreover, when there are multiple equilibria, the equilibrium without bargaining always Pareto dominates the equilibrium with bargaining. Intuitively, the inefficiencies arise because the firms that opt for bargaining do not internalize the damage to the firms that post a wage.

Our analysis of incomplete wage posting falls in the *directed search* tradition pioneered by Moen (1997) and Acemoglu and Shimer (1999a, 1999b), which consider wage posting in a labor market with homogeneous workers. Within the same tradition, Shimer (2001) and Shi (2001, 2002) deal with worker heterogeneity in a context where firms can post fully-contingent wages. The common bottom line is that the hold-up problem makes firms prefer (complete) wage posting to wage bargaining.[8]

[7] Aggregate income net of job creation costs is the standard social welfare measure used in the search literature –see, for example, Pissarides (2000) and Shimer and Smith (2000).

[8] Moreover, wage posting leads to a socially efficient outcome, except in Acemoglu and Shimer

We show, however, that if unverifiable worker qualifications render posted wages incomplete, wage bargaining is likely to prevail, despite its associated social deadweight costs.[9]

Our analysis also relates to the models of directed search of McAfee (1993) and Peters (1997), where buyers have heterogeneous *private* valuations of the exchanged good and sellers can publicly post a pricing mechanism for such good. These papers show that, taking into account sellers' desire to attract buyers as well as to price-discriminate among them, second-price sealed-bid *auctions* are sellers' preferred pricing mechanism.[10] In practice, several factors may limit the applicability of this mechanism in the labor market. Literally taken, those auctions imply that each firm (or vacancy) is bought out by the corresponding winning worker, who pays the firm some sum in advance (the counterpart of the firm's net profits under a standard labor contract) and thereby becomes the residual claimant of the firm's future revenue (the counterpart of his wage). Importantly, if workers' output is not verifiable, as we assume, future claims on such output are unfeasible so workers must be able to pay their bid when they get the job. However, this may be unfeasible if workers are wealth constrained (i.e., cannot advance payments to the firm) or suboptimal if workers are risk-averse (i.e., require a premium for their exposure to business risk), or if having the employer as a residual claimant is convenient for, say, incentive reasons.[11] Since labor relationships tend to emerge precisely when workers are not sufficiently wealthy, risk tolerant, and self-sufficient to become their own employers, we will not consider

(1999a), where workers' risk aversion induces firms to create an excessive number of vacancies.

[9] This implies that directed search models can encompass results that were so far exclusive to *random search* models, where wages are commonly assumed to be bargained —see Pissarides (2000) and the references therein. Ellingsen and Rosen (2000), Camera and Delacroix (2000), and Masters and Muthoo (2001) analyze firms' choice between bargaining and posting in random search models with unverifiable worker heterogeneity. In the absence of directed search, however, the trade-offs involved are very different from ours since the wage setting mechanism plays no role in attracting workers to vacancies.

[10] Building on this result, Shimer (1999) shows that, in a labor market with heterogenous, risk neutral workers, auctions lead to a socially efficient outcome.

[11] See Hart and Moore (1990).

job auctions in our analysis.

The paper is organized as follows. In Section 2 we describe the model. Section 3 elaborates on our notion of labor market equilibrium and provides some important preliminary results. Section 4 characterizes the various possible types of equilibrium. Section 5 analyzes how the existence of each equilibrium relates to the hold-up problem associated with bargaining. Section 6 elaborates on the effects of adverse selection. In Section 7 we compare the various regimes in terms of aggregate income and efficiency. Section 8 discusses possible extensions of the basic model. The conclusions appear in Section 9. The Appendix contains all the technical proofs.

2 The model

We consider a labor market made up of a unit mass of workers and a mass of firms which is endogenously determined by a free entry condition.

2.1 Preferences and technologies

Firms and workers are risk neutral and maximize their expected net income. Each firm can create a job *vacancy* at a cost $c > 0$. Each vacancy becomes a *job* when occupied by a worker. There are two types of workers $i = 0, 1$. Low productivity workers ($i = 0$) represent a fraction $1 - \mu$ of the population and produce an income $y_0 > c$ in the job, while high productivity workers ($i = 1$) represent the remaining fraction μ and produce $y_1 > y_0$. For simplicity we assume that workers earn no income if unemployed and incur no cost in searching for their jobs.

2.2 Information and contracts

Workers know their own productivity type. Such type becomes *observable* to the hiring firm by the end of the hiring process and, thus, it may get reflected in the wage agreed between the firm and the worker at that point. We assume, however, that worker types are *not verifiable* and, therefore, public announcements or contracts that specify hiring policies or wages contingent on such types are not enforceable.[12] Specifically, firms may pre-commit to the wage that they will pay to "whoever is finally hired", but they cannot pre-commit to pay a different wage to the two worker types since no outside authority (say a court) can formally discriminate between them.[13]

Consequently, we assume that each firm can make an announcement $x \in \mathbb{R}_+$ specifying the non-contingent wage that it will pay to whoever it hires. Alternatively the firm can announce that the wage will be bargained with the worker at the end of the hiring process. We denote such announcement by $x_\varnothing$ and assume that the bargained wage is determined according to the Generalized Nash Bargaining Solution where the worker's and the firm's bargaining powers are β and $1-\beta$, respectively.

2.3 Search frictions

Trade in the labor market is subject to search frictions. Firms can costlessly advertise their vacancies among all workers. However, both workers and firms have limited capacities to submit and to process job applications, and to coordinate their decisions. Specifically, each worker can apply for at most one vacancy and each firm can consider at most one (randomly drawn) applicant for its vacancy.[14] In addition, workers cannot coordinate their application decisions: they choose their preferred type of vacancy (possibly using a mixed strategy) and uniformly randomize over the firms opening it. Thus, some firms may receive multiple applications and others none. Analogously, workers face uncertainty on how many other workers will end applying to the same

[12] The debate on the microfoundations of the incomplete contracts literature is still open. For instance, Maskin and Tirole (1999) criticize the logic whereby "observable but not verifiable" information necessarily implies that contracts are incomplete. Segal (1999) and Hart and Moore (1999), among others, provide some formal answers to this criticism.

[13] Since output perfectly identifies a worker's type, we assume that workers' individual output is not verifiable.

[14] For the case in which firms can consider more than one applicant, see Section 8.2.

firm as they do.

To model the effects of the underlying coordination problem, let n denote the *expected number of applicants* for each of the vacancies associated with a given announcement. We assume that the probability that a firm opening one of these vacancies receives at least one applicant is given by an increasing and twice continuously differentiable function $Q(n)$.[15] Clearly, with an average of n applicants per vacancy and a single application per worker, if each firm processes one application with probability $Q(n)$, then the probability with which a worker gets his application processed is $Q(n)/n$.[16] To rule out a "free lunch" whereby increasing n simultaneously raises the probabilities with which vacancies and applicants get occupied, we assume that $Q(n)/n$ is decreasing in n or, equivalently, that the elasticity of $Q(n)$ with respect to n is no greater than one:[17]

$$\varepsilon_Q(n) \equiv \frac{Q'(n)n}{Q(n)} \leq 1. \tag{1}$$

To simplify the analysis, we further assume:

A1. $\lim_{n\to\infty} Q(n) = \lim_{n\to 0} Q(n)/n = 1$

A2. $\varepsilon_Q(n)$ is weakly decreasing in n.

A3. $\lim_{n\to\infty} \varepsilon_Q(n) < 1 - c/y_0$

The boundary conditions in A1 help guarantee the existence of equilibrium. A2 assures uniqueness within each of the types of equilibrium that the model supports. A3 implies that, even if the economy were exclusively populated by low productivity

[15]See Montgomery (1991), Peters (1991), and Burdett et al. (2001) for an explicit probabilistic model of the coordination problem that is consistent with this reduced form.

[16]Indeed, let $P(n)$ denote the probability with which a worker gets his application processed and normalize to one, for simplicity, the measure of available vacancies. Then, by aggregate consistency, the measure of firms with at least one applicant, $Q(n)$, must equal the measure of workers whose applications are processed, $P(n)n$. So $P(n) = Q(n)/n$.

[17]This modelling of search frictions borrows from Acemoglu and Shimer (1999a, 1999b). It can also capture, in a reduced form manner, search frictions stemming from the unsuitability of some workers to certain jobs and vice versa. Actually, there is a one-to-one correspondence between $Q(n)$ and a standard *matching function* a la Pissarides (2000). Matching functions appear in the models of directed search of Moen (1997), Mortensen and Wright (1997), and Acemoglu (2001).

workers, some vacancies could be created at a net social gain in income.[18]

2.4 Wage determination

After a match, the outside options of both the firm (leaving the vacancy unfilled) and the worker (remaining unemployed) are worth zero. Thus, the surplus from the hiring of a worker of type i is $y_i > 0$. Hence, after announcing $x_\varnothing$, Nash bargaining implies that the worker is hired at wage βy_i. Alternatively, if the firm has posted a wage x, the job is created if and only if the profit $y_i - x$ is acceptable to the firm and the wage x is acceptable to the worker: this requires $y_i \geq x \geq 0$.

In order to focus the discussion, we want to rule out the possibility that a firm credibly commits to hire just high productivity workers by posting a wage $x > y_0$.[19] Accordingly we assume:

A4. $y_1 - y_0 < c$.

Under this assumption, even if the firm matches a high productivity worker with probability one, the required wage implies $y_1 - x < y_1 - y_0 < c$, so the firm would suffer losses. Thus firms will never follow this strategy and, hence, in case of posting a wage, will always be willing to hire both high and low productivity workers.

To sum up, let $\widetilde{y}(x) \in \{y_0, y_1\}$ denote the (possibly degenerated) random variable which describes the productivity of the worker that matches with a firm that has announced x.[20] Then, such worker's wage will be

$$\widetilde{w}(x) = \begin{cases} x & \text{if } x \in \mathbb{R}_+, \\ \beta\widetilde{y}(x) & \text{if } x = x_\varnothing. \end{cases} \tag{2}$$

[18] Assumptions A1-A3 are satisfied by the function associated with the explicit urn-ball matching process proposed by Montgomery (1991) and Peters (1991): $Q(n) = 1 - \exp(-n)$. See Blanchard and Diamond (1994), Moen (1999), and Acemoglu and Shimer (2000) for applications of this functional form.

[19] Announcements which rely on unverifiable information such as "the firm will only hire high productivity workers" are not credible. After the firm matches with a low productivity worker both parties have incentives to create the job (and no outside authority can enforce the firm's initial announcement).

[20] Such a worker is randomly drawn from the firm's pool of applicants, which may include both worker types.

Clearly, posted wages are independent of the worker's productivity, while bargained wages increase with it.

2.5 The game

The labor market can be described as a sequential game played by workers and firms. At a first stage, firms simultaneously decide whether to *enter* the market. Entering entails incurring the cost c of creating a vacancy and posting an announcement x chosen from the set of *admissable announcements* $X \equiv \mathbb{R}_+ \cup \{x_\varnothing\}$. The resulting set of posted announcements $X^* \subset X$ and the measure of firms posting each announcement $x \in X^*$ are then observed by all workers.

In a second stage, to which we refer as the *application subgame,* workers simultaneously decide which of the posted announcements $x \in X^*$ they prefer. Each worker then selects randomly one of the firms posting it and submits an application. Workers' decisions produce some expected number of applicants $n(x)$ and a fraction of high productivity applicants $\gamma(x)$ for the vacancies associated with each announcement $x \in X^*$. The matching process then occurs in accordance with the technology described by the function $Q(n)$. If a job is created, production takes place and income is divided as implied by the firm's wage announcement.

3 Equilibrium

The nature of the labor market game allows us to stick to the standard notion of Subgame Perfect Nash Equilibrium (SPNE). To solve for such an equilibrium, we must specify the Nash Equilibrium (NE) of every application subgame that would arise if a firm were unilaterally deviating from its equilibrium vacancy posting strategy. Implicitly, firms use these NE in order to predict the consequences of each of their possible decisions and, therefore, to design their equilibrium strategies.

3.1 Change of variable

Before starting and in order to facilitate the use of diagrams, let the new variable $d \equiv n/Q(n) \in [1, \infty)$ (the inverse of workers' probability of getting the job) describe workers' *demand* for a vacancy whose expected number of applicants is n. Notice that d is a strictly increasing transformation of n, so there is a strictly increasing function $N(d)$ that gives the unique value of n associated with each d. Hence the function $q(d) \equiv Q(N(d))$ will give a firm's probability of filling a vacancy with demand d, while an applicant's probability of occupying such vacancy will just be $1/d$. In addition, it is convenient to define the function

$$\eta(d) \equiv \varepsilon_Q(N(d)) = Q'(N(d))\, d, \tag{3}$$

which takes values lower than one, by (1), and is decreasing in d, by A2.

3.2 Application subgames

Consider an arbitrary application subgame in which, without loss of generality, the set of announcements made by at least one firm in the first stage of the game, $X^* \subset X$, is finite. This application subgame can then be fully described by a mass-measure function v: $X^* \to \mathbb{R}_+$ that specifies the (possibly zero) measure of firms that have posted each of the announcements in the set X^*.[21]

To describe a NE of this subgame we use the functions, d: $X^* \to [1, \infty)$ and γ: $X^* \to [0, 1]$, that specify, respectively, workers' demand and the fraction of high productivity applicants for the vacancies associated with each of the existing announcements, as well as the utilities U_0 and U_1 obtained by low and high productivity workers, respectively.[22]

[21] The results below confirm that the distribution of announcements in the relevant application subgames is always discrete. In models where the equilibrium distribution of posted wages is continuous (e.g., Burdett and Judd, 1983; Burdett and Mortensen, 1998), firms can hire an unlimited number of workers and the continuum of equilibrium wages results from the trade-off between raising the number of workers and reducing the wage paid per worker.

[22] Under our assumption that workers of a given type play identical (mixed) strategies, the demand

From a worker's perspective, other workers' application strategies are only relevant for evaluating, at every $x \in X^*$, the probability $1/d(x)$ that he is hired if his application is sent to a firm announcing x. Each worker takes $d(x)$ as given and selects an announcement that maximizes his expected income. Thus, *equilibrium utilities* satisfy

$$U_i = \max_{x \in X^*} \frac{E_i[\tilde{w}(x)]}{d(x)}, \tag{4}$$

where the operator $E_i(\cdot)$ yields the expected value of its argument when there is a probability i that the relevant worker is of a high productivity type. Since a worker of type i will respond to a given announcement (i.e., $\gamma(x) \neq 1-i$) only if the associated utility matches the utility of his best available alternative (i.e., $E_i[\tilde{w}(x)]/d(x) = U_i$), workers' *optimal application decisions* can be compactly expressed as

$$[1 - i - \gamma(x)]N(d(x)) \left\{ \frac{E_i[\tilde{w}(x)]}{d(x)} - U_i \right\} = 0 \tag{5}$$

for all $x \in X^*$ and $i = 0, 1$.

Additionally, in any NE the masses of workers of a given type applying for the various vacancies should add up to the exogenously given total mass of workers of such type. The resulting *add up constraints* can be compactly written as

$$\sum_{x \in X^*} [1 - i - \gamma(x)]N(d(x))\, v(x) = 1 - i - \mu \tag{6}$$

for $i = 0, 1$, which together with (4) and (5) constitute the conditions for a NE of the considered application subgame.

Lemma 1 *For every application subgame, there is always a unique pair (U_0, U_1), with $U_0 \leq U_1$, and some functions $d(x)$ and $\gamma(x)$, with*

$$d(x) = \begin{cases} \max(1, \frac{\beta y_1}{U_1}) & \text{if } x = x_\varnothing, \\ \max(1, \frac{x}{U_0}) & \text{if } x \in \mathbb{R}_+, \end{cases} \tag{7}$$

that satisfy the NE conditions.

and the applicants composition linked to each announcement x are sufficient statistics for workers' equilibrium strategies.

Equation (7) says that a vacancy that leaves the wage subject to bargaining will attract a positive expected number of workers, $d(x_\varnothing) > 1$, if and only if $\beta y_1 > U_1$, in which case at least high productivity workers will find it attractive.[23] Symmetrically, a vacancy with a posted wage $x \in \mathbb{R}_+$ will attract a positive expected number of workers, $d(x) > 1$, if and only if $x > U_0$ in which case at least low productivity workers will find it attractive. Intuitively, high productivity workers tend to prefer vacancies where the wage is bargained because bargaining translates their greater productivity into a higher wage. Conversely, low productivity workers are more inclined towards posted wages because their fixed nature protects them against their productivity disadvantage.

3.3 The whole game

To save on notation let v: $X^* \rightarrow \mathbb{R}_+$ henceforth describe the application subgame induced by firms' *equilibrium* posting decisions. In the first stage of the game, firms that decide to create a vacancy make an announcement $x \in X$ specifying how the wage will be established in case of hiring. To choose x, each firm must have a prediction on the NE of the application subgame induced by each of its possible choices (and the equilibrium choices of the other firms). As other firms' strategies are taken as given, a firm needs to consider just the minor perturbations that its unilateral deviations cause on the *equilibrium application subgame.* Furthermore, since all firms are infinitesimal, no unilateral deviation alters workers' equilibrium utilities, U_0 and U_1, which together with (7) can be used by the firms to predict workers' demand for any possible vacancy.

When a firm's choice or unilateral deviation consists in either posting no vacancy or posting a vacancy with $x \in X^*$, the function $v(x)$ remains a valid description of the induced application subgame, so the firm can use the NE of the equilibrium subgame to compute the payoff of its choice. When the deviation consists in an announcement not observed in equilibrium, $x \notin X^*$, the function $v(x)$ is still a valid description of

[23] From its definition, d equals one if and only if the expected number of applicants is zero.

the measure of firms posting announcements contained in X^* but the induced application subgame is slightly different because the set of posted announcements now also includes x. So, in general, in order to describe the NE of the relevant perturbations of the equilibrium application subgame, we simply need to extend the domain of the functions $d(x)$ and $\gamma(x)$ from the set of announcements used in equilibrium, X^*, to the whole set of admissable announcements, X.

As the demand for a new vacancy can always be obtained from (7), the only complication is to determine the composition of the pool of applicants for the vacancies with $x \notin X^*$. An indeterminacy arises only if the new vacancy is equally attractive to both types of workers, that is, $E_i[\tilde{w}\,(x')]/d\,(x') = U_i$ for $i = 0, 1$. Otherwise (5) uniquely determines $\gamma(x')$. To resolve the indeterminacy we will assume that firms hold *balanced expectations* about the composition of the pool of applicants for vacancies associated with out-of-equilibrium announcements which are equally attractive to both types of workers. Formally:

Definition 1 *A SPNE features* balanced expectations *if the NE of the subgames induced by adding a vacancy $x' \notin X^*$ to the equilibrium set of posted vacancies X^* satisfy $\gamma(x') = \mu$ whenever $E_i[\tilde{w}\,(x')]/d\,(x') = U_i$ for $i = 0, 1$.*

To characterize firms' *equilibrium posting strategies*, let the (expected) net profit from creating a vacancy associated with an announcement $x \in X$ be given by the function

$$V\,(x) = q\,(d\,(x))\,E_{\gamma(x)}\,[\tilde{y}\,(x) - \tilde{w}\,(x)] - c, \tag{8}$$

where the operator $E_{\gamma(x)}\,(\cdot)$ reflects that the probability that the selected applicant is of the high productivity type equals $\gamma(x)$. Then, firms' profit maximizing behavior and free entry imply:

$$V(x) = 0 \geq V(x'), \quad \text{for all } x \in X^* \text{ and } x' \in X. \tag{9}$$

In words, firms' net profit must be zero under all the announcements observed in equilibrium and no larger than zero under any other possible announcement. With this understanding of the play during the first stage of the labor market game, we adopt the following definition of equilibrium:

Definition 2 *An* equilibrium *of the labor market is a tuple* $\{X^*, v(x), d(x), \gamma(x), (U_0, U_1)\}$ *such that firms' posting strategies and workers' application strategies constitute a SPNE with balanced expectations.*

In the rest of this section we combine the various equilibrium conditions stated so far in order to obtain two important results. First, we derive a useful relationship between workers' equilibrium utilities and composition of the pool of applicants for the (equilibrium and out-of-equilibrium) vacancies with a posted wage. Second we show that, in equilibrium, the set of posted vacancies never includes more than one posted wage.

Lemma 2 *In equilibrium, if* $U_0 = U_1$*, then* $\gamma(x) = \mu$ *for all* $x \in \mathbb{R}_+$*, while if* $U_0 < U_1$*, then* $\gamma(x) = 0$ *for all* $x \in \mathbb{R}_+$*.*

When applying for a vacancy with a posted wage, a worker's payoff is independent of his productivity type. Hence, if $U_0 = U_1$, a vacancy posting a wage x not observed in equilibrium, $x \notin X^*$, is equally attractive to high and low productivity workers so the result that $\gamma(x) = \mu$ follows immediately from the requirement of balanced expectations. More generally, in any SPNE where all workers are equally well-off, the expected fraction of high productivity applicants must be the same across all vacancies with a posted wage. To see this notice that, if the composition were varying across those vacancies, some firms would necessarily be attracting a pool of applicants with a lower average productivity than the population's. As we show in the proof of the lemma, such an outcome can only be consistent with firm's optimization if those firms (pessimistically) expect that they cannot improve the composition of their pool of applicants by announcing some other (out-of-equilibrium) wage. But that would contradict the requirement of balanced expectations.[24] Finally, if $U_0 < U_1$, the result that $\gamma(x) = 0$ for all $x \in \mathbb{R}_+$ follows from (5) and Lemma 1. The intuition is that high productivity workers can achieve a larger utility than low productivity workers only if they apply for vacancies with a bargained wage.

The fact that $\gamma(x)$ is constant for all $x \in \mathbb{R}_+$ leads us to the last result in this section. For given workers' utilities, $V(x)$ is strictly quasi-concave in the $\mathbb{R}_+$ domain and hence at most one posted wage maximizes $V(x)$. Thus:

Lemma 3 *In equilibrium, X^* contains at most one posted wage $x \in \mathbb{R}_+$.*

In graphical terms, having $\gamma(x) = \gamma$ for all $x \in \mathbb{R}_+$ means that a firm's profit from posting a vacancy with demand d and a posted wage w can be written as $V_\gamma = q(d)\left[E_\gamma(\tilde{y}) - w\right] - c$. Under our assumptions A1 and A2, this implies that firms' iso-profit curves are increasing and concave in the (d, w) plane, with a vertical asymptote at $d = 1$.[25] But workers' indifference curves in the (d, w) plane are rays from the origin with slope U. So, given how workers' demand for vacancies with a posted wage is determined, the best wage that a firm can post corresponds to the unique tangency of the relevant iso-profit curve and the indifference line of level U_0. Actually, in equilibria with posting, the value of U_0 can be pinned down by noting that firms' equilibrium profits must be zero under free entry. Figure 1 represents a case with $U_0 = U_1$ and thus $\gamma = \mu$.

4 Candidate equilibrium regimes

Our previous results imply that the equilibrium set of posted announcements, X^*, contains at most two elements, of which only one can be a posted wage. This yields

[24] One can show that all SPNE where firms' expectations are unbalanced are Pareto dominated by a SPNE with balanced expectations.

[25] Higher levels of profits are reached by moving downwards or rightwards, and increasing γ produces vertically parallel upward shifts in the iso-profit curves.

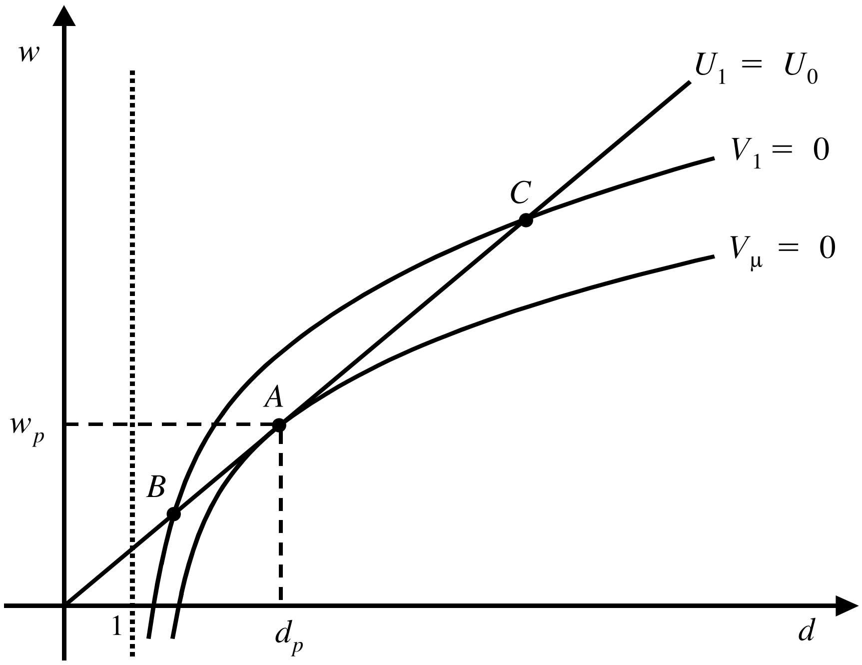

Figure 1: A Pure Posting Equilibrium

three possible equilibrium regimes: (i) *pure posting,* where X^* only contains a posted wage, (ii) *pure bargaining,* where X^* only contains $x_\varnothing$, and (iii) a *mixed regime,* where X^* contains $x_\varnothing$ and a posted wage. In this section we characterize the unique candidate equilibria that emerge within each of these possible regimes and provide necessary and sufficient conditions for their existence. Those conditions will be put together in Section 5 so as to clarify when each candidate equilibrium arises.

4.1 Pure posting

In a pure posting (PP) equilibrium, all vacancies offer the same posted wage $w_p \in \mathbb{R}_+$ and get the same demand d_p, and all workers attain the same utility level $U_p = w_p/d_p$. By Lemma 1, vacancies with a posted wage $x \in \mathbb{R}_+$ have a demand $d(x) = \max(1, x/U_p)$ and attract high productivity applicants in a proportion $\gamma(x) = \mu$.

Firms' optimal choice of w_p implies

$$w_p = \arg\max_{x \in \mathbb{R}_+} q(\frac{x}{U_p})[E_\mu(\tilde{y}) - x] - c,$$

where we adopt the convention that $q(d) = 0$ for $d < 1$. Using the definitions introduced in Section 3.1, the first order condition for the above maximization can be written as:[26]

$$w_p = \eta(d_p) E_\mu(\tilde{y}). \tag{10}$$

But then firms' free entry condition becomes

$$q(d_p)[1 - \eta(d_p)] E_\mu(\tilde{y}) = c, \tag{11}$$

which uniquely determines d_p and, recursively, w_p and U_p. Graphically, the pair (d_p, w_p) corresponds to the tangency point A in Figure 1

Posting a wage w_p is an equilibrium if no firm can make strictly positive profits by posting a vacancy whose wage is subject to bargaining. So we need to check that

$$V(x_\varnothing) = q(\frac{\beta y_1}{U_p})(1 - \beta) y_1 - c \leq 0, \tag{12}$$

where the first equality comes from the fact that vacancies with $x_\varnothing$ would have a demand $d(x_\varnothing) = \max(1, \beta y_1 / U_p)$ and would entice, at most, high productivity workers. Given that (10) implies $U_p = \eta(d_p) E_\mu(\tilde{y}) / d_p$, the above condition can be written as

$$(1 - \beta) q\left(\frac{\beta y_1 d_p}{\eta(d_p) E_\mu(\tilde{y})}\right) \leq \frac{c}{y_1}. \tag{13}$$

Notice that the LHS of this expression measures (as a proportion of y_1) the profits (gross of the creation cost) that a firm would make by posting a vacancy with $x_\varnothing$ in a situation where the attracted workers are of the high productivity type and attain a utility U_p. In Section 5 we discuss the determinants of such profits.

In terms of Figure 1, condition (13) states that PP is an equilibrium when βy_1 is either smaller than the wage associated with point B or greater than the wage

[26] We use the fact that $q'(d) = \frac{\eta(d)}{1-\eta(d)} \frac{q(d)}{d}$ and $U_p = \frac{w_p}{d_p}$.

associated with point C, where B and C correspond to the intersection of workers' indifference curve of level U_p with the zero profit curve of a firm that only attracts high productivity workers, $V_1 = 0$.

4.2 Pure bargaining

In a pure bargaining (PB) equilibrium all firms leave wages subject to bargaining, i.e., announce $x_{\varnothing}$. All vacancies have the same demand d_b and attract workers' types in the same proportions as they exist in the population, so $\gamma(x_{\varnothing}) = \mu$. Given that bargained wages amount to a fraction β of workers' output, firms' zero profit condition is

$$q\left(d_b\right)\left(1-\beta\right)E_{\mu}(\tilde{y}) = c, \tag{14}$$

which uniquely determines d_b. As depicted in Figure 2, d_b is the coordinate at $w = \beta E_{\mu}(\tilde{y})$ of the zero-profit curve of a firm that attracts a balanced proportion of workers of each type (point A in the figure). Clearly, through bargained wages, high productivity workers obtain higher utility, $U_{b1} = \beta y_1/d_b$, than low productivity workers, $U_{b0} = \beta y_0/d_b$.

In a PB equilibrium no firm should find profitable to deviate to a posted wage. Vacancies with a posted wage would have $d(x) = \max(1, x/U_{b0})$, by Lemma 1, and would only attract low productivity workers, by Lemma 2. Thus the best wage that a firm can post is

$$w' = \arg\max_{x\in\mathbb{R}_+} q(\frac{x}{U_{b0}})\left(y_0 - x\right) - c,$$

which is always larger than U_{b0}.[27] Using the definitions in Section 3.1 we can rewrite the first order condition for this maximization as

$$w' = \eta\left(d'\right)y_0, \tag{15}$$

[27]Notice that any $x \in (U_{b0}, y_0)$ produces a net profit strictly larger than $-c$, which is the net profit associated with any $x \leq U_{b0}$.

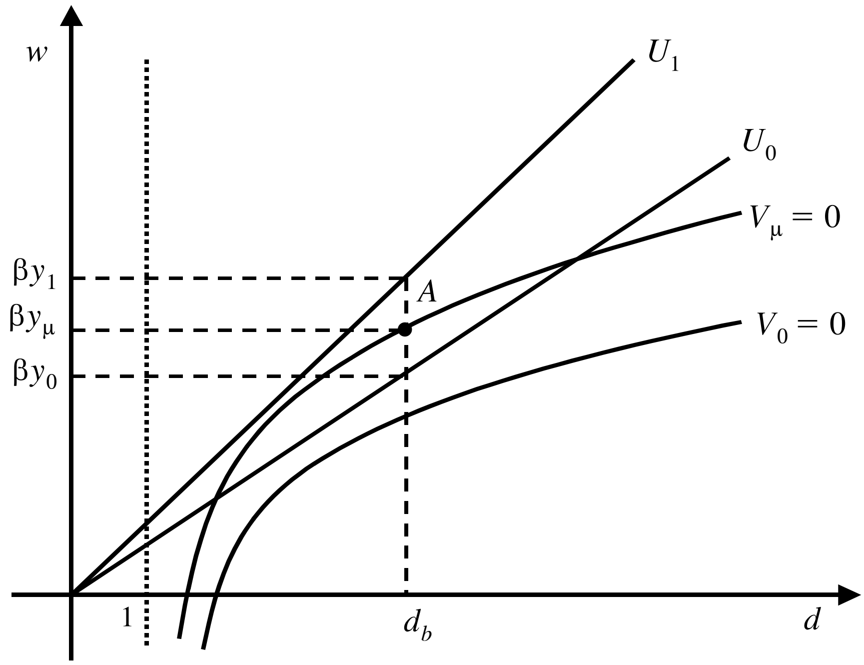

Figure 2: A Pure Bargaining Equilibrium

where $d' = w'/U_{b0}$ is the demand that a vacancy offering w' would have. But given that $U_{b0} = \beta y_0/d_b$, we can write (15) as

$$\beta d' = \eta\,(d')\,d_b, \tag{16}$$

which uniquely determines d'. With this notation, the condition for the absence of a profitable deviation, $V(w') \leq 0$, is equivalent to

$$[1-\eta\,(d')]\,q\,(d') \leq \frac{c}{y_0}. \tag{17}$$

In graphical terms, this condition requires that, as in Figure 2, the workers' indifference curve of level U_{b0} (which identifies the low productivity workers' utility in this regime) does not intersect the zero-profit-line of a firm that only attracts low productivity workers. Otherwise there would be some posted wages which would allow a firm to earn strictly positive profits.

4.3 Mixed regimes

In a mixed regime, some firms post a wage $w_m \in \mathbb{R}_+$ and receive a demand d_{m0}, while the rest leave wages subject to bargaining and receive a demand d_{m1}. In this case, we can only have $U_0 < U_1$ and hence, by Lemma 2, $\gamma(x) = 0$ for all $x \in \mathbb{R}_+$. Indeed, $U_0 = U_1$ would imply $\gamma(x) = \mu$ for all $x \in \mathbb{R}_+$ and hence, by the add up constraints (6), $\gamma(x_\varnothing) = \mu$. But this would contradict $U_0 = U_1$ since, under bargained wages, low productivity workers can never obtain the same utility as high productivity workers.

Accordingly, low productivity workers attain a utility $U_{m0} = w_m/d_{m0}$, where w_m and d_{m0} can be uniquely obtained, as in the case of PP, from the first order condition of the firm's problem

$$w_m = \eta(d_{m0})y_0, \tag{18}$$

and the zero profit condition,

$$q\left(d_{m0}\right)\left[1 - \eta\left(d_{m0}\right)\right]y_0 = c, \tag{19}$$

of the firms posting a wage. Both expressions reflect that these vacancies attract just low productivity workers.

Since all high productivity workers apply for $x_\varnothing$ and at least some low productivity workers apply for w_m, the add-up constraints (6) require that the fraction of high productivity workers among the applicants for $x_\varnothing$ is some $\gamma \in (\mu, 1]$. Free entry in turn requires that the firms opting for bargaining earn zero profits:

$$q\left(d_{m1}\right)\left(1 - \beta\right)E_\gamma(\tilde{y}) = c. \tag{20}$$

Finally, the value of γ must be compatible with workers' optimal application decisions. Notice that a high (low) productivity worker can attain a utility of $U_{m1} = \beta y_1/d_{m1}$ ($\beta y_0/d_{m1}$) by applying for $x_\varnothing$, while any worker can attain a utility U_{m0} by applying for w_m. So two possibilities arise:

1. A semi-separating (SS) equilibrium, where $\gamma \in (\mu, 1]$ and

$$U_{m0} = \frac{\beta y_0}{d_{m1}}, \tag{21}$$

so that low productivity workers are indifferent between applying for w_m and for $x_{\varnothing}$.

2. A fully-separating (FS) equilibrium, where $\gamma = 1$ and

$$\frac{\beta y_0}{d_{m1}} < U_{m0} \leq U_{m1}, \tag{22}$$

so that low productivity workers strictly prefer to apply for a vacancy where the wage is posted.

In terms of Figure 3, a FS equilibrium requires that a worker's utility in point A (which identifies the situation of a high productivity worker who opts for bargaining) is larger than in point B (which corresponds to any worker who opts for posting). In turn, the utility in point B must be greater than in point C (which describes the situation of a low productivity worker who opts for bargaining).

To check when each of these configurations emerges as an equilibrium, notice that if the unique γ which solves (20) for $d_{m1} = \beta y_0 / U_{m0}$, say $\hat{\gamma}$, lies in the interval $(\mu, 1)$ then we have a SS equilibrium.[28] Alternatively, if the unique d_{m1} which solves (20) for $\gamma = 1$, say $\tilde{d}$, also satisfies (22), then it describes a FS equilibrium. Actually, since (20) implies a monotonic increasing relationship between d_{m1} and γ, the first inequality in (22) is satisfied for $d_{m1} = \tilde{d}$ only if $\hat{\gamma} > 1$, which implies that the SS and the FS equilibria never coexist.

5 When does each equilibrium arise?

The emergence of each of our candidate equilibria is driven by the tension between firms' temptation to use bargaining as a means to attract the most productive workers

[28] Notice that $U_{m0} = w_m / d_{m0}$ does not depend on γ since it is entirely determined in the posting segment of the market, where all workers are of the low productivity type.

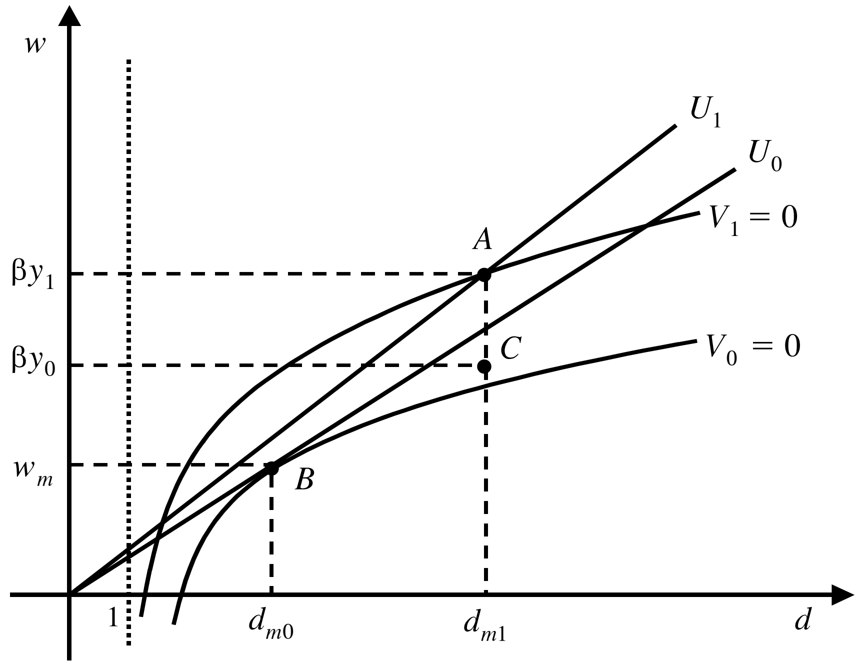

Figure 3: A Fully Separating Equilibrium

and its cost in terms of the *hold-up problem* or problem of inadequate compensation of firms' and workers' ex ante investments in the vacancy. In this section we first develop a metric for measuring this problem through the parameter β. Then we show which of our candidate equilibria arise for each of the admissable values of β.

5.1 A metric for the hold-up problem

When the pool of applicants for a given vacancy has size $N(d)$ and features a fraction α of high productivity workers, the net surplus generated by the vacancy equals the difference between its expected income, $Q(N(d))E_\alpha(\widetilde{y})$, and the expected cost of its pool of applicants, $N(d)U_\alpha$, where $U_\alpha = \alpha U_1 + (1-\alpha)U_0$ measures an average applicant's opportunity cost of applying for such a vacancy —in other words, his ex ante investment in the vacancy. Thus,

$$Q'(N(d))E_\alpha(\widetilde{y}) = U_\alpha \tag{23}$$

is a necessary (and sufficient) condition for worker's demand d to maximize the vacancy's surplus. Interestingly, this condition holds for all vacancies with a posted wage (whether they are a best deviation from an equilibrium with bargaining or an equilibrium outcome).[29]

Under wage bargaining, however, the definition of workers' utilities implies

$$\frac{Q(N(d))}{N(d)}\beta E_{\alpha}(\widetilde{y}) = U_{\alpha},$$

which, compared with (23) and given the definition of $\eta(d)$ in (3), means that workers' demand maximizes surplus if only if $\beta = \eta(d)$. When β is greater (lower) than $\eta(d)$, workers have too much (little) bargaining power and their demand is too high (low) relative to the surplus-maximizing level. Thus under bargaining the marginal return and the marginal cost of an applicant generally differ. This is the result of the hold-up problem associated with bargaining: wages determined once the search process is concluded do not necessarily reflect applicants' ex ante investment in the vacancy.

In the analysis below we measure the severity of the hold-up problem as the distance between the actual value of the bargaining power parameter, β, and the (unique) value that would make (23) hold in a PB regime, $\beta^* = \eta(d_p)$.[30]

5.2 Candidate equilibria and the hold-up problem

To analyze the possibility of a PP equilibrium, let $P(\beta)$ represent the quantity that appears in the LHS of (13) so that PP is an equilibrium when $P(\beta) \leq c/y_1$. As we prove in the Appendix, $P(\beta)$ is a non-negative and quasi-concave function that takes a minimum value of zero when β is close to zero and also when β equals one. In the limit case where $\mu = 1$, this function reaches a maximum value of c/y_1 at $\beta = \beta^*$. As μ decreases, $P(\beta)$ shifts upwards and gives raise to an interval $(p, p') \subset (0, 1)$ of

[29] For example, in a PP equilibrium dividing both sides of equation (10) by d_p and using (3) we obtain the particularization of (23) for the vacancies posting w_p.

[30] To see this notice that if $\beta = \eta(d_p)$, then the average bargained wage in PB is $\eta(d_p)E_{\mu}(\tilde{y})$ which equals w_p by (10). But this means that (14) is solved for, precisely, $d_b = d_p$. Thus an average applicant's utility in PB is $\eta(d_p)E_{\mu}(\tilde{y})/d_p = w_p/d_p = U_p$, as in PP, so (23) holds.

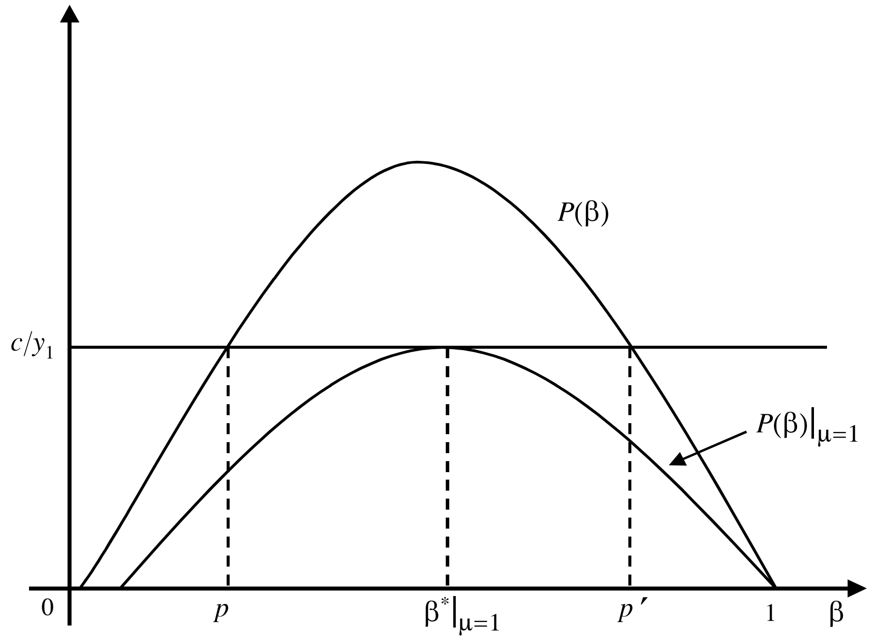

Figure 4: The Prevalence of Pure Posting

values of β which include β^* and for which $P(\beta) > c/y_1$ (see Figure 4).[31] Out of that range (13) holds so:

Proposition 1 *PP is an equilibrium for large levels of the hold-up problem, specifically for* $\beta \notin (p, p') \subset (0, 1)$.

To see the intuition behind this result, notice that $P(\beta)$ measures (as a proportion of y_1 and gross of the creation cost c) the profits that a firm can obtain by posting a vacancy with $x_\varnothing$ in a PP equilibrium –where such an announcement would only attract high productivity workers. Consider first the limit case where the fraction of high productivity workers in the population is one, that is, where deviating to bargaining does not allow the firm to improve on the quality of its applicants. As the adverse selection problem is absent, bargaining only brings about the net costs

[31] To prove that $\beta^* \in (p, p')$, one need to take into account that β^* is, in general, a (non-decreasing) function of μ.

of the hold-up problem. So the deviating firm earns strictly negative profits except if $\beta = \beta^*$, in which case its profits are zero, as under the equilibrium strategy. With $\mu < 1$, however, attracting only high productivity workers implies a gain in terms of the adverse selection problem. Thus, for an interval of values of β around β^*, the hold-up problem is mild enough to make $x_\varnothing$ a profitable deviation and PP ceases to be an equilibrium.

Next, let $B(\beta)$ represent the LHS of inequality (17) so that a PB equilibrium exists if and only $B(\beta) \leq c/y_0$. As proved in the Appendix, $B(\beta)$ is a non-negative and quasi-convex function that reaches a maximum value of $1 - \lim_{d\to\infty} \eta(d)$ both when β equals zero and when β is close to one. With $\mu = 0$, this function takes a minimum value of c/y_0 at $\beta = \beta^*$. As μ increases, $B(\beta)$ shifts downwards and gives raise to a range $[b, b'] \subset (0, 1)$ of values of β which contains β^* and for which $B(\beta) \leq c/y_0$ (see Figure 5). Out of that range, there are wages for which posting constitutes a profitable deviation so:

Proposition 2 *PB is an equilibrium for low levels of the hold-up problem, specifically for* $\beta \in [b, b'] \subset (0, 1)$.

To see the intuition for this result, notice that $B(\beta)$ measures (as a proportion of y_0 and gross of the creation cost c) the maximum profits that a firm may obtain by posting a wage in a PB equilibrium. Such a deviation would only attract low productivity workers and would thus entail an adverse selection cost relative to the equilibrium bargaining strategy. However, in the limit case where the proportion of high productivity workers in the population is zero, the adverse selection cost is nil. In this case PB survives as an equilibrium only for $\beta = \beta^*$, that is, when its underlying hold-up problem is also nil. More generally, with $\mu > 0$, there is a trade-off between the hold-up problem (of bargaining) and the adverse selection problem (of posting) which gets resolved in favor of the existence of PB only when the hold-up problem is mild.

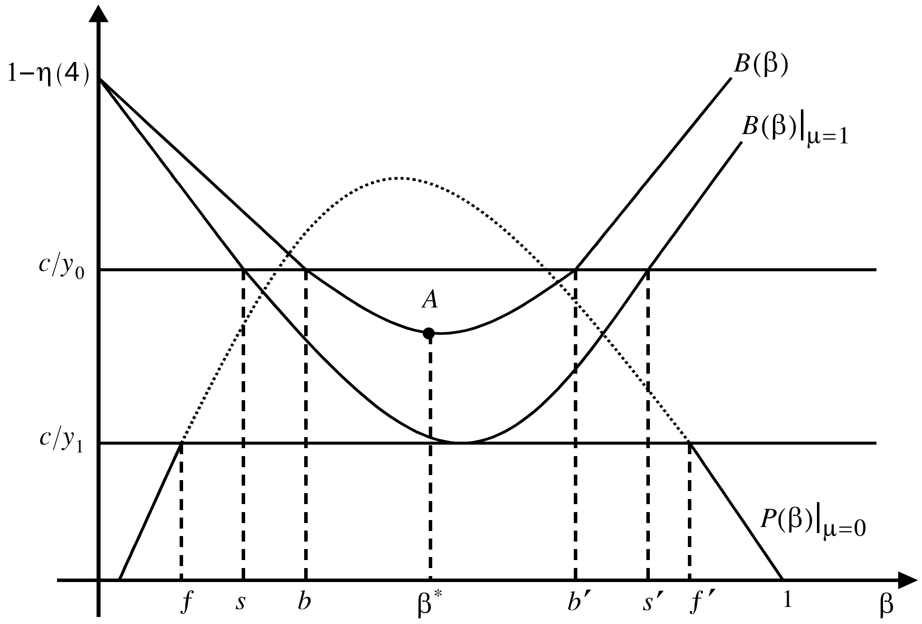

Figure 5: The Prevalence of Bargaining Equilibria

To analyze the existence of a SS equilibrium, notice that firms offering a bargained wage in such an equilibrium face the same situation as those in the PB equilibrium of an "artificial" economy in which the proportion of high productivity workers is some endogenously determined $\gamma \in (\mu, 1]$ rather than μ. Additionally, not only the firms which announce $x_\varnothing$ but also those that post the wage w_m break even. Thus condition (17) of the corresponding artificial economy must hold with equality. Therefore the intersections of $B(\beta)$ and the horizontal line c/y_0 for each of the artificial economies generated by varying the proportion of high productivity in the interval $(\mu, 1]$ identify values of β for which an SS equilibrium exists. At one extreme of the spectrum, the left and right intersections between the graph of $B(\beta)$ for the artificial economy with $\mu = 1$ and the horizontal line c/y_0, say s and s', respectively, identify the values of β that lead to the SS equilibria with $\gamma = 1$. At the other extreme, the PB equilibria that emerge with $\beta = b$ and $\beta = b'$ are degenerated SS equilibria with $\gamma = \mu$. Since increasing μ shifts $B(\beta)$ upwards, we have $s < b$ and $b' < s'$ and we can conclude that an SS equilibrium exists for all $\beta \in [s, b) \cup (b', s']$.

Proposition 3 *SS is an equilibrium for lower-intermediate levels of the hold-up problem, specifically for* $\beta \in [s, b) \cup (b', s']$.

To characterize the region where condition (22) holds and FS is an equilibrium, notice first that with $\beta = s, s'$ the SS equilibrium involves $\gamma(x_\varnothing) = 1$ and fails to constitute a FS equilibrium just because low productivity workers are still indifferent between the two segments of the labor market: $\beta y_0/d_{m1} = U_{m0}$. However, using the fact that d_{m1} is determined by (20) for $\gamma = 1$, one can check that β/d_{m1} is a quasi-concave function of β which reaches a maximum for a β in the interior of the interval $[s, s']$. In contrast, U_{m0} is determined in the posting segment of the market and, thus, is independent of β. Therefore, for values of β right below s or right above s', we have $\beta y_0/d_{m1} < U_{m0}$ and $U_{m1} = \beta y_1/d_{m1} > U_{m0}$ which implies the existence of a FS equilibrium. However, as β moves towards the extremes, U_{m1} becomes closer and eventually equal to U_{m0}. Actually, the case $U_{m1} = U_{m0}$ arises when, in the artificial economy with $\mu = 0$, the condition (13) for the existence of PP holds with equality. Graphically, this occurs at the intersections $\beta = f$ and $\beta = f'$ between the graph of $P(\beta)\mid_{\mu=0}$ and the horizontal line c/y_1 (see Figure 5). Thus:

Proposition 4 *FS is an equilibrium for upper-intermediate levels of the hold-up problem, specifically for* $\beta \in [f, s) \cup (s', f']$.

Intuitively, in order to sustain SS and FS the hold-up problem must be mild enough to convince high productivity workers to opt for bargaining but severe enough to convince (at least some) low productivity workers to opt for posting. As the hold-up problem worsens, less and less (and eventually no) low productivity workers opt for bargaining. When the SS equilibrium ceases to exist, the FS equilibrium emerges.

Interestingly, given that the graph of $P(\beta)$ shifts downwards as μ increases, we have $f < p$ and $p' < f'$, so there is always a range of values of β over which the PP equilibrium and the FS equilibrium coexist. Together with previous results, this implies that over the whole spectrum of bargaining powers at least one (and at most two) of our equilibria exists:[32]

Proposition 5 *An equilibrium always exists. For some levels of the hold-up problem, PP coexists with FS.*

Summing up, when the hold up problem is mild, PB is an equilibrium and PP is not. As the hold-up problem worsens, PP ceases to be an equilibrium and first SS and then FS arise. As the equilibrium moves from PB to SS and eventually to FS, the masses of firms and workers involved in vacancies whose wages are set through bargaining shrink. In other words, as the hold-up problem deteriorates, the incidence of bargaining diminishes. When the hold-up problem is sufficiently severe PP is the only equilibrium.

The possibility of having multiple equilibria is due to the negative externality that wage bargaining imposes on the firms posting a wage. In a PP equilibrium, if a single firm deviates to bargaining, the externality is nil, since a single firm is incapable of affecting workers' utilities and altering the productivity composition of the pool of applicants of the other firms. Consequently, the profitability of wage posting remains unchanged. However, when a positive mass of firms opt for bargaining (as in any of the bargaining regimes) their attraction of high productivity workers damages the productivity composition (and, hence, the profitability) of the vacancies with a posted wage. This explains why the profitability of wage bargaining compared to

[32]Notice that PB and PP (and hence SS and PP) may coexist since it is not generally true that the interval $[b, b']$ ($[s, s']$) is included in the interval (p, p'). Surely PP and PB do not coexist if μ is sufficiently small. To see this, notice that when μ tends to zero the interval $[b, b']$ tends to collapse into the point β^*, while the (positive) length of the interval (p, p'), which contains β^*, tends to its maximum.

wage posting is larger in an equilibrium with bargaining than when a single firm considers a deviation in the PP equilibrium.

6 The effects of adverse selection

In this section we first analyze how the adverse selection problem affects the existence of each type of equilibrium. Secondly we discuss its relation with the cross-subsidization that characterizes pooling equilibria such as PP and PB.

6.1 The effects of workers' productivity dispersion

Adverse selection reduces the incidence of wage posting vis-a-vis wage bargaining. To see this, we analyze the effects of an increase in the dispersion of workers' productivity. Specifically, we consider the experiment of increasing y_1 and decreasing y_0 without changing workers' average productivity $E_\mu(\widetilde{y})$. It follows from (11) that d_p remains unchanged. Thus, in condition (13) for the existence of PP, the LHS rises while the RHS falls, so the inequality is less likely to hold. In terms of Figure 4, the curve $P(\beta)$ shifts upwards while the line c/y_1 shifts downwards, so the interval (p, p') expands.

On the other hand, it follows from (14) that d_b also remains unchanged. Hence, in condition (17) for the existence of PB, the LHS remains constant while the RHS increases, so the inequality is more likely to hold. Graphically in Figure 5, $B(\beta)$ remains unchanged while the line c/y_0 moves upwards, so the interval (b, b') expands. For similar reasons, the thresholds s and f move towards the left, while s' and f' move towards the right. Thus:

Proposition 6 *Mean-preserving spreads in workers' productivity distribution contract the region where PP is an equilibrium and expand the region where PB and, more generally, equilibria with bargaining emerge.*

An implication of this result is that a small increase in workers' unobservable heterogeneity may lead to a large increase in wage dispersion. More specifically, the worsening of the adverse selection problem may induce a regime switch, moving the equilibrium from PP, where all workers are paid the same wage, to one of the equilibria with bargaining, where high productivity workers get higher wages than (at least some of) low productivity workers. Interestingly, the switch may lead to an increase in the wages of high productivity workers and a fall in the wages of low productivity ones. This result may partly explain the simultaneous rise in workers' unobservable heterogeneity and in wage inequality observed in the US over the last twenty years.[33]

6.2 Cross-subsidization and pooling regimes

As the proportion of high productivity workers in the population increases, sustaining pooling equilibria such as PP and PB becomes easier: the income produced by any vacancy that attracts both workers increases, more vacancies are created, and the utilities of both worker types rise. Given this, deviations that attract just one of the worker types become relatively less attractive. In particular, high productivity workers suffer a lower cost when cross-subsidizing the low productivity ones in PP and thus are less tempted to opt for a bargained wage. Similarly, low productivity workers enjoy a larger cross-subsidization in PB and thus are less tempted to opt for a posted wage.

In terms of Figure 4 and 5, increasing μ shifts down the graphs of both $P(\beta)$ and $B(\beta)$, so the interval (p, p') contracts while the interval (b, b') expands, which immediately means that PP and PB are sustainable over larger sets of values of β. On the other hand, since the graphs of $P(\beta)$ and $B(\beta)$ in the artificial economies with $\mu = 0$ and $\mu = 1$, respectively, do not change with μ, the thresholds f, f', s,

[33] The increase in workers' specific wage heterogeneity documented by Juhn, Murphy, and Pierce (1993) is consistent with a regime switch from posting to bargaining. The rise in workers' heterogeneity required to explain that shift may also explain the rise in the demand for screening devices such as temporary help firms and more formal recruitment practices, analyzed by Autor (2001) and Acemoglu (1999), respectively.

and s' remain unaffected. Thus the ranges of values of β where FS is an equilibrium are unchanged, while the ranges where SS emerges shrink due to the expansion of the interval (b, b'). Hence:

Proposition 7 *Increasing the fraction of high productivity workers expands the PP and PB regions, contracts the SS region, and leaves the FS region unaffected.*

Interestingly, this result implies that, by contributing to the sustainability of PP, a large μ favors the existence of multiple equilibria.

7 Efficiency

In this section we compare the various possible equilibria in terms of social welfare. As it is common in the literature, we start identifying social welfare with the sum of all firms' and workers' net income.[34] With this metric, social welfare can simply be computed as the weighted average of the utilities of each worker type, $\mu U_1 + (1-\mu) U_0$, since firms' equilibrium profits are zero.

In order to compute the social welfare W_j attained in the allocations associated with each of our possible equilibrium regimes, j =PP, PB, SS, FS, consider the function

$$G(\beta, \mu) = \mu U_{b1} + (1-\mu) U_{b0} = \frac{\beta E_\mu(\tilde{y})}{d_b},$$

where d_b is implicitly defined by (14). By definition, $G(\beta, \mu)$ yields the level of social welfare in the pure bargaining regime, W_{PB}. As we prove in the Appendix, $G(\beta, \mu)$ is strictly quasi-concave in β and reaches a maximum at $\beta = \beta^*$.[35] Since at $\beta = \beta^*$ the allocations of the PB and the pure posting regimes coincide, we have $W_{\mathrm{PP}} = \max_\beta G(\beta, \mu) \geq W_{\mathrm{PB}}$. In the semi-separating regime, workers' utilities

[34] This is the metric used, among others, by Pissarides (2000) and Shimer and Smith (2000).

[35] Hosios (1990) first proved that, in an economy with search frictions, net income is maximized when bargaining powers reflect the contribution of each side to the creation of matches —which in our setup is measured by $\beta^* \equiv \eta(d_p)$.

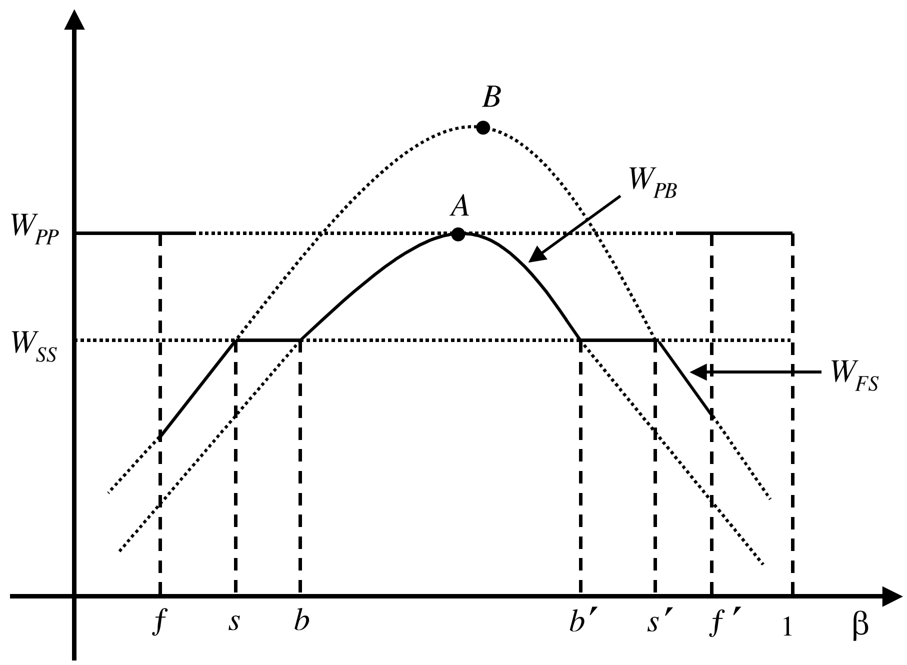

Figure 6: Welfare in the various equilibrium allocations

do not depend of β because U_{m0} is attained in the posting segment of the labor market and, by (21), $U_{m1} = \frac{y_1}{y_0} U_{m0}$. Hence, W_{SS} is independent of β. But since at $\beta = b, b'$ the SS allocation involves $\gamma(x_\varnothing) = \mu$ and, thus, degenerates into a PB allocation, we must have $W_{\mathrm{SS}} = G(b, \mu) = G(b', \mu)$ for all β. Finally, one can easily see that $W_{\mathrm{FS}} = \mu G(\beta, 1) + (1 - \mu) \max_\beta G(\beta, 0)$ since in the fully-separating regime the bargaining segment of the labor market functions like PB in an economy with $\mu = 1$, while the posting segment of the market functions like PP in an economy with $\mu = 0$; so $U_{m1} = G(\beta, 1)$ and $U_{m0} = \max_\beta G(\beta, 0)$. Importantly, for $\beta = s, s'$, we have $W_{\mathrm{FS}} = W_{\mathrm{SS}}$ since the SS allocation involves $\gamma(x_\varnothing) = 1$ and, thus, degenerates into a FS allocation. Furthermore the strict quasi-concavity of $G(\beta, 1)$ implies $W_{\mathrm{FS}} < W_{\mathrm{SS}}$ for all $\beta \in [f, s) \cup (s', f']$ since W_{FS} reaches its maximum at some $\beta^1 \in (s, s')$.[36]

[36] $\beta^1 \equiv \arg\max_\beta G(\beta, 1)$. The Appendix shows that the function $H(\mu) = \max_\beta G(\beta, \mu)$ is strictly convex, which implies $W_{\mathrm{FS}} > W_{\mathrm{PP}}$ at $\beta = \beta^1$, see point B in Figure 6. Interestingly, the FS allocation under $\beta = \beta^1$ reproduces the best possible allocation of an economy with verifiable worker types,

Summarizing these results, Figure 6 depicts the welfare levels associated with the various allocations for each value of β. The solid sections of the curves identify the values of β for which the corresponding allocation can be sustained as an equilibrium. In point A we have $W_{\mathrm{PP}} = W_{\mathrm{PB}} = G(\beta^*, \mu)$. Clearly:

Proposition 8 *PP is the equilibrium that generates the largest net aggregate income. If firms were obliged to post their wages, aggregate net income would never decrease.*

Equilibria with bargaining cope with the adverse selection problem associated with wage posting by allowing high productivity workers to extract higher wages than low productivity workers. This amounts to redistributing income across workers. But, at the aggregate level, the unsolved hold-up problem leads to either excessive vacancies creation (when $\beta < \beta^*$) or excessive unemployment (when $\beta > \beta^*$), so that net aggregate income is, generically, strictly lower in the equilibria with bargaining than in PP.

Interestingly, the welfare costs induced by the hold-up problem can be so large that not only low but also high productivity workers are better off in a PP equilibrium than in an alternative equilibrium with bargaining. Indeed, we prove in the Appendix that:

Proposition 9 *Whenever PP and a bargaining equilibrium coexist, PP is Pareto dominant.*

By Proposition 1, PP and (one of the) equilibria with bargaining tend to coexist only when the hold-up problem is sufficiently severe. In this case the social costs of bargaining are large and high productivity workers' utility is lower than in PP either because their wages are too low (when $\beta < \beta^*$) or because they find too difficult to obtain a job due to the depressed supply of vacancies (when $\beta > \beta^*$).

where firms would post different wages for each type. So the value of W_{FS} at $\beta = \beta^1$ identifies the "first best" level of social welfare. Unfortunately, the hold-up problem is so mild at $\beta = \beta^1$ that FS is not an equilibrium (the equilibrium is either SS or PB, which involve more bargaining).

8 Further discussion

First we analyze how our welfare conclusions change in the presence of endogenous human capital investment decisions. Then we discuss the robustness of our results to the possibility that firms can scrutinize and rank several applicants before hiring one of them.

8.1 Bargaining and the investment in human capital

In our basic model bargaining always reduces net aggregate income because its hold-up problem causes an inefficient level of vacancy creation, while its response to the adverse selection problem produces a mere redistribution of income across workers' productivity types. However, if workers can affect their productivity through endogenous human capital investments, such a redistribution of income affects workers' investment decisions and our welfare conclusions need to be qualified.

To check this, we consider two alternative, relevant scenarios. Suppose first that workers invest in human capital before learning their type (say, during some education stage prior to entering the labor market). Formally, their investment is analogous to a costly entry decision prior to type discovery. A worker's expected utility, conditional on entry and averaged across types, establishes the strength of the worker's incentive to enter the market. Since equilibria with bargaining push this utility down (and below the average social value of labor), bargaining depresses workers' level of participation or, in the alternative interpretation, their investment in human capital. Hence, in this first set-up, our welfare conclusions are, if anything, reinforced.

Suppose, instead, that the investment in human capital increases the chance that the worker acquires the high productivity type or, alternatively, that it increases only the productivity of highly productive types. In this context bargaining would have the virtue of involving a lower level of cross-subsidization (from high to low productivity types) and, thereby, would increase the return from becoming a highly productive worker. The positive incentive effects of the induced wage inequality might offset, at least partially, the previously discussed negative effects of bargaining.

8.2 Bargaining when applicants can be ranked

In our model firms cannot rank their applicants before selecting one of them, since workers' types become observable to the firm at a stage in the hiring process (an interview or, more plausibly, after a probation period) that at most one of the applicants can undergo. Here we comment on how the model logic would be modified if firms could consider two workers at that stage.

In the proposed setting, a firm with two or more applicants would simultaneously consider two candidates for the same job. This would allow the firm to rank the two candidates according to their productivity and, under bargaining, to introduce wage competition between them. As high productivity workers would be ranked first whenever paired with low productivity workers, the probability of getting a given job would now be a function not only of the expected length of the queue of applicants but also of the productivity composition of such a queue. This introduces a new dimension in the analysis of the workers' application subgame, since now vacancies with many high productivity applicants become relatively unattractive to low productivity workers.

Arguably this mechanism could facilitate the sustainability of equilibria in which workers self-select across vacancies with different posted wages. For instance, there could exist two posted wages, say x_0 and x_1, such that high productivity workers do not want to apply for x_0 because wages are too low (or workers' demand is too high), while low productivity workers do not want to apply for x_1 because they fear to compete with high productivity workers. Yet, one can check that, from the perspective of the whole labor market game, these type of equilibria can be sustained only if firms hold the pessimistic belief that if they post a wage different from those observed in equilibrium, they would attract only low productivity workers. In general, when firms' expectations about the NE of out-of-equilibrium application subgames are "balanced" in the sense used in our basic model, the only equilibrium where firms post wages is one where all the posted wages are equal and attract the same composition of worker types.

Bargaining introduces ex-post wage competition between the selected candidates, since the firm can threaten each candidate with hiring the other one. Yet the implied reduction in workers' rents tends to be greater for low than for high productivity workers. Specifically, the threat to hire the other candidate always pushes the wage of a low productivity worker down to zero while a high productivity worker confronted with a low productivity applicant can still appropriate a positive wage which is increasing in the productivity differential $y_1 - y_0$. Thus, it remains true that, on average, the bargained wage is an increasing function of the worker's productivity so that firms are still tempted to use bargaining in order to improve the composition of their pool of applicants. Therefore, by the same forces present in our basic model, if workers productivity is sufficiently dispersed and the hold up problem is mild, posting equilibria cease to exist in favor of equilibria with bargaining.

9 Conclusions

We have presented a tractable search model where firms compete for heterogeneous workers by announcing the wage setting mechanism associated with their vacancies. Since, unverifiable worker qualifications render posted wages incomplete, firms' choices are driven by the trade-off between the adverse selection problem that posting an incomplete wage may involve (attracting mainly low productivity workers) and the hold-up problem that bargaining creates (inducing an inadequate compensation of firms' and workers' ex ante investments in the vacancy).

We predict the prevalence of wage bargaining in those segments of the labor market where the distribution of bargaining power is not extreme and where, after conditioning on workers' verifiable qualifications (such as education and demonstrable years of tenure in a prior employment), some unverifiable qualifications cause a high residual variability on workers' productivity in the job. In this sense our analysis implies that, differences in the functioning of institutions such as the education system and the legal system, which influence the degree of verifiability of workers' relevant qualifications, may explain differences in the prevalence of wage bargaining across countries.

We expect the incidence of wage bargaining to be positively associated with wage dispersion. The reason is twofold. First, under pure wage bargaining, apparently identical workers can be paid differently since their wages reflect qualifications revealed to firms during the recruitment process. Second, if wage posting and wage bargaining coexist, the former attracts the least qualified workers, while the latter attracts the rest, which makes the wage paid in the posting segment of the market likely to be lower than the (average) wage paid elsewhere. Interestingly, *small* (and gradual) changes in workers' unobservable heterogeneity can shift the labor market equilibrium from a posting regime to a bargaining regime and cause a *large* (and sudden) increase in wage inequality. This establishes a plausible theoretical link between the documented increase in workers' unobservable heterogeneity in the US over the last twenty years and the parallel rise in wage inequality. Of course, testing the empirical importance of this link would require examining whether firm's wage setting practices have also changed over the same period. But the field work needed to answer this question makes it a topic for future research.

Appendix

Proof of Lemma 1 The proof is organized in two parts. In the first, we use condition (4) to prove that $\frac{y_0}{y_1}U_1 \leq U_0 \leq U_1$ and the necessity of (7). In the second, we use (7) to substitute for $d(x)$ in the NE conditions (4)-(6) so that the pair (U_0, U_1) and the function $\gamma(x)$ become the only unknowns. We constructively show that there is always a unique (U_0, U_1) (and some compatible $\gamma(x)$) that satisfies the reduced NE conditions.

Part 1. It follows immediately from (2) that $E_0[\tilde{w}(x)] = E_1[\tilde{w}(x)]$ if $x \in \mathbb{R}_+$ and $E_1[\tilde{w}(x_\varnothing)] = \beta y_1 > E_0[\tilde{w}(x_\varnothing)] = \beta y_0$, so (4) yields

$$\frac{y_0}{y_1}U_1 \leq U_0 \leq U_1. \tag{24}$$

To obtain (7), notice that $d(x) = 1$ means that workers do not apply for vacancies which announce x, while, by (4), $d(x) > 1$ requires $E_i[\tilde{w}(x)]/d(x) = U_i$ for at least one worker type i. For $x \in \mathbb{R}_+$ and $x \leq U_0$, we necessarily have $d(x) = 1$ since the alternative $d(x) > 1$ would imply $x/d(x) < U_0 \leq U_1$ which is contradictory with the fact that some workers want to apply for vacancies of this type. For $x \in \mathbb{R}_+$ and $x > U_0$, we prove by contradiction that $d(x) = x/U_0$. Notice that $d(x) < x/U_0$ contradicts (4), while $d(x) > x/U_0 > 1$ together with (24) implies $U_1 \geq U_0 > x/d(x)$ which contradicts $d(x) > 1$. For $x = x_\varnothing$ and $\beta y_1 \leq U_1$, we must have $d(x_\varnothing) = 1$, since for any $d(x_\varnothing) > 1$ we would have $\beta y_1/d(x_\varnothing) < U_1$ and, using (24), $\beta y_0/d(x_\varnothing) < U_0$ as well, which means that no worker would apply for vacancies with $x_\varnothing$. Finally, for $x = x_\varnothing$ and $\beta y_1 > U_1$, we can prove by contradiction that $d(x_\varnothing) = \beta y_1/U_1$, since $d(x_\varnothing) < \beta y_1/U_1$ directly contradicts (4), while $d(x_\varnothing) > \beta y_1/U_1 > 1$ also implies, by (24), $d(x_\varnothing) > \beta y_0/U_0$ but then no worker of either type would want to apply for vacancies with $x_\varnothing$, which contradicts $d(x_\varnothing) > 1$.

Part 2. Before proceeding, define the function

$$g(U) = \sum_{x \in X^* \cap \mathbb{R}_+} N(\max(1, \frac{x}{U}))v(x),$$

which is identically equal to zero if $v(x) = 0$ for all $x \in X^* \cap \mathbb{R}_+$ and is decreasing in U and maps $\mathbb{R}_+$ onto $\mathbb{R}_+$ otherwise. Define also the function

$$h(U) = N(\max(1, \frac{\beta y_1}{U}))v(x_\varnothing),$$

which is identically equal to zero if $v(x_\varnothing) = 0$ and is decreasing in U with image onto $\mathbb{R}_+$ otherwise. Notice that, with a positive mass of vacancies, g and h cannot be identically equal to zero simultaneously. With this notation, substituting (7) into (6) and adding up the two conditions yields:

$$g(U_0) + h(U_1) = 1, \tag{25}$$

which must thus be satisfied by any pair of equilibrium utilities (U_0, U_1). This suggests two classes of subgames whose analysis is trivial. First, if $v(x_\varnothing) = 0$, then (4) and (6) yield $U_0 = U_1$ and (25) reduces to $g(U_0) = 1$, which has a unique solution and determines a unique pair (U_0, U_1) compatible with the NE conditions. Second, if $v(x) = 0$ for all $x \in X^* \cap \mathbb{R}_+$, then $U_0 = \frac{y_0}{y_1}U_1$ and (25) reduces to $h(U_1) = 1$, which determines the only values of U_1 and, recursively, U_0 compatible with the NE conditions. To analyze the remaining classes of subgames, let U^g and U^h be the unique solutions of the equations $g(\frac{y_0}{y_1}U^g) = 1 - \mu$ and $h(U^h) = \mu$, and consider the following intermediate results:

1. There exists a unique (U_0, U_1) with $U_0 = \frac{y_0}{y_1}U_1$ compatible with the NE conditions if and only if $U^g \leq U^h$. If $\frac{y_0}{y_1}U_1 = U_0 < U_1$, after substituting (7) into (5) it follows that $\gamma(x) = 0$ for $x \in \mathbb{R}_+$. This fact together with (6) implies that $g(U_0) \leq 1 - \mu$ and, by (25), $h(U_1) \geq \mu$. Since g and h are not increasing, it must be that $U_0 \geq \frac{y_0}{y_1}U^g$ and $U_1 \leq U^h$, which yields $\frac{y_0}{y_1}U^g \leq U_0 = \frac{y_0}{y_1}U_1 \leq \frac{y_0}{y_1}U^h$ thus implying $U^g \leq U^h$. On the other hand, if $U^g \leq U^h$, the monotonicity of g and h guarantees that there exists a unique $U_1 \in [U^g, U^h]$ such that (25) and the remaining NE conditions are satisfied with $U_0 = \frac{y_0}{y_1}U_1$.

2. There exists a unique (U_0, U_1) with $U_0 = U_1$ compatible with the NE conditions if and only if $U^h \leq \frac{y_0}{y_1}U^g$. If $\frac{y_0}{y_1}U_1 < U_0 = U_1$, after substituting (7) into (5) it follows that $\gamma(x_\varnothing) = 1$. This fact together with (6) implies $h(U_1) \leq \mu$ and, by (25), $g(U_0) \geq 1 - \mu$. Since g and h are not increasing, we must then have $U_1 \geq U^h$ and $U_0 \leq \frac{y_0}{y_1}U^g$, which implies $U^h \leq U_1 = U_0 \leq \frac{y_0}{y_1}U^g$. On the other hand, if $U^h \leq \frac{y_0}{y_1}U^g$, the monotonicity of g and h guarantees that there exists a unique $U_1 \in [U^h, \frac{y_0}{y_1}U^g]$ such that (25) and the remaining NE conditions are satisfied with $U_0 = U_1$.

3. There exists a unique (U_0, U_1) with $U_0 \in (\frac{y_0}{y_1}U_1, U_1)$ compatible with the NE conditions if and only if $U^h \in (\frac{y_0}{y_1}U^g, U^g)$. To prove the necessity part, suppose that $U_0 \in (\frac{y_0}{y_1}U_1, U_1)$. Then after substituting (7) into (5) it follows that $\gamma(x) = 0$

for $x \in \mathbb{R}_+$ and $\gamma(x_\varnothing) = 1$. Hence (6) can be rewritten as $g(U_0) = 1 - \mu$ and $h(U_1) = \mu$, which implies $U_0 = \frac{y_0}{y_1}U^g$ and $U_1 = U^h$. But then $U_0 \in (\frac{y_0}{y_1}U_1, U_1)$ requires $U^h \in (\frac{y_0}{y_1}U^g, U^g)$. To prove sufficiency, notice that the pair $(U_0, U_1) = (\frac{y_0}{y_1}U^g, U^h)$ is of the class $U_0 \in (\frac{y_0}{y_1}U_1, U_1)$ and, by construction, is the only one in this class that satisfies the NE conditions.

As the previous configurations in terms of U^g and U^h do not overlap and are the only ones possible, the previous results prove the existence and uniqueness of the pair (U_0, U_1). Finally, given the unique equilibrium pair (U_0, U_1), the function $d(x)$ can be uniquely obtained from (7), while the (possibly non-unique) function $\gamma(x)$ can be recovered by tracing the application preferences of the workers of each type along the previous discussion.$\|$

Proof of Lemma 2 When $U_0 < U_1$, after substituting (7) into (5) it immediately follows that $\gamma(x) = 0$ for $x \in \mathbb{R}_+$. We next analyze the case with $U_0 = U_1$. If $x \in \mathbb{R}_+ \backslash X^*$, the result follows directly from (7) and the definition of balanced expectations. If $x \in \mathbb{R}_+ \cap X^*$, we argue by contradiction that $\gamma(x) = \mu$. If $\gamma(x) < \mu$, the contradiction is immediate since, by (7) and balanced expectations, an alternative announcement $x' \in \mathbb{R}_+ \backslash X^*$, arbitrarily close to x, would feature $\gamma(x') = \mu$ and thus yield $V(x') > V(x)$. If $\gamma(x) > \mu$, (6) and the fact that, from substituting (7) into (5), $\gamma(x_\varnothing) = 1$ when $U_0 = U_1$, there must exist $x'' \in \mathbb{R}_+ \cap X^*$ with $\gamma(x'') < \mu$. But then the same argument used above would contradict the fact that $x'' \in X^*$ and hence the possibility that $\gamma(x) > \mu$.$\|$

Proof of Lemma 3 Lemma 2 implies that there is constant fraction γ of high productivity applicants for all vacancies with $x \in \mathbb{R}_+$. From (7) and (8), we get

$$V'(x) = q'(d(x))(E_\gamma(\tilde{y}) - x)\frac{1}{U_0} - q(d(x))$$

for all $x \in \mathbb{R}_+$ such that $x \geq U_0$. But (7) also implies that $U_0 = x/d(x)$ so we can group terms using the definitions in Section 3.1 and write

$$V'(x) = q(d(x))\left[\frac{\eta(d(x))}{1 - \eta(d(x))} \cdot \frac{E_\gamma(\tilde{y})}{x} - 1\right].$$

The term in brackets is clearly decreasing in x since $\eta(d)$ is decreasing and $d(x)$ is increasing. Thus, as x increases, the sign of $V'(x)$ shifts from positive to negative at most once. This together with the fact that $V(x)$ is continuous at $x = U_0$ and constant for $x \leq U_0$ proves that $V(x)$ is strictly quasi-concave for all $x \in \mathbb{R}_+$.$\|$

Properties of the function $P(\beta)$

1. *Non-negative.* The fact that $P(\beta)$ is non-negative and $P(\beta) = 0$ for either $\beta \leq \eta(d_p)E_\mu(\widetilde{y})/(y_1 d_p)$ or $\beta = 1$ follows directly from the inspection of the LHS of (13).

2. *Quasi-concave.* Equation (11) allows us to write
$$P'(\beta) = q(z(\beta))\left[\frac{1-\beta}{\beta} \cdot \frac{\eta(z(\beta))}{1-\eta(z(\beta))} - 1\right], \tag{26}$$
where
$$z(\beta) \equiv \frac{\beta y_1 d_p}{\eta(d_p)\, E_\mu(\widetilde{y})} \tag{27}$$
is increasing in β. But then the expression in square brackets is weakly decreasing in β, which implies that, as β increases, the sign of $P'(\beta)$ shifts from positive to negative at most once, as quasi concavity requires.

3. *Maximum at $\widehat{\beta} \leq \beta^*$.* By Property 2, $P(\beta)$ reaches its maximum at the unique value $\widehat{\beta}$ such that $P'(\widehat{\beta}) = 0$. To see that $\widehat{\beta} \leq \beta^* \equiv \eta(d_p)$, notice from (26) and (27) that with $\mu = 1$ we have $z(\beta^*) = d_p$ so $P'(\beta^*) = 0$ and $\widehat{\beta} = \beta^*$. In contrast, with $\mu < 1$ we have $z(\beta^*) > d_p$ which, given that $\eta(\cdot)$ is weakly decreasing, implies $P'(\beta^*) \leq 0$ and, immediately, $\widehat{\beta} \leq \beta^*$.

4. *Strictly decreasing in μ.* Notice that $E_\mu(\widetilde{y})$ is strictly increasing in μ and $\eta(d_p)$ is weakly decreasing in d_p, so that, by (11), d_p is decreasing in μ.

5. *Position relative to c/y_1.* Let β^1 denote the value of $\beta^* \equiv \eta(d_p)$ for the economy with $\mu = 1$. Direct substitution in the LHS of (13) implies that $P(\beta^1) = c/y_1$ when $\mu = 1$. So when $\mu < 1$ we have $P(\beta^1) > c/y_1$ by Property 4. On the other hand, we know that in general $\beta^* \leq \beta^1$ since $\eta(d_p)$ is weakly increasing in μ. But, by Property 3, β^* is to the right of the single peak of $P(\beta)$, so $P(\beta^*) \geq P(\beta^1) > c/y_1$ when $\mu < 1$.

Properties of the function $B(\beta)$

1. *Non-negative.* The fact that $B(\beta)$ is non-negative and $B(\beta) = 1 - \min_{d \geq 1} \eta(d)$ for either $\beta = 0$ or $\beta = 1 - c/E_\mu(\widetilde{y})$ follows directly from the inspection of the LHS of (17) after using A1, (14) and the definition of d'.

2. *Quasi-convex.* Equation (11) allows us to write
$$\frac{d[d_b/\beta]}{d\beta} = \frac{d_b}{\beta^2}\left[\frac{\beta}{1-\beta} \cdot \frac{1-\eta(d_b)}{\eta(d_b)} - 1\right] \tag{28}$$
which, since $\eta(d_b)$ is decreasing in β, immediately implies that d_b/β is quasi convex in β. But, since (16) implies that d' and consequently $q(d')[1-\eta(d')]$ are increasing in d_b/β, it immediately follows that $B(\beta)$ is also quasi-convex.

3. *Minimum at* β^*. Equation (28) implies that d_b/β is globally minimized at $\beta = \beta^* = \eta(d_b)$. But then, since d' and consequently $q(d')[1-\eta(d')]$ are increasing in d_b/β, it follows that $B(\beta)$ also reaches a global minimum at $\beta = \beta^*$.

4. *Decreasing in* μ. Equation (14) implies that d_b is decreasing in μ. This together with the fact that d' and consequently $q(d')[1-\eta(d')]$ are increasing in d_b/β, proves that $B(\beta)$ is decreasing in μ.

5. *Position relative to* c/y_0. When $\beta = \beta^*$, we have $d' = d_b$ and $\eta(d_b) = \beta^*$ so (14) implies $B(\beta^*) = c/E_\mu(\widetilde{y})$. Then, clearly, $B(\beta^*) = c/y_0$ if $\mu = 0$ and $B(\beta^*) < c/y_0$ if $\mu > 0$.

Properties of the function $G(\beta, \mu)$

1. *Quasi-concave in* β. We have already shown, using (28), that d_b/β is quasi convex, which implies that $G(\beta, \mu)$ is quasi-concave in β.

2. *Maximum at* β^*. The fact that $G(\beta, \mu)$ is maximized at $\beta = \beta^*$ follows from the fact that d_b/β is globally minimized at $\beta = \beta^* = \eta(d_p)$, by (28).

3. *The function* $H(\mu) = \max_\beta G(\beta, \mu)$ *is strictly convex.* The envelope theorem implies $H'(\mu) = \partial G(\beta^*, \mu)/\partial\mu$. By differentiating in (14) and using the definition of $G(\beta, \mu)$, we obtain
$$\frac{\partial G(\beta, \mu)}{\partial \mu} = \frac{\beta(y_1 - y_0)}{\eta(d_b) d_b}.$$

But at $\beta = \beta^*$ we have $\beta = \eta(d_p) = \eta(d_b)$ so

$$H'(\mu) = \frac{y_1 - y_0}{d_p},$$

which, given (11), is strictly increasing in μ.

Proof of Proposition 9 As $U_1 \geq U_0$ it is enough to show that when PP coexists with either FS, SS or PB, high productivity workers are not worse off than in a PP equilibrium. Recall that PP is an equilibrium if and only if

$$q\left(\underline{d}\right)(1-\beta)\, y_1 \leq c, \tag{29}$$

where $\underline{d} = \max(1, \beta y_1/U_p)$ from (7) while U_p is the utility achieved by high productivity workers in a PP equilibrium. Instead in a bargaining regime high productivity workers earn $U_1 = \beta y_1/d_\gamma$, where

$$q\left(d_\gamma\right)(1-\beta)\, E_\gamma(\tilde{y}) = c, \tag{30}$$

and $\gamma = \mu$ in PB, $\gamma \in (\mu, 1)$ in SS, and $\gamma = 1$ in FS. Clearly, since $q(d)$ is increasing, we have $d_\gamma \geq d_1$ for all $\gamma \in [\mu, 1]$. Then comparing (29) with (30) for $\gamma = 1$ immediately yields that $d_1 \geq \underline{d}$ and so $d_\gamma \geq \underline{d}$ for all $\gamma \in [\mu, 1]$. But, given the definition of $\underline{d}$, we can conclude that $U_p \geq U_1 = \beta y_1/d_\gamma$ for all $\gamma \in [\mu, 1]$.$\|$

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