TESTING THE ORDER OF INTEGRATION OF THE U.K. UNEMPLOYMENT

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Abstract

Tests proposed by Robinson (1994a) for testing unit roots and other fractionally integrated hypotheses are applied in this article to several measures of the U.K. unemployment. The results clearly reject the trend-stationary I(0) representations, but even the unit roots I(1) hypotheses are also rejected in favour of alternatives with $d > 1$. Thus the standard approach of taking first differences to get I(0) stationary series may be too restrictive, obtaining series with long memory behaviour.

JEL Classification: C22, C5

Key words: Unemployment in UK, Unit roots, Fractionally Integrated Series

1. Introduction

The issue in this article is to report evidence in favour of fractionally integrated models for the U.K. unemployment. A major debate concerning the dynamics properties of macroeconomic time series, (including unemployment), came after the seminal work of Nelson and Plosser (1982). In that paper, they challenged the traditional view that macroeconomic series were stationary around a deterministic function of time. Using statistical techniques developed by Fuller (1976) and Dickey and Fuller (1979), they found no strong evidence against unit roots in US historical annual time series.

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However, unit root models can be viewed as particular specialized cases of a much more general class of processes called long memory processes, due to their ability to display significant dependence between observations widely separated in time. A popular technique to analyse fractionally integrated models is through the fractional differencing operator $(1 - L)^d$, where

$$(1-L)^d = \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} L^k = 1 - d\,L + \frac{d\,(d-1)}{2} L^2 - \frac{d\,(d-1)(d-2)}{6} L^3 + ...$$

and L is the lag operator ($Lx_t = x_{t-1}$). To illustrate this in case of a scalar time series x_t, t=1,2,…, suppose that v_t is an unobservable covariance stationary sequence with spectral density that is bounded and bounded away from zero at any frequency, and

$$(1 - L)^d \, x_t = v_t , \qquad t = 1, 2, \, ... \, . \tag{1}$$

The process v_t could itself be a stationary and invertible ARMA sequence, when its autocovariances decay exponentially, however, they could decay much slower than exponentially. When d = 0 in (1), $x_t = v_t$ and thus x_t is 'weakly autocorrelated', also termed 'weakly dependent'. If $0 < d < 0.5$, x_t is still stationary but its lagged j autocovariance γ_j decreases very slowly, like the power law j^{2d-1} as $j \to \infty$ and so the γ_j are non-summable. We say then that x_t has long memory given that its spectral density $f(\lambda)$ is unbounded at the origin. Finally, as d in (1) increases beyond 0.5 and through 1 (the unit root case), x_t can be viewed as becoming "more nonstationary" in the sense, for example, that the variance of the partial sums increases in magnitude. This is also true for d > 1, so a large class of nonstationary processes may be described by (1) with $d \geq 0.5$. The distinction between I(d) with different values of d is also important from an economic viewpoint: if d < 1, the process is mean-reverting, with shocks affecting to the system, but the system returns to its original level sometime in the future. On the contrary, $d \geq 1$ means that the series is nonstationary and non mean-reverting.

In this paper we propose the use of Robinson's (1994a) tests for testing unit roots and other fractionally integrated hypotheses when modelling the U.K. unemployment. The testing procedure is presented in Section 2. Section 3 applies the tests to different measures of the U.K. unemployment and finally Section 4 contains some concluding remarks.

2. Tests of fractional integration

Robinson (1994a) proposes a very general procedure for testing unit roots and other hypotheses in raw time series. Unlike most unit root tests, embedded in autoregressive (AR) alternatives, Robinson's (1994a) tests can be nested in a fractionally integrated model

$$(1 - L)^{d+\theta} \; x_t \; = \; v_t \,, \qquad t = 1, 2, ... \tag{2}$$

where d is a given real number. v_t is an I(0) process with parametric spectral density f, which is a given function of frequency λ and of unknown parameters, specifically,

$$f(\lambda \,; \sigma^2; \, \tau) \; = \; \frac{\sigma^2}{2\pi} \, g(\lambda ; \tau), \qquad -\pi \; < \; \lambda \; \leq \; \pi,$$

where the scalar σ^2 and the (qx1) vector τ are unknown but g is of known form, and x_t are the errors in the regression model

$$y_t \; = \; \beta' z_t \; + \; x_t \,, \qquad t = 1, \, 2, ... \tag{3}$$

where $\beta = (\beta_1, \beta_2, \ldots, \beta_k)'$ is a vector of unknown parameters; z_t is a (kx1) vector of deterministic variables that might include an intercept or a time trend for example; and y_t is the time series we observe from t = 1,2,….n. Thus under the null hypothesis

$$H_o : \; \theta \; = \; 0, \tag{4}$$

x_t in (2) is I(d), and if d = 1 contains a unit root at the zero frequency. Under (4), the residuals in (2) and (3) are

$$\bar{v}_t \; = (1 - L)^d \; y_t \; - \; \bar{\beta}'(1 - L)^d \; z_t \,,$$

where

$$\overline{\beta} = \left(\sum_{t=1}^{n} w_t \, w_t' \right)^{-1} \sum_{t=1}^{n} w_t \, (1-L)^d \, y_t \,, \qquad w_t = (1-L)^d \, z_t .$$

Unless g is a completely known function, (e.g., g ≡ 1, as when v_t is white noise) we have to estimate the nuisance parameter τ, for example by $\overline{\tau} = \arg\min_{\tau \in T} \sigma^2(\tau)$, where T is a suitable subset of R^q and

$$\sigma^2(\tau) = \frac{2\pi}{n} \sum_{j=1}^{n-1} g(\lambda_j;\tau)^{-1} \, I(\lambda_j), \qquad \text{with}$$

$$I(\lambda_j) = \left| \frac{1}{\sqrt{2\pi n}} \sum_{t=1}^{n} \overline{v}_t \, e^{i\lambda_j t} \right|^2; \qquad \lambda_j = \frac{2\pi j}{n}.$$

The test statistic, which is derived from the Lagrange multiplier (LM) principle is:

$$\overline{r} = \frac{\sqrt{n}}{\overline{\sigma}^2} \, \overline{A}^{-1/2} \, \overline{a}, \tag{5}$$

where

$$\overline{a} = \frac{-2\pi}{n} \sum_{j=1}^{n-1} \psi(\lambda_j) \, g(\lambda_j;\overline{\tau})^{-1} \, I(\lambda_j)$$

$$\overline{A} = \frac{2}{n} \left(\sum_{j=1}^{n-1} \psi(\lambda_j)^2 - \sum_{j=1}^{n-1} \psi(\lambda_j) \overline{\xi}(\lambda_j)' \left(\sum_{j=1}^{n-1} \overline{\xi}(\lambda_j) \overline{\xi}(\lambda_j)' \right)^{-1} \sum_{j=1}^{n-1} \overline{\xi}(\lambda_j) \psi(\lambda_j)' \right)$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|, \qquad \overline{\xi}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j;\overline{\tau}).$$

Based on (4), Robinson (1994a) established under regularity conditions that

$$\bar{r} \quad \rightarrow_d \quad N(0,1) \qquad as \quad n \rightarrow \infty \tag{6}$$

and thus, an approximate one-sided 100α% test of (4) against the alternative H_a: θ > 0 rejects H_o if $\bar{r} > z_\alpha$, where the probability that a standard normal variate exceeds z_α is α. Conversely, a test of (4) against alternatives H_a: θ < 0 rejects H_o if $\bar{r} < -z_\alpha$. He also showed that the tests are efficient in the Pitman sense in that when directed against local alternatives of the form H_1: $\theta = \delta\, n^{-1/2}$ for $\delta \neq 0$, the limit distribution is Normal with variance 1 and mean which cannot (when v_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic. Thus, we are under standard situations, unlike most of unit root tests where a non-standard limit distribution is obtained. Furthermore, the null N(0,1) distribution holds across a wide range of null hypothesized values of d, and across a broad class of exogenous regressors, including, for example, $z_t \equiv 1$ and $z_t = (1,t)'$ in cases of an intercept and a linear time trend respectively.

This version of the tests of Robinson (1994a) was applied to annual macroeconomic time series data in Gil-Alana and Robinson (1997) and Gil-Alana (2000), and other versions of the tests based on seasonal (quarterly and monthly) and cyclical data can be respectively found in Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001c). In this article, they are applied to different measures of unemployment in the United Kingdom.

3. An empirical application to the U.K. unemployment

Four different measures of unemployment were considered in this paper. First we look at the number of people claiming unemployment benefit. This measure is known as the claimant count (CC) and is available monthly. We look at this measure (U_t) and also at its logarithmic transformation (log U_t). Another measure, which is related with the unemployment rate, is the CC as a percentage of the workforce. We also look at this series, (u_t), as well as its logistic transformation:

$$u_t^* = \log\left(\frac{u_t}{1 - u_t}\right).$$

The data are monthly starting in January 1971 and ending in August 1998. These series have been analyzed in Gil-Alana (2001a, b), estimating respectively their orders of integration with parametric (ARFIMA models) and semiparametric techniques. He found in these papers strong evidence in favour of fractional roots of orders about 1.50 and thus, rejecting in practically all cases the hypothesis of a unit root. In this paper, we concentrate on testing the orders of integration of the series for given parametric models, the conclusions here being in line with those in these previous works, finding evidence of something much higher than a unit root. Furthermore, the fact that the unit root is in many cases rejected indicates that the standard approach of taking first differences does not guarantee I(0) stationary residuals but series with long memory behaviour.[1]

The first twenty sample autocorrelation values for the four series are plotted in Table 1a, while the autocorrelations of their first and second differences are plotted in Tables 1b and 1c respectively. In Table 1a the autocorrelations start at around 0.99 and then they decay very slowly. Similarly, the autocorrelations for the first differenced series also decay very slowly, with significant values even at lag 16 in all of them. Taking two differences, in Table 1c, we still see significant autocorrelations, especially at lag-1, with also some apparent slow decay and/or oscillation, which could be indicative of fractional integration greater than a unit root in the original series.

[1] In Gil-Alana (2001a), the models differ from those reported in this paper only with respect to its short run (ARMA) components, while in Gil-Alana (2001b), the models are specified only for its long run properties. Thus, the three papers are connected each other only in relation to its long run behaviour, which is determined by the order of integration of the series.

TABLE 1*

Table 1a: First 20 sample autocorrelations ot the original time series

Series	U_t	Log U_t	u_t	u^*_t
1	**.994**	**.993**	**.994**	**.992**
2	**.998**	**.985**	**.988**	**.984**
3	**.981**	**.976**	**.980**	**.976**
4	**.973**	**.967**	**.971**	**.966**
5	**.963**	**.957**	**.962**	**.956**
6	**.953**	**.947**	**.951**	**.945**
7	**.942**	**.936**	**.939**	**.933**
8	**.930**	**.924**	**.926**	**.920**
9	**.917**	**.912**	**.912**	**.907**
10	**.903**	**.899**	**.898**	**.893**
11	**.889**	**.886**	**.883**	**.879**
12	**.875**	**.874**	**.867**	**.865**
13	**.860**	**.860**	**.851**	**.851**
14	**.844**	**.847**	**.835**	**.836**
15	**.828**	**.834**	**.818**	**.822**
16	**.812**	**.820**	**.800**	**.807**
17	**.795**	**.806**	**.783**	**.792**
18	**.778**	**.792**	**.765**	**.776**
19	**.760**	**.777**	**.746**	**.761**
20	**.743**	**.763**	**.727**	**.745**

Table 1b: First 20 sample autocorrelations ot the first differenced series

Series	U_t	Log U_t	u_t	u^*_t
1	**.858**	**.790**	**.711**	**.669**
2	**.833**	**.717**	**.725**	**.633**
3	**.801**	**.658**	**.720**	**.591**
4	**.780**	**.623**	**.661**	**.571**
5	**.737**	**.612**	**.652**	**.538**
6	**.685**	**.537**	**.606**	**.525**
7	**.646**	**.527**	**.563**	**.462**
8	**.614**	**.446**	**.521**	**.403**
9	**.559**	**.407**	**.464**	**.351**
10	**.506**	**.343**	**.461**	**.323**
11	**.452**	**.273**	**.387**	**.243**
12	**.397**	**.204**	**.339**	**.187**
13	**.379**	**.157**	**.339**	**.181**
14	**.332**	**.136**	**.248**	**.102**
15	**.283**	**.085**	**.263**	**.080**
16	**.254**	**.084**	**.222**	**.064**
17	**.243**	.024	**.185**	.034
18	**.198**	-.007	**.196**	-.012
19	**.162**	**-.62**	**.118**	-.035
20	**.131**	**-.104**	**.121**	**-.086**

Table 1c: First 20 sample autocorrelations ot the original time series

Series	U_t	Log U_t	u_t	u^*_t
1	**-.415**	**-.332**	**-.527**	**-.447**
2	.024	-.039	.035	.014
3	-.041	**-.066**	**.092**	-.035
4	**.082**	-.036	**-.086**	.020
5	.033	**.159**	**.067**	-.030
6	-.043	**-.154**	-.006	**.072**
7	-.031	**.160**	-.003	-.003
8	**.084**	**-.110**	-.025	-.012
9	-.014	**.060**	**-.095**	-.036
10	.007	.025	**.127**	**.077**
11	.009	**.010**	-.044	-.035
12	**-.136**	**-.067**	**-.083**	**-.075**
13	**.104**	**-.062**	**-.153**	**.106**
14	.006	**.081**	**-.178**	**-.082**
15	-.066	**-.103**	**.099**	-.009
16	-.063	**.138**	-.006	.017
17	.117	**-.074**	**-.083**	.029
18	-.030	**.059**	**.153**	-.041
19	-.027	-.046	**-.147**	.048
20	-.082	**-.103**	.006	**-.127**

* In bold: Significant autocorrelation values. The large sample standard error under the null hypothesis of no autocorrelation is $1/n^{1/2}$, or roughly, .054 for series of length considered here.

Denoting any of the series y_t, we employ throughout the model (2) and (3) with $z_t = (1,t)'$, $t \geq 1$, $z_t = (0,0)'$ otherwise, so under the null hypothesis (4)

$$y_t = \beta_1 + \beta_2 t + x_t, \qquad t = 1, 2, \ldots \tag{7}$$

$$(1 - L)^d x_t = v_t, \qquad t = 1, 2, \ldots. \tag{8}$$

We will be treating separately the cases $\beta_1 = \beta_2 = 0$ a priori, β_1 unknown and $\beta_2 = 0$ a priori, and (β_1, β_2) unknown, that is, studying the cases of no regressors, an intercept, and a linear time trend respectively. We will model the I(0) process v_t to be both white noise and to have parametric autocorrelation.

We start with the assumption that v_t in (8) is white noise. Thus, when d = 1, for example, the difference $(1 – L) y_t$ behaves, for t > 1, like a random walk when $\beta_2 = 0$, and a random walk with a drift when $\beta_2 \neq 0$. However, we report test statistics, not merely for the case of d = 1 in (8) but for a variety of values from 0 to 2, including also a test for nostationarity (when d = 0.5) and for I(2) (when d = 2) as well as other fractional possibilities.

The test statistics reported in Tables 2 – 5 are the one-sided tests given by $\bar{r}$ in (5), so that significantly positive values of this are consistent with the alternative H_a: $\theta > 0$, implying that the order of integration should be higher than the value chosen for d. Similarly, significantly negative ones are consistent with the alternative H_a: $\theta < 0$, implying smaller values for d.

A notable feature of Table 2a, in which v_t is taken to be white noise (when the form of $\bar{r}$ significantly simplifies) and $\beta_1 = \beta_2 = 0$ a priori, is the fact that we cannot reject the unit root null hypothesis in any of the transformed series, log U_t and u^*_t, while in the original series, U_t and u_t, this hypothesis is strongly rejected in favour of more nonstationary alternatives, with d > 1. In these two series, the null hypothesis is rejected for all given values of d (though some non-rejections might appear when d is between 1.25 and 1.50 where the sign of the test statistic changes).

TABLE 2									
Table 2a: $\bar{r}$ in (5) with no regressors and white noise u_t									
Series \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
U_t	55.60	49.88	42.73	33.92	18.75	3.86	-3.43	-6.20	-7.36
Log U_t	55.39	31.03	21.37	8.25	**-0.08**	-3.95	-5.85	-6.92	-7.60
u_t	55.27	49.18	41.92	32.20	16.21	2.30	-4.00	-6.39	-7.43
u^*_t	55.22	44.37	26.97	9.40	**-0.07**	-4.04	-5.91	-6.95	-7.62
Table 2b: $\bar{r}$ in (5) with an intercept and white noise u_t									
Series \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
U_t	55.60	52.19	47.44	42.73	34.84	20.97	5.87	-2.65	-6.06
Log U_t	55.39	50.89	44.00	37.90	28.06	14.78	3.47	-2.68	-5.48
u_t	55.27	51.81	46.85	41.03	29.76	12.40	**-0.39**	-5.60	-7.43
u^*_t	55.22	50.74	43.77	36.60	24.59	9.77	**-0.32**	-4.86	-6.74
Table 2c: $\bar{r}$ in (5) with a linear time trend and white noise u_t									
Series \ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
U_t	52.16	50.41	47.24	42.70	34.88	20.93	5.78	-2.72	-6.10
Log U_t	53.50	50.60	45.35	38.24	28.07	14.67	3.27	-2.85	-5.61
u_t	52.28	50.35	46.81	41.06	29.76	12.35	**-0.48**	-5.66	-7.48
u^*_t	53.16	50.21	44.83	36.93	24.60	9.67	**-0.50**	-5.01	-6.88

*: In bold the non-rejection values of the null hypothesis (5) at the 95% significance level.

Tables 2b and 2c give results with, respectively, $\beta_2 = 0$ a priori (no time trend in the undifferenced regression), and both β_1 and β_2 unrestricted, still with white noise v_t. In every case in both tables, (and also in Table 2a), $\bar{r}$ is monotonic with respect to d. This monotonicity is a characteristic of any reasonable statistic, given correct specification and adequate sample size, because, for example, we would wish that if the null (4) is rejected in favour of alternatives with $\theta > 0$ when $d = 0.75$, an even more significant result in this direction would be obtained when $d = 0.50$ is tested. We observe that though there are some significant differences in the values of $\bar{r}$ across Tables 2b and 2c for the same series/d combination, the conclusions suggested by both seem very similar, that on the whole the extreme nonstochastic trends ($d = 0$) are inappropriate. The unit root null hypothesis ($d = 1$) is now rejected in all the series and we observe non-rejection values only for u_t and u^*_t when $d = 1.50$.

Tables 3 follows the same structure as in Table 2 but now we allow v_t to be weakly parametrically autocorrelated. In particular, we assume that v_t follows an AR(1) process. Higher order autoregressions were also performed, obtaining similar results. We only report in the table the results for those cases where monotonicity was achieved, and indicate by "--" the values where we observe lack of this property, which in most cases occur when $d < 1$. This is not surprising given the wide range of null hypothesized values of d, and the difficulties which arise when compounding fractional differencing with autoregressions. For example, if we think that a plausible model for y_t is:

$$(1 - L) y_t = v_t; \qquad v_t = \bar{\tau} v_{t-1} + \varepsilon_t, \qquad t = 1, 2, ...$$

with white noise ε_t and τ close to 0, a very similar model, though with very different statistical properties might be

$$y_t = v_t; \quad v_t = \bar{\tau} v_{t-1} + \varepsilon_t, \qquad t = 1, 2, ...$$

with $\bar{\tau}$ close to 1, and thus, we could expect not to reject H_o (4) in (2) and (3) with AR(1) v_t, either when d = 0, (in which case the estimated AR coefficient should be arbitrarily close to 1), or when d = 1, (with the estimated AR coefficient arbitrarily close to 0), but reject H_o perhaps for values of d ranging between these two values.

TABLE 3

Table 3a: $\bar{r}$ in (5) with no regressors and AR(1) u_t

Series\ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
U_t	--	--	--	--	--	4.39	2.44	**-1.86**	-4.21
Log U_t	--	--	--	--	**--**	2.05	**-1.22**	-3.27	-4.66
u_t	--	--	--	--	--	5.28	**1.42**	-2.46	-4.54
u^*_t	--	--	--	--	**--**	**1.74**	**-1.42**	-3.39	-4.72

Table 3b: $\bar{r}$ in (5) with an intercept and AR(1) u_t

Series\ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
U_t	--	--	--	--	--	--	6.66	**1.44**	-2.88
Log U_t	--	--	--	--	--	2.98	2.62	**-0.88**	-3.53
u_t	--	--	--	--	--	9.82	5.16	**-1.21**	-4.77
u^*_t	--	--	--	--	--	6.01	2.49	**-1.79**	-4.34

Table 3c: $\bar{r}$ in (5) with a linear time trend and AR(1) u_t

Series\ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
U_t	--	--	--	--	--	--	6.63	**1.37**	-2.96
Log U_t	--	--	--	--	--	3.03	2.63	**-0.95**	-3.71
u_t	--	--	--	--	--	9.81	5.09	**-1.33**	-4.90
u^*_t	--	--	--	--	--	6.01	2.42	**-1.92**	-4.60

*: In bold the non-rejection values of the null hypothesis (5) at the 95% significance level.

--: We do not achieve monotonicity in the value of the test statistic with respect to d.

TABLE 4

Table 4a: $\bar{r}$ in (5) with no regressors and AR(1) u_t

Series\ d	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
U_t	5.81	5.01	4.32	2.44	**0.47**	**-1.17**	-2.46	-3.45	-4.21
Log U_t	2.94	**1.26**	**-0.10**	**-1.22**	-2.15	-2.93	-3.59	-4.16	-4.66
u_t	5.91	5.03	3.44	**1.42**	**-0.39**	**-1.85**	-2.99	-3.86	-4.54
u^*_t	2.59	**0.96**	**-0.34**	**-1.42**	-2.31	-3.06	-3.69	-4.25	-4.72

Table 4b: $\bar{r}$ in (5) with an intercept and AR(1) u_t

Series\ d	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
U_t	--	8.05	7.47	6.66	4.79	2.55	**0.40**	**-1.43**	-2.88
Log U_t	3.64	3.63	3.60	2.62	**1.23**	**-0.20**	**-1.52**	-2.63	-3.53
u_t	9.39	9.19	7.83	5.16	2.34	**-0.15**	-2.15	-3.66	-4.77
u^*_t	5.92	5.70	4.33	2.49	**0.60**	**-1.06**	-2.44	-3.52	-4.34

Table 4c: $\bar{r}$ in (5) with a linear time trend and AR(1) u_t

Series\ d	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
U_t	--	7.56	7.47	6.63	4.74	2.48	**0.32**	**-1.51**	-2.96
Log U_t	3.69	3.67	3.62	2.63	**1.23**	**-0.24**	**-1.62**	-2.78	-3.71
u_t	9.39	9.37	7.79	5.09	2.25	**-0.26**	-2.27	-3.79	-4.90
u^*_t	5.92	5.68	4.29	2.42	**0.52**	**-1.17**	-2.60	-3.74	-4.60

*: In bold the non-rejection values of the null hypothesis (5) at the 95% significance level.

--: We do not achieve monotonicity in the value of the test statistic with respect to d.

We observe across Table 3 that monotonicity is only achieved when d > 1. If there are no regressors, (in Table 3a), the non-rejections appear when d = 1.75 for U_t; when d = 1.50 for log U_t and u_t; and when d = 1.25 and 1.50 for u^*_t. Including an intercept or a linear time trend, (in Tables 3b and 3c), the results are more precise, with the null hypothesis being rejected in all the series for all values of d except when d = 1.75. In order to be more accurate about the order of integration of the series in this context of autoregressive v_t, we report, in Tables 4, the same statistics as above but for a range of values of d from 1.20 …(0.10) … 2.00. We see that if v_t is AR(1) and there are no regressors, the non-rejection values of d change slightly depending on the series.

Thus, H_o (4) is not rejected when d = 1.30, 1.40 and 1.50 for log U_t and u^*_t, i.e., the logarithmic and the logistic transformations of the CC and of the CC as percentage of the workforce respectively. For u_t, H_o is not rejected if d = 1.50, 1.60 and 1.70, while for U_t, d = 1.60 and 1.70 are the only non-rejection values of d. Including an intercept (in Table 4b) or a linear time trend (in Table 4c), the results are similar in both cases, with the non-rejection cases occurring for the same d/series combination: when d = 1.80 and 1.90 for U_t; when d = 1.60, 1.70 and 1.80 for log U_t; d = 1.70 is the only case where H_o is not rejected for u_t; and d = 1.60 and 1.70 for u^*_t.

TABLE 5

Table 5a: $\bar{r}$ in (5) with no regressors and seasonal AR(1) u_t

Series\ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
U_t	42.82	37.44	31.17	24.44	14.68	3.66	-3.55	-6.20	-7.38
Log U_t	38.65	19.68	13.48	6.22	**-0.08**	-3.98	-5.87	-6.93	-7.61
u_t	42.74	36.79	30.27	22.94	12.83	**1.94**	-3.98	-6.40	-7.45
U^*_t	39.23	20.99	13.25	6.63	**-0.11**	-4.05	-5.91	-6.96	-7.63

Table 5b: $\bar{r}$ in (5) with an intercept and seasonal AR(1) u_t

Series\ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
U_t	42.82	36.53	34.45	33.07	27.86	18.91	6.35	-2.44	-6.05
Log U_t	38.65	31.21	30.33	30.09	25.10	14.72	3.62	-2.72	-5.55
u_t	42.74	36.67	34.01	31.03	23.39	11.40	**-0.20**	-5.56	-7.44
U^*_t	39.23	32.20	30.70	29.32	21.87	9.77	**-0.21**	-4.86	-6.77

Table 5c: $\bar{r}$ in (5) with a linear time trend and seasonal AR(1) u_t

Series\ d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
U_t	37.74	37.05	36.05	33.04	27.86	18.89	6.25	-2.51	-6.09
Log U_t	36.97	35.90	33.51	30.72	25.11	14.62	3.42	-2.86	-5.67
u_t	38.21	37.97	35.39	31.02	23.39	11.37	**-0.29**	-5.62	-7.48
U^*_t	37.16	35.95	33.22	29.44	21.88	9.67	**-0.38**	-5.00	-6.90

*: In bold the non-rejection values of the null hypothesis (5) at the 95% significance level.

We also performed the tests allowing seasonal AR processes for the disturbances. Table 5 reports values of $\bar{r}$ in (5) when v_t follows a seasonal AR(1) process of form

$$v_t \;=\; \tau\, v_{t-12} \;+\; \varepsilon_t\,, \qquad\qquad t \;=\; 1, 2, ...$$

for the same range of values of d as in Tables 2 and 3. Again here, higher order autoregressions were computed and the results were very similar to those reported in Table 5 for the AR(1) case. Comparing the results in this table with those in Table 2 (when v_t was white noise), the results are rather similar. In fact, all the non-rejection d's in Table 2 form a proper subset of those in Table 5. If there are no regressors, the non-rejection values of d are 1 for log U_t and u^*_t, and 1.25 for u_t. Including an intercept and a linear time trend, the only non-rejections occur when d = 1.50 for u_t and u^*_t, i.e., for the same d/series combination as in Table 2.

Graph 1. Evolution of employment and unemployment in UK 1900-2000 (thousands of people)

Note: Left axis represents Unemployment and right axis correspond to Labour Force and Employment. Elaborated by Euro-American Assoc. of Economic Development Studies, from several international sources.

4. Conclusions

The tests of Robinson (1994a) for testing unit roots and other fractionally integrated hypotheses were applied in this article to several measures of the U.K. unemployment. The tests were performed for different regression models, including no regressors, an intercept, and a linear time trend, and for different models of the I(0) disturbances v_t, in particular, white noise and seasonal and non-seasonal AR processes. These tests were applied to the number of unemployees (U_t); its logarithmic transformation (log U_t); the unemployment rate (u_t) and its logistic transformation (u^*_t), and the conclusions can be summarized as follows:

The test statistics clearly reject the trend-stationary I(0) representations, but even the unit root I(1) hypotheses are also rejected in practically all cases in favour of alternatives with $d > 1$. The results vary slightly depending on the series and the way of modelling the I(0) disturbances.

The orders of integration seem to be slightly greater for the original series (U_t and u_t) than for the transformed ones (log U_t and u^*_t). Modelling u_t as white noise or as seasonal AR processes, the orders of integration fluctuates around 1 if we do not include regressors but they are much higher (around 1.50) when including an intercept or a linear trend. Modelling u_t as non-seasonal autoregressions, the orders of integration are again higher than 1, ranging now between 1.20 and 2.

It would be worthwhile proceeding to get point estimates of the orders of integration of each of the series. However, we should note that the approach used in this paper simply generates computed diagnostics from departures from real orders of integration and thus, it is not surprising that different models may result non-rejected. On the other hand, these results are completely consistent with those found in Gil-Alana (2001a, b). In the first of these articles, fractionally integrated ARMA (ARFIMA) models were estimated by maximum likelihood for the four series of unemployment, finding orders of integration ranging between 1 and 2.

In Gil-Alana (2001b), the orders of integration of the series were estimated by means of semi-parametric techniques. Using a battery of test statistics proposed by Robinson (eg. 1994b, c and 1995a, b), the conclusions suggest orders of integrations of around 1.50. Our results here also indicate that the orders of integration are higher than 1 but smaller than 2 and thus, the standard approach of taking first differences does not lead to I(0) stationary residuals but series with long memory behaviour.

These results suggest that the unemployment in the U.K. is a highly persistent variable. For instance, if we concentrate on the 'log U_t' series, we observe that even taking first differences, the growth rate series still presents a strong degree of dependence between the observations and thus, in order to correctly analyse the series, its order of integration should be estimated rather than being imposed to be zero.

Finally, the fact that the unit root null hypothesis is practically always rejected in favour of more nonstationary alternatives suggests that any shock affecting the series will have a permanent effect, implying that policy action should be required to bring the variables back to its original level.

Several other lines of research are under way which should prove relevant to the analysis of these and other macroeconomic data. Tests for fractional and non-fractional cyclical models (see eg, Gil-Alana, 2001c) for unemployment are being implemented. Work is also proceeding on the Bloomfield (1973) exponential spectral model for modelling the I(0) disturbances v_t.

This is a non-parametric approach for modelling the stationary disturbances which has found to be relevant in several econometric applications. Finally, modelling unemployment in terms of exogenous regressors in this fractionally integrated context is another line of future research.

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Appendix

All calculations were obtained using FORTRAN. A diskette containing the codes for the programs is available from the author upon request.