APPLICATION OF LANGEVIN EQUATION IN ECONOMETRICS TO THE INTERACTION BETWEEN THE EXCHANGE RATES OF JAPAN AND SOUTH KOREA

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Abstract

This article presents an application of statistical physics to economic relationships, based on the fluctuation-dissipation theorem and the anomalous fluctuation theorem. In the framework of time series that follow the Langevin equation, the interaction of two time series can be treated. The application to the won-dollar and yen-dollar rates shows that the former fluctuates under the influence of the latter.

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1. Introduction

Statistical mechanics provides us some methods to deal with some economic random phenomena such as the fluctuation of the exchange rate. The physical analysis of a random phenomenon has the origin in the study of the Brownian motion by Uhlenbeck and Ornstein(1930). They handled an equation of motion whose left hand side is the time derivative of the velocity multiplied by the mass of the particle. The right hand side is the sum of the friction and the random force. The equation with a random force is said to be the Langevin equation.

When we postulate that the random force is white, the variance of the velocity of the particle is calculated. If the Brownian system is in a heat equilibrium, the physical law of equipatition exists. Then there comes a relation between the magnitude of the

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white noise and the friction constant. This relation is called the fluctuation-dissipation theorem.

The variety of the Langevin equation for a physical or economic quantity depends on the property of the right hand side of the equation. If we replace the term of the friction with a polynomial of higher order, it is observed that the fluctuation of the quantity has a maximum when the quantity reaches a special point. This phenomenon is knows as the anomalous fluctuation in physics.

In the section 2, we will examine whether this phenomenon is observed in the time series data of the exchange rate, and in the section 3, we introduce the interaction term in the Langevin equation and using this generalized equation, we will show that the behavior of the won-dollar exchange rate is well explained under the interaction of the yen-dollar exchange rate.

2. Analysis of anomalous fluctuation in the exchange rate

Following the statistical mechanics by Suzuki (1976), we decompose the variable $x(t)$, which represents the exchange rate, by a term $y(t)$ which expresses a trend and a term $\xi(t)$ which expresses the fluctuation:

$$x(t) = y(t) + \sqrt{\varepsilon}\xi(t) \qquad (1)$$

where ε is a scale parameter.

The variance for the linear case: $\dot{y}(t) = ay(t)$, where the left side is the derivative of $y(t)$ with respect to time, is given by:

$$\sigma^2(t) \sim e^{2at} \qquad (2)$$

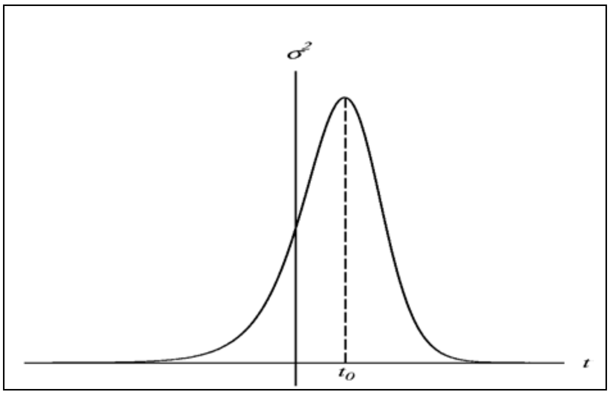
When $a > 0$, the point $y = 0$ is the unstable equilibrium point. The variance goes to infinity as $t \to \infty$. When $a < 0$ the point $y = 0$ is the stable equilibrium point and the variance goes to zero as $t \to \infty$. The variance for the nonlinear case:

$$\dot{y}(t) = ay(t) - by(t)^3 \ (a > 0 \text{ and } b > 0).$$

In this case the shape of $\sigma^2(t)$ is given by Graph 1.

The maximum fluctuation occurs in this case at $y = \sqrt{a/(3b)}$ which is not any equilibrium point, and this phenomenon is said to be the anomalous fluctuation in statistical mechanics.

Graph 1. Shape of $\sigma^2(t)$



We will take the data of daily exchange rate of yen to US dollar from January 2, 2001 to September 24, 2002[1], and examine whether the exchange rate of yen has this kind of anomalous fluctuation. The exchange rate x_t oscillates so frequently that some

[1] Source: Pacific Exchange Rate Service, http://pacific.commerce.ubc.ca/xr

processes of smoothing are needed. First we take the moving average of order r :

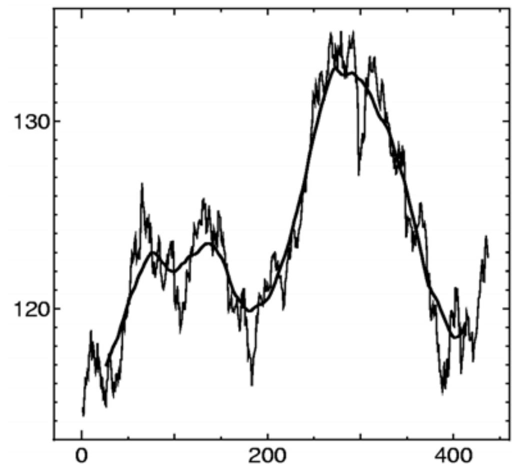
$$z_t = \frac{1}{r}\sum_{i=0}^{r-1} x_{t-i} \qquad (3)$$

In order to get a well smoothed z_t we take r =50. The smoothed z_t and the original x_t are plotted in Graph 2.

Finally we apply a polynomial regression of order six for z_t , being y_t the fitted values of z_t in the estimated polynomial:

$$y_t = \sum_{k=0}^{6} c_k t^k . \qquad (4)$$

Graph 2. Moving average z_t and original data x_t

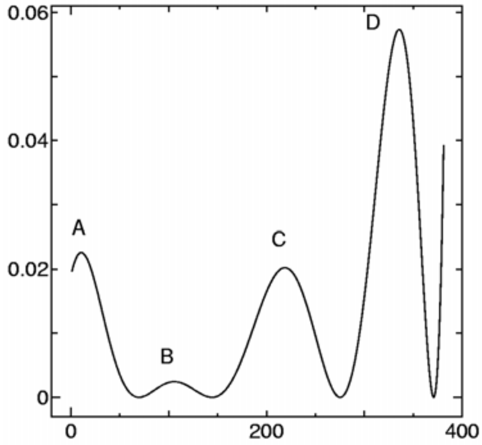


The coefficients of the best fit polynomial for z_t are in Table 1. The polynomial y_t is supposed to be the first term of the right hand side of (1). The variance of the exchange rate y_t is shown in Graph 3, and we notice that there are four local maxima: A, B, C, and D which represent anomalous fluctuations.

Table 1. Coefficients of the polynomial

Coefficient	Estimated	Coefficient	Estimated
c_0	116.5	c_4	3.58E-07
c_1	1378	c_5	-1.31E-09
c_2	1.27E-03	c_6	1.63E-12
c_3	-4.72E-05		

Graph 3. Variance of exchange rate y_t



3. Langevin equation with interaction between won and yen exchange rates

It is useful to consider the generalized Langevin equation (Kubo et al. (1995)):

$$\frac{dx(t)}{dt} = -\int_{-\infty}^{t} f(t-\tau)x(\tau)d\tau + \lambda w(t) + \mu\eta(t) \qquad (5)$$

Here $f(t)$ is the friction retardation in mechanics, but in economics, a kind of weight, λ and μ are numerical constants, and $w(t)$ is an external force and $\eta(t)$ is a random force.

Korea and Japan are economically related: their currencies should be interdependent. We will take $x(t)$ and $w(t)$ as won-dollar rate and yen-dollar rate, respectively. The data of the daily exchange rate of won to US dollar are taken from the same source in the previous section. We will see if won will oscillate under the influence of yen. In the numerical analysis, the functions $x(t)$ and $w(t)$ are approximated by the series x_t and w_t as before. The integration of (5) will be transformed to a weighted sum:

$$\int_{-\infty}^{t} f(t-\tau)x(\tau)d\tau \;\rightarrow\; \sum_{\tau=-\infty}^{t} f_{t-\tau}x_\tau \qquad (6)$$

To get good convergence, we postulate that

$$|f_0| > |f_1| > |f_2| > \cdots \qquad (7)$$

We also transform the derivative to a difference :

$$\frac{dx(t)}{dt} \;\rightarrow\; x_t - x_{t-1}$$

Now the integral-differential equation (5) is transformed to a difference equation:

$$x_t = \sum_{k=1}^{N} a_k x_{t-k} + \lambda' w_t + \eta'_t$$

$$a_1 = \frac{1+f_1}{1-f_0},\; a_k = \frac{f_k}{1-f_0} \;\; (k \ge 2),\; \lambda' = \frac{\lambda}{1-f_0}.$$

The coefficients can be determined by the ordinary least square method. Neglecting the terms of $(k \geq 2)$, we have the first order approximation. The regression equation is

$$x_t = a_1 x_{t-1} + \lambda' w_t + \eta'_t \qquad (8)$$

The numerical results are given in the Table 2.

Table 2. First order approximation

Adjusted R^2	0.980			
d-w statistic	2.038			
Akaike info criteria	6.496			
Mean of dependent variable	1275.0			
Mean of external force	123.5			
Coefficient	Estimated value	St. error	t-statistic	p-value
a_1	0.9861	9.40E-03	154.1	0.000
λ'	0.1421	6.60E-02	2.153	0.032

In the equation (8), there is one estimated coefficient: a_1. We have two unknown constants: f_0 and f_1. Therefore we put a relation between f_0 and f_1 satisfying (7):

$$f_0 = \delta - 1, \;\; f_1 = -\delta(\delta - 1), \;\; 0 < \delta < 1.$$

From Table 2, we have the values of the coefficients:

$$f_0 = -0.1248, \;\; f_1 = 0.1092, \;\; \lambda = 0.1598, \;\; \delta = 0.8751.$$

In the second and third order approximation, the regression equations are

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + \lambda' w_t + \eta'_t$$
$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + a_3 x_{t-3} + \lambda' w_t + \eta'_t$$

The results of the regression analysis for those equations are given in Tables 3 and 4.

Table 3. Second order approximation

Adjusted R^2	0.980			
d-w statistic	1.981			
Akaike info criteria	6.500			
Coefficient	Estimated value	St. error	t-statistic	p-value
a_1	0.954	4.820E-02	19.760	0.000
a_2	0.033	4.800E-02	0.680	0.497
λ'	0.134	6.640E-02	2.018	0.044

Table 4. Third order approximation

Adjusted R^2	0.980			
d-w statistic	1.986			
Akaike info criteria	6.504			
Coefficient	Estimated value	St. error	t-statistic	p-value
a_1	0.9509	4.82E-02	19.64	0.000
a_2	0.0273	4.80E-02	0.040	0.967
a_3	0.0338	6.64E-02	0.699	0.484
λ'	0.1277	6.67E-02	1.912	0.056

From the tables, AIC, t-statistic, and p-value suggest that the first order approximation is the best estimation. As a comparison, let us estimate the non-interacting model:

$$x_t = a_1 x_{t-1} + \eta'_t \,.$$

The result is as follows:

$$a_1 = 0.9999, \ \text{AIC} = 6.502, \ \text{Adjusted } R^2 = 0.979.$$

In the analysis of x_t, the model AR(1) without interaction will fit the data, and this will be second best.

If we take the Langevin equation with interaction (8) as the best model, we will be able to examine the symmetry between the variables x_t and w_t. The reversed equation is

$$w_t = a_1 w_{t-1} + a_2 w_{t-2} + a_3 w_{t-3} + \cdots + \lambda' x_t + \eta'_t.$$

Taking into account of t-statistic, p-value, and AIC, we know that the best fit function is the first order approximation:

$$\begin{aligned} & w_t = a_1 w_{t-1} + \lambda' x_t + \eta'_t \\ & a_1 = 0.9762, \ \lambda' = 2.310E-02 \end{aligned} \qquad (9)$$

The value of λ' in (9) is 10% smaller than that of Table 2 even though we take into account of the ratio of the mean value of yen-dollar rates to that of won-dollar rates. Therefore there is no symmetry between yen and won exchange rate. The oscillation of yen will not be affected by won.

4. Conclusion

In this paper it is revealed that the Langevin equation is useful to analyze the economic data. We get the following results.

1) Applying the Uhlenbeck-Ornstein type equation to the economic data, such as an exchange rate, we get the fluctuation-dissipation relation in economics.

2) From Suzuki type equation, we find the anomalous fluctuation of the variance without using the physical idea, the scaling law. In the economic data, the variance of the exchange rate has some local maxima. The orbit on the phase diagram is many-valued, and there is no equilibrium point on it.

3) The interaction of the two time series is analyzed by the introduction of the interaction term in the Langevin equation. The won exchange rate oscillates under the influence of yen. The symmetry of won and yen is not observed.

Bibliography

Uhlenbeck, G. E. and Ornstein, L.S.(1930). "On the Theory of the Brownian Motion". *Physical Review,* Vol. 36, pp. 823-841.

Kubo, R.; Toda, M. and Hashisume, N.(1995). "*Statistical Physics* II". Springer, New York.

Suzuki, M.(1976). "Scaling Theory of Non-Equilibrium Systems near the Instability Point I". *Progress of Theoretical Physics,* Vol. 56, pp. 77-94.