EMPIRICAL ANALYSIS OF THE DEMAND ELASTICITY FOR TUNISIAN EXPORTS

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Abstract
The objective of this paper is to estimate the demand elasticity for Tunisian exports using recently developed non-stationary panel methodologies. We consider quarterly data of Tunisian exports to the major European trading partners from 1987 to 2004. Our estimation results of the price and foreign income elasticities of demand for Tunisian exports suggest a significant relationship between the real exchange rate index and Tunisian export demand, both in the long-run and in the short-run. The results also suggest that currency devaluation policy alone may not be very effective in promoting Tunisian exports which are found to be inelastic with respect to real exchange rate.
Keywords: Export-demand elasticity, fully-modified OLS, panel cointegration tests.
Jel Classification: C23, C33, F14

1. Introduction

Several developing countries rely heavily on exports to stimulate their economic growth. This is because, it has been suggested that the specialization of production in commodities in which a country has a comparative advantage can increase the efficiency of resource utilisation by increasing the rate of capital formation and improving the growth rate of total factor productivity. An empirical investigation conducted by Chee-Keong *et al.* (2003) showed evidence to support the export-led growth hypothesis in the Malaysian economy. But without the implementation of effective Macroeconomic policy that aims at stabilizing trade balance, the reliance on heavy trade alone may not be effective in stimulating growth due to changes in international markets that may not be in favour of the local economy. For this reason, it is crucial to have accurate measures of price and income elasticities of exports because they are of considerable importance in choosing the appropriate trade policy. From international trade theory, it was argued that local currency depreciation would improve the economy's trade balance only if the sum of export-demand elasticity and import-demand elasticity are greater than unity. It is therefore important to get accurate estimates of these elasticities.

In their paper, Senhadji and Montenegro (1999) discussed the importance of export-demand elasticity and argued that the higher income elasticity of the export demand, the more powerful exports will be, as an engine of growth. The higher the price elasticity, the more competitive is the international market for exports of that particular country, and thus the more successful will be a real devaluation in promoting export revenues. Thus, price and income elasticities of export demand are indicative on whether trade deficit of a single country could be reduced by devaluation. It is well-known that currency devaluation affects a country's trade balance via its impact on relative prices (elasticity approach), spending behaviour (absorption approach), and the purchasing power of money balances (monetary approach). Furthermore, the Marshall-Lerner condition, which refers

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to the elasticity approach to devaluation, suggests that currency devaluation policy works best at improving a country's trade balance when demand elasticities are high, and over the long-run the effect will be more pronounced (Bahmani-Oskooe and Niroomand, 1998).

Thus, our objective in this paper is to estimate the foreign demand elasticities for Tunisian exports in order to assess the impact of real devaluation of the Tunisian currency (the *dinar*) and whether this policy would spur export promotion and trade deficit reduction.

Despite the existence of numerous empirical studies on the estimation of price and income elasticities, only a few attempts using sound econometric methods to measure these elasticities for Tunisian exports are found in the literature. In particular, in their paper, Senhadji and Montenegro use the fully-modified OLS approach to estimate export demand elasticities for 74 countries including Tunisia, for the period 1960-1993. They found that for Tunisia, in the long-run, a decrease of 10 % in real exchange rate index and 10% in real foreign income lead respectively to, a decrease of 7.8 % and 24.3% for the real export demand. In the short run, the same conclusion has been found but with a decrease of respectively, 3.4% and 17.3% for the export demand. However, in this paper, we use non-stationary panel methodology to present a more comprehensive analysis of the sensitivity of the demand for Tunisian exports based on alternative efficient estimation methods. Therefore, our results are more general.

Empirical studies regarding other countries include Rose (1990, 1991). In the paper, the author applied the Engel and Granger cointegration procedure in order to examine the trade-exchange rate relationship for several developing countries. It is found that a real depreciation does not have a significant effect on the trade balance. Bahmani-Oskooee (1991, 1998) obtained a similar result. The author employed the same econometric method to test whether the U.S. trade balance is cointegrated with the exchange rate. The test results could not confirm the existence of such a relationship in the long-run. Arize (1994) used maximum likelihood estimation procedure to study the long-run relationship between trade balance and exchange rate for nine Asian developing countries from 1973 to 1991. The author found evidence of cointegration between the two variables for all countries.

In section 2 of this paper, we present the model and the data and we discuss our econometric results and their implications. Some concluding remarks are given in section 3.

2. The Model and the Data.

We collect quarterly data of Tunisian export series to six European countries. The major trading partners for Tunisia considered in this paper are France, Italy, Germany, Belgium, the Netherlands, and Spain. The data covers the period from 1987 to 2004, with 1995 being the base year. The data sources are the IMF database and the financial statistics obtained from the Tunisian Central Bank. Our empirical analysis is based on the imperfect-substitutes model as in Goldstein and Khan (1985). The main assumption of this model is that neither imports nor exports are perfect substitutes for domestic products. The imperfect-substitutes model stipulated that the representative agent maximizes his/her lifetime utility subject to a lifetime utility budget constraint. In this case, export demand function may be specified as a function of the real exchange rate and the rest-of world real incomes. Thus, the export demand equation is given by:

$$\log(RX)_{i,t} = \delta_i + \alpha_1 \log\left(RGDP\right)_{i,t} + \alpha_2 \log\left(RERI\right)_{i,t} + \varepsilon_{i,t} \tag{1}$$

Where *i* denotes the country index, and *t* is the time index.
RX denotes the volume of Tunisian exports to country *i* deflated by the aggregate export price index.RGDP is the Real Gross Domestic Product of country *i*, used as a proxy for the foreign income.RER denotes the bilateral real exchange rate of country *i*, and computed as follow:RERI is the real exchange rate index defined as RER divided by its value in the base year (1995).

We have $RER_{i,t} = \dfrac{P_d NE_i}{P_i}$, where NE_i is the nominal exchange rate defined as the number of units of country *i*'s currency per unit of *dinar*, P_d is the domestic price level and P_i is the price level in *i*.

Given the log-linear form of equation (1), α_1 and α_2 are respectively the real foreign income elasticity and the real exchange rate elasticity of export demand.
From the theory, we expect a positive value for α_1 which means that when foreign income increases then demand for exports will rise, and α_2 is expected to be positive, which implies that currency devaluation would improve trade balance provided that the Marshall-Lerner condition is satisfied. Following this result currency devaluation becomes an important economic stabilization policy sought by governments to increase exports and thus improve domestic output.
However the difference between the short-run and the long-run elasticities is crucial and should be emphasized since it leads to the J-curve effects and empirically it implies that devaluation will worsen the current account balance in the short-run as exports are price-inelastic in the short-run but in the long-run several empirical studies found that elasticity increases and trade balance may improve. Recently Shieh (2006) introduced disposable income as an argument in the export function and shows that the modified Romer-Hsing-Taylor model can solve the inconsistency between empirical findings and theoretical results for the effects of currency devaluation, without the assumption that the Marshall-Lerner condition is always met.

In this paper we use a panel cointegration approach in order to present an empirical analysis to assess the demand elasticity for Tunisian exports and therefore the trade policy effectiveness of the devaluation of the Tunisian currency.
In the econometric literature, there has been extensive work on unit root testing with panel data. It is now well-known that standard unit root and cointegration tests may have low power against stationary alternative hypotheses as argued in Campbell and Perron (1991). With panel data, the time series dimension is enhanced by the cross section and therefore the results rely on a much broader information set leading to gains in power as indicated in Baltagi and Kao (2000) and Baltagi (2003), and consequently, more reliable results may be obtained. To test for the existence of a unit root in a panel data setting, we use alternative tests which were initially developed by Breitung and Mayer (1994) (henceforth BM), Levin, Lin and Chu (2002) (henceforth LLC), Im, Pesaran and Shin (1997, 2003) (henceforth IPS) and Hadri (2000) (henceforth HAD). In these tests, the null hypothesis is of unit root except in HAD where, under the null we assume stationary se-

ries. We compute the tests to check for the presence of a unit root for all the variables in level and in first difference. In the appendix, we describe briefly the theoretical formulation of these tests.We begin with a panel autoregressive equation for each export series:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + \gamma_i t + \sum_{j=1}^{p_i} \theta_{i,j} \Delta y_{i,t-j} + \varepsilon_{i,t} \tag{2}$$

We consider alternative panel-unit root tests that we compute from (2). Table 1 shows the results of the panel unit tests for the all the level series. All the variables are log-transformed.

We consider two unit root regression specifications. In the first, we include individual constants in equation (2). The results are shown in the upper-panel of Table 1.

Table 1: Panel unit root tests for level series

	Lag length	BM*	LLC*	IPS*	*HAD***
Without Trend					
LRX	0	-0.54188	-6.64705	-5.01834	*12.7332*
	1	0.01578	-2.73604	-1.43221	*12.7332*
	2	0.56349	-1.91165	-0.71409	*12.7332*
	3	1.23129	-1.12991	-0.11928	*12.7332*
LRERI	0	-2.19978	1.12157	0.07819	*1.75752*
	1	-0.97822	2.12553	0.89816	*1.75752*
	2	-1.01395	2.20621	0.77711	*1.75752*
	3	-1.27350	1.90690	0.38874	*1.75752*
LRGDP	0	4.26310	-4.24129	-0.72221	*13.6493*
	1	2.86743	-3.09894	0.25458	*13.6493*
	2	2.37649	-2.88645	0.41553	*13.6493*
	3	1.96752	-2.63373	0.66267	*13.6493*
With Trend					
LRX	0	-3.18116	-14.3470	-13.0314	*5.00376*
	1	-2.17909	-5.71480	-6.44118	*5.00376*
	2	-1.10020	-1.86728	-4.02955	*5.00376*
	3	0.56481	3.88588	1.17813	*5.00376*
LRERI	0	2.86496	2.06478	2.00666	*4.00891*
	1	3.57236	2.99854	3.41055	*4.00891*
	2	3.62283	2.90212	3.70875	*4.00891*
	3	3.36120	2.46216	3.52148	*4.00891*
LRGDP	0	-4.29352	0.23291	-1.69567	*6.26398*
	1	-3.30578	1.64801	1.59128	*6.26398*
	2	0.20133	0.24219	-2.91126	*6.26398*
	3	*-2.52218*	*0.78779*	*0.47879*	*6.26398*

* The 5% critical value is - 1.645. ** The 5% critical value is 1.645.

As an alternative specification, we include constants and linear individual trends and the test results are presented in the lower-panel of Table 1. Overall, the results cast some evidence of unit root in the series. With different lag length specifications the panel unit root test employed fail to reject the null of a unit root for all the export series in level. Therefore, we conclude that all the series in our study are I (1). The panel unit-root test results for the first differenced export series in, are reported in Table 1a in the appendix. It is shown that BM and IPS tests reject the unit root null for all lag length alternatives, under both regression specifications.

Next, we test the existence of cointegrated relationships between the variables.Following Pedroni (2000, 2001), we use panel cointegration techniques which give researchers the advantage of selectively pooling information regarding common long-run relationships from across the panel while allowing the associated short-run dynamics and fixed effects to be heterogeneous across different members of the panel, in our case the trading partners of Tunisia. These techniques are more useful when time series dimension is relatively large, since otherwise it would be difficult to model dynamic flexibility, because serial correlation properties differ across members of panel, and thus it requires sufficient time series length for each member to account for member specific dynamics.

In Table 2, we report the panel cointegration test results following Pedroni's method. In the first four tests, we run individual cointegrating regression for each member, we collect estimated residuals, and we compute pooled within dimension tests. In the last three tests, we compute group-mean unit root tests from the estimated residuals of individual cointegration regressions. It should be noted that the main difference between the two sets of tests is that the residuals are grouped rather than pooled in the group-mean tests, which are preferred since they allow more flexibility under alternative hypotheses.

All these tests allow heterogeneous dynamics, heterogeneous cointegration vectors, endogeneity, and they are normally distributed. For the v-statistic test large positive values (greater than 1.65) imply rejection of the no-cointegration null hypothesis. For the other tests, large negative values (less than -1.65) imply rejection on the no-cointegration null. All the test results suggest evidence of cointegration relationships between the variables.

Table 2: Pedroni's Panel Cointegration Tests*

a) Without heterogeneous trend

Test statistic	**Panel v-stat**	**Panel rho-stat**	**Panel pp-stat**	**Panel adf-stat**	**Group rho-stat**	**Group pp-stat**	***Group adf-stat***
Test value	*5.267*	*-17.536*	*-13.725*	*-6.3802*	*-17.357*	*-17.193*	*-5.256*

b) With heterogeneous trend

Test statistic	**Panel v-stat**	**Panel rho-stat**	**Panel pp-stat**	**Panel adf-stat**	**Group Rho-stat**	**Group pp-stat**	***Group adf-stat***
Test value	*2.511*	*-15.045*	*-14.839*	*-6.373*	*-13.75*	*-16.883*	*-4.290*

* The tests are computed in RATS *pancoint.src* procedure.

Alternatively, we compute the panel-cointegration test developed by Larsson, Lyhagen, and Lothgren (2001). They suggest a likelihood-based test of cointegration rank in panels based on the LR test of Johansen (1996). Consider a p-dimensional time series as follows:

$$\mathrm{Y}_{\mathrm{i},t} = \sum_{\mathrm{j=1}}^{\mathrm{k_i}} \Pi_{ij} Y_{i,t-j} + \mu_i + \varepsilon_{i,t} \tag{3}$$

where *i =1,…, N*, *t =1,…, T*, $\varepsilon_{i,t}$ is a series of *IID* innovations, and Π_{ij} are *p*-dimensional square matrices of parameters
From (3), write the corresponding heterogeneous error correction model,

$$\Delta\mathrm{Y}_{\mathrm{i},t} = \Pi_i Y_{i,t-1} + \sum_{\mathrm{j=1}}^{\mathrm{k_i}-1} \Phi_{ij} \Delta Y_{i,t-j} + \mu_i + \varepsilon_{i,t} \tag{4}$$

where Π_i is a $p \times p$ matrix of rank $r_i \le p$ and Φ_{ij} are p-dimensional square matrices calculated from Π_{ij}. The cointegration rank hypothesis is formulated by: H_0: rank (Π_i) $\le r_i$. H_1: rank (Π_i) $= p$, for all *i*.The likelihood-trace statistic for the above hypothesis is:

$$LR_i = -T_i \sum_{s=r_i+1}^{p} \log(1-\lambda_{\mathrm{is}}), \tag{5}$$

where λ_{is} are eigenvalues defined as in Johansen (1996). The panel test of Larsson, Lyhagen, and Lothgren (denoted by LLL) is given by the average of the individual LR trace statistics, similarly to Im, Pesaran, and Shin (2003).

In Table 3, we report the results of LLL test and the individual Johansen cointegration tests.

Table 3: LLL and Johansen cointegration tests

Johansen Test Values				
Country	**r = 0**	**r ≤ 1**	**r ≤ 2**	**CI- rank**
France	40.257	11.272	2.518	1
Italy	38.940	8.132	2.030	1
Germany	44.423	10.667	2.405	1
Belgium	12.751	4.496	1.374	0
Netherlands	32.267	8.911	3.247	1
Spain	41.955	11.825	0.658	1
Johansen 5% c-value	29.68	15.41	3.76	
LLL test	9.919	2.362	1.485	2

We estimate model equation (1) and the reported Johansen cointegration test results for individual countries show evidence of rejection the hypothesis that there is at most one cointegration relationship between the variables for only one country, which is Belgium. For the other countries, we conclude the existence of one cointegration relationship. However, the LLL panel test indicates two long-run relationships between exports, GDP, and the real exchange rate index. We use mean-group (MGE) and fully-modified OLS estimation methods to compute the elasticity parameters in the long-run equation (1). The results are as follows:

Table 4: Long-run elasticity estimation results

	MGE	FMOLS
LRERI	0.1622 (0.821)	0.1901 (2.012)
LRGDP	2.9596 (31.532)	2.9710 (37.051)

Next, we apply the generalized method of moments (Arellano and Bond, 1991) to estimate the panel vector error-correction model. The estimated error-correction model is given by:

$$\Delta(LRX)_{i,t} = \sum_{j}\left[\alpha_l \Delta(LRX)_{i,t-j} + \gamma_l \Delta(X)'_{i,t-j}\right] + \lambda(LRX_{i,t-1} - \hat{\delta}_i - \hat{\theta}_1 X'_{i,t-1}) + \varepsilon_{i,t}$$

Table 5: Panel VECM estimation results

Variables	Estimates	t-statistics
$\Delta LRX_{i,t-1}$	- 0.2327	- 16.655
$\Delta LRX_{i,t-2}$	- 0.1124	- 9.788
$\Delta LRGDP_{i,t-1}$	1.0765	8.873
$\Delta LRGDP_{i,t-2}$	1.8944	20.157
$\Delta LRERI_{i,t-1}$	0.7597	11.902
$\Delta LRERI_{i,t-2}$	0.2758	5.969
Error correction term	- 0.5037	- 16.511
Constant	0.0021	1.361

The results show that exchange rate and income elasticity are significant. In fact, the estimates indicate a positive long-run relationship between real exchange rate and real export demand. From the tables, it is also found that foreign income is elastic and that real exchange rate is inelastic, both in the long-run and in short-run, with an error correction speed equals to 50.37%. Furthermore, it is shown that in the long-run a 10 % increase in the foreign income leads to 29.7 % increase in exports, whereas a similar 10% depreciation in local currency generates less than 2% increase in exports and consequently may have a depressing effect on domestic output. Thus, a devaluation of the Tunisian currency, with everything else being unchanged slightly improves export demand but may be quite costly.

3. Concluding Remarks

In this paper, we present an empirical study to estimate foreign income and price elasticities of the demand for Tunisian exports. We use panel cointegration framework based on mean-group estimator and the fully-modified OLS. Our theoretical basis is the imperfect-substitute model of Goldstein and Khan (1985). In the empirical analysis, we perform panel unit root and panel cointegration tests, and our test results show evidence of cointegration between the variables of interest. Specifically, we estimate the long-run relationship between real export demand, real income, and real exchange rate index and we identify the error correction model that relates the short-run dynamics of these variables. The estimation results suggest that aggregate export demand is income-elastic and real exchange rate is inelastic. The relationship between the real exchange rate index and the real export demand is positive in the long-run but the econometric results prove that Tunisian exports are inelastic with respect to real exchange rate. This finding is very important as it implies that currency devaluation policy can only have limited effects in promoting export growth. Competitiveness and restructuring of the exporting sector may be more efficient alternatives.

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Appendix on line at the journal web site: http://www.usc.es/economet/aeid.htm

Appendix

Table 1a: Panel unit root tests for first-differenced series.

	Lag length	BM*	LLC*	IPS*	HAD**
Without Trend					
LRX	0	-9.17278	-34.1615	-32.2490	1.00829
	1	-9.30050	-16.4931	-20.3631	1.00829
	2	-8.12075	-16.3237	-24.6098	1.00829
	3	-4.27417	-9.75448	-11.6331	1.00829
LRERI	0	-8.53200	-22.7149	-20.9014	0.23920
	1	-10.6732	-10.7903	-11.7989	0.23920
	2	-8.70945	-4.75649	-7.69678	0.23920
	3	-4.89228	-0.25236	-4.68839	0.23920
LRGDP	0	-12.1524	-31.2957	-31.3498	2.36482
	1	-5.85722	-4.34787	-8.52549	2.36482
	2	-5.29046	-2.10750	-8.59946	2.36482
	3	-3.44501	-4.39978	-3.27031	2.36482
With Trend					
LRX	0	-9.39262	-35.0528	-33.1275	3.71032
	1	-9.00005	-15.9237	-20.2399	3.71032
	2	-7.58763	-15.4155	-25.4602	3.71032
	3	-4.38668	13.4278	-11.4900	3.71032
LRERI	0	-7.21893	-23.9528	-21.2738	3.77280
	1	-7.86806	-11.0408	-11.2311	3.77280
	2	-7.53080	-4.38410	-6.70703	3.77280
	3	-5.85875	0.61482	-3.25275	3.77280
LRGDP	0	-10.8371	-33.4415	-32.7802	2.96891
	1	-5.91873	-4.09909	-7.83706	2.96891
	2	-4.93473	-1.54412	-7.99087	2.96891
	3	-3.05886	-5.92931	-2.00320	2.96891

* Critical value at 5% level: - 1.645
** Critical value at 5% level: 1.645

Review of Panel Unit-Root and Cointegration Tests

Consider the pooled-ADF regression:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t\text{-}1} + \gamma_i t + \sum_{j=1}^{p_i} \theta_{i,j} \Delta y_{i,t-j} + \varepsilon_{i,t} \tag{A1}$$

1) Levin, Lin, and Chu test

Step 1: transform the data as follows:

$$y^*_{i,t} = y_{i,t} - \frac{1}{N}\sum_{i=1}^{N} y_{i,t}.$$

Step 2: Apply ADF test to individual series and normalize the residuals from the following auxiliary regressions:

$$\Delta y_{i,t} = \alpha_{1i} + \rho_{1i} y_{i,t\text{-}1} + \gamma_{1i} t + \sum_{j=1}^{p_i} \theta_{1i,j} \Delta y_{i,t-j} + e_{i,t} \tag{A2}$$

$$y_{i,t\text{-}1} = \alpha_{2i} + \rho_{2i} y_{i,t\text{-}1} + \gamma_{2i} t + \sum_{j=1}^{p_i} \theta_{2i,j} \Delta y_{i,t-j} + v_{i,t} \tag{A3}$$

Denote the residual series by $\hat{e}_{i,t}$ and $\hat{v}_{i,t}$, respectively.

Step 3: Consider the following regression: $\hat{e}_{i,t} = \rho_i \hat{v}_{i,t} + \varepsilon_{i,t}$, which gives the OLS estimator of ρ_i denoted by $\hat{\rho}_i$.

In order to deal with heteroscedasticity, Levin, Lin and Chu suggest a normalisation of the errors that would allow heterogeneity within the individual dimension:

$$\hat{\sigma}^2_{ei} = \frac{1}{T - \rho_i - 1} \sum_{t=\rho_i+2}^{T} \left(\tilde{e}_{i,t} - \hat{\rho}_i \tilde{v}_{i,t-1}\right)^2$$

$\tilde{e}_{i,t}$ and $\tilde{v}_{i,t-1}$ are given by: $\tilde{e}_{i,t} = \dfrac{\hat{e}_{i,t}}{\hat{\sigma}_{ei}}$, $\tilde{v}_{i,t-1} = \dfrac{\hat{v}_{i,t-1}}{\hat{\sigma}_{ei}}$.

Step 4: Compute the ratio of the short-run and the long-run standard deviations:

$\hat{S}_{NT} = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} \dfrac{\hat{\sigma}_{yi}}{\hat{\sigma}_{ei}}$, where the estimated long-run variance $\hat{\sigma}^2_{yi}$ is given by:

$$\hat{\sigma}^2_{yi} = \frac{1}{T-1}\sum_{t=2}^{T} \Delta y^2_{i,t} + 2\sum_{L=1}^{\bar{k}} w_{\overline{K}L}\left(\frac{1}{T-1}\sum_{t=l+2}^{T} \Delta y_{i,t} \Delta y_{i,t-1}\right),$$

where $\overline{K}$ is a lag truncation with $w_{\overline{K}L} = \dfrac{1-L}{K+1}$; $k = 3.2T^{\frac{1}{3}}$

Step 5: Under the null hypothesis, $\tilde{e}_{i,t}$ and $\tilde{v}_{i,t-1}$ are independent across groups and the asymptotic normality test statistics is given by: $t_{\ell=0} = \dfrac{\hat{\rho}}{RSE(\hat{\rho})}$, where $\hat{\rho}$ is the OLS estimator obtained from the regression $\tilde{e}_{i,t} = \rho \tilde{v}_{i,t-1} + \tilde{\varepsilon}_{i,t}$

and $RSE(\hat{\rho})=\hat{\sigma}_{\varepsilon}\left(\sum_{i=1}^{N}\sum_{t=\rho_i+2}^{T}\tilde{v}_{i,t-1}^{2}\right)^{-\frac{1}{2}}$, with $\hat{\sigma}_{\varepsilon}^{2}=\frac{1}{N\tilde{T}}\sum_{i=1}^{N}\sum_{t=\rho_i+2}^{T}\left(\tilde{e}_{i,t}-\hat{\rho}\tilde{v}_{i,t-1}\right)^{2}$ and

$\tilde{T}=\left(T-\bar{\rho}-1\right)$, $\overline{\mathrm{p}}=\frac{1}{\mathrm{N}}\sum_{i=1}^{N}\rho_i$. $\bar{\rho}$ is the mean of lags used in the individual's ADF regression.

Now we compute the LLC adjusted *t*-statistic:

$$t_{\rho*}=\frac{t_{\rho=0}-N\tilde{T}\hat{S}_{NT}\hat{\sigma}_{\varepsilon}^{-2}RSE(\hat{\rho})\mu*_{\tilde{T}}}{\sigma_{\tilde{T}}^{*}}$$

The mean $\mu_{\tilde{T}}^{*}$ and the standard deviation $\sigma_{\tilde{T}}^{*}$ are calculated using Monte Carlo simulations. Under the null hypothesis, $H_0:\rho=0$, t_{ρ}^{*} is normally distributed.

2) Breitung and Mayer test

Breitung and Meyer developed a panel unit root test for fixed T and when $\mathrm{N}\rightarrow\infty$. Their method differs from LLC in two distinct ways:

Firstly, only the autoregressive component is removed when constructing the standardized differenced series:

$$\Delta\tilde{y}_{i,t}=\frac{(\Delta y_{i,t}-\sum_{j=1}^{P_i}\hat{\theta}_{ij}\Delta y_{i,t-j})}{s_i}\text{, and }\tilde{y}_{i,t-1}=\frac{(y_{i,t}-\sum_{j=1}^{P_i}\hat{\theta}_{ij}\Delta y_{i,t-j})}{s_i}$$

Secondly, we transform $\Delta y_{i,t}$ as follows:

$$\Delta y_{i,t}^{*}=\sqrt{\frac{T-t}{T-t+1}}(\Delta\tilde{y}_{i,t}-\frac{\sum_{j=1}^{T}\Delta y_{i,t+j}}{T-t})\text{, where }y_{i,t}^{*}=\tilde{y}_{i,t-1}-c_{i,t}$$

$$\begin{aligned} c_{i,t} &= 0, && \text{If no intercept and no trend}\\ &=\tilde{y}_{i,1}, && \text{with intercept}\\ &=\tilde{y}_{i,1}-(\frac{t-1}{T})\tilde{y}_{i,T}, && \text{with intercept and trend}\end{aligned}$$

The parameter ρ is estimated from the following regression: $\Delta y_{i,t}^{*}=\rho y_{i,t-1}^{*}+v_{i,t}$.

Breitung and Mayer showed that under the null hypothesis, the resulting estimator ρ^{*} is asymptotically normally distributed.

3) Hadri's test

Hadri's panel-unit root test is similar to KPSS test. Under the null hypothesis of no unit root, the test statistic is computed from residuals obtained from individual OLS regressions of $y_{i,t}$ on a constant, or alternatively on a constant and a trend. For example, if we include both the constant and a trend we have:

$$y_{i,t} = \delta_i + \eta_i t + \varepsilon_{i,t} \tag{A4}$$

Given the residuals from the individual regressions, we form the LM statistic:

$$LM_1 = \frac{1}{N}\left(\sum_{i=1}^{N}\left(\sum_t S_i(t)^2 / T^2\right) / \widetilde{f}_0\right)$$

where $S_i(t)$ are the cumulative sums of residuals, $S_i(t) = \sum_{t=1}^{T} \hat{\varepsilon}_{i,t}$

$\widetilde{f}_0$ is the average of estimators of the residual spectrum at frequency zero,

$$\widetilde{f}_0 = \frac{1}{N}\sum_{i=1}^{N} f_{i,0}$$

An alternative form of the LM statistic that counts for heteroskedasticity across groups is given by:

$$LM_2 = \frac{1}{N}\left(\sum_{i=1}^{N}\left(\sum_t S_i(t)^2 / T^2\right) / f_{i,0}\right)$$

Under some assumptions, the test is asymptotically standard normally distributed,

$$H = \frac{\sqrt{N}(LM - \xi)}{\varsigma} \Rightarrow N(0,1)$$

where ξ=1/6 and ς=1/45, if the model includes only a constant, and ξ=1/15 and ς=11/6300, otherwise.

4) Im, Pesaran and Shin test

For each panel series $y_{i,t}$, the authors estimate the following model:

$$y_{i,t} = \rho_i y_{it-1} + \sum_{j=1}^{p_i} \alpha_{ij} \Delta y_{it-j} + z'_{it}\gamma + \varepsilon_{it} \tag{A5}$$

The null and the alternative hypothesis are:

$H_0 : \rho_i = 0$ for all i

$H_1 : \rho_i < 0$ for i=1,2,…, N_1, and ρ_i=0 for i= N_1+1, N_1+2,…, N

The test statistics if given by the average of individual ADF statistics:

$$\bar{t} = \frac{1}{N}\sum_{i=1}^{N} t_{\rho i} \tag{A6}$$

where $t_{\rho_i} \Rightarrow \frac{\int_0^1 W_{iz} dW_{iz}}{\left[\int_0^1 W_{iz}^2\right]^{1/2}} = t_{iT}$, (A7)

we have $\frac{\sqrt{N}\left(\frac{1}{N}\sum_{i=1}^{N} t_{iT} - E[t_{iT} / \rho_i = 1\right)}{\sqrt{Var[t_{iT} / \rho_{i=1}]}} \underset{N\to\infty}{\Rightarrow} N(0,1)$ (A8)

Under some assumptions, the authors prove that their test statistics is standard normally distributed:

$$t_{IPS} = \frac{\sqrt{N}\left(\bar{t} - E[t_{iT} / \rho_i = 1\right)}{\sqrt{VAR[t_{iT} / \rho_{i=1}]}} \underset{T,N\to\infty}{\Rightarrow} N(0,1) \tag{A9}$$

The values of $E\lfloor t_{i,T} / \rho_i = 1\rfloor$ and $Var\lfloor t_{i,T} / \rho_i = 1\rfloor$ are simulated.

5) Pedroni's panel cointegration tests

These tests allow the fixed effects and the cointegrated vectors to differ across members of the panel, and for heterogeneity in the errors across the cross-section units.
The cointegration equation is given by:

$$y_{it} = \alpha_i + \delta_i t + \gamma_t + \beta_{1i} X_{2i,t} + \ldots + \beta_{Mi} X_{Mi,t} + e_{it} \tag{A10}$$

where $i = 1,\ldots,N$; $\mathrm{t} = 1,\ldots,\mathrm{T}$, and $\mathrm{m} = 1,\ldots,M$.

α_i : the individual specific effects

γ_t : the temporal specific effects

$\delta_i t$: the deterministic trend which may vary across groups

H_0 : the variables are not cointegrated for each group of the panel.

H_1 : for each member of the panel, there exists a cointegration relashionship.

Pedroni developed seven test statistics that could be listed into two categories:
The first category include tests based on the within dimension.

- *Panel v-test*: $T^2 N^{3/2} Z_{\hat{\upsilon} N,T} = T^2 N^{3/2} \left(\sum_{i=1}^{N}\sum_{t=1}^{T} \hat{L}_{11i}^{-2} \hat{e}_{i,t-1}^2 \right)^{-1}$

-Panel ρ *-*

$$test: T\sqrt{N}\, Z_{\hat{\rho}\, N,T} = T\sqrt{N}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\,\hat{e}_{i,t-1}^{2}\right)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\left(\hat{e}_{i,t-1}\,\Delta\hat{e}_{i,t}-\hat{\lambda}_i\right)$$

$$\text{-Panel pp-test: } Z_{t\,N,T} = \left(\tilde{\sigma}_{N,T}^{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\,\hat{e}_{i,t-1}^{2}\right)^{-1/2}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\left(\hat{e}_{i,t-1}\,\Delta\hat{e}_{i,t}-\hat{\lambda}_i\right)$$

$$\text{-Panel adf-test: } Z_{t\,N,T}^{*} = \left(\tilde{s}_{N,T}^{*2}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\,\hat{e}_{i,t-1}^{*2}\right)^{-1/2}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{L}_{11i}^{-2}\left(\hat{e}_{i,t-1}^{*}\,\Delta\hat{e}_{i,t}^{*}\right)$$

The second category include tests which based on the between dimension

$$\text{-Group } \rho\text{-test: } T\,N^{-1/2}\,\tilde{Z}_{\tilde{\rho}\,N,T-1} = T\,N^{-1/2}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{e}_{i,t-1}^{2}\right)^{-1}\sum_{t=1}^{T}\left(\hat{e}_{i,t-1}\,\Delta\hat{e}_{i,t}-\hat{\lambda}_i\right)$$

$$\text{-Group pp-test: } N^{-1/2}\,\tilde{Z}_{t\,N,T-1} = N^{-1/2}\sum_{i=1}^{N}\left(\hat{\sigma}_i^{2}\sum_{t=1}^{T}\hat{e}_{i,t-1}^{2}\right)^{-1/2}\sum_{t=1}^{T}\left(\hat{e}_{i,t-1}\,\Delta\hat{e}_{i,t}-\hat{\lambda}_i\right)$$

$$\text{-Group adf-test: } N^{-1/2}\,\tilde{Z}_{tN,T}^{*} = N^{-1/2}\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{s}_i^{*2}\,\hat{e}_{i,t-1}^{*2}\right)^{-1/2}\sum_{t=1}^{T}\left(\hat{e}_{i,t-1}^{*}\,\Delta\hat{e}_{i,t}^{*}\right)$$

6) Larsson *et al* cointegration test

The procedure developed by Pedroni is applicable to test only for a single cointegration relationship. In order to test multiple cointegration relationships between the variables, one can use the test developed by Larsson, Lyhagen and Lothgren (2001). The objective is to test the hypothesis that all groups in the panel have the same (maximum) number of cointegrated vectors among the system of k variables in a p-dimensional vector error-correction model. The test is based on the maximum likelihood method as described by Johansen (1996).

Let the data generating process be represented by a vector auto-regressive of order p_i :

$$y_{it} = \sum_{j=1}^{\rho_i}\Pi_{ij}\,y_{it-j} + \varepsilon_{it}\,,\quad i=1,\ldots,N,\ t=1,\ldots,T$$

The VECM representation is:

$$\Delta y_{it} = \Pi_i\,y_{it-1} + \sum_{\ell=1}^{\rho_i}\Gamma_j\,y_{it-j} + \varepsilon_{it}$$

where Π_i is a $k\times k$ matrix, and k is the number of variables in each group.

Write $\Pi_i = \alpha_i \beta_i'$, where α_i and β_i are two $k \times r_i$ matrices that represent the adjustment space and the cointegration space, respectively. The null and the alternative hypotheses are given as follows:

$H_0(r) : rank(\Pi_i) \leq r$, and $H_1(k) : rank(\Pi_i) = k$.

Compute the test statistics $LR_{NT} = \frac{1}{N}\sum_{i=1}^{N} LR_{iT}$ and standardize it by:

$$\gamma_{LR} = \frac{\sqrt{N}(LR_{NT} - E(Z_K))}{\sqrt{Var(Z_K)}} \underset{N,T\to\infty}{\Rightarrow} N(0,1)$$

where $E(Z_K)$ and $Var(Z_K)$ are the asymptotic mean and the asymptotic variance of the test statistic.