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# Twin Default Crises\*

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## Abstract

Twin Default Crises are rare and severe episodes of borrower and bank defaults. We build a quantitative model that links borrower and bank solvency. This is crucial to reproduce key features of the data both in normal times and in Twin Default Crises. Specialization exposes banks to non-diversifiable borrowers' default risk. Fluctuations in the non-diversifiable component of credit risk and bank leverage are important determinants of Twin Default Crises. Capturing the frequency and severity of Twin Default Crises is key for the correct calibration of bank capital requirements. Our framework implies higher capital requirements than alternative frameworks that do not model the link between borrower and bank default.

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# 1. Introduction

Banking crises are commonly described as systemic events involving investors' loss of confidence in banks, bank failures, and the need for massive government rescue plans (e.g. [Laeven and Valencia, 2013](#); [Schularick and Taylor, 2012](#); [Jorda, Schularick and Taylor, 2016](#)). A recent paper by [Baron, Verner and Xiong \(2019\)](#) points to bank equity losses as the key driver of the economy into these events. Large declines in the value of banks' equity are found to pre-date the occurrence of other crisis symptoms such as the loss of confidence among investors and to predict subsequent contractions in bank credit and aggregate economic activity. What causes large bank equity declines? How are these declines transmitted to the real economy? And can bank capital regulation reduce the occurrence and severity of banking crises? These are the questions we investigate in this paper.

Losses on the asset side of banks' balance sheet are the main driver of large bank equity declines. Explorations into banking crises of the 20th century (e.g. [BIS, 2004](#)) attribute a prominent role to the realization of credit losses due to above-average borrower defaults. In fact, loans are still the main asset on banks' balance sheets and the primary object of concern for prudential authorities.<sup>1</sup> Thus, understanding the link between the default of banks and of their borrowers is crucial for the calibration of bank capital requirements, which are the main policy tools to protect the economy against bank insolvencies.

In this paper, we build and estimate with euro area (EA) data a macroeconomic model in which bank equity declines and their most extreme manifestation, bank default, happen as a result of credit losses due to the default of borrowing firms. Banks fail because part of borrowers' default risk is not diversifiable at the bank level. We assume that banks operate on islands and borrowers in the same island are exposed to common productivity shocks (island-idiosyncratic shocks). The segmentation of credit markets in islands is a short-cut to capture limits to banks' diversification across local, sectorial or business-model specific sources of risk.<sup>2</sup> These limits to diversification make banks more vulnerable to borrowers' defaults. Since banks cannot lend across islands, island-idiosyncratic shocks generate heterogeneity in banks' performance and default outcomes.

Borrowers in our model also face idiosyncratic shocks to their productivity (firm-idiosyncratic shocks) that banks can diversify away by virtue of lending to all the firms on the island. Finally, banks are also exposed to aggregate total-factor-productivity (TFP) shocks, as well as

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<sup>1</sup>Consistent with this view, the treatment of credit risk in the banking books was the focus of the first Basel agreement on capital requirements in 1988. Credit losses due to non-performing loans are still the main source of potential capital losses for European banks as revealed by various stress testing exercises ([European Banking Authority, 2018](#); [European Central Bank, 2018](#)). Although banks' trading books have grown in recent decades, they largely consist of securitized loan receivables and other debt securities and, hence, are also highly exposed to credit risk.

<sup>2</sup>In Europe, banks operate largely within national borders and many specialize in lending to particular industries and sectors (see [Guiso, Sapienza and Zingales, 2004](#); [De Bonis, Pozzolo and Stacchini, 2011](#); [Behr and Schmidt, 2016](#); [De Jonghe et al., 2019](#)). Geographical and sectorial specialization is also a feature of US small and medium-sized banks ([Deyoung et al., 2015](#); [Regehr and Sengupta, 2016](#)) and banks in Peru ([Paravisini, Rappoport and Schnabl, 2015](#)).

firm- and island-risk shocks which affect the dispersion of firm- and island-idiosyncratic shocks, respectively.

The modelling of bank assets as portfolios of risky loans distinguishes our approach from the standard financial accelerator literature which abstracts from banks (e.g. [Bernanke and Gertler, 1989](#); [Kiyotaki and Moore, 1997](#); [Bernanke, Gertler and Gilchrist, 1999](#)) and subsequent developments where banks directly hold productive assets (e.g. [Gertler, Kiyotaki and Queralto, 2012](#); [Brunnermeier and Sannikov, 2014](#); [He and Krishnamurthy, 2019](#); [Gertler, Kiyotaki and Prestipino, 2016](#)). In such frameworks, abstracting from banks’ role as credit intermediaries precludes investigating borrowers’ default as a driver of bank equity declines and bank defaults. This is an important limitation for the analysis of bank capital requirements.

A growing number of papers also build general equilibrium models in which banks extend risky loans and feature endogenous bank failure risk (e.g. [Elenev, Landvoigt and Nieuwerburgh, 2018](#); [Mendicino et al., 2019](#); [Jermann, 2019](#)).<sup>3</sup> However, for tractability, in these models, heterogeneity in bank asset returns results from multiplicative log-normal shocks affecting banks’ loan portfolios independently of the performance of the individual borrowers. Such a formulation makes bank asset returns (and their implications for equity returns and bank solvency) akin to the classical Merton model of corporate default ([Merton, 1974](#)). As pointed by [Gornall and Strebulaev \(2018\)](#) and [Nagel and Purnanandam \(2019\)](#), the Merton approach to asset returns is inadequate to describe the distribution of returns of a portfolio of defaultable loans, which involves bounded returns at the upper tail and a thicker and longer lower tail than in the log-normal distribution.<sup>4</sup>

Differently from other existing frameworks, our island approach implies that banks hold a portfolio of loans with returns that have limited upside but significant downside risks. Capturing the non-linearity in bank asset returns allows our model to match three important features of the data: the unconditional positive correlation between firm and bank defaults; the fact that during episodes of abnormally high firm and bank defaults (that we call Twin Default Crises) macroeconomic performance is substantially worse; and the frequency and severity of these episodes. Importantly, both in the data and in the model, the correlation between firm and bank default is larger at higher quantiles of bank default. Further, the model implies a strong negative link between GDP growth and bank default at lower quantiles of GDP growth. This is consistent with recent findings on the importance of financing conditions as a determinant of the economy’s downside risk ([Adrian, Boyarchenko and Giannone, 2019](#)). The Merton approach instead generates a zero conditional and unconditional correlation between firm and

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<sup>3</sup>We share with these papers the focus on lending to firms. This is consistent with the important role of EA banks in lending to non financial corporations (NFCs) and the importance of NFC defaults as drivers of credit losses in Europe ([European Banking Authority, 2018](#)). Our model could be adapted to consider the case in which bank borrowers are households wishing to finance house purchases with mortgages. However such setup would be less relevant in the EA since the recourse nature of most European mortgages makes the default rates of this class of loans very low even in bad times.

<sup>4</sup>Our approach is related to the single risk factor model of [Vasicek \(2002\)](#) (see [Gordy, 2003](#)) where all loans in a bank’s loan portfolio are exposed to a common risk factor. In such a partial equilibrium formulation, the single factor can be interpreted as a source of aggregate risk or as a source of risk to all the loans in a particular portfolio. Our general equilibrium formulation distinguishes between these two sources of risk.

bank defaults and therefore fails to reproduce the frequency and severity of Twin Default Crises observed in the data. As we will show, the Merton-type version of our model misrepresents the extent to which capital requirements can reduce the costs associated with bank failures and generates lower prescriptions for the optimal level of capital requirements than our model.

After documenting that the model is able to reproduce the Twin Default Crises observed in the data, we investigate the factors that engender these crises in the model. Our results show that firm- and island-risk shocks are important in originating Twin Default Crises. Such crises appear as a result of sequences of small negative island-risk shocks which get increasingly amplified as the probability of bank failure grows. Intuitively, the non-linearity in bank asset returns implies that, once banks have a high risk of failure, the marginal impact of additional credit losses on banks' solvency and the macroeconomy is much larger than in normal times.

To properly capture these non-linearities it is essential to solve the model using a high order approximation.<sup>5</sup> In particular, we solve the model using a third order approximation to the underlying policy functions and estimate its parameters using the Generalized Method of Moments (GMM).<sup>6</sup> For the estimation, we target a large set of unconditional moments in the macro, banking and financial EA data, including the large and positive unconditional correlation between firm and bank default risk in the data. Importantly, no conditional moment is targeted and yet the model reproduces the relative frequencies and conditional moments of normal times, Twin Default Crises, and episodes with abnormally High Firm Default but low bank default. Computing the generalized impulse response functions (GIRFs) as proposed in [Andreasen, Fernandez-Villaverde and Rubio-Ramrez \(2017\)](#) allows us to show the strong non-linear effects of island-risk shocks. When firm and bank default are already high, an island-risk shock of limited size disproportionately increases bank default and leads to a large drop in output.

Finally, we document the role of bank leverage in the propagation of island-risk shocks and examine the policy implications. In the model, bank deposits are fully insured and, thus, bank leverage is determined by the regulatory capital requirement. When this requirement is higher (and hence bank leverage is lower), the effect of island-risk shocks on bank solvency gets significantly dampened as bank equity returns become less sensitive to credit losses. From a normative perspective, however, capital regulation faces a clear trade-off in our model. On the one hand, a higher capital requirement makes banks less vulnerable to credit losses, reducing their default risk and the associated deadweight losses. On the other hand, a higher capital requirement also means a higher average cost of funding for banks and, hence, a higher cost of borrowing for firms. Hence preventing or reducing the severity of Twin Default Crises comes along with a reduction of credit and economic activity in normal times.

The net effect of the bank capital requirement on economic activity and welfare depends on which of the two forces dominates. We find that a fifteen percent bank capital requirement

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<sup>5</sup>If solved to a first order approximation, the model cannot reproduce the state-dependent co-movements of borrower and bank defaults and GDP growth. In addition, the linear version of our model only generates Twin Default Crises with the help of implausibly large realizations of the island-risk shocks.

<sup>6</sup>We use pruning to ensure that the model moments exist (see [Andreasen, Fernandez-Villaverde and Rubio-Ramrez, 2017](#)).

maximizes the expected utility of the representative household in our model, which is about five percentage points higher than the requirement prescribed by a Merton-type version of our model. This underlines the importance of capturing the frequency and severity of Twin Default Crises for the model’s policy implications.

**Other related literature** In the first part of this introduction we have already referred to differences and similarities with the financial accelerator tradition and the literature developing DSGE models with banks to which our paper belongs. Our focus on non-linearities associated with the very nature of credit losses brings a different perspective to the literature that emphasises the highly nonlinear aspects of financial crises through a number of other potential sources of non-linearities: occasionally binding constraints ([Bianchi and Mendoza, 2011](#); [Boissay, Collard and Smets, 2016](#)), asset prices feedback loops ([He and Krishnamurthy, 2013](#); [Brunnermeier and Sannikov, 2014](#)), sovereign defaults ([Bocola, 2016](#)), liquidity problems ([De Fiore, Hoerova and Uhlig, 2018](#)) and bank panics ([Gertler, Kiyotaki and Prestipino, 2019](#)).

By showing the importance of a proper distinction between the diversifiable and non-diversifiable drivers of credit losses, as well as capturing the non-linearities that drive the economy into Twin Default Crises, we also contribute to the growing literature that assesses the determinants of the optimal level of bank capital requirements emphasizing the liquidity provision role of banks ([Van Den Heuvel, 2008](#); [Begenau and Landvoigt, 2017](#); [Begenau, 2019](#)), systemic risk taking ([Martinez-Miera and Suarez, 2014](#)), imperfect competition ([Corbae and D’Erasmus, 2019](#)) and deadweight losses from bank default ([Clerc et al., 2015](#); [Begenau and Landvoigt, 2017](#); [Corbae and D’Erasmus, 2019](#); [Mendicino et al., 2018](#); [Begenau, 2019](#)).

## 2. The Model

The model economy is populated by a representative household (the ‘household’ hereafter) that provides consumption insurance to three types of members: workers, entrepreneurs and bankers. Workers supply labor to production sector. Entrepreneurs and bankers supply equity to entrepreneurial firms and banks. Entrepreneurial firms pay for the input of production in advance. They use (inside) equity and bank loans to rent labor from workers and purchase capital used to produce the final good. Banks use (inside) equity and deposits from the household to provide loans to firms. Both entrepreneurial firms and banks operate under limited liability and live for one period. Deposits are insured by a deposit guarantee scheme (DGS).

### 2.1 The Household

In each period some workers become either entrepreneurs or bankers and the same measure of entrepreneurs and bankers retire and become workers again.<sup>7</sup> At the beginning of each period entrepreneurs receive payments from last period entrepreneurial firms and stays active with

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<sup>7</sup>This assumption guarantees that entrepreneurs and bankers do not accumulate enough net worth such that entrepreneurial firms and banks can be entirely financed with internal funds (see [Gertler and Kiyotaki, 2010](#)).

probability  $\theta_e$  or retire otherwise. Upon retirement entrepreneurs transfer any accumulated net worth to the household. At the same time, a mass  $(1 - \theta_e)$  of workers becomes entrepreneurs. The new entrepreneurs receive aggregate endowments  $\iota_{e,t}$  from the household. Similarly, at the beginning of each period bankers receive payments from banks that lived last period and stays active with probability  $\theta_b$  and retire otherwise. Upon retirement they transfer any accumulated net worth to the household. At the same time, a mass  $(1 - \theta_b)$  of workers becomes bankers. The new bankers receive aggregate endowments  $\iota_{b,t}$  from the household and pay lump-sum taxes to the DGS so as to cover the losses on the insured deposits in banks which failed in the previous period.

The household chooses consumption,  $C_t$ , hours worked,  $H_t$ , and investments in insured bank deposits,  $D_t$ , to maximize the present discounted value of utility

$$\mathbb{E}_t \sum_{s=t}^{\infty} \left( \beta^s \log(C_s) - \frac{\varphi}{1+\eta} H_s^{1+\eta} \right)$$

subject to the budget constraint

$$C_t + D_t = w_t H_t + R_{d,t-1} D_{t-1} + \Upsilon_t + \Xi_t \quad (1)$$

where  $\eta$  is the inverse of the Frisch elasticity of labor supply,  $\varphi$  is the weight of labor supply in the utility of households,  $w_t$  is the real hourly wage and  $R_{d,t-1}$  is the gross rate of deposits. In addition  $\Upsilon_t$  are aggregate net transfers from entrepreneurs and bankers to households, and  $\Xi_t$  are profits from the capital producing firms that households own. We are interested in the symmetric equilibrium, hence we assume that the household invests its deposits symmetrically in all the (symmetric) banks in the economy. All the variables in the problem of the household represent aggregate variables.<sup>8</sup>

The household's problem yields the following FOCs with respect to consumption,

$$U_{C_t} = \lambda_t, \quad (2)$$

labor supply,

$$-U_{H_t} = w_t \lambda_t, \quad (3)$$

and demand for the portfolio of insured deposits,

$$1 = \mathbb{E}_t (\Lambda_{t+1}) R_{d,t}, \quad (4)$$

where  $\lambda_t$  is the Lagrange multiplier of the budget constraint and  $\Lambda_{t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}$  is the household's stochastic discount factor.

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<sup>8</sup>As will see below, there are both aggregated and idiosyncratic shocks. All the conditional expectations are taken with respect the aggregated shocks.

## 2.2 Entrepreneurs, Entrepreneurial Firms, Bankers, and Banks

Entrepreneurs and bankers provide equity to entrepreneurial firms and banks, respectively. Both entrepreneurial firms and banks live for one period, operate under limited liability (they can default) and obtain external financing by issuing non-recourse uncontingent debt: bank loans and deposits, respectively. They both operate in a continuum of measure one of islands. In each island there is a continuum of measure one of entrepreneurial firms and a representative bank.

Entrepreneurial firms produce the final good and are subject to both firm- and island-idiosyncratic shocks, whose realizations affect their terminal net worth. Non-defaulted entrepreneurial firms and banks pay their terminal net worth to entrepreneurs and bankers respectively. Each representative bank diversifies its lending across entrepreneurial firms in its island but not across islands so its terminal net worth depends on the island-idiosyncratic shock. Entrepreneurial firms and banks default when their terminal asset value is lower than their debt repayment obligations. In this case, their lenders take possession of their assets at a cost, equal to a proportion  $\mu_f$  of assets.

### 2.2.1 Entrepreneurs

Let  $V_{e,t}(n_{e,t}(i))$  be the value of being entrepreneur  $i \in (0, 1)$  with net worth  $n_{e,t}(i)$  at period  $t$  from the perspective of the household to which she belongs. Every period, entrepreneur  $i$  decides how much of her net worth,  $n_{e,t}(i)$ , to invest in a portfolio of equity of the continuum of entrepreneurial firms living in period  $t$ ,  $eq_{e,t}(i)$ , and how much to pay back to the household in the form of dividends,  $dv_{e,t}(i)$ , to maximize

$$V_{e,t}(n_{e,t}(i)) = \max_{eq_{e,t}(i), dv_{e,t}(i)} \{dv_{e,t}(i) + \mathbb{E}_t \Lambda_{t+1} [(1 - \theta_e) n_{e,t+1}(i) + \theta_e V_{e,t+1}(n_{e,t+1}(i))]\} \quad (5)$$

subject to

$$\begin{aligned} eq_{e,t}(i) + dv_{e,t}(i) &= n_{e,t}(i), \\ n_{e,t+1}(i) &= \theta_e \rho_{e,t+1} eq_{e,t}(i), \text{ and} \\ dv_{e,t}(i) &\geq 0. \end{aligned} \quad (6)$$

The portfolio invested in entrepreneurial equity in period  $t$  pays at the beginning of period  $t+1$ . From the point of view of the entrepreneur maximizing the valued function described above,  $\rho_{e,t+1}$  is an exogenous random variable. In equilibrium,  $\rho_{e,t+1}$  equals the gross rate of return of the portfolio of entrepreneurial equity. A detailed expression for the equilibrium value of  $\rho_{e,t+1}$  is provided below. We are interested in the symmetric equilibrium, hence we assume that each entrepreneur invests symmetrically in all the (symmetric) entrepreneurial firms of the economy. The constraint  $dv_{e,t}(i) \geq 0$  reflects that entrepreneurs can freely pay positive dividends back to the household but the household cannot provide further net worth to the entrepreneurs. All



the variables in the problem of the entrepreneur represent per capita variables.<sup>9</sup>

As in [Gertler and Kiyotaki \(2010\)](#), we guess that the value function of being an entrepreneur is linear in her net worth

$$V_{e,t}(n_{e,t}(i)) = \nu_{e,t} n_{e,t}(i),$$

where  $\nu_{e,t}$  is the shadow value of one unit of entrepreneurial net worth. Then we can write the Bellman Equation (5) as

$$\nu_{e,t} n_{e,t}(i) = \max_{eq_{e,t}(i), dv_{e,t}(i)} [dv_{e,t}(i) + \mathbb{E}_t \Lambda_{t+1} (1 - \theta_e + \theta_e \nu_{e,t+1}) n_{e,t+1}(i)]. \quad (7)$$

We guess and later verify that, in the proximity of the steady state,  $\nu_{e,t} \geq 1$ . From the Envelope Theorem  $dv_{e,t}(i) = 0$  whenever  $\nu_{e,t} > 1$ . Under our parameter values  $\nu_{e,t} = 1$  with a probability close to zero. As a result, we impose  $dv_{e,t}(i) = 0$  such that the Bellman equation (7) reduces to

$$\nu_{e,t} = \mathbb{E}_t \Lambda_{t+1} (1 - \theta_e + \theta_e \nu_{e,t+1}) \rho_{e,t+1} \quad (8)$$

and the evolution of an entrepreneur's net worth is

$$n_{e,t+1}(i) = \rho_{e,t+1} n_{e,t}(i). \quad (9)$$

Intuitively, we focus on parameterizations for which the inability of the household to transfer as much net worth to entrepreneurs as it wants, creates an aggregate shortage of entrepreneurial equity which keeps the risk adjusted expected return to entrepreneurial equity greater than the risk-free rate. As a result, the shadow value of funds in the hands of entrepreneurs,  $\nu_{e,t}$ , is greater than unity (that is, if the household were able to transfer more funds to entrepreneurs, it would do so). The entrepreneur responds to this by retaining all her net worth and paying no dividends until she retires. Equation (8) allows us to define entrepreneurs' stochastic discount factor for later use as

$$\Lambda_{e,t+1} = \Lambda_{t+1} (1 - \theta_e + \theta_e \nu_{e,t+1}). \quad (10)$$

We need an index  $i$  for individual entrepreneurs because each of them has been an entrepreneur for a different length of time and has therefore accumulated a different level of net worth. However, since individual entrepreneur value functions and policy functions are linear in own net worth, the distribution of entrepreneurial wealth is irrelevant for aggregate outcomes. As we will see below, aggregate investment and credit demand depend only on the aggregate wealth of the entrepreneurial sector.

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<sup>9</sup>The rate of return  $\rho_{e,t+1}$  is a function of time  $t$  endogenous aggregate state variables and time  $t+1$  exogenous aggregate state variables (i.e. aggregate shocks). Hence, the value function  $V_{e,t}$  is not only function of the individual state variable,  $n_{e,t}(i)$  but also the aggregate state variables. For this reason the individual entrepreneur needs to guess rules to forecast the aggregate state variables. In equilibrium, those rules need to be coherent with behaviour. To simplify notation, we only describe the dependence of the value function with respect to the individual state variable.

### 2.2.2 Entrepreneurial firms

Entrepreneurial firms live for a single period and pay out their terminal net worth, if positive, to entrepreneurs. If their terminal net worth is negative they default. Entrepreneurial firms living at period  $t$  produce the final good,  $y_{t+1}$ , using labor,  $h_t$ , and capital,  $k_t$

$$y_{t+1} = A_{t+1} k_t^\alpha h_t^{1-\alpha}, \quad (11)$$

where  $A_t$  is an aggregate TFP shock. At the beginning of the period entrepreneurial firms buy capital from capital producers at price  $q_t$ . In period  $t$ , they pay  $w_t h_t + q_t k_t$  using equity from entrepreneurs,  $EQ_{e,t}$ , and loans from their island bank,  $B_{f,t}$ , with gross loan interest rate  $R_{f,t}$ . At the beginning of period  $t + 1$ , the final good is produced and sold to the households and the depreciated capital,  $(1 - \delta) k_t$ , is sold back to the capital producers at price  $q_{t+1}$ . Although entrepreneurial firms can only borrow from island-specific banks, final goods, labor, and capital can move freely across islands.

Since entrepreneurial firms have constant returns, the scale of an individual firm is indeterminate. Without loss of generality, we assume that there is a continuum of measure one of firms and each firm receives an identical amount of equity from entrepreneurs (which is also equal to the aggregate wealth of entrepreneurs). Also, as we will see below, we make the same assumption about individual banks. Given that loans are extended before any shock is realized, borrowers are identical and hence the loan amount received by each entrepreneurial firms will be the same and equal to aggregate bank loans.

**The idiosyncratic shocks** Entrepreneurial firms living at period  $t$  face a firm-idiosyncratic shock,  $\omega_i$ , and an island-idiosyncratic shock,  $\omega_j$  to the terminal value of their assets (output plus the market value of un-depreciated capital). These shocks are independent and they are realized at the same time. We assume that  $\omega_i$  and  $\omega_j$  are log-normally distributed

$$\log(\omega_\vartheta) \sim \mathcal{N}\left(-\frac{\sigma_{\omega_\vartheta,t+1}^2}{2}, \sigma_{\omega_\vartheta,t+1}^2\right),$$

for  $\vartheta = i, j$ . The standard deviation of the idiosyncratic shocks is time-varying. The law of motion of these two aggregate shocks will be introduced below. There will be persistent shocks which will change the standard deviations of both the firm and island idiosyncratic shocks. These shocks to the risk facing firms and banks will be an important source of aggregate risk in the model and will be vital to generate defaults. We denote the CDFs of  $\omega_i$  and  $\omega_j$  by  $F_{i,t+1}$  and  $F_{j,t+1}$ , respectively. The subscript  $t + 1$  captures the dependence of the CDFs on the aggregate risk shocks.

**Terminal net worth of an entrepreneurial firm** Entrepreneurial firms default if their terminal assets are insufficient to pay back their loan,  $R_{f,t} B_{f,t}$ , in full. In other words, the terminal net worth of an entrepreneurial firm experiencing shocks  $(\omega_i, \omega_j)$  at the beginning of

period  $t + 1$  equals

$$\Pi_{i,j,t+1}(\omega_i, \omega_j) = \omega_i \omega_j [q_{t+1}(1 - \delta)k_t + y_{t+1}] - R_{f,t}B_{f,t}, \quad (12)$$

and the entrepreneurial firm defaults on its loan when  $\Pi_{i,j,t+1}(\omega_i, \omega_j) < 0$ .

From Equation (12), it is useful to define the threshold value for the firm-idiosyncratic shock  $\omega_i$  below which entrepreneurial firms in an island experiencing an island-idiosyncratic shock  $\omega_j$  default

$$\bar{\omega}_{t+1}(\omega_j) = \frac{R_{f,t}B_{f,t}}{\omega_j(q_{t+1}(1 - \delta)k_t + y_{t+1})}. \quad (13)$$

Note that a low realization of the island-idiosyncratic shock increases the threshold for which firms default on their bank. This, in turn, will ensure that the default rate on the portfolio of the bank on island  $j$  depends on the realization of its island-idiosyncratic shock. This is going to be a key source of default for the banks in our framework.

The payoff that an entrepreneur would receive from the equity invested across all entrepreneurial firms in the economy is

$$\Pi_{f,t+1} = \int_0^\infty \int_{\bar{\omega}_{t+1}(\omega_j)}^\infty \Pi_{i,j,t+1}(\omega_i, \omega_j) dF_{i,t+1}(\omega_i) dF_{j,t+1}(\omega_j) \quad (14)$$

Entrepreneurial firms choose capital, hours worked, the loans amount, and the gross loan rate to maximize

$$\max_{k_t, h_t, B_{f,t}, R_{f,t}} \mathbb{E}_t \Lambda_{e,t+1} \Pi_{f,t+1}$$

subject to

$$B_{f,t} + EQ_{e,t} = w_t h_t + q_t k_t, \text{ and} \quad (15)$$

$$\mathbb{E}_t \Lambda_{b,t+1} \Pi_{b,t+1} \geq \nu_{b,t} \phi B_{f,t}, \quad (16)$$

where Equation (15) is the entrepreneurial firm's budget constraint and Equation (16) is the bankers' participation constraint, which reflects the competitive pricing of the loans that banks are willing to offer for different leverage and productive choices by entrepreneurial firms. As explained and fully specified in detail below,  $\Lambda_{b,t+1}$  is bankers' stochastic discount factor,  $\Pi_{b,t+1}$  is the payoff that bankers would receive from equity invested across banks that provide the corresponding bank loans to all the firms operating in their island (that will be defined below),  $\nu_{b,t}$  is the shadow value of bankers' net worth, and  $\phi$  is the regulatory capital requirement that determines the fraction of the loan amount  $B_{f,t}$  which must be funded with equity at period  $t$ .

The entrepreneurial firm's problem yields the following FOCs with respect to capital,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial k_t} + \zeta_{f,t} q_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial k_t} = 0, \quad (17)$$

labor demand,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial h_t} + \zeta_{f,t} w_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial h_t} = 0, \quad (18)$$

loans,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial B_{f,t}} - \zeta_{f,t} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial B_{f,t}} + \xi_{f,t} \nu_{b,t} \phi = 0, \quad (19)$$

and the gross loan rate,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial R_{f,t}} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial R_{f,t}} = 0, \quad (20)$$

where  $\zeta_{f,t}$  is the Lagrange multiplier of the entrepreneurial firm's budget constraint, and  $\xi_{f,t}$  is the Lagrange multiplier of the bankers' participation constraint.

Finally, the gross rate of return on the portfolio of equity of an entrepreneur that symmetrically invests in all entrepreneurial firms is

$$\rho_{e,t+1} = \frac{\Pi_{f,t+1}}{EQ_{e,t}}. \quad (21)$$

### 2.2.3 Bankers

Let  $V_{b,t}(n_{b,t}(j))$  be the value of being banker  $j \in (0, 1)$  with net worth  $n_{b,t}(j)$  at period  $t$  from the perspective of the household to which she belongs. Every period, banker  $j$  decides how much of her net worth,  $n_{b,t}(j)$ , to invest in a portfolio of equity of the continuum of banks,  $eq_{b,t}(j)$ , and how much to pay back to the household in the form of dividends,  $dv_{b,t}(j)$ , to maximize

$$V_{b,t}(n_{b,t}(j)) = \max_{eq_{b,t}(j), dv_{b,t}(j)} \{dv_{b,t}(j) + \mathbb{E}_t \Lambda_{t+1} [(1 - \theta_b) n_{b,t+1}(j) + \theta_b V_{b,t+1}(n_{b,t+1}(j))]\} \quad (22)$$

subject to

$$\begin{aligned} eq_{b,t}(j) + dv_{b,t}(j) &= n_{b,t}(j), \\ n_{b,t+1}(j) &= \rho_{b,t+1} eq_{b,t}(j), \text{ and} \\ dv_{b,t}(j) &\geq 0. \end{aligned} \quad (23)$$

The portfolio invested in bank equity in period  $t$  pays at the beginning of period  $t + 1$ . From the point of view of the banker maximizing the valued function described above,  $\rho_{b,t+1}$  is an exogenous random variable. In equilibrium,  $\rho_{b,t+1}$  equals the gross rate of return of the portfolio of entrepreneurial equity. A detailed expression for the equilibrium value of  $\rho_{b,t+1}$  is provided below. We are interested in the symmetric equilibrium, hence we assume that each banker invests symmetrically in all the (symmetric) banks of the economy. The constraint  $dv_{b,t}(i) \geq 0$  reflects that bankers can freely pay positive dividends back to the household but the household cannot provide further net worth to the bankers. All the variables in the problem of the banker

represent per capita variables.

As in the case of entrepreneurs, we guess that bankers' value function is linear

$$V_{b,t}(n_{b,t}(j)) = \nu_{b,t} n_{b,t}(j),$$

where  $\nu_{b,t}$  is the shadow value of a unit of banker net worth. The Bellman equation in (22) becomes

$$\nu_{b,t} n_{b,t}(j) = \max_{eq_{b,t}(j), dv_{b,t}(j)} [dv_{b,t}(j) + \mathbb{E}_t \Lambda_{t+1} (1 - \theta_b + \theta_b \nu_{b,t+1}) n_{b,t+1}(j)]. \quad (24)$$

As before, we guess and later verify that, in the proximity of the steady state,  $\nu_{b,t} \geq 1$ . From the Envelope Theorem  $dv_{b,t}(j) = 0$  whenever  $\nu_{b,t} > 1$ . Under our parameter values  $\nu_{b,t} = 1$  with probability close to zero. Hence we impose  $dv_{b,t}(j) = 0$  to have

$$\nu_{b,t} = \mathbb{E}_t \Lambda_{t+1} (1 - \theta_b + \theta_b \nu_{b,t+1}) \rho_{b,t+1} \quad (25)$$

and the evolution of an banker's net worth is

$$n_{b,t+1}(j) = \rho_{b,t+1} n_{e,t}(j). \quad (26)$$

Again, just as in the case of entrepreneurs, the inability of the household to transfer as much net worth to bankers as it wants, creates an aggregate shortage of bank equity which keeps the risk adjusted expected return to bank equity greater than the risk-free rate. As a result, the shadow value of funds in the hands of bankers,  $\nu_{b,t}$ , is greater than unity. Bankers respond to this by retaining all of their net worth and paying no dividends until they retire.

Equation (24) allows us to define bankers' stochastic discount factor as

$$\Lambda_{b,t+1} = \Lambda_{t+1} (1 - \theta_b + \theta_b \nu_{b,t+1}). \quad (27)$$

As it was the case with entrepreneurs, we need an index  $j$  for individual bankers because each of them has been an banker for a different length of time and has therefore accumulated a different level of net worth. However, since individual banker value functions and policy functions are linear in own net worth, the distribution of bankers' wealth is irrelevant for aggregate outcomes. As we will see below, aggregate investment and credit demand depend only on the aggregate wealth of the banking sector.

#### 2.2.4 Banks

As entrepreneurial firms, banks live for a single period and pay out their terminal net worth, if positive, to bankers. If their terminal net worth is negative they default. At period  $t$  banks use equity  $EQ_{b,t}$  from bankers and deposits  $d_t$  from households in order to provide loans  $b_{f,t}$  to entrepreneurial firms operating in their island. Hence they face the following balance sheet constraint

$$b_{f,t} = EQ_{b,t} + d_t. \quad (28)$$

We assume that banks invest symmetrically in all the (symmetric) entrepreneurial firms in their island. Banks also face the following regulatory capital constraint

$$EQ_{b,t} \geq \phi b_{f,t}$$

where  $\phi$  is the capital requirement on loans. As explained below, in equilibrium, the capital constraint is always binding because funding the bank with deposits is always cheaper than with equity, hence

$$EQ_{b,t} = \phi b_{f,t} \quad (29)$$

**Gross rate of return on assets of the bank in island  $j$**  Banks operate under constant returns to scale hence their individual loan supply is perfectly elastic as long as the loan rate and the decisions of the borrowing firm satisfy bankers' participation constraint. This constraint plays the role of the zero profit condition in standard production theory and it stipulates that the loan must guarantee the bankers the equilibrium expected rate of return on banker equity. Because the bank is a levered institution with the possibility to default at time  $t + 1$ , the expected equity return also includes the value of limited liability which allows shareholders to avoid negative returns. In the remainder of this section, we cover the steps needed to derive a detailed expression for bankers' payoffs.

We start by computing the terminal asset value of the representative bank living in island  $j$ . At the beginning of period  $t + 1$ , firm- and island-idiosyncratic shocks,  $\omega_i$  and  $\omega_j$ , hit the entrepreneurial firms living at period  $t$ . As derived in Equation (13), conditional on the island-idiosyncratic shock  $\omega_j$ , an entrepreneurial firm pays back its loan in full when it experiences a firm-idiosyncratic shock no lower than  $\bar{\omega}_{t+1}(\omega_j)$ . Entrepreneurial firms with firm-idiosyncratic shocks smaller than  $\bar{\omega}_{t+1}(\omega_j)$  default on their loans and the bank only recovers a fraction  $1 - \mu_f$  of the terminal value of the entrepreneurial firm's assets. So the total repayment from defaulting entrepreneurial firms in an island experiencing an island-idiosyncratic shock  $\omega_j$  is

$$(1 - \mu_f) \omega_j [q_{t+1} (1 - \delta) k_t + y_{t+1}] \int_0^{\bar{\omega}_{t+1}(\omega_j)} \omega_i dF_{i,t+1}(\omega_i),$$

In turn, the total repayment from non-defaulting entrepreneurial firms in an island experiencing an island-idiosyncratic shock  $\omega_j$  is

$$R_{f,t} b_{f,t} \int_{\bar{\omega}_{t+1}(\omega_j)}^{\infty} dF_{i,t+1}(\omega_i).$$

So the gross rate of return of assets of bank in island  $j$  is

$$\tilde{R}_{f,t+1}(\omega_j) = \frac{(1 - \mu_f) \omega_j [q_{t+1} (1 - \delta) k_t + y_{t+1}]}{b_{f,t}} \int_0^{\bar{\omega}_{t+1}(\omega_j)} \omega_i dF_{i,t+1}(\omega_i) + R_{f,t} \int_{\bar{\omega}_{t+1}(\omega_j)}^{\infty} dF_{i,t+1}(\omega_i). \quad (30)$$

Following [Bernanke, Gertler and Gilchrist \(1999\)](#) it is useful to define

$$\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j)) = \int_0^{\bar{\omega}_{t+1}(\omega_j)} \omega_i dF_{i,t+1}(\omega_i) + \bar{\omega}_{t+1}(\omega_j) \int_{\bar{\omega}_{t+1}(\omega_j)}^{\infty} dF_{i,t+1}(\omega_i)$$

and

$$G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j)) = \int_0^{\bar{\omega}_{t+1}(\omega_j)} \omega_i dF_{i,t+1}(\omega_i).$$

Then Equation (30) can be rewritten more compactly as

$$\tilde{R}_{f,t+1}(\omega_j) = [\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j)) - \mu_f G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))] \frac{\omega_j [q_{t+1}(1 - \delta)k_t + y_{t+1}]}{b_{f,t}}. \quad (31)$$

Yet again, it is worth noting that the return of bank in island  $j$  is a function of the island-idiosyncratic shock  $\omega_j$ . The mechanism works through the impact of  $\omega_j$  on the default rate of entrepreneurial firms on island  $j$  as well as on the recovery value of the assets of defaulted entrepreneurial firms.

Equation (31) shows that the bank's loan portfolio return is a non-linear function of  $\omega_j$  and hence is not log-normal even if  $\omega_j$  itself is. Thus our model departs from the standard Merton approach ([Merton, 1974](#)) where it is assumed that bank asset returns follow a log-normal process. Our choice complicates the model solution with respect to the Merton's approach. More details will be provided in Section 3.1.

We will analyze extensively the implications of this feature of our model in Section 4.1. Most importantly, our approach captures the limited upside of loan payoffs together with the significant downside risks posed by borrower defaults. This feature distinguishes our model from model bank default using the Merton approach, including the most recent DSGE literature on bank defaults (e.g. [Clerc et al., 2015](#); [Begenau and Landvoigt, 2017](#); [Elenev, Landvoigt and Nieuwerburgh, 2018](#); [Mendicino et al., 2019](#); [Begenau, 2019](#); [Jermann, 2019](#)). As we will see our model captures important features of the data, including the conditional and unconditional positive correlation between firm and bank defaults and the frequency and severity of Twin Default Crises. In contrast, the Merton-type version of our model fails to reproduce them. This will imply that the Merton-type model generates lower prescriptions for the optimal level of capital requirements than our model.

**Terminal net worth of a bank** Banks default on their deposits if their revenue at the beginning of period  $t + 1$ ,  $\tilde{R}_{f,t+1}(\omega_j) b_{f,t}$ , is insufficient to pay the promised repayment,  $R_{d,t}d_t$ , in full. In other words, the terminal net worth of a bank experiencing shock  $\omega_j$  at the beginning of period  $t + 1$  equals

$$\Pi_{b,t+1}(\omega_j) = \tilde{R}_{f,t+1}(\omega_j) b_{f,t} - R_{d,t}d_t \quad (32)$$

and the bank defaults on its deposits when  $\Pi_{b,t+1}(\omega_j) < 0$ .

**Gross rate of return on the portfolio of bank equity** From Equation (32), it is useful to define a threshold value for the island-specific shock  $\omega_j$  below which the bank in island  $j$  defaults. This is implicitly done in the next equation

$$\tilde{R}_{f,t+1}(\bar{\omega}_{j,t+1}) b_{f,t} - R_{d,t} d_t = 0. \quad (33)$$

Equation (33) implies that banks' failure rate at the beginning of period  $t + 1$  is  $F_{j,t+1}(\bar{\omega}_{j,t+1})$ . Thus, the aggregate payoffs of a portfolio containing equity of all banks is then

$$\Pi_{b,t+1} = \int_{\bar{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}(\omega_j) b_{f,t} dF_{j,t+1}(\omega_j) - R_{d,t} d_t (1 - F_{j,t+1}(\bar{\omega}_{j,t+1})) \quad (34)$$

and the gross rate of return on the portfolio of equity of a banker that symmetrically invests in all banks is

$$\rho_{b,t+1} = \frac{\Pi_{b,t+1}}{EQ_{b,t}} = \frac{\Pi_{b,t+1}}{\phi b_{f,t}}. \quad (35)$$

## 2.3 Capital Production

Capital producers combine the final good,  $I_t$ , with the last period capital goods,  $K_{t-1}$ , in order to produce new capital goods which competitively sell to entrepreneurial firms at price  $q_t$ . The representative capital producing firm is owned by the household and maximizes the expected discounted value of profits

$$\max_{I_{t+j}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[ q_{t+j} S \left( \frac{I_{t+j}}{K_{t+j-1}} \right) K_{t+j-1} - I_{t+j} \right],$$

where  $S \left( \frac{I_{t+j}}{K_{t+j-1}} \right) K_{t+j-1}$  gives the units of new capital produced by investing  $I_{t+j}$  and using  $K_{t+j-1}$ . The increasing and concave function  $S(\cdot)$  captures the existence of adjustment costs, which we specify as in [Jermann \(1998\)](#)

$$S \left( \frac{I_{k,t}}{K_{t-1}} \right) = \frac{a_{k,1}}{1 - \frac{1}{\psi_k}} \left( \frac{I_t}{K_{t-1}} \right)^{1 - \frac{1}{\psi_k}} + a_{k,2},$$

where  $a_{k,1}$  and  $a_{k,2}$  are chosen to guarantee that in the steady state the investment-to-capital ratio is equal to the depreciation rate and  $S'(I_t/K_{t-1})$  equals one.

The FOC of this problem is

$$q_t = \left[ S' \left( \frac{I_t}{K_{t-1}} \right) \right]^{-1}. \quad (36)$$



## 2.4 Deposit Guarantee Scheme

The DGS has to balance his budget period-by-period. Thus, the lump-sum taxes have to cover the cost of guaranteeing deposits every period. Hence the total lump sum tax  $T_t$  imposed on bankers to balance the agency's budget is

$$T_t = F_{j,t}(\bar{\omega}_{j,t}) R_{d,t-1} d_{t-1} - (1 - \mu_j) \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j) b_{f,t} dF_{j,t}(\omega_j), \quad (37)$$

where the first part is the deposits lost because of bank defaults and second part is the total repayment from defaulting banks. We can write the above expression as

$$T_t = \Omega_t d_{t-1}, \quad (38)$$

where

$$\Omega_t = F_{j,t}(\bar{\omega}_{j,t}) R_{d,t-1} - \frac{1 - \mu_j}{1 - \phi} \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j) dF_{j,t}(\omega_j) \quad (39)$$

is the expected loss rate per unit of deposits, which has been written using the fact that  $d_t = (1 - \phi) b_{f,t}$ . As mentioned, the lump-sum taxes to the DGS are paid by the new bankers.

## 2.5 Aggregate Shocks

We assume the following AR(1) law of motion for the TFP shock

$$\log(A_{t+1}) = \rho_A \log(A_t) + \sigma_A \epsilon_{A,t+1}, \quad (40)$$

where  $\epsilon_{A,t+1}$  is normally distributed with mean zero and variance one.

The standard deviation of the distribution of each idiosyncratic shock is time-varying and evolves as an AR(1) process

$$\log\left(\frac{\sigma_{\omega_{\vartheta},t+1}}{\bar{\sigma}_{\omega_{\vartheta}}}\right) = \rho_{\sigma_i} \log\left(\frac{\sigma_{\omega_{\vartheta},t}}{\bar{\sigma}_{\omega_{\vartheta}}}\right) + \sigma_{\vartheta} \epsilon_{\omega_{\vartheta},t+1} \quad (41)$$

for  $\vartheta = i, j$ , where  $\epsilon_{\omega_{\vartheta},t+1}$  is normally distributed with mean zero and variance one.<sup>10</sup> We will call these shocks firm- and island-risk shocks.

## 2.6 Aggregation, Market Clearing and Equilibrium Conditions

Aggregation and market clearing conditions are described in Appendix B, whereas the exhaustive list of equilibrium conditions of the model are reported in Appendix C. It is easy to see that the states variables of the model are those included in the following vector

$$\mathbf{w}_t = (D_t, K_t, H_t, N_{e,t}, N_{b,t}, q_t, w_t, R_{f,t}, R_{d,t}, A_{t-1}, \sigma_{\omega_j,t-1}, \sigma_{\omega_i,t-1}). \quad (42)$$

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<sup>10</sup>This specification is similar to the one adopted in [Christiano, Motto and Rostagno \(2014\)](#).

### 3. Solution, Estimation and Quantitative Results

In this section we present the solution of the model, the estimation of the parameters and the quantitative results.

#### 3.1 Solving the Model

We solve the system of stochastic difference equations implied by the equilibrium conditions using a pruned state-space system for the third-order approximation around the steady-state as defined in [Andreasen, Fernndez-Villaverde and Rubio-Ramrez \(2017\)](#). This approach eliminates explosive sample paths and greatly facilitates inference using any method of moments approach. In particular, it ensures the existence of unconditional moments so that we use to estimate the parameters of the model by applying the GMM.

In order to use perturbation methods to approximate the solution to the model we need to compute the aggregate loan returns that banks generate conditional on not defaulting, defined here as  $R_{p,t+1}$ . These returns are the first term in Equation (34), namely

$$R_{p,t+1} \equiv \int_{\tilde{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}(\omega_j) dF_{j,t+1}(\omega_j). \quad (43)$$

As we mentioned before, the bank's loan return  $\tilde{R}_{f,t+1}(\omega_j)$  is not log-normally distributed because  $\omega_j$  enters non-linearly in its definition. This introduces a complication: the integral in Equation (43) as well as its derivatives cannot be written as an explicit function of the state variables. We overcome this challenge by (i) splitting this integral into the sum of integrals taken over smaller intervals, (ii) computing a series of quadratic Taylor approximations of  $\tilde{R}_{f,t+1}(\omega_j)$  around a mid-point of each interval. Due to the fact that powers of log-normally distributed variables are themselves log-normally distributed, the quadratic approximation to the bank profit function is itself approximately log-normally distributed and the expected profits as well as its derivatives can be computed as explicit functions of the state variables. The method is tractable and highly accurate. More details are provided in Appendix E.

#### 3.2 Model Estimation

The estimation of the model is a two-step procedure. First, some parameters are calibrated to standard values prior to the estimation procedure since there is a general consensus about their value in the literature. Second, we estimate the rest of the parameters using quarterly Euro Area (EA) data between 1992:Q1 and 2016:Q4.

Since we use quarterly data, the discount factor of the households,  $\beta$ , is set to 0.995. We set the Frisch elasticity of labor supply,  $\eta$ , equal to one, the value of capital depreciation,  $\delta$ , to 0.025, and the capital-share parameter of the production function,  $\alpha$ , to 0.30. The cost parameters  $\mu_f$  and  $\mu_j$  are set a common value equal to 0.30.<sup>11</sup> Capital requirement level,  $\phi$ , is set to be

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<sup>11</sup>Similar values are used, among others, in [Carlstrom and Fuerst \(1997\)](#), which refers to the evidence in

0.08 which was the regulatory minimum in the Basel II regime. We set both  $\theta_e$  and  $\theta_b$  to 0.975, implying that bankers and entrepreneurs remain active for ten years on average. Finally, the labor utility parameter,  $\varphi$ , which only affects the scale of the economy, is normalized to one.

In the second step, we target the unconditional moments described in Table 1.<sup>12</sup> Appendix A describes the data counterpart of these model variables. The resulting estimated parameters are summarized in Table 2. As evidenced in Table 1, our model matches the targets reasonably well. We target a number of macroeconomic, financial and banking moments. We target the standard deviations of GDP, investment and consumption growth, the mean ratio of corporate loans to GDP ( $B_t/GDP_t$  in the model) along with the standard deviation of loan growth, the mean and standard deviation of the loans spread ( $R_{f,t} - R_t$  in the model), the mean and standard deviation of ROE of banks ( $\rho_{b,t}$  in the model). Additionally, our moment targets the mean and standard deviation of the conditional expectation of entrepreneurial firm defaults ('firm default')

$$DF_t = \mathbb{E}_t \left( \int_0^\infty \int_0^{\bar{\omega}_{t+1}(\omega_j)} dF_{i,t+1}(\omega_i) dF_{j,t+1}(\omega_j) \right),$$

and the conditional expectation of the bank default rate ('bank default')

$$DB_t = \mathbb{E}_t \left( \int_0^{\bar{\omega}_{j,t+1}} dF_{j,t+1}(\omega_j) \right).$$

Importantly, our model is able to replicate an important feature of the relationship between firm and bank default. As Table 1 shows, the unconditional correlation between firm and bank default in data is both large and positive and our model reproduces it (0.64 in the data versus 0.76 in the model).<sup>13</sup> Matching this correlation turns out to be of first order importance when drawing conclusions about optimal bank capital requirements.

### 3.3 Model Validation

Since we are interested in banking crises, the relationship between firm and bank defaults and their implications for GDP growth are crucial aspects of the data our model should be able to capture. As shown in Table 1, the model is able to match the unconditional moments related to defaults and macroeconomic variables. In this section we perform model validation by comparing the model's implications for important untargeted moments of firm and bank defaults and GDP growth conditional on high and low default realizations. This ensures that

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[Alderson and Betker \(1995\)](#), where estimated liquidation costs are as high as 36 percent of asset value. Among non-listed bank-dependent firms these cost can be expected to be larger than among the highly levered publicly traded US corporations studied in [Andrade and Kaplan \(1998\)](#), where estimated financial distress costs fall in the range from 10 percent to 23 percent. Our choice of 30 percent is consistent with the large foreclosure, reorganization and liquidation costs found in some of the countries analyzed by [Djankov et al. \(2008\)](#).

<sup>12</sup>To avoid the impact of the resource costs of default on the measurement of output, we define  $GDP_t$  as  $GDP_t = C_t + I_t$ .

<sup>13</sup>A similar degree of correlation can be observed in US data.

the model is able to characterize the behavior of the economy in normal and crisis times.

### 3.3.1 Defaults and Economic Performance in the Data

We first of all describe the nature of the co-movements between firm and bank default. Figure 1 is a scatter plot of the Moody’s Expected Default Frequency (EDF) of firms and banks in the EA over the period 1992:M1 to 2016:M12. The x-axis displays the firm EDF and the y-axis displays the bank EDF.<sup>14</sup>

The two default series are positively correlated as successfully matched in the estimation. However, we can also see that the overall positive correlation hides substantial shifts in the relationship between these two series over time. Broadly speaking we can identify three different regimes in the relationship between firm and bank defaults. In the most frequent regime, the default rates of both firms and banks are low. We think of this as “normal times” or “Low Default” regime. In another regime, the firm default rate is high but bank default is modest. We think of this as the “High Firm Default” regime. The last regime is one in which the default rates of both firms and banks are elevated. We call this regime “Twin Defaults.” Following the standard definitions of a systemic banking crisis (see [Laeven and Valencia, 2013](#)), we think of the Twin Defaults regime as corresponding to a financial crisis regime or crisis times.<sup>15</sup>

Our working definition of “high” default is when the EDF of either banks or firms is above the 90th percentile of observations. We therefore define Twin Defaults as the simultaneous occurrence of firm and bank EDFs above their respective 90th percentiles. High Firm Default periods are those where firm EDF is above the 90th percentile and bank EDF is below the 90th percentile. In the Low Default regime, both EDFs are below the 90th percentile.

Figure 1 shows that the correlation between firm and bank default is not only positive and large but also highly state-dependent. There are periods in which the correlation is very low because banks remain very stable despite large fluctuations in the corporate EDF. There are other periods when the co-movement in the two EDF series becomes positive and quantitatively larger. High Firm Default can occur with or without high bank default, whereas there is no evidence of the opposite in the data.<sup>16</sup>

Another way of representing the state-contingent relationship between firm and bank default risk is through quantile regressions of the following form:

$$\text{BankDef}_t(\tau) = \zeta_\tau \text{FirmDef}_t, \quad (44)$$

where  $\text{FirmDef}_t$  is firm EDF and  $\text{BankDef}_t$  is bank EDF. We use quarterly data from 1992:Q1 until 2016:Q2.

The left panel of Figure 2 (red line) plots the quantile regression coefficients  $\zeta_\tau$  in equa-

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<sup>14</sup>Each dot represents a monthly average of the corresponding EDF over one year. The underlying EDFs are provided by Moody’s. See Appendix A for more details on the data.

<sup>15</sup>By definition, there is a fourth regime in the data and in the model, where the bank default rate is high but firm default is modest. We do not comment on that regime because the default rate of firms is similar to the one in the Twin Defaults regime for both the data and the model.

<sup>16</sup>The same pattern can be observed in the US.

tion (44). The non-linearity in the relationship between the two defaults is clearly visible and highly statistically significant. At higher levels of bank default risk, the coefficient obtained by regressing bank on firm defaults is higher. Because the variance of firm and bank defaults is roughly constant across bank default quantiles, the quantile regression coefficients indicate that the correlation between firm and bank default is state dependent and increasing with the bank default rate.

The other feature of the data we are interested in is the relationship between firm and bank defaults on the one hand and aggregate economic activity on the other hand. A simple way to demonstrate this relationship is to look at GDP growth during the different firm and bank default regimes we discussed with the help of Figure 1 above.

Table 3 shows for the EA, US and a number of European countries the average GDP growth rate within the High Firm Default regime (first column) and Twin Defaults regime (second column). Average growth rates have been demeaned using the unconditional mean of GDP growth for each country. The table clearly shows that, for all countries, GDP growth is much lower in the Twin Defaults regime.

Once again, quantile regressions offer another way to represent the non-linear relationship between defaults and macroeconomic outcomes. We investigate the relationship between firm and bank defaults and GDP growth using quantile regressions of the following form

$$\Delta y_t(\tau) = \beta_\tau \text{Def}_{t-1} + \gamma_\tau \Delta y_{t-1}, \quad (45)$$

where  $\text{Def}_{t-1}$  can either be  $\text{FirmDef}_{t-1}$  or  $\text{BankDef}_{t-1}$  and  $\Delta y_t$  represents GDP growth. As before, we use the quarterly data from 1992:Q1 until 2016:Q2.

This quantile regression exercise is similar in spirit to the exercise performed in [Adrian, Boyarchenko and Giannone \(2019\)](#) who run a quantile regression of GDP growth on lagged GDP growth and an index of financial conditions. It is natural for us to use firm and bank defaults as these are the main proxies for financial conditions in our framework. This is why we regress GDP growth on lagged quarter of GDP growth and the lagged level of default ( $\text{Def}_{t-1}$ ) of either firm or bank. The right panel of Figure 2 plots the coefficients from the quantile regression estimated on EA data for either firm (the dashed red lines) or bank (the solid red lines) default. We would like to highlight three key features of the non-linear relationship between defaults and real activity. First, the link between both defaults and economic growth is weak for GDP growth quantiles close to the median. This suggests that defaults (whether bank or firm) have only a weak correlation with GDP growth in normal times.

Second, the negative relationship between bank default and GDP growth becomes quantitatively more negative for the bottom quantiles. Increases in bank defaults have a larger (negative) impact on GDP growth when the economy is already in a recession (i.e. at the bottom quantile for GDP growth).

Third, the above relationship does not hold for firm default. In sharp contrast to the non-linear pattern between bank default and economic activity, the impact of corporate defaults on GDP growth is small and flat across all GDP growth quantiles. Thus, Figure 2 (right panel) clearly shows that it is the risk of bank failures which is driving the deterioration in macro-economic performance during periods of Twin Defaults identified in Table 3. This link

between bank default and economic performance during Twin Default Crises will explain the importance of capital regulation in mitigating this downside risk to the real economy.

### 3.3.2 Model implications

Section 3.3.1 established a number of important facts regarding the state-dependent co-movement between default rates and GDP growth. We learned that the marginal impact of corporate failures on bank solvency is stronger when banks are weaker. We saw that Twin Defaults are associated with deeper recessions. Finally, our results established that the correlation of bank (but not firm) defaults with real activity grows in recessions. These important facts were not targeted in estimation and we now want to compare our model’s implications for these data moments.

In the previous section, we used a 90th percentile-based criterion to identify the Low Default, High Firm Default and Twin Defaults regimes in the EA data. Here, we use  $DF_t$  and  $DB_t$  as the model counterparts for firm and bank EDFs respectively and we employ the same criterion to split the model-simulated time series into the three regimes.

Table 4 compares the model and data averages for firm default, bank default and GDP growth within each regime for the EA data and the model simulated data.<sup>17</sup> The model does a good job of reproducing these untargeted conditional moments. First, it reproduces remarkably well the frequency of the three default regimes. Second, the model reproduces the clear ranking in terms of the performance of GDP growth in the three regimes. The Twin Default regime features by far the worst GDP growth realizations whereas the High Firm Default regime features a relatively mild recession despite the fact that firms’ default rates are very similar.

In the previous section we also used quantile regressions to characterize the non-linear relationships among the two default series and GDP growth. The black lines in Figure 2 show that our model can replicate both quantile regressions remarkably well.<sup>18</sup> The model is qualitatively and quantitatively consistent with all the key facts identified in our description of the quantile regressions on EA data. The correlation between firm and bank default is higher when banks are more fragile and have a higher probability of default. During times of average GDP growth, neither firm nor bank defaults affect economic performance in a significant manner. Bank (but not firm) defaults have a large and negative impact on GDP growth when the economy is already in recession. Despite the fact that these moments were not targeted in estimation, the model delivers a very close match.

## 4. Understanding the Model

In the previous section we saw that our model was able to replicate the co-movement of firm and bank defaults and GDP growth both in the different regimes. Here we dig into the underlying

<sup>17</sup>Table 4 also reports the results for the Merton’s approach model. We will comment on those later.

<sup>18</sup>Regression coefficients for the model are obtained using simulation of the model for 100,000 periods. As before, we use  $DF_t$  and  $DB_t$  as the model counterparts for firm and bank EDFs, respectively.

mechanism in order to highlight the role of several important features of our framework in replicating different features of the EA data.

We start in Section 4.1 with the most important innovation of our paper - the modelling of bank assets as debt claims with limited upside. We explain in more detail how our approach works and how it differs from previous studies before showing that it is essential in matching the data. Then, in Section 4.2, we examine the importance of other model ingredients: the firm and island-idiosyncratic shocks, the two risk shocks (which widen the dispersion of the island and firm-idiosyncratic shocks) and the approximation order to which we solve the model.

## 4.1 Bank Asset Returns

Following Merton (1974), a common approach in modelling default (which we call the Merton approach) has been to use the Black-Merton-Scholes log-normal real options framework. Initially developed to model corporate defaults, this approach assumes that the value of a firm's assets follows a log-normal process and then apply the standard Black-Scholes formula to evaluate the likelihood that the value of the firm falls below the value of debt, triggering default. This approach has also been used in modelling bank default by a number of recent papers where loan portfolios are subject to log-normal shocks (e.g. Clerc et al., 2015; Bege-nau and Landvoigt, 2017; Elenev, Landvoigt and Nieuwerburgh, 2018; Mendicino et al., 2019; Bege-nau, 2019; Jermann, 2019).<sup>19</sup> Assuming log-normally distributed bank asset returns is attractive because it makes the banking problem very tractable. But, as two recent papers (See Nagel and Purnanandam, 2019; Gornall and Strebulaev, 2018) have shown, such a modelling strategy is not appropriate for banks even if the banks are financing underlying projects with log-normal returns. This is because financial intermediaries hold portfolios of loans with asymmetrically distributed payoffs. If borrowers repay, they repay a fixed contractual amount. If they default, the recovery value on loans is limited to a fraction of firms' asset values. Such a downwardly-skewed payoff structure is not consistent with a log-normal distribution.

In an important contribution to the modeling of bank default in macroeconomic models, our paper departs from the Merton approach. Instead, we model banks as holding portfolios of defaultable loans with correlated outcomes, which deliver returns that are endogenously not log-normally distributed.<sup>20</sup> Crucially, we embed this realistic banking framework into an otherwise standard DSGE model. Here we explain how our approach differs from what currently exists in the literature and why using it is important.

To create a Merton-type version of our model that is in line with current literature, we modify Equation (31) in two ways. First, we remove the impact of the island-idiosyncratic shocks by setting them to unity at all times  $\omega_j = 1$ . This is equivalent to assuming that banks are perfectly diversified across borrowers. Second, in order to introduce ex post heterogeneity

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<sup>19</sup>This the modelling of bank assets as equity claims is also shared by models in which banks hold productive capital directly (e.g. Gertler, Kiyotaki and Queralto, 2012; Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2019).

<sup>20</sup>This approach is similar to the single risk factor model formulation of Vasicek (2002): in our setup the island-idiosyncratic shock,  $\omega_j$ , is a risk factor affecting all borrowers in an island. In addition our model also accommodates aggregate productivity shocks that affect borrowers in all islands.



in bank default outcomes, we include a log-normally distributed bank-idiosyncratic shock to bank revenues  $\omega_b$ . The loan portfolio returns under this specification are determined by

$$\tilde{R}_{f,t+1}^M(\omega_b) = [\Gamma_{i,t+1}(\bar{\omega}_{t+1}(1)) - \mu_f G_{i,t+1}(\bar{\omega}_{t+1}(1))] \frac{\omega_b [q_{t+1}(1 - \delta)k_t + y_{t+1}]}{b_{f,t}}. \quad (46)$$

Identically to the island-idiosyncratic shock, also the standard deviation of the distribution of bank-idiosyncratic shock  $\omega_b$  is time-varying and evolves as in Equation 41. We keep the rest of the model identical.<sup>21</sup>

We are now ready to compare our model with the approach taken in Merton-type models. Figures 3 shows the crucial differences between  $\tilde{R}_{f,t+1}(\omega_j)$  and  $\tilde{R}_{f,t+1}^M(\omega_b)$ . In the top left panel of Figure 3 we report  $\tilde{R}_{f,t+1}(\omega_j)$  on the y-axis as a function of the island-idiosyncratic shock  $\omega_j$  on the x-axis. In the top right panel, we depict the distribution of  $\tilde{R}_{f,t+1}(\omega_j)$ . In the bottom left panel of Figure 3 we similarly report,  $\tilde{R}_{f,t+1}^M(\omega_b)$  as a function of the idiosyncratic shock to bank loan revenues and in the bottom right plot we depict the distribution of  $\tilde{R}_{f,t+1}^M(\omega_b)$ .<sup>22</sup>

The top left panel of Figure 3 clearly shows that bank asset returns are highly non-linear in the island-idiosyncratic shock ( $\omega_j$ ). When  $\omega_j$  is very high, all borrowers repay and the bank receives the promised repayment including interest from all its borrowers. But the upside is limited for the lender as is naturally the case under a standard debt contract. However, the presence of default creates downside for the financial intermediary. As the island idiosyncratic shock takes very low values, some firms start to default and bank asset returns begin to decline in a highly non-linear fashion.

The top right panel of Figure 3 shows the distribution of  $\tilde{R}_{f,t+1}(\omega_j)$  which is clearly not log-normal. There is a large spike which occurs when all borrowers repay. Bank asset returns are left-skewed with a fat left tail of low return realizations caused by firm defaults.

In contrast, the bottom panels of the Figure 3 show that the Merton approach produces very different bank asset returns as a function of the exogenous bank-idiosyncratic shock to loan revenues. Since  $\tilde{R}_{f,t+1}^M(\omega_b)$  is linear in  $\omega_b$ , it looks like the returns on an equity portfolio in the sense that the bank experience upside and downside shocks in a fully symmetric fashion. The distribution of returns looks very different too. Since  $\omega_b$  is log-normal,  $\tilde{R}_{f,t+1}^M(\omega_b)$  is log-normal too. Banks in the Merton world are exposed to less risk due to the far smaller left tail. This is important for the model's implications regarding the level of capital requirement required to make banks safe.

Characterizing bank asset returns in an accurate manner is also essential when studying the relationship between firm and bank defaults. The top right panel of Figure 4 plots the scatter plot of firm and bank defaults in the model when we use  $\tilde{R}_{f,t+1}^M(\omega_b)$  instead of  $\tilde{R}_{f,t+1}(\omega_j)$  and compares it to what our model delivers (top left panel). As one can see the correlation between

<sup>21</sup>We estimate the parameters of the Merton-type version of our model to match the set of moments presented in Table 1.

<sup>22</sup>For these figures, we have fixed  $q_{t+1}, k_t, y_{t+1}, b_{f,t}, R_{d,t}, d_t$  to their steady-state values obtained with the parameter values described in Section 3.2. We set  $\sigma_{\omega_b, t+1}$  such that the expected bank default equals its targeted value from Table 1. We use 10,000,000 draws of  $\omega_j$  to plot the histograms.



firm and bank failures is zero when we use  $\tilde{R}_{f,t+1}^M(\omega_b)$  instead of  $\tilde{R}_{f,t+1}(\omega_j)$ .

This should not be a surprise since the shocks used to generate firm and bank default are orthogonal to one another. In contrast, our mechanism treats the two defaults as intimately linked thus helping to endogenously generate an empirically realistic relationship close to that in the data.

Another way to examine the relationship between firm and bank defaults is through quantile regressions. The bottom panels of Figure 4 compare the quantile regression coefficients for Equations (44) and (45) in our model (black line) to those obtained from a Merton-type version of our model (blue line). For completeness, we also include the relationship in the EA data (red line). Yet again, in the Merton-type approach the relationship between firm and bank defaults is close to zero at all quantiles of bank default. Although the Merton-type version of the model does a better job matching the quantile regression coefficients in Equation (45), it still fails to match the correlation between GDP growth and bank default at both the top and the bottom quantiles of GDP growth.

Failing to capture the non-linear relationship between firm and bank default has important implications for the frequency and severity of the Twin Default Regime. In Table 4 we compare the performance of our baseline model with the Merton-type model in terms of the untargeted conditional moments in the three default regimes. There are two main data features which the Merton-type approach cannot reproduce. First, the frequency of the Twin Defaults regime implied by the Merton-type model is significantly lower than the one observed in the data and in our baseline model. Second, the Merton approach underestimates (overestimates) the severity of the Twin Defaults (High Firm Default) regime in terms of GDP growth.

## 4.2 Other Important Features of the Model

Our model with its endogenous connection between firm and bank solvency features a number of idiosyncratic and aggregate risk shocks which are important for the transmission of firm defaults to bank defaults and to the macroeconomy at large. In this section we investigate the importance of each of these shocks. We do this by removing them on an individual basis and then examining the extent to which this deteriorates the model's performance in replicating the quantile regressions in Section 3.1.

### 4.2.1 Island-idiosyncratic and island-risk shocks

We start with the island-idiosyncratic and island-risk shocks. In our framework banks default when they experience abnormally low realizations of the island-idiosyncratic shock. Our model also allows aggregate fluctuations in the non-diversifiable (island) risk by means of island-risk shocks, i.e. shocks to the dispersion of the island-idiosyncratic risk. These shocks increase the probability of very low realizations of the island-idiosyncratic shocks, making banks more vulnerable.

The results of eliminating island-idiosyncratic and island-risk shocks are shown in the top panels of Figure 5. The figure presents the quantile regression coefficients for Equations (44)

and (45) for the model without island-idiosyncratic shocks (green line), i.e. when the island-idiosyncratic shock is set to one, and without the island-risk shocks (blue line). The red and black lines correspond to our baseline model and the data, respectively.<sup>23</sup>

The figure shows that both island-idiosyncratic and island-risk shocks are vital in generating realistic sensitivity of bank to firm defaults and of GDP growth to bank defaults. In the model without island-idiosyncratic shocks the quantile regression coefficients go to zero because banks are perfectly diversified and their loan portfolio returns are very stable. Firms continue to default because of the firm-idiosyncratic shocks but banks are diversified against these shocks. And while aggregate shocks induce some fluctuations in firm default, these are too small to make banks fail since our banks' solvency is protected by their equity buffers. Thus, if the bank is fully diversified, bank defaults do not happen and cannot possibly affect GDP growth. The model without island-risk shocks shows that, although eliminating this shock does not lead to fully diversified banks, keeping the non-diversifiable risk (and hence the probability of bank default) low and relatively constant reduces the capability of the model of matching the sensitivity of bank to firm defaults and of GDP growth to bank defaults. Clearly, the model without island-risk shocks, although it does a better job than the model without island-idiosyncratic shocks, fails to generate the state dependent relationship between firm and bank defaults and economic activity that we see in the data.

This experiment clearly indicates the importance of both island-idiosyncratic and island-risk shocks in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When the non-diversifiable risk is constant (no island-risk shocks), bank defaults are rare, they are mostly unaffected by firm defaults and they do not affect real economic activity. When non-diversifiable risk is absent (no island-idiosyncratic shocks) banks do not default.

#### 4.2.2 Firm-idiosyncratic and firm-risk shocks

The other source of risk to firms in our model comes from firm-idiosyncratic and firm-risk shocks, i.e. shocks to the dispersion of the firm-idiosyncratic risk. These shocks capture risks to individual firms which are diversifiable at the individual bank level. The firm-risk shocks increase firm defaults but they affect different banks evenly rather than concentrating the bulk of losses on a few unlucky banks as for island-risk shocks.

In this section we investigate how the ability of the model to replicate the quantile regression coefficients for Equations (44) and (45) changes when we eliminate the firm-idiosyncratic and -risk shocks. The middle panels of Figure 5 show the results. This time the green line displays the quantile regression coefficients in the model where we set the firm-idiosyncratic shock equal to unity for all firms while the blue presents the results from the model where firm-risk shocks are shut down.

Both the green and blue lines display a relationship between firm and bank defaults. Intuitively, the green lines in middle panels of Figure 5 correspond to an economy with fully non-diversified banks in which the defaults of banks and firms are almost perfectly correlated.

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<sup>23</sup>Note that when we eliminate the island-idiosyncratic shocks, the island-risk shocks became irrelevant.

This makes the sensitivity of bank to firm default very large and rather constant over states. The impact of shutting down the firm-risk is qualitatively similar to the elimination of the firm-idiosyncratic shocks but not as quantitatively large with respect to the the quantile regression coefficients for Equations (44). The right middle panel shows that eliminating either firm-idiosyncratic or the firm-risk shocks generates the state dependent relationship between bank defaults and economic activity that are too weak compared with the data and our baseline model.

This experiment clearly indicates the importance of both firm and island shocks in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When we eliminate non-diversifiable risk (no island shocks) the conditional and unconditional correlation between firm and bank default is too small. Instead, when we eliminate diversifiable risk (no firm shocks) the conditional and unconditional correlation between firm and bank default is too large. In both instances the conditional and unconditional correlation between bank default and economic activity is too low.

### 4.2.3 Approximation order

Finally, we investigate the role of our solution method by comparing the quantile regressions implied by our baseline model (which is solved using third order approximation) with the quantile regressions implied by first-order (green lines) or second-order (blue lines) approximate solutions.<sup>24</sup> The bottom panels of Figure 5 shows the results.

Both the linear and the second-order model clearly fail to match the non-linearities found in the data. They generate flat quantile regression coefficients in both panels. Intuitively, a model solved to first or second order works well in normal times but fails to generate the sharp and non-linear deterioration of economic and financial conditions in crises or recessions. In contrast, a third order approximation captures the non-linearity in the co-movements of firm and bank defaults and economic activity.

We already discussed in Section 4.1 that modelling bank portfolios as consisting of defaultable loans introduces an important non-linearity into bank asset returns and hence into bank default realizations. It is therefore natural that a non-linear solution method is needed to capture such non-linearities in an accurate manner. Our results show that a third-order solution is sufficient for this purpose.

Table 4 also reports the performance of the linear approximated version of the model in terms of untargeted conditional moments in the three default regimes. As we pointed out before, capturing the non-linear relationship between firm and bank default has important implications for the frequency and severity of the Twin Default Regime. Indeed, the frequency of the Twin Defaults regime reproduced by the first-order model is somewhat lower than the one observed in the data and in our baseline model. In addition, it underestimates (overestimates) the severity of the Twin Defaults (High Firm Default) regime in terms of GDP growth.

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<sup>24</sup>We estimate the parameters of the first and second-order approximation versions of our model to match the set of moments presented in Table 1.

## 5. The Anatomy of Twin Default Crises

In what follows we investigate how the model generates episodes of Twin Defaults. The results confirm the importance of island-risk shocks and reveal the ability of the model to endogenously propagate them. In fact, under high leverage and a solution method that captures non-linearities, the model does not require large shocks to generate severe but rare episodes of Twin Defaults Crises. This analysis will lead us to the policy section where we examine the role of in the capital requirement in helping preventing them.

### 5.1 Average Path to Crisis

Figure 6 shows the average path leading to High Firm Default (blue line) and Twin Default (red line) regimes. The figure is generated by simulating the model for 1.000.000 periods, identifying periods in which defaults are above the 90th percentile and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods.

Several messages stand out from the figure. First of all, the model implies that Twin Default episodes generate output falls which are twice as large compared to High Firm Default events. Second, the model captures remarkably well the evolution of bank defaults for both regimes. Bank defaults rise above four percent during Twin Defaults, which is very close to what we observe in the EA data during the recent financial crisis. In contrast, bank failures remain below one percent in the episodes of High Firm Defaults. Both cases are very close to the evidence reported in Table 4. The declines in output, investment and lending are more pronounced in the case of Twin Defaults when compared with High Firm Defaults.

The model is also consistent with the empirical finding ([Schularick and Taylor, 2012](#); [Jorda, Schularick and Taylor, 2016](#)) that financial crises tend to be preceded by above average economic activity and lending. In the Twin Defaults regime, output, bank lending, bank capital and consumption reach a cyclical peak around 8-10 quarters before the crisis event and fall sharply as it approaches. In our model, the pre-crisis boom is mainly driven by good (below average) realizations of the firm-risk shock. As a result, corporate leverage is elevated, making firms (and hence banks) more vulnerable to subsequent adverse shocks.

The figure also shows that, on average, TFP remains broadly unchanged in both types of episodes. The two risk-shocks instead play an important role. The rise in the firm-risk shocks plays a role in both High Firm Defaults and Twin Defaults, while the rise in the island-risk shocks plays a key role in generating Twin Default Crises.

Finally, the model does not need very large risk shocks in order to generate a Twin Default Crisis. These episodes occur following a sequence of small and positive risk shocks that accumulate into a 1.5 standard deviation increase. As the figures shows the island-risk shocks are crucial to generate bank defaults and, therefore, Twin Default Crisis. Firm-risk shocks by themselves can only create High Firm Default events. The fact that our baseline model does not need very large risk shocks explains why our model can match the frequency of the Twin Default regime in the data.

## 5.2 Generalized Impulse Response Functions to an Island-risk Shock

We now use the GIRFs to show that the economy “accelerates” into a Twin Default event as the impact of additional island-risk shocks grows. This internal propagation helps the model generate Twin Default crises without the need for huge shocks. Figure 7 reports three sets of GIRFs to a one standard deviation island-risk shock. The solid line shows the unconditional GIRF, the blue dashed line shows the GIRF conditional on the economy being at a High Firm Defaults episode, and the red dashed line shows the GIRF conditional on the economy being in a Twin Default episode. Details on how to compute both conditional and unconditional GIRFs can be found in Appendix F.

Clearly, the shock has a much larger impact when conditioning on either a Twin Default or a High Firm Defaults episode.<sup>25</sup> The GDP drop is much larger than the effect in the unconditional GIRF. The same is true for the investment and capital price drop and for the impact on firm and bank defaults. This shows how the model solved with a third order approximation is able to amplify island-risk shocks during crisis times differently than during normal times. In our model, once the economy finds itself in a situation of High Firm Default, it becomes very vulnerable to island-risk shocks. In any case the GIRFs conditional on only High Firm Default shows much less amplification than when we condition on a Twin Default episode.

## 5.3 Non-linear Effects

What are the key aspects of the model and its solution that help us generate a path to crisis as shown in Figure 6? In this section, we focus our attention on a key elements of our setup: the non-linear solution method. The importance of non-linearities is confirmed by the comparison, in Figure 8, of the baseline path to crisis (solid lines) with that obtained with a first-order approximation (dashed lines).<sup>26</sup>

The figure shows that the first-order approximation needs very large (and therefore highly unlikely) island-risk shocks needs to increase by three standard deviations in the first-order approximation rather than only 1.5 standard deviations in the baseline. It is also the case that, despite the large shocks, the first-order approximation cannot generate a realistic increase in the probability of bank default. This large shocks explain why the linear model can only produce Twin defaults with four percent probability, while the frequency implied by the baseline model is very closed to the observed one of seven percent.

## 5.4 The Role of Bank Leverage

Bank leverage is the second key aspect of the model we want to focus on. Figure 8 also compares the baseline path to crisis (solid lines) – that has a eight percent bank capital requirements –

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<sup>25</sup>It is important to note that the traditional linear IRFs are independent of the state of the economy.

<sup>26</sup>As it was the case in Table 4, the thresholds used to define the three regimes are always the ones determined by the baseline model.

with a path implied by a model with higher bank capital requirement of fifteen percent (dashed line).<sup>27</sup>

The figure clearly shows that the model with a fifteen percent bank capital requirement needs a 3.5 standard deviations increase in the island-risk shock in order to generate a Twin Default crisis. Table 4 shows the performance of the model with higher capital requirements in terms of untargeted conditional moments in the three default regimes. The fact that the model requires such a large shocks explains why the frequency of the Twin Defaults (High Firm Default) regime is dramatically lower (higher) under the fifteen per cent capital requirement. In the following section, we will see that imposing such a capital requirement is welfare improving due to the high costs associated with Twin Default Crises.

## 6. Implications for Capital Requirements

We conclude our analysis by examining the model’s implications for the socially optimal level of bank capital requirements. Figure 9 shows the implications of different values of the capital requirement on the mean of the ergodic distribution of selected variables for our baseline model.<sup>28</sup>

The figure shows that the imposition of higher capital requirements faces a trade-off between reducing the probability of Twin Default Crises, and maintaining the supply of bank credit in normal times. Higher capital requirements make bank equity returns better protected against the island-risk shocks. When banks are less levered, non-diversifiable risk has a lower impact on banks’ equity and this reduces banks default, as well as, the correlation between firm and bank default. As a result, Twin Default crises become less frequent and deadweight losses associated with asset repossession costs decline. However, higher capital requirements are also costly for the economy. They increase the relative scarcity of bank net worth and the average cost of bank funding. This implies higher borrowing costs, reduced bank credit, and lower investment.

The trade-off is reflected in the overall effects of higher capital requirements on welfare. The solid black line in Figure 10 reports the ergodic mean of household welfare as a function of the level of bank capital requirement. The optimal bank capital requirement is around fifteen percent which is associated with welfare gains of approximately 0.1 percent in certainty equivalent consumption terms relative to the baseline model where the bank capital requirement was calibrated to be eight percent.

Starting from the eight percent capital requirement, welfare first increases because the gains from the reduction in the probability of bank default outweigh the losses from imposing higher funding costs on banks. At the optimum, the probability of bank default is below 0.1 percent and further reductions in bank failures have a limited impact on welfare. For a capital requirement above fifteen percent, the negative effect of elevated borrowing costs for firms dominates and welfare declines.

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<sup>27</sup>As it was the case in Table 4, the thresholds used to define the three regimes are always the ones determined by the baseline model.

<sup>28</sup>We consider changes in the capital requirement  $\phi$  while keeping all other parameters unchanged.



In what follows we explore how the prescriptions regarding the capital requirement depend on the importance of island-idiosyncratic shocks and on the adoption of the island approach instead of a Merton-type framework. As shown above, these are crucial elements for the occurrence of Twin Default Crises.

To highlight the importance of non-diversifiable island-idiosyncratic shocks on the results, we increase the average standard deviation of the island-idiosyncratic shock and reduce the average standard deviation of the firm-idiosyncratic shock while keeping the probability of firm default unchanged. In this way, we reduce the extent to which banks can diversify away the firm default risk.<sup>29</sup> The red dashed line in Figure 10 shows household welfare when the island-idiosyncratic shock is more important in the determination of firm default and bank failures are, thus, more frequent. The optimal capital requirement is substantially higher; close to twenty per cent.

Figure 10 also reports welfare as a function of the capital requirement under a Merton-type specification of bank asset returns (blue dashed line). This version of the model underestimates the gains from having higher capital requirements with respect to our baseline model. The optimal capital requirement is significantly lower under the Merton-type formulation (around eleven percent). We calibrate the Merton-type model so as to ensure that the mean and standard deviation of bank default are the same as in our baseline model. This implies that the mean deadweight loss due to bank default is the same in both models. However, as shown in Section 4., the correlation between firm and bank default in this version of the model is substantially lower. This means that high bank defaults are not necessarily accompanied by firm defaults and the overall macroeconomic losses (and consumption declines) associated with bank default are lower. As a result, the Merton-type framework implies a lower optimal level of the bank capital requirement.

## 7. Conclusions

We build a quantitative macro model with banks which micro-founds the link between borrower and bank defaults. The model reproduces key Euro Area data moments and in particular the unconditional and conditional correlation of firm and bank defaults and their relationship with output. Moreover, the model is able to match the frequency and severity of Twin Default Crises characterized by very high firm and bank defaults as well as deep recessions. Aggregate shocks to non-diversifiable sources of borrowers' default risk for banks play an important role in driving the economy into such crises.

We make an important contribution to the modelling of banks in DSGE by properly characterizing the non-linear nature of loan portfolio returns featuring a limited upside but downside risks due to borrower default risk. These non-linearities (captured by solving the model using

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<sup>29</sup>The average standard deviation of the island-idiosyncratic shock is increased by 10 percent, whereas the average standard deviation of the firm-idiosyncratic shock is reduced by 6.3 percent. While the average probability of firm default remains equal to 2.25 percent, the probability of bank default increases from 0.59 percent to 1.03 percent. The probability of Twin Default Crises increases from 5.8 percent to 8.8 percent.

a third order approximation) turn out to be key to generating Twin Default Crises of realistic frequency and severity.

Turning to the model’s policy implications, we show that bank leverage is crucial for the frequency and severity of financial crises. Crisis frequency can therefore be reduced by imposing a higher capital requirement on banks at the cost of reducing credit and output in normal times. We find that a capital requirement of fifteen per cent optimally trades off lower crisis frequency (benefits of higher capital) against lower credit in normal times (costs of higher capital).

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Table 1: Targeted Moments: Baseline Model

Variable	Data	Model	Variable	Data	Model
Std GDP growth	0.6877	0.6217	Std Cons. growth	0.5617	0.4912
Mean Loans/GDP	2.442	2.6386	Std Loan growth	1.1965	0.5936
Mean Loan spread	1.2443	1.0058	Std Loan spread	0.6828	0.7535
Mean Firm default	2.6469	2.2497	Std Firm default	1.0989	2.4384
Mean Bank default	0.6646	0.5860	Std Bank default	0.8438	1.2320
Mean ROE banks	6.4154	6.4652	Std ROE banks	4.1273	3.7971
Corr (FD & BD)	0.6421	0.7648	Std Inv. growth	1.3908	2.0106

*Notes:* Interest rates, equity returns, default rates, and spreads are reported in annualized percentage points. The standard deviation (Std) of GDP growth, Investment (Inv), and Loan growth are reported in quarterly percentage points.

Table 2: Estimated Parameters

Par.	Description	Value	Par.	Description	Value
$\chi_b$	Bankers' endowment	0.6520	$\chi_e$	Entrepreneurs' endowment	0.5435
$\bar{\sigma}_{\omega_i}$	Mean firm-risk shock	0.3447	$\bar{\sigma}_{\omega_j}$	Mean island-risk shock	0.2625
$\sigma_A$	Std TFP shock	0.0044	$\rho_A$	Persistence TFP shock	0.9804
$\sigma_i$	Std firm-risk shock	0.0729	$\rho_{\sigma_i}$	Persistence firm-risk shock	0.9141
$\sigma_j$	Std island-risk shock	0.0936	$\rho_{\sigma_j}$	Persistence island-risk shock	0.7539
$\psi_k$	Capital adjustment cost	1.9942			

*Notes:* The reader should note that  $\sigma_i$  is not the standard deviation of firm-risk shock which is  $\frac{\sigma_i}{\sqrt{1-\rho_{\sigma_i}^2}}$ . The

same applies for the standard deviation of island-risk shock.

Table 3: Average Quarterly GDP growth (demeaned)

	High Firm Def.	Twin Defaults
Euro Area	-0.0466	-0.5842
Germany	-0.2550	-0.6690
France	-0.0718	-0.6605
Italy	-0.0242	-0.5471
Netherlands	-0.5043	-2.1904
Belgium	-0.3645	-0.4051
US	-0.0781	-0.9790

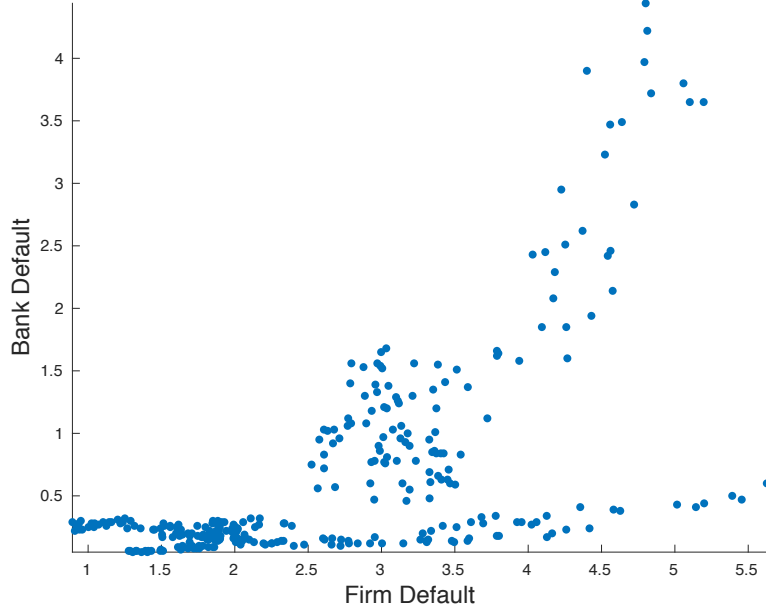
*Notes:* First column refers to periods of high firm defaults and low bank defaults, whereas the second column uses periods of Twin Defaults. Growth rates are reported in quarterly rates. Sample: EA 1992Q1-2016Q4, US: 1940:Q1-2016:Q4.

Table 4: Three Defaults Regimes

	Frequency	GDP growth	Bank default	Firm default
Low Default Regime				
Data	86.0%	0.0923	0.4346	2.3480
Baseline Model	85.9%	0.0588	0.2388	1.5154
Merton-type Model	82.1%	0.0522	0.3351	1.7118
1st Order App.	87.3%	0.0318	0.3130	1.7113
Higher Cap. Req.	91.5%	0.0293	0.0453	1.5406
High Firm Default Regime				
Data	4.0%	-0.0466	0.4033	4.8500
Baseline Model	4.0%	-0.1344	0.9270	6.8463
Merton-type Model	7.5%	-0.1887	0.3980	7.4093
1st Order App.	1.0%	-0.1556	1.4211	5.5756
Higher Cap. Req.	8.3%	-0.3079	0.3387	7.693
Twin Defaults Regime				
Data	7.0%	-0.8189	3.0224	4.6076
Baseline Model	5.9%	-0.5737	4.0225	8.6266
Merton-type Model	1.1%	-0.5399	3.2624	7.7483
1st Order App.	4.1%	-0.2735	2.4816	6.0247
Higher Cap. Req.	0.1%	-0.8990	2.2445	11.833

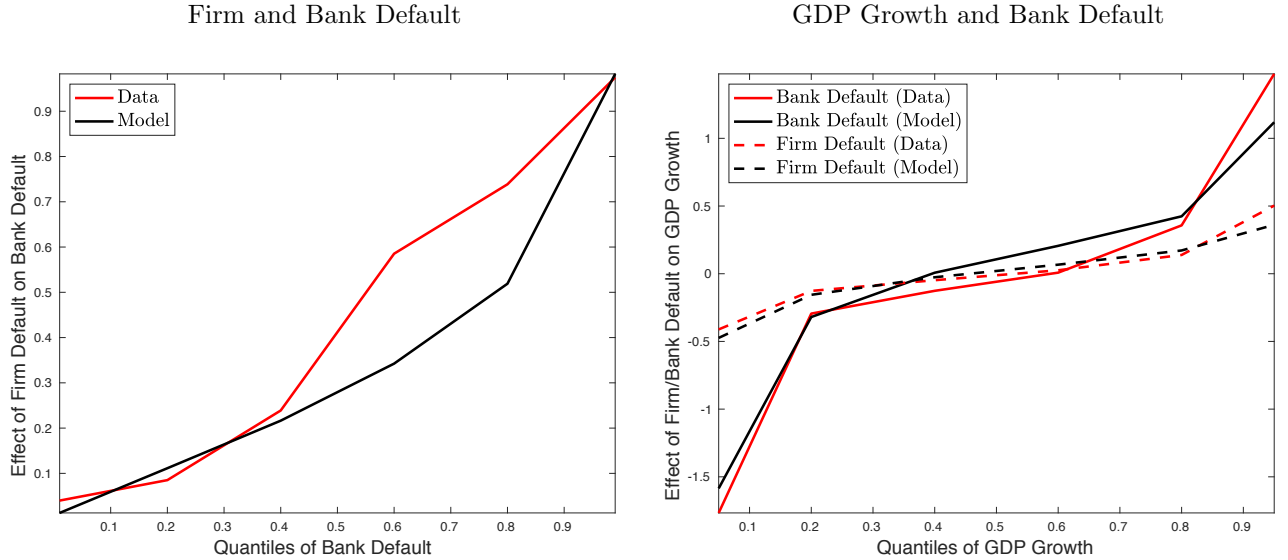
*Notes:* This Table compares the model and data averages for firm default, bank default and GDP growth within three default regimes for the EA data and the simulated data from different models. Baseline model corresponds to capital requirement set to 8 percent ( $\phi = 0.08$ ) and solved with third order perturbation. Merton-type Model corresponds to the model in which the Merton-type specification of bank asset returns is adopted. 1st Order App. corresponds to the model solved with first order perturbation methods. Higher Cap. Req. corresponds to the model with capital requirement set to 15 percent ( $\phi = 0.15$ ). Twin Defaults episodes are defined as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High Firm Default are episodes with firm default above the 90th percentile and bank default below the 90th percentile. In Low Default episodes, both bank and firm default are below the 90th percentile. The default thresholds used to define the three regimes in the Merton-type model and the 1st Order App. model are the ones determined by the baseline model. Model results are based on 1,000,000 simulations. GDP growth is demeaned.

Figure 1: Firm and Bank Default Rates



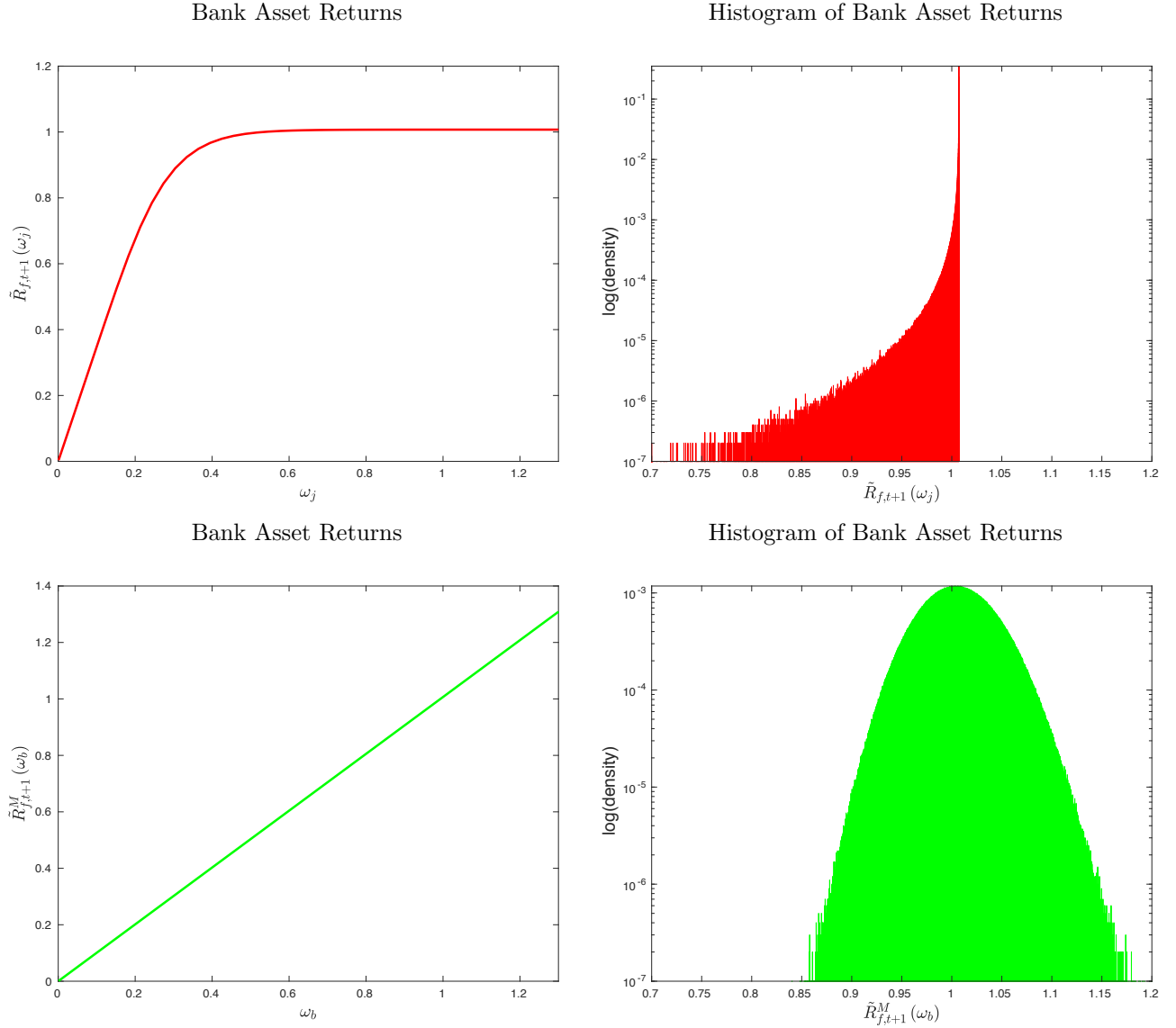
Notes: Scatter plot of Moody's cross-sectional average of expected default frequencies (EDF) within one year for a sample of firms (non-financial corporations) and banks in the EA during the period 1992:M1 to 2016:M12 at a monthly frequency. Both series are in percent.

Figure 2: Quantile Regression: Data vs Model



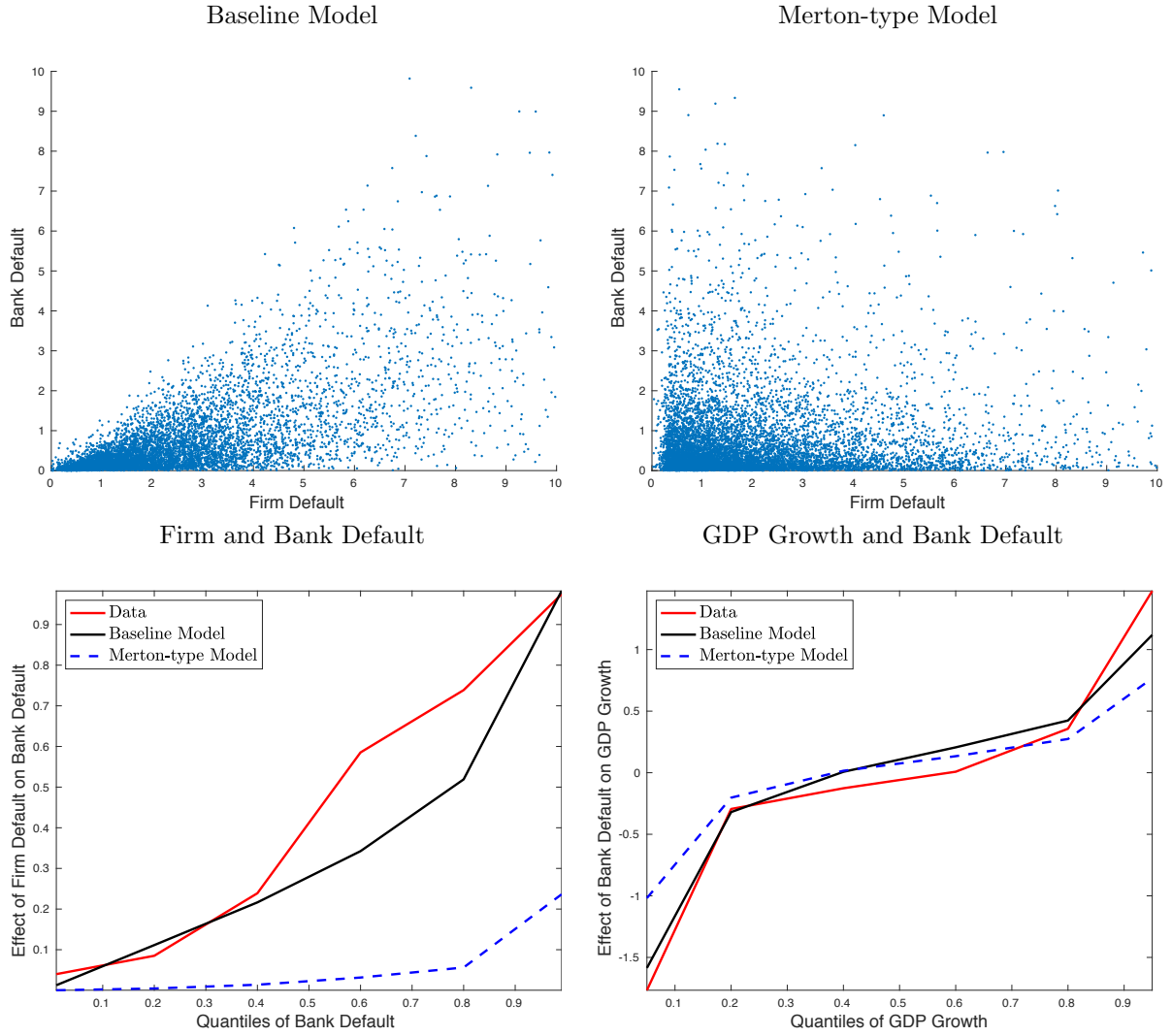
Notes: Left panel of this figure presents coefficients  $\zeta_\tau$  from the quantile regression in Equation (44). Right panel of this figure presents coefficients  $\beta_\tau$  from the quantile regression in Equation (45). Both equations are estimated on EA data (1992-2016) and on simulated data from the baseline model.

Figure 3: Bank Asset Returns: Baseline vs Merton-type Model



Notes: Top panels of this figure present bank asset returns as a function the non-diversifiable island shock  $\omega_j$  (left plot) and the histogram of bank asset returns (right plot) in the baseline model. Bottom panels of this figure present bank asset returns as a of function the bank-idiosyncratic shock  $\omega_b$  (left plot) and the histogram of bank asset returns (right plot) in the Merton-type version of our model.

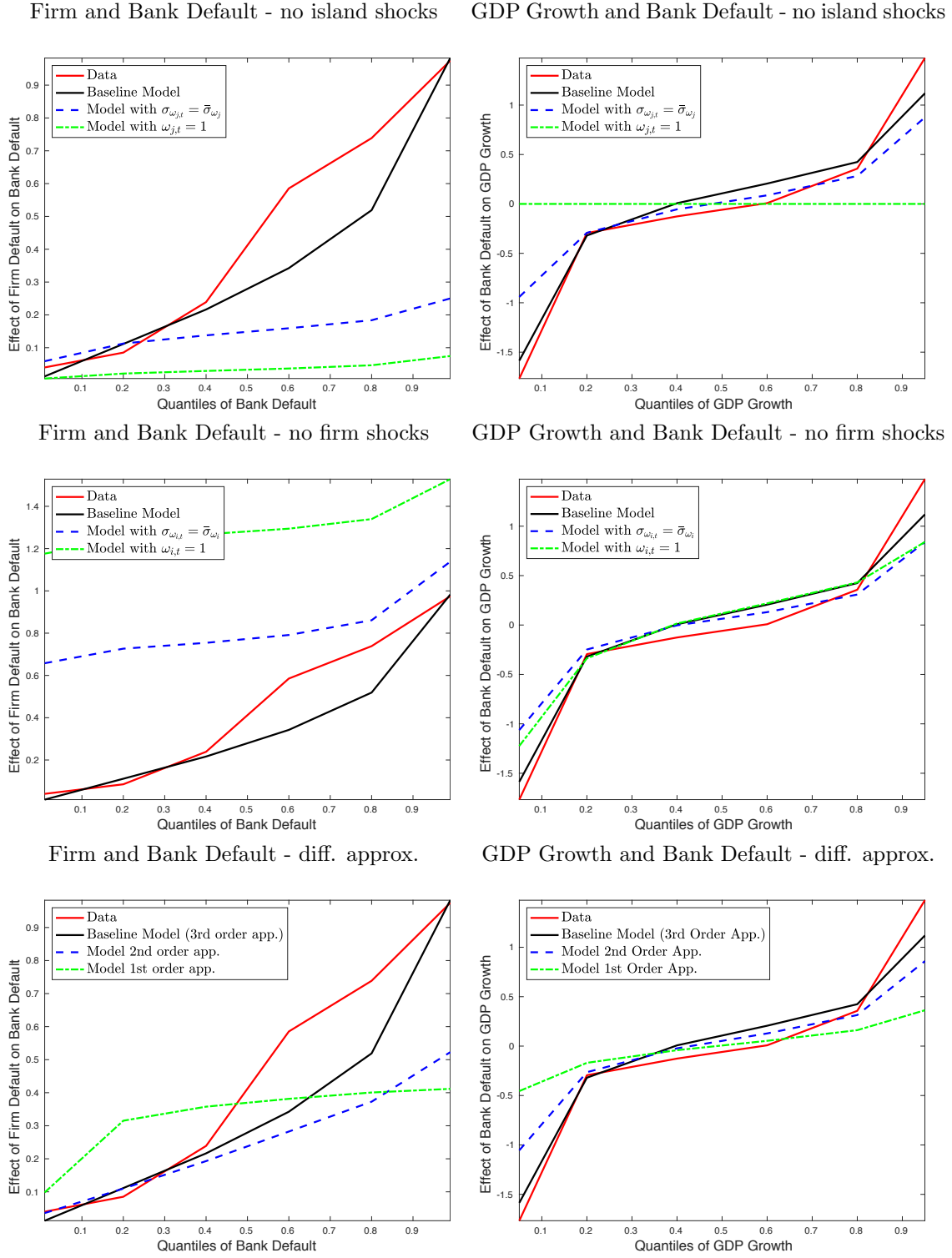
Figure 4: Scatter Plots and Quantile Regressions: Baseline vs Merton-type Model



Notes: Top panels display the scatter plot of firm and bank default produced with the baseline model (top left plot) and the Merton-type version of our model (top right). Bottom left panel presents coefficients  $\zeta_\tau$  from the quantile regression in Equation (44), whereas bottom right panel presents coefficients  $\beta_\tau$  from the quantile regression in Equation (45) for the Merton-type model (blue line), the data (red line) as well as the baseline model (black line).

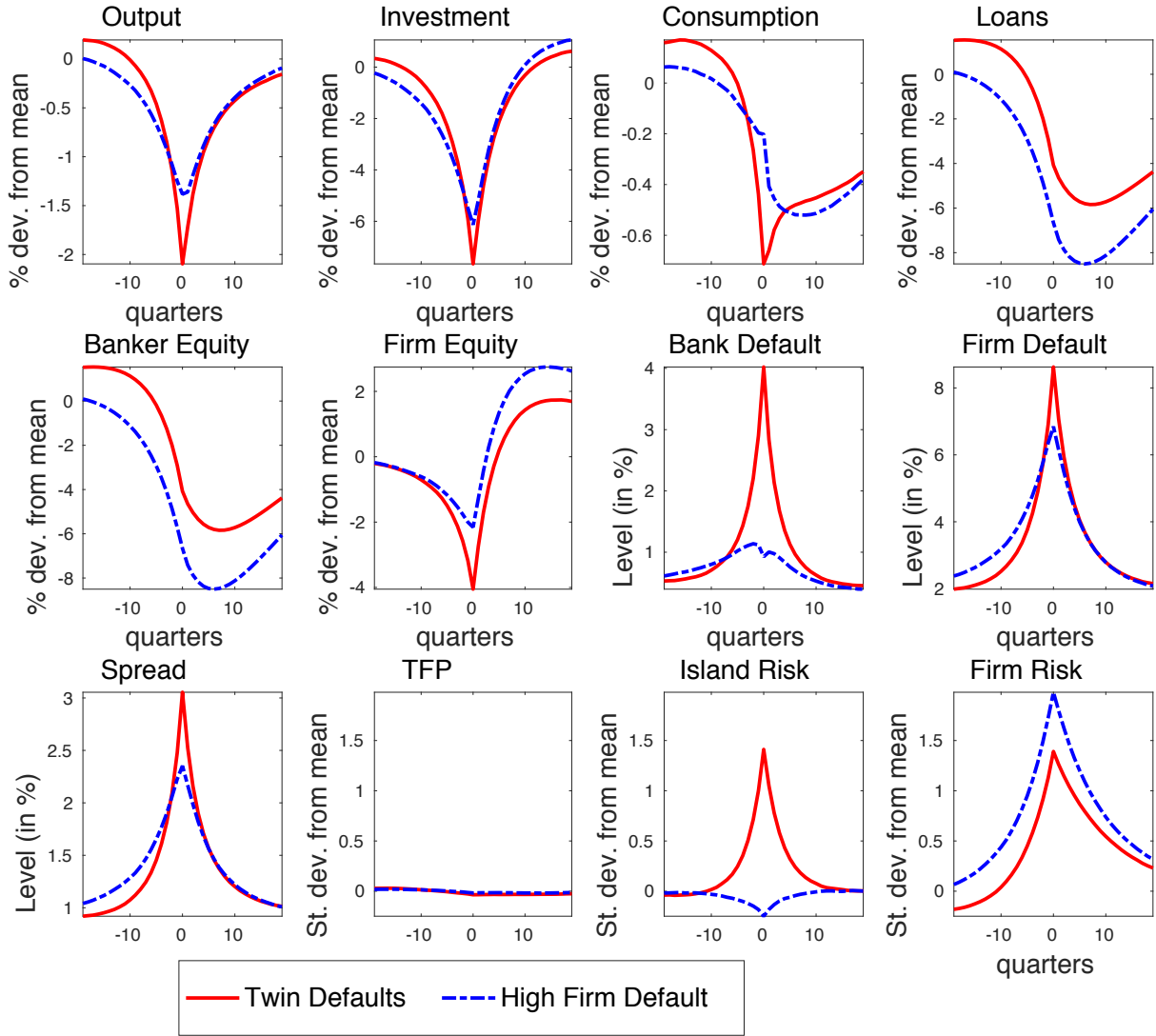


Figure 5: Quantile Regressions: Key Model Features



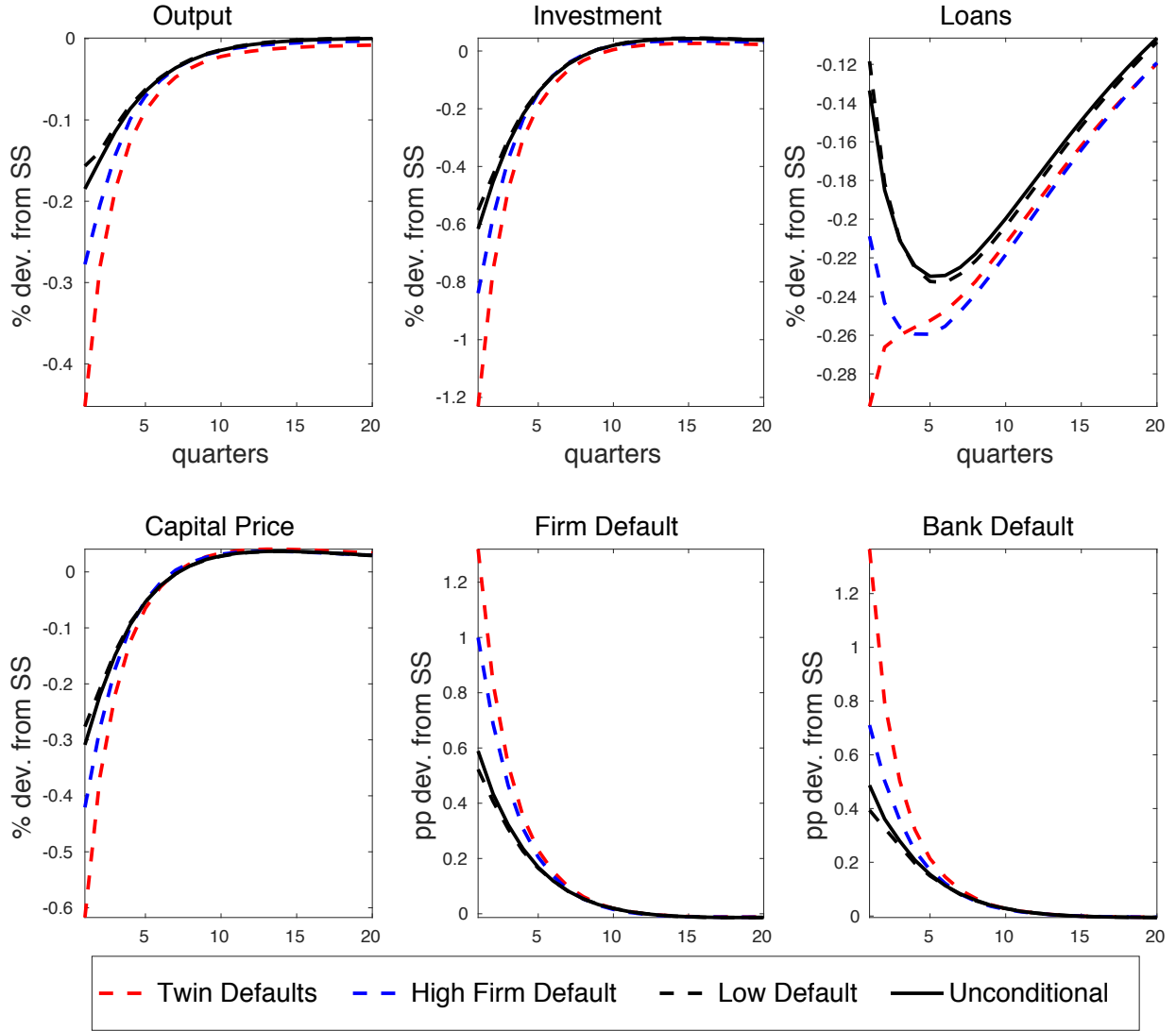
Notes: The figure explores the importance of non-diversifiable risk (top panels), diversifiable risk (middle-panels) and approximation order (bottom panels). Right column presents coefficients  $\zeta_\tau$  from the quantile regression in Equation (44), while right column presents coefficients  $\beta_\tau$  from the quantile regression in Equation (45).

Figure 6: Paths to Crises



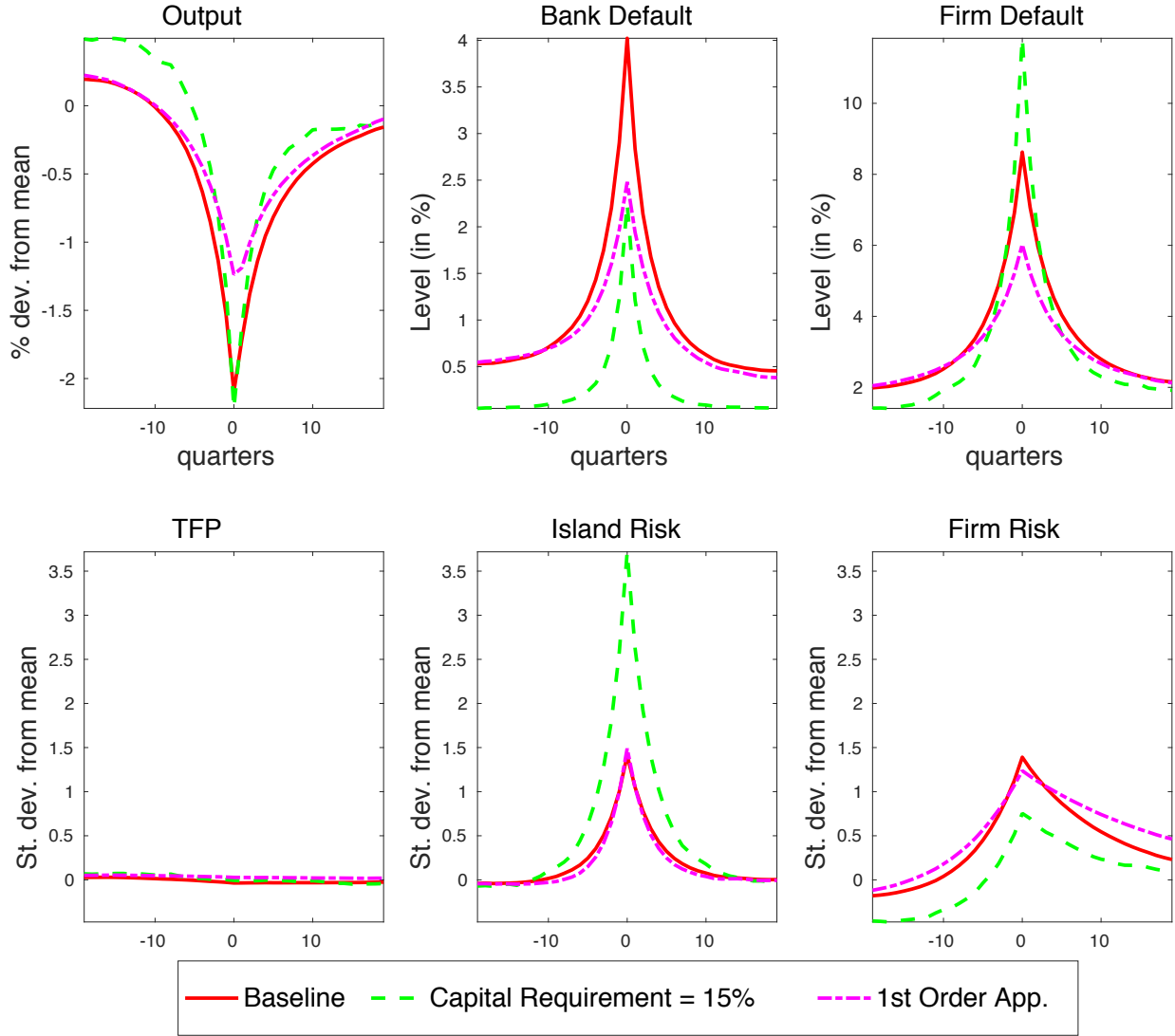
Notes: This figure shows the average path leading to High Firm Default (blue dashed line) and Twin Defaults (red solid line) regimes. The figure is generated by simulating the model for 1.000.000 periods, identifying periods in which defaults are above the 90th percentile and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define Twin Defaults as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High Firm Default periods are those where firm default is above the 90th percentile and bank default is below the 90th percentile. TFP, Island Risk and Firm Risk represent the level of  $A_t$ ,  $\frac{\sigma_{\omega_j, t+1}}{\bar{\sigma}_{\omega_j}}$  and  $\frac{\sigma_{\omega_i, t+1}}{\bar{\sigma}_{\omega_i}}$  in their respective standard deviation units.

Figure 7: Conditional Impulse Response Functions: Island Risk Shock



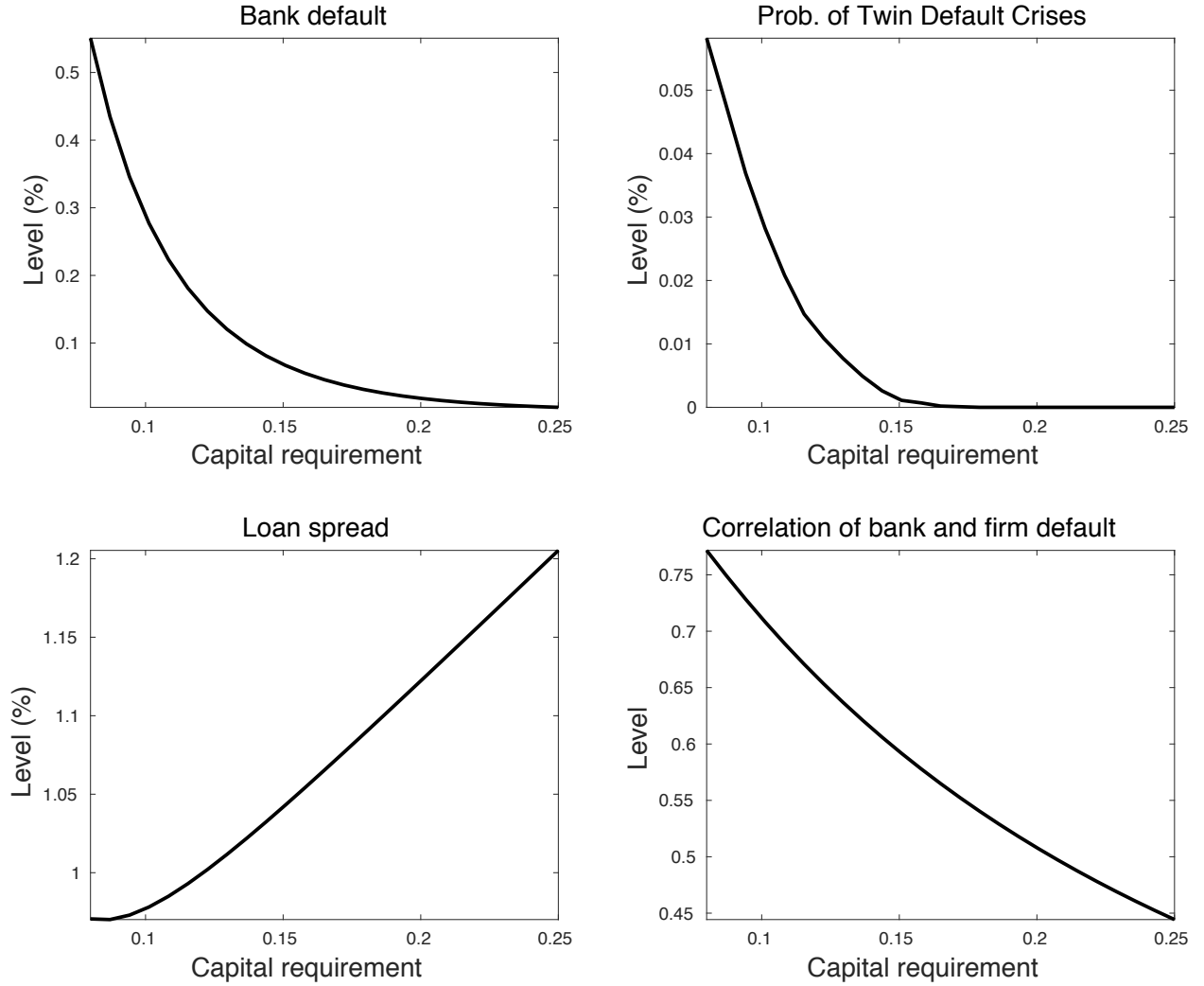
Notes: This figure reports three sets of generalized impulse response functions (GIRFs) to a one standard deviation island-risk shock. The solid black line shows the unconditional GIRF, the black dashed line shows the GIRF conditional on the economy being in a Low Default episode, the blue dashed line shows the GIRF conditional on the economy being at a High Firm Default episode, and the red dashed line shows the GIRF conditional on the economy being in a Twin Defaults episode. We define a Twin Defaults episode as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High Firm Default episodes are those where firm default is above the 90th percentile and bank default is below the 90th percentile. In the Low Default regime, both bank and firm default are below the 90th percentile. Details on how to compute both conditional and unconditional GIRFs can be found in Appendix F.

Figure 8: Path to Twin Defaults in Different Scenarios



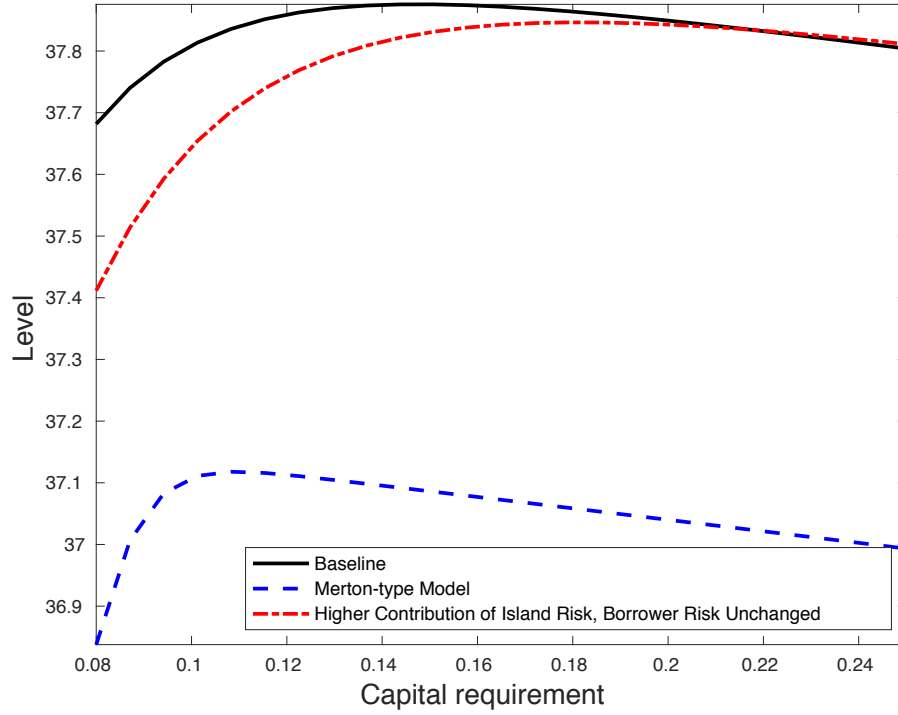
Notes: This figure shows the average path leading to a Twin Defaults episode under different model assumptions. Baseline case (red, solid line) corresponds to our baseline model with capital requirement set to 8 percent ( $\phi = 0.08$ ) and solved with third order perturbation methods. Capital Requirement = 15% case (green, dashed line) corresponds to the model with capital requirement set to 15 percent ( $\phi = 0.15$ ). 1st Order App. case (pink, dashed-dotted line) corresponds to the model solved with first order perturbation methods. The figure is generated by simulating the model for 1.000.000 periods, identifying periods of Twin Defaults and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define a Twin Defaults episode as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. The 90th percentile default thresholds used to define the three regimes in the four models are always the ones determined by the baseline model. TFP, Island Risk and Firm Risk represent the level of  $A_t$ ,  $\frac{\sigma_{\omega_{j,t+1}}}{\sigma_{\omega_j}}$  and  $\frac{\sigma_{\omega_{i,t+1}}}{\sigma_{\omega_i}}$  in their respective standard deviation units.

Figure 9: Comparative Statics with Respect to Capital Requirement Level



Notes: This Figure shows the implications of different values of the capital requirement  $\phi$  on the mean of the ergodic distribution of selected variables for our baseline model.

Figure 10: Welfare Effects of the Capital Requirement in Different Scenarios



Notes: This figure reports the ergodic mean of household welfare as a function of the level of bank capital requirement in different scenarios. Baseline (black solid line) corresponds to our baseline model. Merton-type model (blue dashed line) corresponds to the model in which Merton-type specification of bank asset returns is adopted. Higher Contribution of Island Risk, Borrower Risk Unchanged (red dashed-dotted line) corresponds to the model in which we increase the average standard deviation of the island-idiosyncratic shock by 10 percent and reduce the average standard deviation of the firm-idiosyncratic shock by 6.3 percent while keeping the probability of firm default unchanged. While the average probability of firm default remains equal to 2.25 percent, the probability of bank default increases from 0.59 percent to 1.03 percent and the probability of a Twin Defaults Crisis increases from 5.8 percent to 8.8 percent.

## Appendix A Data

- Investment: Gross Fixed Capital Formation, Millions of euros, Chain linked volume, Calendar and seasonally adjusted data, Reference year 1995, Source: the Area Wide Model (AWM) dataset.
- Gross Domestic Product (GDP): we define the GDP as the sum of Consumption and Investment.
- Loans: Outstanding amounts of loans at the end of quarter (stock) extended to non-financial corporations by Monetary and Financial Institution (MFIs) in EA, Source: MFI Balance Sheet Items Statistics (BSI Statistics), Monetary and Financial Statistics (S/MFS), European Central Bank.
- Loan Spread: Spread between the composite interest rate on loans and the composite risk free rate. We compute this spread in two steps.
  1. Firstly, we compute the composite loan interest rate as the weighted average of interest rates at each maturity range (up to 1 year, 1-5 years, over 5 years).
  2. Secondly, we compute corresponding composite risk free rates that take into account the maturity breakdown of loans. The maturity-adjusted risk-free rate is the weighted average (with the same weights as in case of composite loan interest rate) of the following risk-free rates chosen for maturity ranges:
    - 3 month EURIBOR (up to 1 year).
    - German Bund 3 year yield (1-5 years).
    - German Bund 10 year yield (over 5 years for commercial loans).
    - German Bund 7 year yield (5-10 years for housing loans).
    - German Bund 20 year yield (over 10 years for housing loans).

Source: MFI Interest Rate Statistics of the European Central Bank, Bloomberg.

- Return on equity of banks: Bank Equity Return (after tax), EA. Source: Global Financial Development, World Bank.
- Expected default of Banks: Asset weighted average of EDF within one year for the sample of banks in EA. The data comes on the monthly basis. We aggregate it to quarterly series by averaging the monthly series within a quarter. <sup>30</sup> Source: Moody's KMV.
- Expected default of non-financial firms: we compute it using Moody's EDF series for a sample of non-financial corporations in the EA. Since in the Moody's dataset we have an over-representation of large firms and under-representation of small and medium-sized enterprises (SMEs) compared to the loan portfolio of bank in the EA, we proceed in two

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<sup>30</sup>See detailed EDF description on the Moody's [webpage](#).

steps.<sup>31</sup> Firstly, we construct two separate EDF indices: i) for SMEs, ii) for large firms.<sup>32</sup> Secondly, we build an aggregate default series for non-financial firms as a weighted average of EDF indices for SMEs and large firms. As weights we use the share of loans extended by banks in EA to SMEs and large firms respectively.<sup>33</sup> The data comes on the monthly basis. We aggregate it to quarterly series by averaging the monthly series within a quarter.

## Appendix B Model Aggregation and Market Clearing

In this subsection we describe model aggregation and market clearing conditions.

**Final good** The clearing of the market for final good requires

$$Y_t = y_t, \quad (47)$$

where aggregate output  $Y_t$  equals household consumption,  $C_t$ , plus the investment in the production of new capital,  $I_t$ , plus the resources absorbed by the costs of repossessing assets from defaulting entrepreneurial firms and banks

$$Y_t = C_t + I_t + \Sigma_{b,t} + \Sigma_{e,t}, \quad (48)$$

where

$$\begin{aligned} \Sigma_{b,t} &= \mu_j \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j) B_{f,t-1} dF_{j,t}(\omega_j) \text{ and} \\ \Sigma_{e,t} &= \mu_f \int_0^\infty \int_0^{\bar{\omega}_t(\omega_j)} \omega_i \omega_j [q_t(1-\delta) K_{t-1} + Y_t] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j). \end{aligned}$$

**Labor** The clearing of the labor market requires

$$H_t = h_t. \quad (49)$$

**Physical capital** The clearing of the market for physical capital requires

$$K_t = k_t. \quad (50)$$

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<sup>31</sup>We define SMEs as firms with average total assets below €43 m within the sample period in the database (as in the definition of the European Commission)

<sup>32</sup>EDF indices are constructed as asset weighted average of EDF within one year for the sample of non-financial firms within the size category

<sup>33</sup>We obtain the data on the share of SMEs loans in total loans from Financing SMEs and Entrepreneurs database of OECD.



**Equity** The clearing of the market for equity requires

$$EQ_{e,t} = \int_0^\infty eq_{e,t}(i)di \text{ and} \quad (51)$$

$$EQ_{b,t} = \int_0^\infty eq_{b,t}(j)dj. \quad (52)$$

**Loans** The clearing of the market for loans, requires

$$B_{f,t} = b_{f,t}. \quad (53)$$

**Bank deposits** The clearing of the market for bank deposits, requires

$$D_t = d_{f,t}. \quad (54)$$

**Law of motion of capital** Finally, the law of motion of capital is given by

$$K_t = (1 - \delta) K_{t-1} + S \left( \frac{I_t}{K_{t-1}} \right) K_{t-1}. \quad (55)$$

**Law of motion of entrepreneurs' aggregate net worth** Let  $N_{e,t}$  be the aggregate net wealth of entrepreneurs at period  $t$ . Then

$$N_{e,t} = \theta_e \rho_{e,t} N_{e,t-1} + \iota_{e,t}, \quad (56)$$

which reflects the retention of net worth by the fraction  $\theta_e$  of non-retiring entrepreneurs, whom individual net worth evolves as described by Equation (9), the aggregate endowment  $\iota_{e,t}$  added by the entering entrepreneurs, and the fact that aggregate net wealth equals individual net wealth

$$N_{e,t} = \int_0^\infty n_{e,t}(i)di. \quad (57)$$

**Law of motion of bankers aggregate net worth** Let  $N_{b,t}$  be the aggregate net wealth of bankers at period  $t$ . Then

$$N_{b,t} = \theta_b \rho_{b,t} N_{b,t-1} + \iota_{b,t} - T_t, \quad (58)$$

which reflects the retention of the aggregate net worth by the fraction  $\theta_b$  of non-retiring bankers, whom individual net worth evolves as described by Equation (26), the aggregate endowment  $\iota_{b,t+1}$  received by the entering bankers, and the fact that aggregate net wealth equals individual net wealth

$$\int_0^\infty N_{b,t} = n_{b,t}(j)dj. \quad (59)$$

**Endowments of entering entrepreneurs and bankers** We model the aggregate endowment of entering entrepreneurs as a proportion  $\chi_e$  of the aggregate net worth of the retiring entrepreneurs

$$\iota_{e,t} = \chi_e(1 - \theta_e)\rho_{e,t}N_{e,t-1}. \quad (60)$$

Akin to the case of entrepreneurs, we model the aggregate endowment of entering bankers as a proportion  $\chi_b$  of the aggregate net worth of the retiring bankers

$$\iota_{b,t} = \chi_b(1 - \theta_b)\rho_{b,t}N_{b,t-1}. \quad (61)$$

**Net transfers from entrepreneurs and bankers to the household** Let the net transfers received by the household from entrepreneurs and bankers at period  $t$  be  $\Upsilon_{e,t}$  and  $\Upsilon_{b,t}$  respectively. Then, we have

$$\Upsilon_{e,t} = (1 - \theta_e)\rho_{e,t}N_{e,t-1} - \iota_{e,t}, \quad (62)$$

which reflects the the aggregate worth of the fraction  $1 - \theta_e$  of retiring entrepreneurs minus the aggregate endowment  $\iota_{e,t}$  added by the entering entrepreneurs. Equivalently, we also have that

$$\Upsilon_{b,t} = (1 - \theta_b)\rho_{b,t}N_{b,t-1} - \iota_{b,t}, \quad (63)$$

which reflects the the aggregate worth of the fraction  $1 - \theta_e$  of retiring bankers minus the aggregate endowment  $\iota_{b,t}$  added by the entering bankers. Thus, we have that the sum of net transfers from entrepreneurs and bankers to the household is

$$\Upsilon_t = (1 - \theta_e)\rho_{e,t}N_{e,t-1} - \iota_{e,t} + (1 - \theta_b)\rho_{b,t}N_{b,t-1} - \iota_{b,t}. \quad (64)$$

**Profits from capital production** Profits received by households from capital producing firms are

$$\Xi_t = q_t S \left( \frac{I_t}{K_{t-1}} \right) K_{t-1} - I_t. \quad (65)$$

**Deposit insurance costs** Using  $D_t = d_{f,t}$ , Equation (38) can be written as

$$T_t = \Omega_t D_{t-1}. \quad (66)$$

## Appendix C Model Equilibrium Conditions

We provide the equilibrium conditions for our model. We begin with the equilibrium conditions related to the households, then entrepreneurs and entrepreneurial firms, then bankers and banks, then the capital production sector, and finally the market clearing conditions.

**The household** Using Equations (2) and (3) we obtain

$$-\frac{U_{H_t}}{U_{C_t}} = w_t, \quad (67)$$

Equation (4) is part of the equilibrium conditions. Hence, we have

$$1 = \mathbb{E}_t \Lambda_{t+1} R_{d,t}. \quad (68)$$

**Entrepreneurs** Equations (8) and (10) gives us

$$\nu_{e,t} = \mathbb{E}_t \Lambda_{e,t+1} \rho_{e,t+1}. \quad (69)$$

The elements of the law of motion of entrepreneurs' net worth reflected in Equations (56) and (60) are also part of the equilibrium. Hence, we have

$$N_{e,t+1} = \theta_e \rho_{e,t+1} N_{e,t} + \iota_{e,t+1} \text{ and} \quad (70)$$

$$\iota_{e,t+1} = \chi_e (1 - \theta_e) \rho_{e,t+1} N_{e,t}. \quad (71)$$

**Entrepreneurial Firm** Equations (11)-(21) from the entrepreneurial firms' problem are also part of the equilibrium conditions. Hence, we have

$$Y_{t+1} = A_{t+1} K_t^\alpha (H_t)^{(1-\alpha)}, \quad (72)$$

$$\Pi_{i,j,t+1} (\omega_i, \omega_j) = \omega_i \omega_j (q_{t+1} (1 - \delta) K_t + Y_{t+1}) - R_{f,t} B_{f,t}, \quad (73)$$

$$\bar{\omega}_{t+1} (\omega_j) = \frac{R_{f,t} B_{f,t}}{\omega_j (q_{t+1} (1 - \delta) K_t + Y_{t+1})}, \quad (74)$$

$$\Pi_{f,t+1} = \int_0^\infty \int_{\bar{\omega}_{t+1}(\omega_j)}^\infty \Pi_{i,j,t+1} (\omega_i, \omega_j) dF_{i,t+1} (\omega_i) dF_{j,t+1} (\omega_j), \text{ and} \quad (75)$$

$$B_{f,t} + N_{e,t} = w_t H_t + q_t K_t, \quad (76)$$

$$\mathbb{E}_t \Lambda_{b,t+1} \Pi_{b,t+1} = \nu_{b,t} \phi B_{f,t}, \quad (77)$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial K_t} + \zeta_{f,t} q_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial K_t} = 0, \quad (78)$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial H_t} + \zeta_{f,t} w_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial H_t} = 0, \quad (79)$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial B_{f,t}} - \zeta_{f,t} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial B_{f,t}} + \xi_{f,t} \nu_{b,t} \phi = 0, \quad (80)$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial R_{f,t}} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial R_{f,t}} = 0, \quad (81)$$

$$\rho_{e,t+1} = \frac{\Pi_{f,t+1}}{N_{e,t}}, \quad (82)$$

where we have also used the clearing of the market for the final good, labor, physical capital, and the entrepreneurial firms' equity and the fact that aggregate net wealth equals individual net wealth, i.e. Equations (47), (49), (50), (51), and (57) and the balance sheet of the entrepreneurs, i.e. Equation (6), together with  $dv_{e,t} = 0$ .<sup>34</sup>

**Bankers** Equations (25) and (27) gives us

$$\nu_{b,t} = \mathbb{E}_t \Lambda_{b,t+1} \rho_{b,t+1}. \quad (83)$$

The laws of motion of bankers net worth reflected in Equations (58) and (61) also part of the equilibrium. Hence, we have

$$N_{b,t+1} = \theta_b \rho_{b,t+1} N_{b,t} + \iota_{b,t+1} - T_t \text{ and} \quad (84)$$

$$\iota_{b,t+1} = \chi_b (1 - \theta_b) \rho_{b,t+1} N_{b,t}. \quad (85)$$

**Banks** Equations (28), (29), (31), (33), (34), and (35) from the banks' problem are also part of the equilibrium conditions. Hence, we have

$$B_{f,t} = N_{b,t} + D_t, \quad (86)$$

$$N_{b,t} = \phi B_{f,t}, \quad (87)$$

$$\tilde{R}_{f,t+1}(\omega_j) = [\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j)) - \mu_f G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))] \frac{\omega_j [q_{t+1}(1 - \delta) K_t + Y_{t+1}]}{B_{f,t}}, \quad (88)$$

$$\tilde{R}_{f,t+1}(\bar{\omega}_{j,t+1}) B_{f,t} - R_{d,t} D_t = 0, \quad (89)$$

$$\Pi_{b,t+1} = \int_{\bar{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}(\omega_j) B_{f,t} dF_{j,t+1}(\omega_j) - R_{d,t} D_t (1 - F_{j,t+1}(\bar{\omega}_{j,t+1})), \text{ and} \quad (90)$$

$$\rho_{b,t+1} = \frac{\Pi_{b,t+1}}{\phi B_{f,t}}, \quad (91)$$

where we have also used the clearing of the market for the final good, physical capital, and banks' equity and the fact that aggregate net wealth equals individual net wealth, i.e. Equations (47), (50), (52), and (59) and the balance sheet of the bankers, i.e. Equation (23), together with  $dv_{b,t} = 0$ .

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<sup>34</sup>To simplify notation, we say that the derivative  $\frac{\partial f}{\partial X}$  is evaluated at  $X$ , while the derivative  $\frac{\partial f}{\partial x}$  is evaluated at  $x$ .

**Capital production** The evolution of capital is controlled by the FOC of the capital producer and the law of motion of capital, i.e. Equations (36) and (55)

$$q_t = \left[ S' \left( \frac{I_t}{K_{t-1}} \right) \right]^{-1} \text{ and} \quad (92)$$

$$K_t = (1 - \delta) K_{t-1} + S \left( \frac{I_t}{K_{t-1}} \right) K_{t-1}. \quad (93)$$

**Deposit insurance costs** The deposit insurance cost Equation (66)

$$T_t = \Omega_t D_{t-1}. \quad (94)$$

**Market clearing** The aggregate resource constraint Equation (48) can be written as,

$$Y_t = C_t + I_t + \Sigma_{b,t} + \Sigma_{e,t}, \quad (95)$$

where we have also used the clearing of the market for the final good, labor, and physical capital, i.e. Equations (47), (49), and (50).<sup>35</sup>

## Appendix D Walras Law

This section shows that given the equilibrium conditions (67)-(93), the consumer budget constraint in Equation (1) is equivalent to the resource constraint in Equation (95).

Using Equations (21), (56), (62) and several clearing market conditions is easy to see that profits of entrepreneurial firms,  $\Pi_{f,t}$ , equals to the aggregate net worth of entrepreneurs after net transfers from households,  $N_{e,t}$ , plus the net transfers from the entrepreneurs to households,  $\Upsilon_{e,t}$ . Thus,

$$N_{e,t} + \Upsilon_{e,t} = \Pi_{f,t} = \int_0^\infty \int_{\bar{\omega}_{t+1}(\omega_j)}^\infty \{ \omega_i \omega_j [q_{t+1} (1 - \delta) k_t + y_{t+1}] - R_{f,t} B_{f,t} \} dF_{i,t}(\omega_i) dF_{j,t}(\omega_j). \quad (96)$$

Using Equations (35), (58), (63) and several clearing market conditions is easy that profits of banks,  $\Pi_{b,t}$ , equals to the aggregate net worth of bankers after net transfers from households,  $N_{b,t}$ , plus the net transfers from the bankers to households,  $\Upsilon_{b,t}$ , plus lump-sum taxes  $T_t$ . Thus,

$$N_{b,t} + \Upsilon_{b,t} = \Pi_{b,t} - T_t = \int_{\bar{\omega}_{j,t}}^\infty \tilde{R}_{f,t}(\omega_j) b_{f,t-1} dF_{j,t}(\omega_j) - R_{d,t-1} D_{t-1} (1 - F_{j,t}(\bar{\omega}_{j,t})) - T_t, \quad (97)$$

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<sup>35</sup>Appendix D shows that the aggregate resource constraint in Equation (95) implies the household budget constraint in Equation (1).

where we have used several clearing market conditions. From Equation (37) and several clearing market conditions, we also have that

$$T_t = F_{j,t}(\bar{\omega}_{j,t}) R_{d,t-1} D_{t-1} + (1 - \mu_j) \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j) B_{f,t-1} dF_{j,t}(\omega_j), \quad (98)$$

and therefore

$$\Pi_{b,t} + R_{d,t-1} D_{t-1} - T_t = \int_0^\infty \tilde{R}_{f,t}(\omega_j) B_{f,t-1} dF_{j,t}(\omega_j) - \Sigma_{b,t}. \quad (99)$$

The loan return on of bank in island  $j$  is given by

$$\begin{aligned} \tilde{R}_{f,t}(\omega_j) B_{f,t-1} &= \int_0^\infty \min \{ \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t], R_{f,t-1} B_{f,t-1} \} dF_{i,t}(\omega_i) \\ &\quad - \mu_f \int_0^{\bar{\omega}_t(\omega_j)} \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t] dF_{i,t}(\omega_i). \end{aligned} \quad (100)$$

If we substitute Equation (100) into Equation (99), then we get

$$\begin{aligned} &\Pi_{b,t} + R_{d,t-1} D_{t-1} - T_t \\ &= \int_0^\infty \int_0^\infty \min \{ \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t], R_{f,t-1} B_{f,t-1} \} dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) \\ &\quad - \mu_f \int_0^\infty \int_0^{\bar{\omega}_t(\omega_j)} \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) - \Sigma_{b,t} \\ &= \int_0^\infty \int_0^\infty \min \{ \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t], R_{f,t-1} B_{f,t-1} \} dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) - \Sigma_{e,t} - \Sigma_{b,t}. \end{aligned} \quad (101)$$

Clearly, it is the case that

$$\Pi_{f,t} = \int_0^\infty \int_0^\infty \max \{ \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t], R_{f,t-1} B_{f,t-1} \} dF_{i,t}(\omega_i) dF_{j,t}(\omega_j). \quad (102)$$

Adding (101) and (102), we get

$$\begin{aligned} &\Pi_{f,t} + \Pi_{b,t} + R_{d,t-1} D_{t-1} - T_t \\ &= \int_0^\infty \int_0^\infty \omega_i \omega_j [q_t (1 - \delta) K_{t-1} + Y_t] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j) - \Sigma_{e,t} - \Sigma_{b,t} \\ &= q_t (1 - \delta) K_{t-1} + Y_t - \Sigma_{e,t} - \Sigma_{b,t}. \end{aligned} \quad (103)$$

Using the fact that  $N_{e,t} + \Upsilon_{e,t} = \Pi_{f,t}$ ,  $N_{b,t} + \Upsilon_{b,t} = \Pi_{b,t} - T_t$ , and  $\Upsilon_t = \Upsilon_{b,t} + \Upsilon_{e,t}$  in Equation (103), we get

$$N_{e,t} + N_{b,t} + \Upsilon_t = q_t (1 - \delta) K_{t-1} + Y_t - R_{d,t-1} D_{t-1} - \Sigma_{e,t} - \Sigma_{b,t}. \quad (104)$$

Equation (104) can be written as

$$N_{b,t} + N_{e,t} = Y_t + q_t (1 - \delta) K_{t-1} - R_{d,t-1} D_{t-1} - \Sigma_{e,t} - \Sigma_{b,t} - \Upsilon_t. \quad (105)$$

Using Equations (76) and (86) we obtain

$$N_{b,t} + N_{e,t} - q_t K_t = w_t H_t - D_t. \quad (106)$$

Substituting Equation (106) into Equation (1), we get

$$C_t + q_t K_t = N_{b,t} + N_{e,t} + R_{d,t-1} D_{t-1} + \Upsilon_t + \Xi_t, \quad (107)$$

but since  $\Upsilon_t = \Upsilon_{e,t} + \Upsilon_{b,t}$ , if we substitute Equation (105) into Equation (107), we get

$$C_t + q_t K_t = Y_t + q_t (1 - \delta) K_{t-1} - \Sigma_{e,t} - \Sigma_{b,t} + \Xi_t. \quad (108)$$

But we have that

$$\Xi_t = q_t S \left( \frac{I_t}{K_{t-1}} \right) K_{t-1} - I_t, \quad (109)$$

but using Equation (55) we obtain that

$$\Xi_t = q_t (K_t - (1 - \delta) K_{t-1}) - I_t. \quad (110)$$

If we substitute Equation (110) into Equation (108) we obtain

$$C_t + I_t = Y_t - \Sigma_{e,t} - \Sigma_{b,t} \quad (111)$$

that it is equal to Equation (95) by the definitions of  $\Sigma_{e,t}$  and  $\Sigma_{b,t}$ .

## Appendix E Approximating Banks' Expected Profits

As mentioned, in order to use perturbation methods to approximate the solution to the model we need to compute bank's expected return on the loan portfolio (conditional on not defaulting), defined here as  $R_{p,t+1}$ , which is part of Equation (34) and is given by the integral defined in Equation (43).

In the subsequent analysis, we take  $q_{t+1}, k_t, y_{t+1}, b_{f,t}, R_{d,t}, d_t$  as given and use the notation of  $\tilde{R}_{f,t+1}$  to be the function of island shock,  $\omega_j$ , only. From the analysis in Section 4.1, it should be clear that the bank's loan return  $\tilde{R}_{f,t+1}(\omega_j)$  is not log-normally distributed. Mathematically, this is due to the fact that  $\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))$  and  $G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))$  which enter into  $\tilde{R}_{f,t+1}(\omega_j)$  are both non-linear functions of  $\omega_j$ . As a result of highly non-linear shape of  $\tilde{R}_{f,t+1}(\omega_j)$ , the integral in Equation (43) cannot be computed as explicit function of the state variables and perturbation methods cannot be applied. We overcome this challenge by (i) splitting this integral into the sum of integrals taken over smaller intervals, (ii) computing a series of

quadratic Taylor approximations of  $\tilde{R}_{f,t+1}(\omega_j)$  around a mid-point of each interval.

Formally, we split the domain of  $\omega_j$  into  $N$  intervals of equal length defined on  $N + 1$  points  $x_k$  ranging from  $x_1 = \bar{\omega}_{j,t+1}$  to  $x_{N+1} = \omega_j^{\max}$  where the highest point  $\omega_j^{\max}$  is chosen such that  $\tilde{R}_{f,t+1}(\omega_j^{\max}) = R_{f,t}$  almost surely. Given those assumptions,  $R_{p,t+1}$  is approximately given by:

$$R_{p,t+1} \approx \sum_{k=1}^N \left( \int_{x_k}^{x_{k+1}} \Theta^k(\omega_j) dF_{j,t+1}(\omega_j) \right) + [1 - F_{j,t+1}(x_{N+1})] R_{f,t} \quad (112)$$

where  $\Theta^k(\omega_j)$  is a Taylor approximation of  $\tilde{R}_{f,t+1}(\omega_j)$  around a point  $\omega_j = \bar{x}_k \equiv \frac{x_{k+1} + x_k}{2}$  and is given by

$$\Theta^k(\omega_j) = \tilde{R}_{f,t+1}(\bar{x}_k) + \tilde{R}'_{f,t+1}(\bar{x}_k)(\omega_j - \bar{x}_k) + \frac{1}{2} \tilde{R}''_{f,t+1}(\bar{x}_k)(\omega_j - \bar{x}_k)^2 \quad (113)$$

All the derivatives of  $\tilde{R}_{f,t+1}$  are with respect to  $\omega_j$  and can be computed as an explicit functions of the state variables. Using the simplified expression for  $\Theta^k(\omega_j)$  we can rewrite  $\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j) dF_{j,t+1}$  as follows:

$$\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j) dF_{j,t+1} = Q_0(\bar{x}_k) + Q_1(\bar{x}_k) \int_{x_k}^{x_{k+1}} \omega_j dF_{j,t+1} + Q_2(\bar{x}_k) \int_{x_k}^{x_{k+1}} \omega_j^2 dF_{j,t+1} \quad (114)$$

where:  $Q_i(\bar{x}_k)$  are just constants given by:

$$\begin{aligned} Q_0(\bar{x}_k) &= [F_{j,t+1}(x_{k+1}) - F_{j,t+1}(x_k)] \left[ \tilde{R}_{f,t+1}(\bar{x}_k) - \bar{x}_k \tilde{R}'_{f,t+1}(\bar{x}_k) + \frac{1}{2} \bar{x}_k^2 \tilde{R}''_{f,t+1}(\bar{x}_k) \right], \\ Q_1(\bar{x}_k) &= [F_{j,t+1}(x_{k+1}) - F_{j,t+1}(x_k)] \left[ \tilde{R}'_{f,t+1}(\bar{x}_k) - \frac{1}{2} \bar{x}_k \tilde{R}''_{f,t+1}(\bar{x}_k) \right], \\ Q_2(\bar{x}_k) &= [F_{j,t+1}(x_{k+1}) - F_{j,t+1}(x_k)] \left[ \frac{1}{2} \tilde{R}''_{f,t+1}(\bar{x}_k) \right], \end{aligned}$$

Given our assumption of log-normally distributed island shock,  $\omega_j$ , we have expressions for  $\int_{x_k}^{x_{k+1}} \omega_j dF_{j,t+1}$  and  $\int_{x_k}^{x_{k+1}} \omega_j^2 dF_{j,t+1}$  as explicit functions of the state variables. Consequently, we can easily derive very accurate, the approximation of  $R_{p,t+1}$  in Equation (112) as an explicit function of the state variables.

## Appendix F IRFs

We consider the GIRF proposed by [Koop, Pesaran, and Potter \(1996\)](#). The GIRF for any variable in the model  $var$  in period  $t + l$  following a disturbance to the  $n^{th}$  shock of size  $\nu_n$  in period  $t + 1$  is defined as

$$GIRF_{var}(l, \epsilon_{n,t+1} = \nu, \mathbf{w}_t) = \mathbb{E}[var_{t+l} | \mathbf{w}_t, \epsilon_{n,t+1} = \nu] - \mathbb{E}[var_{t+l} | \mathbf{w}_t], \quad (115)$$



where  $\tilde{\mathbf{z}}_t$  are the value of the state variables of the model at time  $t$  (see Equation (42)) and  $n \in \{A, \delta, i, j\}$ . Hence, the GIRF depend on the value of the state variables when the shocks hits. For example,

$$GIRF_{\Delta \log Y_t}(4, \epsilon_{A,t+1} = -3, (1.1D_{ss}, 0.9K_{ss}, \dots, 1.01\bar{\sigma}_{\omega_i}))$$

is the GIRF of GDP growth,  $\Delta \log Y_t$ , at period  $t + 4$ , after a TFP shock of value  $-3$  in period  $t + 1$ , when  $D_t$  was 10 percent above the steady state,  $K_t$  was 10 percent below the steady state,  $\dots$ , and  $\sigma_{\omega_i,t}$  was one percent above steady state.

But GIRF defined in Equation (115) are conditioned on the value of the state variables when the shocks hits. In what follows, instead we want to compute GIRFs that are conditioned on the values of observables when the shocks hits. For example, we would condition on the expected default rate of firms,  $ED_{f,t}$ , to be above one percent at the time of the shock. In this case, we want to compute the following GIRF

$$GIRF_{var}(l, \epsilon_{n,t+1} = \nu, ED_{f,t} > 0.01) = \int \mathbf{1}_{\{ED_{f,t} > 0.01\}}(\mathbf{w}_t) GIRF_{var}(l, \epsilon_{n,t+1} = \nu, \mathbf{w}_t) f(\mathbf{w}_t) d\mathbf{w}_t, \quad (116)$$

where  $\mathbf{1}_{\{ED_{f,t} > 0.01\}}(\mathbf{w}_t)$  takes a value equal to one if the state variables at time  $t$  are such that he expected default rate of firms is above one percent at time  $t$  and zero otherwise and where  $f(\mathbf{w}_t)$  is the unconditional density of the state variables. Of course Equation (116) needs to be computed by simulation.

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