

A TIME SERIES TEST OF REGIONAL CONVERGENCE IN THE USA WITH DYNAMIC PANEL MODELS, 1972 - 1998

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Abstract

A good deal of controversy surrounds the empirical regularity of convergence. If capital's share is taken to be $1/3$, as in national accounts, then convergence should occur at a much faster rate than observed. Problems are worse if the economy is open. With perfect capital mobility convergence should occur at an infinite rate. Convergence estimates appear to be as slow for state economies as for national economies, even though the assumption of perfect capital mobility is a closer approximation of reality for these economies. Some argue that other variables, most prominently human capital, must be included in any cross sectional estimation of convergence. Supposedly, this addition of variables can bring the implied rate of convergence in line with empirical estimates by controlling for differences in the steady state level of per capita income. This paper extends the analysis of Islam (1995) to US states by estimating dynamic panel data models. This is a more appropriate method of allowing for different steady states. We find that the data suggests states converge very quickly, implying a high degree of capital mobility, if each state economy is allowed to have its own steady state captured through its own fixed effect. These results demonstrate the pitfalls of applying closed economy models to study growth in very open economies and the dangers of adding variables to the estimation which have, at best, only a weak relationship to differential steady states.

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1. Introduction

The growth model developed by Solow (1956) has dominated most economists' understanding of the growth process for more than four decades. Unfortunately, the empirical evidence does not appear to favor this neoclassical approach in one important way. There exists a significant discrepancy between the actual factor shares in national income accounts and implied factor shares as calculated from the estimate of the convergence coefficient.

Given the standard Cobb-Douglas production function it is straight forward to calculate the rate of convergence to the steady state, b . Typically, b is found to be in the neighborhood of 2% to 2.5% in studies involving countries, U.S. States, regions of Western Europe, Canadian provinces, and Japanese prefectures, (Barro, 1997). Given reasonable estimates of population growth, n , technological change, x , and depreciation of capital, d , of 2%, 2%, and 5% respectively (see Barro and Sala-i-Martin 1995 and Romer, 1996), the slow rate of convergence implies that capital's share, a , is in the neighborhood of 72% of value added. This is approximately double the value obtained from the actual national accounts, which suggests capital's share is close to 33%.

The most popular explanation of this slow rate of convergence considered in the literature is the lack of accounting for human capital as a factor of production. Mankiw, Romer, and Weil (M-R-W) (1992) attempt to patch up the model and bring empirical estimates of b in line with observed factor shares by augmenting the neoclassical production function with human capital. Holtz-Eakin (1993) extends the analysis to states. While these approaches represent important contributions, the addition of these variables is not the best way of approaching the problem if an estimate of the rate of convergence is desired. This paper argues that using a panel data approach is the preferred method of estimation.

2. Related Literature

The standard neoclassical growth model considers a closed economy. Begin with a Cobb-Douglas production function with two factors of production, Labor (L) and Capital (K). Technology is assumed to augment labor and is measured by the parameter A. The growth rate of this parameter is exogenous and equal to x , while labor grows at a constant rate equal to n .

$$Y = K^a (AL)^{1-a} \quad (1)$$

With a constant savings rate, s , capital accumulates according to the simple identity:

$$\dot{K} = sY - dK \quad (2)$$

Equation 1 is expressed in "intensive" form by dividing both sides by

AL. Equation 1 becomes $\hat{y} = \hat{k}^a$. The lower case indicates a per capita measure and the "hat" implies that the variable is measured in terms of effective labor, AL. From this framework the well known empirical growth framework is derived (see appendix 1).

$$\frac{\ln(y_i(T)) / y_i(0))}{T} = a_i - \frac{(1 - e^{-bT})}{T} \ln(y_i(0)) + e_i \quad (3)$$

$$a_i = x + \frac{(1 - e^{-bT})}{T} \frac{a}{1 - a} [\ln s_i^* - \ln(n + x + d)] \quad (4)$$

$$b = (1 - l)(x + n + d) \quad (5)$$

i indexes economies, and s_i^* is the steady state savings rate. As it stands, equation 3 performs very poorly in empirical work unless the researcher is careful to select a sample of similar countries or regions such as OECD nations, oil producing countries, US states etc (Barro and Sala-i-Martin, 1991, 1995; Mankiw Romer and Weil, 1992; De

Long, 1998). The obvious problem with directly estimating equation 3 is that it implicitly assumes $a_i = a_j$ for all $i \neq j$. In other words each economy is converging to the same steady state. This notion of absolute convergence, of course, is not what neoclassical growth theory predicts.

Not properly accounting for these differences in steady states biases the estimate of convergence downward if steady state per capita income is positively correlated with the initial level of per capita income. If the omitted variables are related to the initial level of productivity such that

$$a_i = b_0 + b_1 \ln(y_i(0)) + e_i \quad (6)$$

then the estimation of the coefficient on $\ln(y_i(0))$ in equation 3 captures two effects. First, it captures the direct effect of the initial income on the subsequent growth rate, the convergence effect. It also captures an indirect effect due to its relation to the omitted variable. If b_1 is positive, and higher steady states are positively related to higher levels of initial productivity, then the coefficient from which the convergence estimate is calculated is biased upward, causing a downward bias in the estimation of the convergence coefficient, b .

Adding variables such as investment as a percentage of GDP, the population growth rates, schooling, and others often leads to a significant estimate of b . Once variables are added b can be brought into the range of 2% to 3%, depending on the sample period and group of economies studied. Problems remain with this estimate of conditional convergence, however, since it is too low. According to equation 5 an estimate of b of 2.5% and a reasonable estimate of $(n + x + d)$ of 9% implies that a capital's share, a , is equal to 73%. Estimates of capital's share from national income accounts suggest a more accurate measure of capital's share is 33%.

The most notable attempt to account for the slow rate of convergence and provide empirical support for the quantitative implications of the neoclassical growth model is Mankiw et al.(1992). They argue that, in order to test the neoclassical model, the concept of capital must be expanded to include human capital, as in the following production function:

$$Y = K^a H^l (AL)^{1-a-l} \quad (7)$$

where H represents human capital. After augmenting the production function, it is straightforward to show that the convergence coefficient is $b = (1 - a - l)(n + x + d)$.

Thus, accounting for human capital suggests a slower rate of convergence by the value of λ . Mankiw et al.(1992) claim that for a group of 98 countries, after controlling for human capital, the rate of convergence across countries should be around 2.5%, as estimated by most studies. Thus, by adding human capital to the production function, the authors appear to bring the slow rate of convergence in line with a rate necessary to bring predicted factor shares in line with the actual figures.

Further difficulties arise, however, when the model is augmented to include the portion of human capital accumulated through learning by doing. Persson and Malmberg (1996) directly extend the model by including variables to control for the demographic structure of the population, arguing that growth should be positively related to the proportion of the population who are of working age.

They test the implications of this model using data for US states, and find that growth is positively related to the percentage of the population aged 25-44 years old and 45-65 years old indicating an important role for the learning by doing component of overall human capital. If the production function is amended to include human capital, it is important to include both schooling and training in the estimation.

This addition increases the rate of convergence to about 5.8%. This somewhat faster rate of convergence is no longer consistent with the neoclassical model augmented for human capital. A convergence coefficient of 5.8% suggests that human capital's share, given the parameter values outlined in Mankiw et al. (1992), is equal to -30%! In their calculation of factor shares, $(n + x + \delta)$ is taken to be .06, while α is .33. Given the parameter estimates used in the current paper of $(n + x + \delta)$ and α equalling 9% and 33% respectively, human capital's share is 2.6%. Either estimate is far from the value between 33% and 50% Mankiw et al. suggest.

Finally, the whole debate concerning the speed of convergence becomes more complicated if an open economy is specified. The rate of convergence increases to infinity as capital becomes perfectly mobile in a small open economy. This is easy to see, since a small economy's interest rate is determined by its marginal product of capital. If r is an (exogenous) world interest rate and $r_i = f' - d$ then free capital flows ensure that $r_i = r$ and the capital to labor ratio converges instantly to the world capital to labor ratio. If the capital to labor ratio is too low r_i will be greater than r and the difference will spark large capital flows until capital to labor ratios are equated.

Thus, even rates of convergence in line with factor shares after accounting for human capital suffer from the same criticisms once an open economy is specified. There are many reasons why capital is not perfectly mobil, and they undoubtedly apply to states as well as nations, albeit to a lesser degree. Cohen and Sachs (1986) show that convergence is not infinite if there is the potential for one economy to default on what it owes to another.

Capital mobility can be added in a straightforward way for a small economy. Equation 2, expressed in intensive form, is augmented with capital inflows (if $f' - d > r$) and outflows (if $f' - d < r$).

$$\frac{\dot{\hat{k}}}{\hat{k}} = s\hat{k}^{a-1} - (n + x + d) + y(a\hat{k}^{a-1} - d - r), \quad 0 \leq y \leq \infty \quad (8)$$

Expressing equation 8 in terms of $\ln(\hat{k})$, taking a Taylor series expansion around the steady state, and noting that $s^* e^{-(1-a)\ln(\hat{k}^*)} + y a e^{-(1-a)\ln(\hat{k}^*)} = (n + x + d) + y(r + d)$ leads directly to the following modification of the convergence coefficient.

$$b = (1 - a)(n + x + (1 + y)d + yr) \quad (9)$$

y is a measure of capital mobility. If $y = 0$ then the economy is closed (only intertemporal trade is allowed in a one sector model). In this case equation 9 is equivalent to equation 5. If $y = \infty$ then $b = \infty$ as well and convergence is instantaneous.

There likely exists varying degrees of capital mobility across different samples of economies. This presents yet another problem for slow convergence estimates across US states. The assumption of perfect capital mobility should be a better approximation of capital mobility across states than across nations, but convergence appears to occur at the standard 2.5% rate in most conditional convergence studies across states.

3. Empirical Results

The rate of convergence is estimated using data collected for the 48 continental US States and the District of Columbia (DC). Alaska and Hawaii are excluded to make the results of the analysis comparable with other studies of growth across US states. (Barro and Sala-i-Martin, 1991, 1992, 1995; Persson and Malmberg, 1996). The data spans the years 1972 to 1998. These years are chosen because reliable Gross State Product measures do not date earlier than 1972. 1998 represents the most recent year that allows for our division of the data.

The data is divided into three sub-periods each of an equal number of years (1972-1980, 1981-1989, 1990-1998). There is nothing special about the periods chosen. The productivity measure is Gross State Product per worker. A schooling variable is included, the percentage of the state's population over age 25 with a college degree,

to illustrate impact of allowing for state fixed effects. To avoid endogeneity problems this variable is measured in 1970, 1980, and 1989.

Considering equation 3, it is clear that a method of estimation in a cross section must, in some way, account for the differing values of a_i across economies. Examination of equation 4 shows that, while a_i can vary across from one economy to another; it is theoretically constant for a particular economy.

All parameters included in equation 4 are theoretically measured at the steady state where, by definition, output per unit of effective worker is constant. The most common method of estimating conditional convergence proceeds by adding variables to the estimation of equation 3.

Supposedly these variables are correlated with the long run level of an economy's production function. This is the methodology advocated by M-R-W (1992) and extended in many growth studies.

There are two serious problems in proceeding in this manner. To illustrate arguments consider the Solow model augmented with human capital via equation 7. In this case

$$\ln(y^*) = \frac{a}{1-a-l} \ln s_K^* + \frac{l}{1-a-l} \ln s_h^* - \frac{a+l}{1-a-l} \ln(n+x+d)$$

where s_h^* and s_K^* are the steady state measures of the percentage of output used to produce physical capital and human capital respectively. An updated version of equation 3 is:

$$\begin{aligned} \frac{\ln(y_i(T))/y_i(0))}{T} = & x + \frac{(1-e^{-bT})}{T} \frac{a}{1-a-l} \ln s_{i,K}^* + \frac{(1-e^{-bT})}{T} \frac{l}{1-a-l} \ln s_{i,H}^* \\ & - \frac{(1-e^{-bT})}{T} \frac{a+l}{1-a-l} \ln(n+x+d) - \frac{(1-e^{-bT})}{T} \ln(y_i(0)) + e_i \end{aligned} \quad (10)$$

In an attempt to control for *steady state* differences across the economies, variables to control for differences in *steady state* rates of population growth, *steady state* differences in the percentage of output devoted to equipment investment, *steady state* differences in the percentage of output devoted to human capital investment, as well as a kitchen sink full of other variables, are often included.

The first problem with this approach is obvious. Current measures of schooling and investment may be very poor indicators of the values these variables eventually take in the long run. If the Solow model is extended to a framework that includes optimization (Ramsey, 1928) it is clear that savings rates can rise or fall as the economy transitions to the steady state. It is unclear exactly what role measures of current savings rates play in the estimation of equation 10 since no a priori relationship can be established between current savings rates, s , and steady state savings rates, $s_{i,j}^*$ (i.e. $s_{i,j} > s_{i,j}^*$ or $s_{i,j} < s_{i,j}^*$). Any variable added to the regression potentially suffers from this criticism.

This is particularly true of human capital measures since they are, at best, poor ad-hoc proxies for the *steady state* percentage of output devoted to human capital formation.

The second major problem relates to the restrictive assumptions of the model. The implications of opening up the closed economy model are discussed above. If the data used in estimating equation 10 are generated from open economies then measures of investment may have little relationship to the parameters in equation 10. Consider including a measure of the percentage of the *current* population with a college degree (School) to the regression in an attempt to control for

differences in s_H^* , the *steady state* proportion of output invested in human capital. If human capital migrates then the School variable tells little about any particular economy's domestic investment in human capital.

This common approach to modeling is, of course, necessary if an investigator's purpose is to understand why steady states differ. This is a worthy but difficult research agenda. If, however, an answer to the question of how fast economies converge is desired then an alternative approach is preferred. Islam (1995) reports stronger evidence of convergence across countries when each is allowed to have their own intercept term in a panel regression. His estimates of b increase from the standard 2.5% estimate to somewhere in the range of 4% to 6%.

Augmentation of Solow's model and the addition of ad-hoc variables are unnecessary. These estimates are in accord with factor shares if capital is highly immobile across national borders. Similarly, Sedgley and Elmslie (2003) show evidence of absolute convergence across OECD economies when the steady state is allowed to shift over time.

Equation 3 is estimated directly using the panel data set on state per worker Gross State Product. It is expected that the rate of convergence should increase and take a value greater than the 4% to 6% value reported by Islam if capital (financial and human) is more mobile across states than across the nations he studies. To date the evidence suggests that states converge at the same standard 2.5% rate reported in most cross country studies, as outlined in the literature review.

Equation 3 is estimated using both a one way and a two way fixed effects specification. The results are reported in Table 1. The top rows report results based on partitioned least squares. The bottom rows report the results using two-stage partitioned least squares. The instruments are fitted values of first stage regressions. Fitted values of the first stage regressions are based on regressions of each right hand

side variable on lagged values of all variables included in the stage two regression.

The lagged value of per capita income is significant in each set of regression results, implying catch up is important across state economies. The first model, without fixed effects or period effects, suggests a rate of convergence between 1.8% and 3.9%, slower than the 4% to 6% value found across nations in Islam's (1995) results. While it is encouraging to find evidence of convergence this slow speed of convergence is inconsistent with the intuition that capital and labor are more mobile across states than across nations.

The half-life to convergence measures the amount of time it takes an economy to close half of the gap between the current level of productivity and the steady state level of productivity. The half-life is calculated from $\ln(y) = (1 - e^{-Bt})\ln y^* + e^{-Bt} \ln(y(0))$, where y^* is the steady state value of output per worker. At the time t when y is halfway between $y(0)$ and y^* it must be true that $(1 - e^{-Bt}) = e^{-Bt}$. Solving for t gives a formula for the half-life of $t = \ln(2) / b$.

The results of the first model are consistent with most estimates of convergence in the sense that convergence is significant but slow. The Table reports that the half-life to convergence based on a specification absent of fixed and/or period effects is likely to be from 20 to 40 years across US states.

The F test in the last column tests the significance of the additional fixed or period effects as they are added to the model. When partitioned least squares is used the fixed effects are statistically significant if schooling is included, while the period effects are significant if schooling is excluded. The schooling variable is insignificant in the two way model. When two stage least squares is used as the estimation technique the results become more consistent. This estimator is preferred since it accounts for possible endogeneity and measurement error. We focus, therefore, on the 2SLS estimates.

Allowing for fixed effects increases the estimate of convergence and lowers the half-life dramatically. The model including schooling suggests that the half-life falls from 20 years to only 4 years as the rate of convergence jumps from 3.4% to 17.7%. The F test suggests the addition of the fixed effects significantly increases the R-squared. A study of why steady states might shift is the domain of new growth theory. If a period effect is included these shifts can be tested for significance. The addition of period effects is significant but has little impact on the estimated rate of convergence to the steady state. Not surprisingly the ad-hoc measure of human capital, School, becomes insignificant once equation 3 is estimated directly using a panel data approach.

When convergence is estimated without the additional unneeded School variable it is clear that the estimate of convergence is impacted dramatically by properly accounting for the fixed effects and period effects across states. The rate of convergence increases from 1.8% to 16.2% while the half-life falls from 39 years to just 4 years. It is very important to find evidence that states converge so much more quickly than nations given simple intuition concerning capital and labor mobility. The time to close half the gap toward the steady state is a mere 1/3 of the fastest estimates across nations implied by Islam's (1995) results, who uses the same methodology.

4. Conclusion

The neoclassical growth framework, still a workhorse of growth theory, is often criticized because empirical estimates of the convergence parameter are not in line with expectations formed on the basis of capital's share in national income accounts. More specifically, the rate of convergence is too slow to correspond with the capital share of 33% typically reported for a narrow concept of physical capital.

The most significant attempt to account for this empirical anomaly is provided by Mankiw et al (1992) who "take Solow seriously" and augment the neoclassical model to include a broader concept of capital that encompasses human capital. They then show that, in a cross

section of countries, schooling is significant and rate of convergence remains within the 2% to 3% range typically reported. Problems remain, however, since convergence across states appears to occur at the same lethargic 2.5% rate reported in most cross-country studies. This does not match up with intuition since the relatively high degree of capital and labor mobility across states suggests a much higher rate of convergence across states than across countries.

We follow Islam (1995) in using panel data methods to estimate the rate of convergence. The fixed effects allow each economy to have a unique steady state without depending on ad-hoc proxies for eventual steady state values of important parameters. State economies may differ in their long run production functions due to industry concentration, natural resource endowments, historical access to trade and waterways, and many other historical factors. Allowing for fixed effects appears to solve an important anomaly; state economies appear to converge at a substantially faster rate than national economies.

This makes sense due to the very open nature of trade, labor mobility, and capital mobility across states. The data suggests that half the gap between the steady state level of productivity and the current level of productivity is closed in only four years. Earlier studies suggest that it takes much longer, perhaps 30 years, to move half way to the steady state.

Appendix 1.

First note that the growth of output can be expressed as:

$$\frac{\dot{\hat{y}}}{\hat{y}} = a \frac{\dot{\hat{k}}}{\hat{k}}$$

and according to equation (2)

$$\frac{\dot{\hat{k}}}{\hat{k}} = se^{(a-1)\ln(\hat{k})} - (n + x + s)$$

a Taylor series expansion around the steady state (denoted with an *) yields:

$$\frac{\dot{\hat{k}}}{\hat{k}} = s e^{(a-1) \ln(\hat{k}^*)} (a-1) [\ln(\hat{k}) - \ln(\hat{k}^*)]$$

In the steady state with $\frac{\dot{\hat{k}}^*}{\hat{k}^*}$ equal to zero the above equation is:

$$\frac{\dot{\hat{k}}}{\hat{k}} = (n + x + s)(a-1) [\ln(\hat{k}) - \ln(\hat{k}^*)]$$

This together with $\frac{\dot{\hat{y}}}{\hat{y}} = a \frac{\dot{\hat{k}}}{\hat{k}}$ implies:

$$\frac{\dot{\hat{y}}}{\hat{y}} = (n + x + s)(a-1) [\ln(\hat{y}) - \ln(\hat{y}^*)]$$

This is a differential equation with the following solution:

$$\ln(\hat{y}) = (1 - e^{-bt}) \ln(\hat{y}^*) + e^{-bt} \ln(\hat{y}(0))$$

The following steps then derive the empirical growth framework.

$$\frac{1}{T} \ln(\hat{y} / \hat{y}(0)) = \frac{e^{-BT}}{T} \ln \hat{y}(0) - \frac{1}{T} \ln \hat{y}(0) + \frac{(1 - e^{-BT})}{T} \ln \hat{y}^*$$

$$\frac{1}{T} \ln(y / y(0)) = x + \frac{(1 - e^{-BT})}{T} \ln(\hat{y}^* / \hat{y}(0)), \text{ where}$$

$$y = y / A, \hat{y}^* = y^* / A,^*$$

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$y(0) = y(0) / A(0)$, and normalizing $A(0)=1$.

$$\frac{1}{T} \ln(y / y(0)) = x + \frac{(1 - e^{-bT})}{T} \ln(\hat{y}^*) - \frac{(1 - e^{-bT})}{T} \ln(y(0))$$

Which is identical to equation (3).

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