

**RANKING SOCIAL DECISIONS WITHOUT INDIVIDUAL
PREFERENCES ON THE BASIS OF OPPORTUNITIES***

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** C. Herrero: University of Alicante and Instituto Valenciano de Investigaciones Económicas, I. Iturbe: University of Alicante, J. Nieto: Universidad Pública de Navarra.

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A B S T R A C T

In this paper we study several methods of ranking profiles of opportunity sets by taking as a primary notion equality of opportunity, understood as equality of choice sets. Each of these social decision rules looks first at the size of the common opportunity set available to all members of the society. If this is not decisive, additional criteria are used in a lexicographic procedure. Axiomatic characterizations of each method are also provided.

KEYWORDS: Equality of Opportunity, Ranking Profiles of Opportunity Sets.

1.- INTRODUCTION

The notion of *equality of opportunity*, as an expression of social justice, has played a major role in theoretical and applied economics and indeed it is almost ubiquitous in public economics.

The most immediate interpretation of equality of opportunity is *equality of individual choice sets* [see Kolm (1973) (1995), Thomson (1992)]. According to this, equality of opportunity has mostly to do with equality of accessibility, and not with outcome equality [see also Arneson (1989), Bossert (1995), Cohen (1989), Dworkin (1981), Herrero (1995) or Roemer (1994) for different models in which personal responsibility justifiably restricts the degree of outcome equality]. Yet, there is a clear conflict between the ethical value of previous proposal and its practical relevance: identical choice sets is a rather unlikely event even for twins, and a theory focusing on this strict approach will leave a major class of situations unsolved. The relevant question is, therefore, how to rank social decisions when in the associated profiles, individual opportunity sets are not identical.

This problem has been treated from a variety of viewpoints. We take the less demanding approach in informational terms: the planner does not take into account any information on individual preferences. To the best of our knowledge the first attempt to rank distributions of opportunity sets in this setting is made in Kranich (1994).

Apart from the possible planner's lack of information about individual preferences, two other difficulties appear related to the use of private information in evaluating opportunity sets. The first one refers to dynamic considerations and changing tastes leading to the *preference for flexibility* model [see Kreps (1979)]. The second one is related to *adapting preferences*, that is, to the idea that people tend to adjust their aspirations to their possibilities, as argued by Elster by means of the *sour grapes paradox* [see Elster (1982), pp.219].

In Kolm's words [Kolm (1995)],

"freedom is the end-value of justice. This is equivalent to considering individuals' preferences as irrelevant for justice"

Kranich (1994) suggests a way of evaluating profiles of (finite) opportunity sets faced by a group of agents. He proposes to consider only the difference in the number of elements faced by the agents as a measure of the degree of inequality of opportunity. That is, *equality in the cardinality* of the choice sets is all that matters.

There is a source of discomfort in Kranich's proposal, namely that only the *relative size* of the opportunity sets faced by the agents matters, while neither the size of the *common possibility set*, nor the *absolute size* of the opportunity sets are taken into account. For instance, by means of such a procedure, a profile **A**, in which both opportunity sets have a single (and different) element is declared as indifferent to a profile **B**, in which both opportunity sets have many (and identical) elements.

The main proposal of this paper is to take as a basic approach of equality of opportunity the size of the *common opportunity set*, and declare above in the ranking a situation having a bigger common set. A problem with this criterion is that there are too many situations which are not distinguishable and it seems sensible to take into account also additional characteristics of the opportunity profiles. Thus, as a second step, and in the case we face a couple of situations such that the size of the common sets are identical, we may look at those additional properties.

Since additional properties play a subsidiary role, we propose some *lexicographic method* as a proper way of ranking profiles of opportunity sets: first we look at the common opportunity set. If this is not decisive, we look at those additional properties. With this in mind, three alternative methods are studied in this paper, each stressing in different ways the trade off between pure equality of opportunity and efficiency. For the sake of simplicity we only deal with the two-person case.

In a first place we study the *common opportunity relation*, where only equality of opportunity issues are taking into account. In this case, the Kranich's cardinality difference relation is taken as a subsidiary criterion. In a second place, we propose the *lexmin opportunity relation*, where at a second step we look at the sizes of the individual opportunity sets, starting by the smallest ones. Finally, we also study the *utilitarian opportunity relation*. Now, the complementary criterion is the size of the aggregate opportunity set.

Section 2 is devoted to notation and preliminaries. Sections 3 to 5 have a common structure. We present a particular social relation, then we provide with an axiomatic characterization of the relation, and we prove the independence of the axioms used. Finally, Section 6, with some final comments, closes the paper.

2.- PRELIMINARIES: NOTATION AND DEFINITIONS

Let $N = \{1, 2, \dots, n\}$ denote the set of agents, and L be an infinite set of *opportunities*. We denote the set of nonempty, finite subsets of L by \mathcal{L} , and consider lists or profiles of opportunity sets of the form $\mathbf{A} = (A^1, \dots, A^n)$, where $A^i \in \mathcal{L}$ for all $i \in N$. Let $\mathcal{L}^n = \prod_{i \in N} \mathcal{L}$ denote the set of all profiles.

A *social relation* is a complete, reflexive and transitive relation \geq defined on \mathcal{L}^n that ranks distributions of opportunities on the basis of some fairness criteria. For $\mathbf{A}, \mathbf{B} \in \mathcal{L}^n$, we write $\mathbf{A} \geq \mathbf{B}$ meaning that profile \mathbf{A} is socially considered at least as good as profile \mathbf{B} . As usual, \succ and \sim are the asymmetric and reflexive parts, respectively, of \geq .

If $A, B \in \mathcal{L}$, $A \subset B$, $A \subseteq B$ denotes strict and weak inclusion; $A \setminus B = \{x \in A \mid x \notin B\}$; $\# A$ = number of elements in set A ; For $\mathbf{A} \in \mathcal{L}^n$, $\sigma(\mathbf{A})$ denotes a permutation of N such that $\# A^{\sigma(1)} \leq \# A^{\sigma(2)} \leq \dots \leq \# A^{\sigma(n)}$; $\# \mathbf{A} = (\# A^1, \dots, \# A^n)$. Vector inequalities are denoted \gg , $>$ and \geq with the usual interpretation [$\mathbf{a} \gg \mathbf{b} \Leftrightarrow a_i > b_i$ for all i ; $\mathbf{a} > \mathbf{b} \Leftrightarrow a_i \geq b_i$ for all i and $\mathbf{a} \neq \mathbf{b}$; $\mathbf{a} \geq \mathbf{b} \Leftrightarrow a_i \geq b_i$ for all i]. The lexicographic ordering L in \mathbb{R}^n is defined as usually: for any $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$, $\mathbf{a} \succ_L \mathbf{b} \Leftrightarrow a_i > b_i$, and $a_k = b_k$ for all $k < i$.

Consider now the two-agents case. Thus, a profile is a pair $\mathbf{A} = (A^1, A^2)$, $A^i \in \mathcal{L}$. Now, for a *social relation* \geq on \mathcal{L}^2 , the following properties will be used in the sequel:

ANONYMITY (AN): For any $A \in \mathcal{L}^2$, $A = (A^1, A^2)$, $A \sim (A^2, A^1)$.

UNIFORMATION OF OPPORTUNITY SETS (UNI): For any $A \in \mathcal{L}^2$, $A = (A^1, A^2)$,

$X \subseteq A^1 \setminus A^2$, $Y \subseteq A^2 \setminus A^1$, $x \notin A^1 \cap A^2$, then $[(A^1 \setminus X) \cup \{x\}, (A^2 \setminus Y) \cup \{x\}] \succ A$.

COMMON REPLACING (REP): For any $A \in \mathcal{L}^2$, $A = (A^1, A^2)$, if $x \in A^1 \cap A^2$, $y \in L$,

$y \notin (A^1 \setminus \{x\}) \cup (A^2 \setminus \{x\})$, then $[(A^1 \setminus \{x\}) \cup \{y\}, (A^2 \setminus \{x\}) \cup \{y\}] \sim A$.

UNCOMMON REPLACING (UR): For any $A \in \mathcal{L}^2$, $A = (A^1, A^2)$, if $x^2 \in A^2 \setminus A^1$,

$x \notin A^2 \setminus \{x^2\}$, then $[A^1, (A^2 \setminus \{x^2\}) \cup \{x\}] \succeq A$.

IRRELEVANT EXPANSIONS (IE): For any $A \in \mathcal{L}^2$, $A = (A^1, A^2)$, if $x, y \notin A^1 \cup A^2$,

$x \neq y$, then $[A^1 \cup \{x\}, A^2 \cup \{y\}] \sim A$.

NEGATIVE MONOTONICITY (NM): For any $A \in \mathcal{L}^2$, $A = (A^1, A^2)$ if $A^1 \subseteq A^2 \subset B^2$,

then $A \succ (A^1, B^2)$.

MONOTONICITY (MON): For any $A \in \mathcal{L}^2$, $A = (A^1, A^2)$ if $A^1 \subseteq B^1$, $A^2 \subseteq B^2$,

$A^1 \cup A^2 \subset B^1 \cup B^2$, then $B \succ A$.

TRANSFER (TR): For any $A \in \mathcal{L}^2$, $A = (A^1, A^2)$, $X \subset A^2 \setminus A^1$, $x \notin A^1$. If

$\#(A^1 \cup \{x\}) \leq \#(A^2 \setminus X)$, then $(A^1 \cup \{x\}, A^2 \setminus X) \succ A$.

IRRELEVANT TRANSFERS (IT): Let $A = (A^1, A^2) \in \mathcal{L}^2$, and $x \in A^1 \setminus A^2$. Then,

$(A^1 \setminus \{x\}, A^2 \cup \{x\}) \sim A$.

Previous properties have different status. A first group of properties

have to do either with the setting or with our basic principle in ranking profiles of opportunity sets, and are fulfilled by all the proposed relations. A second group of properties are related with possible alternative complementary principles, and they play the role of differentiating among the relations we propose.

Within the first group of properties, AN is a minimal fairness principle. It says that agents' identities play no role on the social ordering. UNI is the expression of the priority of the common opportunity set. It says that whenever opportunity sets become more similar, then the new situation is ranked above the previous one. As for REP and UR, they reflect the planner's lack of information on individual preferences and/or perceptions of the alternatives. So, REP has to do with the way the planner looks at the common set. Whenever something common is replaced by something else, both situations are ranked equally. On the other hand, UR says that if we change a non common element by a different one, the new situation is not ranked below, reflecting not only the lack of information but also the possibility of strict improvements due to a possible enlargement of the common opportunity set.

In the second group of axioms, IE and NM follow the line of taking only care of the pure concept of equality of opportunity. Adding different opportunities to both agents does not affect the ranking (IE), and if we enlarge the opportunities of the agent facing the biggest set, then the degree of equality of opportunity decreases (NM). TR deals with the *transfer principle* as a way of decreasing inequalities: whenever we transfer opportunities from those agents facing more opportunities to those

facing less, the degree of inequality decreases. Both MON and IT deal with different aspects of efficiency. MON means that whenever the opportunity set of an agent expands, the situation becomes better. IT says that we rank equally any situation having identical common opportunities irrespective of who is the agent enjoying non common opportunities. Notice that IT implies AN.

Consider now the following social relations:

DEFINITION 0 [Kranich (1994)]: For $A, B \in \mathcal{L}^2$, the *cardinality difference relation*, \geq_{cd} is defined by:

$$A \geq_{cd} B \Leftrightarrow \# A^{\sigma(1)} - \# A^{\sigma(2)} \leq \# B^{\sigma(1)} - \# B^{\sigma(2)}$$

DEFINITION 1: For $A, B \in \mathcal{L}^2$, the *common opportunity relation*, \geq_{co} is defined by: $A \geq_{co} B \Leftrightarrow$

$$[\# (A^1 \cap A^2), \# A^{\sigma(1)} - \# A^{\sigma(2)}] \geq_L [\# (B^1 \cap B^2), \# B^{\sigma(1)} - \# B^{\sigma(2)}]$$

DEFINITION 2: For $A, B \in \mathcal{L}^2$, the *utilitarian opportunity relation*, \geq_u is defined by:

$$A \geq_u B \Leftrightarrow [\# (A^1 \cap A^2), \# (A^1 \cup A^2)] \geq_L [\# (B^1 \cap B^2), \# (B^1 \cup B^2)]$$

DEFINITION 3: For $A, B \in \mathcal{L}^2$, the *lexmin opportunity relation*, \geq_{lo} is defined by:

$$A \geq_{lo} B \Leftrightarrow [\# (A^1 \cap A^2), \# \sigma(A)] \geq_L [\# (B^1 \cap B^2), \# \sigma(B)]$$

Previous social relations rank profiles of opportunity sets in different ways. For the cardinality difference relation all that matters is whether opportunity sets have equal size. In the remaining relations we consider that the most important criterion deals with the *common* opportunity set. With such a spirit, we start by considering the common opportunity relation, which ranks profiles on the basis of the size of the common opportunity set, and, at a second step, by looking at the differences in the size. The utilitarian opportunity relation looks at a second step at the size of the aggregate opportunities set. Finally, the lexmin opportunity relation uses as a complementary criterion the sizes of both individual opportunity sets, starting by the smallest one.

3. AXIOMATIC CHARACTERIZATION OF THE COMMON OPPORTUNITY RELATION

THEOREM 1.- A weak preorder \geq on \mathcal{L}^2 satisfies AN, UNI, REP, UR, IE and NM if and only if \geq coincides with \geq_{co} .

PROOF:

Obviously, \geq_{co} satisfies all the properties. Let us see the sufficiency part. By AN, we may assume, for any $A \in \mathcal{L}^2$ that $A = \sigma(A)$.

(i) Let $A, B \in \mathcal{L}^2$ such that $[\#(A^1 \cap A^2), \#A] = [\#(B^1 \cap B^2), \#B]$. Then $A \sim B$.

Choose $x \in A^1 \cap A^2$, $y \in B^1 \cap B^2$. Thus, by REP, $[(A^1 \setminus \{x\}) \cup \{y\}, (A^2 \setminus \{x\}) \cup \{y\}] \sim A$.

By repeating the procedure a suitable number of times, and by transitivity, we obtain that $C = (C^1, C^2) \sim A$, where $C^1 = [A^1 \setminus (A^1 \cap A^2)] \cup (B^1 \cap B^2)$.

Now, take $x \in A^1 \setminus A^2$, $y \in B^1 \setminus B^2$. Thus, by construction, $x \in C^1 \setminus C^2$, and by UR, $[(C^1 \setminus \{x\}) \cup \{y\}, C^2] \geq C$. By suitably repetition of the procedure and transitivity, we get that $(B^1, C^2) \geq C$.

Apply a symmetric procedure to the previous one for $x \in A^2 \setminus A^1$, $y \in B^2 \setminus B^1$. Thus, $[B^1, (C^2 \setminus \{y\}) \cup \{x\}] \geq (B^1, C^2)$. By repeating the procedure, $B \geq (B^1, C^2)$. Then, by transitivity, $B \geq A$.

By a similar procedure, starting by B , we also obtain $A \geq B$, and therefore, $A \sim B$.

(ii) Let $A = (A^1, A^2)$, $B = (B^1, B^2) \in \mathcal{L}^2$ such that $\#(A^1 \cap A^2) = \#(B^1 \cap B^2)$, $\#A^1 - \#A^2 = \#B^1 - \#B^2$. To proof: $A \sim B$.

Assume, without loss of generality, that $\#A^1 > \#B^1$. Then, by repeated application of IE, $A \sim B$.

As a consequence, for any $A \in \mathcal{L}^2$, we may find $B \sim A$, $B = (B^1, B^2)$, with $B^1 \subseteq B^2$.

(iii) Let $\mathbf{A} = (A^1, A^2)$, $\mathbf{B} = (B^1, B^2) \in \mathcal{L}^2$ such that $\# (A^1 \cap A^2) = \# (B^1 \cap B^2)$, $\# A^1 - \# A^2 > \# B^1 - \# B^2$. To proof: $\mathbf{A} \succ \mathbf{B}$. Choose \mathbf{A} , \mathbf{B} such that $A^1 = B^1$, $A^1 \subseteq A^2 \subset B^2$. Then, by NM, $\mathbf{A} \succ \mathbf{B}$.

(iv) Let $\mathbf{A} = (A^1, A^2)$, $\mathbf{B} = (B^1, B^2) \in \mathcal{L}^2$ such that $\# (A^1 \cap A^2) > \# (B^1 \cap B^2)$. To proof: $\mathbf{A} \succ \mathbf{B}$.

(iv.a) If moreover, $\# A^1 - \# A^2 \geq \# B^1 - \# B^2$, construct $\mathbf{C} = (B^1 \cup X, B^2 \cup X)$, where X is chosen such that $\# X = \# (B^1 \cap B^2) - \# (A^1 \cap A^2)$, $X \cap (B^1 \cup B^2) = \emptyset$. Thus, $\# (C^1 \cap C^2) = \# (A^1 \cap A^2)$, $\# C^1 - \# C^2 = \# B^1 - \# B^2$. By UNI, $\mathbf{C} \succ \mathbf{B}$, and by (iii), $\mathbf{A} \succ \mathbf{C}$. Thus, by transitivity, $\mathbf{A} \succ \mathbf{B}$.

(iv.b) Take now the case $\# A^1 - \# A^2 < \# B^1 - \# B^2$. BY (i), we may assume that $B^1 \subseteq B^2 \subset A^1 \subset A^2$. Now, take $X = A^2 \setminus B^1$, and Y such that $\# Y = \# X$, and $Y \cap A^2 = \emptyset$. By applying IE a suitable number of times ($\# X$), we obtain that $(B^1 \cup Y, B^2 \cup X) = (B^1 \cup Y, A^2) \sim \mathbf{B}$.

Now take $Z = A^1 \setminus B^1$. Then, by UNI, $\mathbf{A} = [(B^1 \cup Y) \setminus Y] \cup Z, A^2 \succ (B^1 \cup Y, A^2)$, and by transitivity, $\mathbf{A} \succ \mathbf{B}$. ■

THEOREM 2.- AN, UNI, REP, UR, IE and NM are independent.

PROOF:

We provide with examples of weak preorders satisfying all the properties but one at any time:

(AN) Take \succeq_1 defined by $(A^1, A^2) \succeq_1 (B^1, B^2) \Leftrightarrow [\# (A^1 \cap A^2), \# A^1 - \# A^2] \succeq_L [\# (B^1 \cap B^2), \# B^1 - \# B^2]$. It satisfies REP, UNI, UR, IE and NM, but it does not satisfy AN: $(\{x, y, z\}, \{x, y\}) \succ_1 (\{x, y\}, \{x, y, z\})$.

(UNI) Consider $\succeq_2 = \succeq_{cd}$. It satisfies AN, REP, UR, IE and NM, but it does not satisfy UNI: $(\{a\}, \{b\}) \sim (\{a\}, \{a\})$.

(REP) Consider a fixed set $K \subset L$, Now, $A \succ_3 B \Leftrightarrow A \succ_{co} B$, and if $A \sim_{co} B$, when $\# (A^1 \cap A^2 \cap K) > \# (B^1 \cap B^2 \cap K)$. It satisfies AN, UNI, UR, IE and NM, but does not satisfy REP: Let A be such that $x \in A^1 \cap A^2$, $x \notin K$, $y \in K$. Thus, $[(A^1 \setminus \{x\}) \cup \{y\}, (A^2 \setminus \{x\}) \cup \{y\}] \succ_3 A$.

(UR) Take a fixed set $K \subset L$. Now, $A \succ_4 B \Leftrightarrow A \succ_{co} B$, and if $A \sim_{co} B$, when $\# [(A^1 \cup A^2) \cap K] > \# [(B^1 \cup B^2) \cap K]$. It satisfies AN, UNI, REP, IE and NM, but does not satisfy UR: Let A be such that $x \in A^2 \setminus A^1$, $x \in K$, $y \notin K$, $y \notin A^2 \setminus \{x\}$. Thus, $A \succ_4 [A^1, (A^2 \setminus \{x\}) \cup \{y\}]$.

(IE) Take \succeq_5 defined as follows: $(A^1, A^2) \succeq_5 (B^1, B^2) \Leftrightarrow$

$$\left[\# (A^1 \cap A^2), \frac{\# A^{\sigma(1)}}{\# A^{\sigma(2)}} \right] \succeq_L \left[\# (B^1 \cap B^2), \frac{\# B^{\sigma(1)}}{\# B^{\sigma(2)}} \right].$$

It satisfies AN, UNI,

REP, UR, and NM, but it does not satisfy IE:

$$(\{x, z\}, \{x, y, t\}) \succ_5 (\{x\}, \{x, y\}).$$

(NM) Consider \succeq_6 defined as follows: $(A^1, A^2) \succeq_6 (B^1, B^2) \Leftrightarrow$

$$[\# (A^1 \cap A^2), \# A^{\sigma(2)} - \# A^{\sigma(1)}] \succeq_L [\# (B^1 \cap B^2), \# B^{\sigma(2)} - \# B^{\sigma(1)}].$$

It

satisfies AN, UNI, REP, UR, IE but does not fulfill NM:

$$(\{x\}, \{x, y\}) \succ_6 (\{x\}, \{x\}). \blacksquare$$

4. THE UTILITARIAN OPPORTUNITY RELATION

THEOREM 3.- A weak preorder \succeq on \mathcal{L}^2 satisfies UNI, REP, UR, MON and IT if and only if \succeq coincides with \succeq_u .

PROOF:

Since \succeq_u obviously satisfies all the properties, we only deal with the sufficiency part.

(i) Let $A, B \in \mathcal{L}^2$, $A = (A^1, A^2)$, $B = (B^1, B^2)$ such that $\#(A^1 \cap A^2) = \#(B^1 \cap B^2)$, $\#(A^1 \cup A^2) = \#(B^1 \cup B^2)$. By IT, we may assume that $A^1 \subseteq A^2$, $B^1 \subseteq B^2$. Assume furthermore that $A^i \cap B^j = \emptyset$ for all $i, j = 1, 2$. Now, by repeated application of REP, $[A^1 \setminus (A^1 \cap A^2) \cup (B^1 \cap B^2), A^2 \setminus (A^1 \cap A^2) \cup (B^1 \cap B^2)] \sim A$. Then, by UR, $B \succeq A$. By a similar argument, $A \succeq B$, and in consequence, $A \sim B$.

(ii) Let $A, B \in \mathcal{L}^2$, $A = (A^1, A^2)$, $B = (B^1, B^2)$ be such that $\#(A^1 \cap A^2) = \#(B^1 \cap B^2)$, $\#(A^1 \cup A^2) > \#(B^1 \cup B^2)$. By IT, we may consider that $A^1 \subseteq A^2$, $B^1 \subseteq B^2$, $\#A^1 = \#B^1$, $\#A^2 > \#B^2$. Consider $C^2 \supset B^2$ such that $\#C^2 = \#A^2$. Then, by MON, $(B^1, C^2) \succ B$, and by (i), $(B^1, C^2) \sim A$. Thus, by transitivity, $A \succ B$.

(iii) Let $A, B \in \mathcal{L}^2$, $A = (A^1, A^2)$, $B = (B^1, B^2)$ be such that $\#(A^1 \cap A^2) > \#(B^1 \cap B^2)$. Again, by IT, assume that $A^1 \subseteq A^2$, $B^1 \subseteq B^2$. If moreover, $\#A^2 \geq \#B^2$, take $C^1 \supset B^1$, $C^2 \supseteq B^2$ such that $\#C^1 = \#A^1$, $\#C^2 = \#A^2$. Then, by MON, $C \succ B$, and by (i), $A \sim C$. Thus, by transitivity, $A \succ B$.

Consider now the case $\#A^2 < \#B^2$. Let $p = \#A^1 - \#B^1$. Take $C^2 \supset B^2$ such that $\#C^2 > \#A^2 + p$. Thus, by MON, $(B^1, C^2) \succ B$. Take now $x \in C^2 \setminus B^1$,

$\emptyset \subset B^1$, and $y \in L$, $y \notin B^1$. Thus, by UNI, $[B^1 \cup \{y\}, (C^2 \setminus \{x\}) \cup \{y\}] \succ (B^1, C^2)$. Applying UNI p times, and choosing appropriately $X \subset C^2 \setminus B^1$ in the last step, we end up at some $D = (D^1, D^2) \succ (B^1, C^2)$, such that $D^1 \subseteq D^2$, $\# A^1 = \# D^1$, $\# A^2 = \# D^2$. Thus, by (i), $A \sim D$, and by transitivity, $A \succ B$. ■

THEOREM 4.- UNI, REP, UR, MON and IT are independent.

PROOF:

Again, we provide with examples of weak preorders such that satisfy all but one of previous properties at any time.

(UNI) Consider \succeq_1 defined by $A \succeq_1 B$ iff $\# (A^1 \cup A^2) \geq \# (B^1 \cup B^2)$. It satisfies REP, UR, MON and IT, but it fails to satisfy UNI: $((a, e, c), \{a, f, d\}) \succ_1 ((a, b, c), \{a, b, d\})$.

(REP) Consider \succeq_2 defined as follows: Take a fixed set $K \subset L$. Now, $A \succ_2 B \iff A \succ_u B$, and if $A \sim_u B$, when $\# (A^1 \cap A^2 \cap K) > \# (B^1 \cap B^2 \cap K)$. It satisfies UNI, UR, MON and IT, but does not satisfy REP: Let A be such that $x \in A^1 \cap A^2$, $x \notin K$, $y \in K$. Thus, $[(A^1 \setminus \{x\}) \cup \{y\}, (A^2 \setminus \{x\}) \cup \{y\}] \succ_2 A$.

(UR) Consider \succeq_3 defined as follows: Take a fixed set $K \subset L$, Now, $A \succ_3 B \iff A \succ_u B$, and if $A \sim_u B$, when $\# [(A^1 \cup A^2) \cap K] > \# [(B^1 \cup B^2) \cap K]$. It satisfies UNI, REP, MON and IT, but does not satisfy UR: Let A be such that $x \in A^2 \setminus A^1$, $x \in K$, $y \notin K$, $y \notin A^2 \setminus \{x\}$. Thus, $A \succ_3 [(A^1, (A^2 \setminus \{x\}) \cup \{y\})]$.

(MON) Consider \succeq_4 defined by $A \succeq_4 B$ iff $\# (A^1 \cap A^2) \geq \# (B^1 \cap B^2)$. It satisfies UNI, REP, UR and IT, but it fails to fulfill MON:

$$(\{a, b\}, \{a, b, c\}) \sim_4 (\{a, b\}, \{a, b\}).$$

(IT) Consider \succeq_5 defined by $\succeq_5 = \succeq_{10}$. It satisfies UNI, REP, UR and MON, but it fails to fulfill IT: $(\{a, b\}, \{a, c, d\}) \succ (\{a\}, \{a, b, c, d\})$. ■

5. THE LEXIMIN OPPORTUNITY RELATION

THEOREM 5.- A weak preorder \geq on \mathcal{L}^2 satisfies AN, UNI, REP, UR, MON, and TR if and only if \geq coincides with \geq_{io} .

PROOF:

By AN, Consider $A = \sigma(A)$, $B = \sigma(B)$. Without loss of generality, we may consider $A = (A^1, A^2)$, $B = (B^1, B^2)$ such that $A^i \cap B^j = \emptyset$, for all $i, j = 1, 2$.

(i) Let $A, B \in \mathcal{L}^2$ such that $[\#(A^1 \cap A^2), \#A] = [\#(B^1 \cap B^2), \#B]$. Then $A \sim B$. Notice that this is exactly case (i) in Theorem 1. The same proof applies.

(ii) Let $A, B \in \mathcal{L}^2$ be such that $\#(A^1 \cap A^2) = \#(B^1 \cap B^2)$; $\#A^1 = \#B^1$, $\#A^2 < \#B^2$. Then $B \succ A$.

Choose $C^2 \supset A^2$ such that $\#C^2 = \#B^2$ and $A^1 \cap A^2 = A^1 \cap C^2$. By MON, $(A^1, C^2) \succ A$, and by (i), $(A^1, C^2) \sim B$. Thus, by transitivity, $B \succ A$.

(iii) Let $A, B \in \mathcal{L}^2$ be such that $\#(A^1 \cap A^2) = \#(B^1 \cap B^2)$; $\#A^1 < \#B^1$. Then $B \succ A$.

If moreover $\#A^2 \leq \#B^2$, Take $C^1 \supset A^1$, $C^2 \supseteq A^2$ such that $A^1 \cap A^2 = C^1 \cap C^2$, $\#C^1 = \#B^1$, $\#C^2 = \#B^2$. Then, by MON, $C \succ A$, and by (i), $C \sim B$. Thus, by transitivity, $B \succ A$.

Consider now the case $\#A^2 > \#B^2$. Take $X \subseteq A^2 \setminus A^1$ such that $\#B^2 \geq \#(A^2 \setminus X) \geq \#A^1 + 1$, and choose $x \notin A^1 \cup A^2$. Then, by TR, $(A^1 \cup \{x\}, A^2 \setminus X) \succ A$. But $\#(A^1 \cup \{x\}) \leq \#B^1$, $\#(A^2 \setminus X) \leq \#B^2$, and $\#(A^1 \cup \{x\}) \cap (A^2 \setminus X) = \#(B^1 \cap B^2)$. In consequence, $B \geq (A^1 \cup \{x\}, A^2 \setminus X)$, and by transitivity, $B \succ A$.

(iv) Let $A, B \in \mathcal{L}^2$ be such that $\#(A^1 \cap A^2) < \#(B^1 \cap B^2)$. Then $B \succ A$.

Consider first the case $A^1 \cap A^2 = \emptyset$. Then, $\#(B^1 \cap B^2) \geq 1$. Take $x \in B^1 \cap B^2$. By UNI, $(\{x\}, \{x\}) \succ A$. If $B^1 = B^2 = \{x\}$, we are done. Otherwise, by MON, $B \succ (\{x\}, \{x\})$, and by transitivity, $B \succ A$.

Suppose now that $A^1 \cap A^2 \neq \emptyset$, and $\#(B^1 \cap B^2) = \#(A^1 \cap A^2) + 1$. Take $X \subset A^1$ such that $\#(A^1 \setminus X) < \#B^1$, and $x \notin A^1 \cap A^2$. Then, by UNI, $[(A^1 \setminus X) \cup \{x\}, A^2 \cup \{x\}] \succ A$. Furthermore, $\#[(A^1 \setminus X) \cup \{x\}] \cap (A^2 \cup \{x\}) = \#(B^1 \cap B^2)$, $\#[(A^1 \setminus X) \cup \{x\}] < \#B^1$. Thus, by (iii), $B \succ [(A^1 \setminus X) \cup \{x\}, A^2 \cup \{x\}]$, and by transitivity, $B \succ A$.

Finally, assume that $A^1 \cap A^2 \neq \emptyset$, and $\#(B^1 \cap B^2) = \#(A^1 \cap A^2) + p$, $p \geq 2$. Construct $C^1 = A^1 \cup Y$, $C^2 = A^2 \cup Y$, such that $\#(C^1 \cap C^2) = \#(B^1 \cap B^2) - 1$. By MON, $C \succ A$. Now, we are in the previous case and may act as before. ■

THEOREM 6.- AN, UNI, REP, UR, MON, and TR are independent.

PROOF:

We provide with examples of equality relations fulfilling all but one property at any time:

(AN) Define \succeq_1 in the following way: $A \succeq_1 B \Leftrightarrow (\#(A^1 \cap A^2), \#A) \succeq_L (\#(B^1 \cap B^2), \#B)$. It satisfies UNI, REP, UR, MON and TR but it fails to satisfy AN: $(\{a, b\}, \{c\}) \succ_1 (\{c\}, \{a, b\})$.

(UNI) Define \succeq_2 in the following way: $A \succeq_2 B \Leftrightarrow [\#A^{\sigma(1)}, \#(A^1 \cup A^2)] \succeq_L [\#B^{\sigma(1)}, \#(B^1 \cup B^2)]$. It satisfies AN, REP, UR, MON and TR but it does not satisfy UNI: $(\{a\}, \{b, c\}) \succ_2 (\{d\}, \{d\})$.

(REP) Consider \succeq_3 defined as follows: Take a fixed set $K \subset L$. Now, $A \succ_3 B \Leftrightarrow A \succ_{10} B$, and if $A \sim_{10} B$, when $\#(A^1 \cap A^2 \cap K) > \#(B^1 \cap B^2 \cap K)$. It

satisfies AN, UNI, UR, MON and TR, but it does not satisfy REP: Let A be such that $x \in A^1 \cap A^2$, $x \notin K$, $y \in K$. Thus, $[(A^1 \setminus \{x\}) \cup \{y\}, (A^2 \setminus \{x\}) \cup \{y\}] \succ_3 A$.

(UR) Consider \succeq_4 defined as follows: Take a fixed set $K \subset L$. Now, $A \succ_4 B \iff A \succ_{10} B$, and if $A \sim_{10} B$, when $\#(A^1 \cup A^2) \cap K > \#(B^1 \cup B^2) \cap K$. It satisfies AN, UNI, REP, MON and TR, but it does not satisfy UR: Let A be such that $x \in A^2 \setminus A^1$, $x \in K$, $y \notin K$, $y \notin A^2 \setminus \{x\}$. Thus, $A \succ_4 [(A^1, (A^2 \setminus \{x\}) \cup \{y\})]$.

(MON) Take \succeq_5 defined as follows: $(A^1, A^2) \succeq_5 (B^1, B^2) \iff [\#(A^1 \cap A^2), \#A^{\sigma(1)}] \succeq_L [\#(B^1 \cap B^2), \#B^{\sigma(1)}]$. It fulfills AN, UNI, REP, UR and TR, but it does not satisfy MON: $(\{a\}, \{a, b\}) \sim_5 (\{a\}, \{a\})$.

(TR) Take \succeq_6 to be the utilitarian opportunity relation. It satisfies AN, UNI, REP, UR and MON, but it does not satisfy TR:

$(\{a\}, \{a, b, c, d\}) \succ_6 (\{a, b\}, \{a, b\})$. ■

6. FINAL REMARKS

We start by providing a summary of results in Table 1. The different columns refer to the different social relations studied in this paper. The last column corresponds to the cardinality difference relation presented in Kranich (1994). We summarize the behavior of the aforementioned relations with respect to the properties presented in Section 2. The starred properties correspond to the properties used in the characterization results offered in the paper.

	\geq_{co}	\geq_u	\geq_{lo}	\geq_{cd}
AN	yes*	yes	yes*	yes
UNI	yes*	yes*	yes*	no
REP	yes*	yes*	yes*	yes
UR	yes*	yes*	yes*	yes
IE	yes*	no	no	yes
NM	yes*	no	no	yes
MON	no	yes*	yes*	no
TR	yes	no	yes*	yes
IT	no	yes*	no	no

Table 1

Some remarks are in order:

(1) Among all the properties presented, AN and NM appear in Kranich (1994). Moreover, IE shares the same spirit than Kranich's Independence of Common Expansions.

(2) The cardinality difference relation satisfies TR, but it does not satisfy UNI. Nonetheless, UNI is satisfied by all our relations.

(3) As for the remaining properties, REP and UR are also satisfied by all the relations; MON is satisfied by all of them but \geq_{co} and TR is satisfied by all of them but \geq_u .

(4) Relations \geq_u and \geq_{lo} extend straightforwardly to the n -person case, whereas \geq_{co} presents analogous difficulties to \geq_{cd} .

(5) Throughout the paper we assumed that all information on individual preferences is irrelevant for the social ranking. A hidden assumption somehow included in the previous one is that the possible diversity or similarity between the basic opportunities is also neglected. This can be a matter of discussion. When we compare two individual choice sets, we do not pay attention to the similarity between the alternatives they contain. In the case we would like to take these similarities into account, we should deal, at the same time, with a binary relation describing such similarities. When this similarity is conceived as an equivalence relation, a simple strategy for extending our results may be to reconstruct our social relations over the quotient set. In the case the similarity relation is less structured the situation becomes more complex and deserves additional research.

REFERENCES

- ARNESON, R. (1989), Equality of opportunity for welfare, *Philosophical Studies*, 56, pp. 77-93.
- BOSSERT, W. (1995), Redistribution mechanisms based on individual characteristics, *Mathematical Social Sciences*, 29, pp. 1-17.
- COHEN, G.A. (1989), On the Currency of Egalitarian Justice, *Ethics*, 99, pp. 906-944.
- DWORKIN, R. (1981), Equality of Welfare, *Philosophy and Public Affairs*, 10, pp. 185-246.
- ELSTER, J. (1982), Sour grapes—Utilitarianism and the Genesis of Wants, in A. Sen & B. Williams (eds), *Utilitarianism and beyond*, Cambridge: Cambridge University Press.
- HERRERO, C. (1995), Capabilities and Utilities, *A Discusión*, WP-AD 95-06, Universidad de Alicante.
- KOLM, S. (1973), Super-équité, *Kyklos*, 26, pp. 841-843.
- KOLM, S. (1995), The Economics of Social Sentiments: the case of Envy, *The Japanese Economic Review*, 46 (1), pp. 63-87.

KRANICH, L. (1994), *Equitable Opportunities: An Axiomatic Approach*, Mimeo, Universidad Carlos III de Madrid.

KREPS, D. (1979), A Representation Theorem for "Preference for Flexibility", *Econometrica*, 47, pp. 565-577.

ROEMER, J. (1994), *On a Mechanism implementing Egalitarianism with Responsibility*, Mimeo, University of California, Davis.

THOMSON, W. (1992), *Notions of Equal, or Equivalent Opportunities*, Mimeo, University of Rochester.

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