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MEAN-VARIANCE STOCHASTIC GOAL PROGRAMMING FOR SUSTAINABLE MUTUAL FUNDS' PORTFOLIO SELECTION

ANA GARCIA-BERNABEU
angarber@esp.upv.es
Universidad Politécnica de Valencia

DAVID PLA
dplasan@esp.upv.es
Universidad Politécnica de Valencia

MILA BRAVO
mibrasel@epsa.upv,es
Universidad Politécnica de Valencia

BLANCA PEREZ-GLADISH
bperez@uniovi.es
Universidad de Oviedo

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ABSTRACT: Mean-Variance Stochastic Goal Programming models (MV-SGP) provide satisficing investment solutions in uncertain contexts. In this work, an MV-SGP model is proposed for portfolio selection which includes goals with regards to traditional and sustainable assets. The proposed approach is based on a two-step procedure. In the first step, sustainability and/or financial screens are applied to a set of assets (mutual funds) previously evaluated with TOPSIS to determine the opportunity set. In a second step, satisficing portfolios of assets are obtained using a Goal Programming approach. Two different goals are considered. The first goal reflects only the purely financial side of the target while the second goal is referred to the sustainable side. Aversion to Risk Absolute (ARA) coefficients are estimated and incorporated in our investment decision making approach using two different approaches.

Keywords : Mean-Variance portfolio selection model; TOPSIS; Stochastic Goal Programming; Expected Utility Theory; ARA coefficients; Mutual Funds.

RESUMEN: Los modelos media-varianza de Programación por Metas Estocásticas (MV-SGP) proporcionan soluciones de inversión satisfactorias en contextos de incertidumbre. En este trabajo, proponemos un modelo MV-SGP que incluye metas financieras y de sostenibilidad. El enfoque que proponemos se basa en un procedimiento en dos etapas. En la primera etapa se obtiene el conjunto de oportunidades de inversión aplicando filtros de sostenibilidad y/o financieros a un conjunto de activos financieros previamente evaluado y ordenado mediante el método TOPSIS. En la segunda etapa se obtienen carteras de activos "satisfactorias" mediante la aplicación del método de Programación por Metas. Hemos considerado dos metas. La primera meta refleja únicamente aspectos financieros mientras que la segunda representa la vertiente referida a la sostenibilidad. El modelo incluye estimaciones de la aversión al riesgo absoluta del inversor para cuya obtención se proponen dos enfoques diferentes.

Palabras clave: Modelo de selección de carteras media-varianza; TOPSIS; Programación por Metas Estocásticas; Teoría de la Utilidad Esperada; Coeficientes de aversión al riesgo absoluto; Fondos de Inversión.

1. Introduction

There is no universally agreed definition on what sustainability means. The origin of the term comes from the definition of sustainable development which was popularized in Our Common Future, a report published by the World Commission on Environment and Development in 1987 (WCED, 1987), also known as the Brundtland report. In this report sustainable development is defined as the "development which meets the needs of the present without compromising the ability of future generations to meet their own needs."

In the context of investment decision making, sustainable, responsible and impact investing (SRI) is an investment discipline that considers environmental, social and corporate governance (ESG) criteria to generate long-term competitive financial returns and positive societal impact (http://www.ussif.org/).

The US SIF Foundation's *Report on Sustainable and Responsible Investing Trends in the United States* identified $6.57 trillion in total assets under management at the end of 2013 using one or more sustainable, responsible and impact investing strategies.

From 2012 to 2014, sustainable, responsible and impact investing enjoyed a growth rate of more than 76 percent, increasing from $3.74 trillion in 2012. More than one out of every six dollars under professional management in the United States today is involved in SRI (USSIF, 2014).

SRI investors comprise individuals, including average retail investors to very high net worth individuals and family offices, as well as institutions, such as universities, foundations, pension funds, nonprofit organizations and religious institutions.

Mutual funds are one of the most dynamic instruments of SRI. The number of socially responsible and sustainable mutual funds in the United States grew from 333 to 456 in the last reported two years, and assets increased from $641 billion to $1.93 trillion, an over 200 percent increase.

SRI include a wide range of financial products and asset classes, as public equity investments (stocks), cash, fixed income and alternative investments, such as private equity, venture capital and real estate. SRI investors, like conventional investors, seek a competitive financial return on their investments (USSIF, 2014).

Several research studies have demonstrated that companies with strong corporate social responsibility policies and practices are sound investments. Many companies report on sustainability, including through Corporate Social Responsibility (CSR) reporting.

More than 1,500 organizations from 60 countries have used the guidelines from the Global Reporting Initiative (which works in cooperation with the United Nations Global Compact) to produce sustainability reports.

To help investors evaluate funds based on environmental, social, and governance (ESG) factors, Morningstar recently introduced the Morningstar Sustainability Rating™ for funds. The new rating allows the evaluation of mutual funds based on how well the companies held in their funds are managing their ESG risks and opportunities.

Morningstar calculates the rating based on the underlying fund holdings using company-level ESG ratings from Sustainalytics, an independent rating agency for responsible investment strategies.

Morningstar's initial analysis of the ratings reveals that funds with explicit sustainable or responsible investment policies are "generally practicing what they preach".

In this paper, we address the problem of sustainable funds' portfolio selection using the Mean-Variance Stochastic Goal Programming (MV-SGP) approach and Morningstar's Sustainability Rating. This approach is capable of providing "satisficing" solutions in the uncertainty case from the standard expected utility theory (Ballestero, 2001).

The proposed model involves two steps. In the first step, the opportunity set of assets is determined using a well-known multiple criteria decision analysis (MCDA) method, TOPSIS (Technique for Order Preference by Similarity to Ideal Solution).

TOPSIS is a simple ranking method which identifies the alternatives which simultaneously have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution (see Behzadian et al. 2012 for a state-of-the-art survey of TOPSIS applications).

The positive ideal solution maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria. TOPSIS and differentiating assets that have been rated in terms of sustainability from those which have been rated only financially. In the second step, satisficing portfolios from a financial and sustainability point of view are obtained using a MV-SGP approach.

2. Determining the opportunity set of assets

We will start with an opportunity set S of m assets, which is split as follows (see Figure 1):

(a) A subset S* of *h* sustainable assets, which are characterized by sustainable and financial criteria

(b) A subset S** of the (*m*-*h*) remaining assets, which are characterized by financial criteria only.

Notation is F_i $(i=1,\ldots,h)$ for subset S* and F_i $(i=h+1,\ldots,m)$ for subset S**.

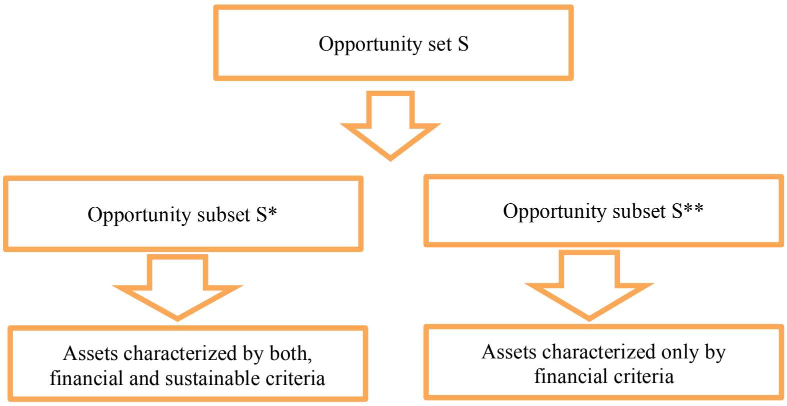

Figure 1. Description of the investor's opportunity set

In order to obtain subset S** we will rely on well-known financial ratings as Morningstar's Analyst Rating which assigns funds three positive ratings, Gold, Silver, and Bronze, a Neutral rating, and a Negative rating. The Analyst Rating is based on the analyst's conviction in the fund's ability to outperform its peer group and/or relevant benchmark on a risk-adjusted basis over the long term. If a fund receives a positive rating of Gold, Silver, or Bronze, it means Morningstar analysts think highly of the fund and expect it to outperform over a full market cycle of at least five years (see Table 1).

Table 1. Funds' financial rating from analysts of Morningstar

Rating	Score	Description
Gold	5	Best-of-breed fund that distinguishes itself across all the relevant rating pillars and has garnered the analysts' highest level of conviction.
Silver	4	Fund with advantages that outweigh the disadvantages across the analysts' pillars and with sufficient level of analyst conviction to warrant a positive rating.
Bronze	3	Fund with notable advantages across several, but perhaps not all, of the analyst's pillars—strengths that give the analysts a high level of conviction.
Neutral	2	Fund that isn't likely to deliver standout returns but also isn't likely to significantly underperform, according to the analysts.
Negative	1	Fund that has at least one flaw likely to significantly hamper future performance and that is considered by analysts an inferior offering to its peers.

Source: Based on Morningstar Direct

Opportunity set S** will be composed of funds with a rating of Gold, Silver, or Bronze. This means, the analysts think highly of the funds in terms of five different pillars: What is the fund's strategy and does management have a competitive advantage enabling it to execute the process well and consistently over time? Is the fund's performance pattern logical given its process? Has the fund earned its keep with strong risk-adjusted returns over relevant time periods? What is Morningstar's assessment of the manager's talent, tenure, and resources? What priorities prevail at the firm? Stewardship or salesmanship? Is the fund a good value proposition compared with similar funds sold through similar channels? (www.morningstar.com).

Opportunity set S* will be composed by funds with a good sustainable performance. We will rely, as described in the introduction, on The Morningstar Sustainability Rating™ which is a measure of how well the holdings in a portfolio (in this case a mutual fund) are managing their environmental, social, and governance (ESG) risks and opportunities relative to their Morningstar Category peers. The Morningstar Sustainability Rating is derived from the Morningstar® Portfolio Sustainability Score™, which is calculated based on company-level ESG scores and company involvement in ESG-related controversies. In what follows we will describe how Morningstar calculates its sustainability scores and ratings for mutual funds (www.morningstar.com).

The portfolio sustainability score is calculated as described in Figure 2.

Portfolio Sustainability Score = Portfolio ESG Score – Portfolio Controversy Deduction

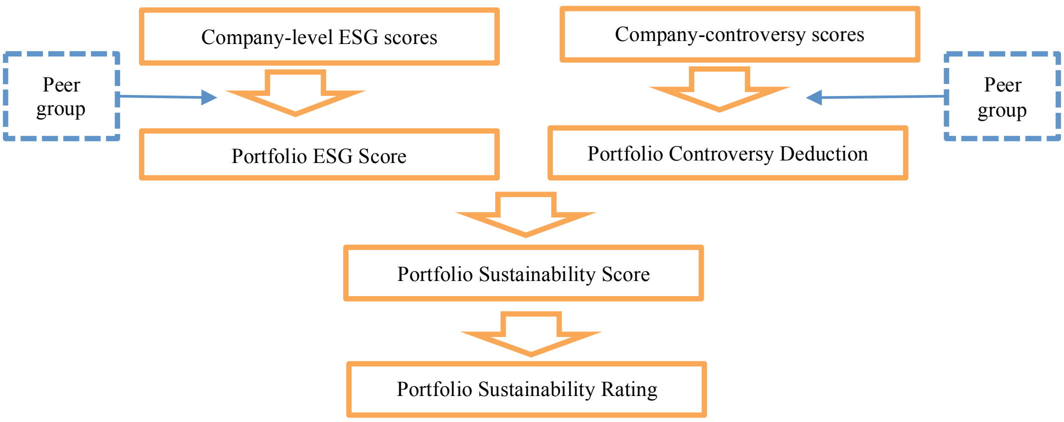

Figure 2. Morningstar Sustainability Rating

The Portfolio ESG Score is a weighted average of normalized company-level ESG scores obtained from Sustainalytics. The company-level ESG scores reflect how well a firm is addressing ESG issues. Companies' performance on ESG issues relative to other firms in the same global industry peer group is measured using a 0-100 scale. In each peer group different indicators are used depending on the specific nature of the companies' activity. To make the ESG scores comparable across peer groups, Morningstar normalizes the scores of each peer group using a z-score transformation, as follows:

$$Z_{peer} = \frac{ESG_i - \mu_{peer}}{\sigma_{peer}}, \quad i = 1,\ldots,m \tag{1}$$

where:

ESG_i: Sustainalytics company ESG score for company i

μ_{peer}: Peer-group mean ESG score

σ_{peer}: Peer-group standard deviation score

The z-scores are used to create the normalized ESG scores on a 0-100 scale, with a mean of 50, as follows:

$$ESGnormilized = 50 + \left(Z_{peer} \times 10\right) \quad (2)$$

Normalized company ESG scores can be interpreted as follows:

70 or more = Company scores at least two standard deviations above average in its peer group
60 = Company scores one standard deviation above average in its peer group
50 = Company scores at peer group average
40 = Company scores one standard deviation below average in its peer group
30 or least = Company scores at least two standard deviations below average in its peer group

Once the company ESG scores are normalized, they are aggregated to a portfolio ESG score. To receive a portfolio ESG score, at least 50% of a portfolio's assets under management must have a company ESG score.

The percentage of assets under management of the covered securities is rescaled to 100% before calculating the portfolio ESG score.

Sustainalytics also scores companies based on the highest level of current involvement in ESG-related controversies, on a 0-100 scale (see Table 2).

Table 2. Sustainalytics Controversy Score

Category	Impact on environment or society	Risk to company	Company controversy score
5	Severe	Serious	0
4	High	Significant	20
3	Significant	Moderate	50
2	Moderate	Minimal	80
1	Low	Negligible	99
0	No evidence of controversy	None	100

Source: Morningstar Ltd

Because the presence of controversy is a negative contributor to a company's overall sustainability performance, Morningstar reverses the scale of company controversy scores when aggregating them to create a Morningstar Portfolio Controversy Score, as follows:

$$PortControversy = 100 - \sum_{i=1}^{m} Controversy \times Weightadj \quad (3)$$

To calculate the Morningstar Portfolio Sustainability Score, the portfolio controversy score is rescaled, creating the portfolio controversy deduction, as follows:

Table 3. Portfolio controversy deduction

	Score	Deduction
Best	0	0
	1	0.2
	20	4
	50	10
	80	16
Worse	100	20

Source: Morningstar Ltd

Based on their portfolio ESG scores and controversy scores, funds are assigned ranks within their Morningstar Categories (Tables 4 and 5).

Table 4. Portfolio ESG scores

Distribution	Score	Descriptive rank
Highest 10% (best)	5	High
Next 22.5%	4	Above average
Next 35%	3	Average
Next 22.5%	2	Below average
Lowest 10% (worse)	1	Low

Source: Morningstar Ltd

Table 5. Portfolio controversy scores

Distribution	Score	Descriptive rank
Lowest 10% (best)	5	High
Next 22.5%	4	Above average
Next 35%	3	Average
Next 22.5%	2	Below average
Highest 10% (worse)	1	Low

Source: Morningstar Ltd

Based on their portfolio sustainability scores, funds are assigned ranks within their Morningstar Categories. A fund's Morningstar Sustainability Rating is its normally distributed ordinal score and descriptive rank relative to the fund's category.

The opportunity set S* will contain funds with a good financial and sustainability performance. Funds will be rated using TOPSIS. TOPSIS, developed by Hwang and Yoon in 1981, is a simple ranking method in conception and application. The standard TOPSIS method attempts to choose alternatives that simultaneously have the shortest distance from the positive ideal solution and the farther distance from the negative-ideal solution. The positive ideal solution maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria.

Table 6. Morningstar sustainability rating

Distribution	Score	Descriptive rank
Lowest 10%	5	High
Next 22.5%	4	Above average
Next 35%	3	Average
Next 22.5%	2	Below average
Highest 10%	1	Low

Source: Morningstar Ltd

The stepwise procedure followed for the evaluation of the funds in terms of sustainability and financial performance is the following one (Hwang and Yoon, 1981):

STEP 1. Determine the decision matrix D, where the number of criteria, financial and sustainable, is *n=2* and the number of alternatives, funds, is *m*, $D = \left(x_{ij}\right)_{mxn}$.

Financial criterion will be based on Morningstar Analyst's Scores and sustainability criterion will be based on Morningstar Sustainability scores. As both criteria use the same scoring scale, 0-5, it is not necessary to normalize the criteria.

STEP 2. Determine the weighted decision matrix. The weighted normalised value $\overline{v}_{ij}$ is calculated as:

$$v_{ij} = w_j r_{ij}, \quad i = 1,\ldots,m, \quad j = 1,2 \tag{4}$$

where w_j is the weight associated to each criterion. In this work, equal weights will be assigned to the financial and sustainability criteria.

STEP 3. Determine the positive ideal (PIS) and negative ideal solutions (NIS). The positive ideal value is fixed by the investor as a score equal to 5 for both criteria, financial and sustainability. The negative ideal value is fixed by the investor as a score equal to 1.

STEP 4. Calculate the separation measures. Calculation of the separation of each mutual fund with respect to the PIS and NIS, respectively:

$$d_i^+ = \left\{ \sum_{j=1}^{2} \left(v_{ij} - v_j^+ \right)^2 \right\}^{\frac{1}{2}} \quad i = 1, 2, \ldots, m \tag{5}$$

$$d_i^- = \left\{ \sum_{j=1}^{2} \left(v_{ij} - v_j^- \right)^2 \right\}^{\frac{1}{2}} \quad i = 1, 2, \ldots, m \tag{6}$$

STEP 5. Calculate the relative proximity to the ideal solution. Calculation of the relative proximity of each alternative to the PIS and NIS using the proximity index.

$$R_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad i = 1, \ldots, m \tag{7}$$

The R_i value lies between 0 and 1. If $R_i = 1 \rightarrow A_i = A^+$ and if $R_i = 0 \rightarrow A_i = A^-$. The closer the R_i value is to 1 the higher the priority of the *i-th* alternative is.

STEP 6. Rank the preference order. Rank the best alternatives according to R_i in descending order. Subset S* will be composed by those funds ranking in the first quartile.

3. Description of the goals

Once the opportunity set S is obtained and assets in each subset, S* and S** have been determined, let us formulate the goals of our problem under uncertainty. We will consider two different goals, Goal 1 and Goal 2. Goal 1 will be the goal of a traditionally investor not concerned at all about sustainability ratings. Goal 2 will be the goal of an extremely social conscious investor, willing to invest only in those assets with a positive sustainability rating. A third type of investor will be considered in this paper, an intermediate investor between the traditional and the extremely social conscious. This investor faces goals, Goal 1 and Goal 2 (see Figure 3).

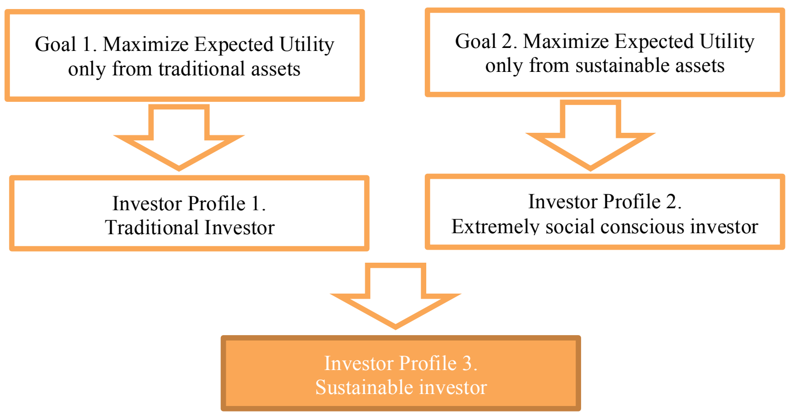

Figure 3. Description of the goals

The choice of sustainable portfolios relies on classical normative $Eu(\hat{R})$ utility theory under uncertainty (Von Neumann and Morgenstern, 1947; Arrow, 1965). As well-known, $\hat{R}$ denotes random returns while $Eu(\hat{R})$ is the expected utility of these returns. According to this classical theory, the higher the expected utility the better the investment.

Goal 1 is defined as follows:

$$
\begin{aligned}
& Eu_1(\hat{R}_1) \rightarrow u_1\left(\bar{R}_1\right) \\
& \bar{R}_1 \geq g_0 \\
& \hat{R}_1 = \sum_{i=1}^{m} \hat{f}_i x_i \\
& \sum_{i=1}^{m} x_i = 1, \quad x_i \geq 0, \quad i = 1,\ldots,m
\end{aligned} \tag{8}
$$

Goal 2 is defined as follows:

$$
\begin{aligned}
& Eu_2(\hat{R}_2) \rightarrow u_2\left(\bar{R}_2\right) \\
& \bar{R}_2 \geq e_0 \\
& \hat{R}_2 = \sum_{i=1}^{h} \hat{f}_i x_i + \sum_{i=h+1}^{m} \hat{\varphi}_i x_i \\
& \sum_{i=1}^{m} x_i = 1, x_i \geq 0, \quad i = 1,\ldots,m
\end{aligned} \tag{9}
$$

where

u_1, u_2 are the investor's utilities from goals 1 and 2, respectively.

Eu_1, Eu_2 are the expected utilities for u_1, u_2, respectively.

$\hat{R}_1, \hat{R}_2$ are random returns on each portfolio.

$\bar{R}_1, \bar{R}_2$ are expected returns.

g_0, e_0 are investor's targets or aspiration levels with respect to each goal.

$\hat{f}_i$ is the weekly random return on the i^{th} asset.

$\hat{\varphi}_i$ is the weekly random return on the i^{th} asset for those assets characterized only financially.

x_i is the i^{th} portfolio weight.

$\rightarrow$ means that expected utility should approximate its respective upper limit (utility) as close as possible.

Let us highlight the meaning of each goal. Traditionally, most investors are merely interested in expected return and risk, namely, their primary objective is financial, no matter sustainability. Any sustainability objectives falls outside the scope of these traditional investors. Conversely, the extremely social conscious investor looks for a compromise between the financial and sustainable goals as follows.

Goal 1. It reflects the purely financial side of the question. It is a classical issue in financial analysis, namely, the $Eu(\hat{R})$ objective of traditional investors, who consider series of historical returns in the recent past as the best guidance to invest.

Goal 2. It reflects the sustainability side. It is the $Eu(\hat{R})$objective of an extremely social conscious investor. Although several degrees of commitment with sustainability are possible, we will consider an investor with total sustainability commitment. This means that this investor is not willing to invest in the assets included in S**. Therefore, we will make $\hat{\varphi}_i = 0 \quad (i = h+1,\ldots,m)$ in (8), namely, every random return is replaced by a fictitious return equal to zero. This means that assets Fi ($i=h+1,\ldots,m$) have no value for the extremely social conscious investor. Mathematically, this based statement is more convenient than the alternative statement of removing the "non-sustainable" set of assets from Goal 2. In fact, the based statement allows us to define both goals in a similar way.

In our paper, the investor is neither a traditional investor nor an extremely sustainability conscious investor, but a decision maker who seeks a satisfying solution from these two goals.

4. The MV-SGP model

Ballestero (2001, 2005) proved that system (8)-(9) is equivalent to the following MV-SGP parametric and quadratic model:

$$\begin{aligned}&\min XV^{TX}\\&s.t.\\&\bar{R}_1 = \sum_{i=1}^{m} \bar{f}_i x_i \geq g_0\\&\bar{R}_2 = \sum_{i=1}^{h} \bar{f}_i x_i \geq e_0\\&\sum_{i=1}^{m} x_i = 1\\&x_i \geq 0\end{aligned} \tag{10}$$

where:

X is the row vector $(x_1,...,x_m)$.

X^T is the transpose vector of X.

V is a mxm matrix summarizing variability of the returns which can be stated as $V = r_1V_1 + r_2V_2$, where r_1, r_2 are the Arrow's absolute risk aversion (ARA) coefficients for each goal, while V_1, V_2 are covariance matrices expressing variability of returns for Goals 1 and 2, respectively.

$\bar{f}_i$ is the expected return of the i^{th} asset.

Let us first, define the sustainability target as

$$e_0 = \lambda \bar{f}_{e\max} \tag{11}$$

where $\bar{f}_{e\max} = \max \bar{f}_i \left(i = 1,...,h\right)$ while parameter $0 \leq \lambda \leq 1$ increases as the investor's aspiration level for the sustainability goal increases. As usual in M-V models, target g_0 is treated as a parameter moving on a feasible interval.

Suppose first, that the above maximum expected return, namely, the mean value in (11), is positive.

Case 1. Suppose $\lambda > 1$. Then, no feasible solution to model (10) can be found.

Case 2. Suppose $\lambda = 1$. Then, there is only one solution, namely,
$x_i = 1$ if $i = p$ where p is the sustainable asset of maximum expected return in (11),
$x_i = 0$ if $i \neq p$.

This non-diversified solution corresponds to an extremely social conscious investor who maximizes the expected return.

Case 3. Suppose $0 \leq \lambda < 1$. Then, the higher the value of λ the higher the sustainability target e_0 Consider a value $\lambda = \lambda_0$. This leads to solutions such as the following ones:

$$\sum_{i=1}^{h} x_i = q \geq \lambda_0; \quad q \leq 1; \quad \sum_{i=1}^{h} x_i = 1 - q.$$

Case 4. Suppose $\lambda < 0$. Then, target e_0 given by (11) would be less than zero, which has little sense because even the extremely social conscious investor does not like negative expected returns.

Now, suppose that the maximum expected return in (11) is negative. Then, to invest in sustainable assets is not advisable, as the social conscious investors are not satisfied with negative expected returns either.

5. Estimating ARA coefficients

As well-known, risk aversion does not mean risk at all. It is a psychological concept describing the investor's psychological attitude towards risk (this attitude may or may not be influenced by a risk perception). Many investors behave as risk averse, namely, they prefer portfolios of low volatility, other things being equal. Other investors are risk neutral, while a few are risk lovers. To estimate the ARA coefficients in our context the following two approaches can be alternatively used.

First approach. Coefficients r_1, r_2 are straightforwardly elicited by comparing the investor's attitude towards risk in a framework unrelated to Arrow's risk aversion theory. An advantage of this approach is simplicity. However, there is a major drawback that ignores Arrow's risk aversion equation. This will be illustrated below.

Second approach. Comparison of r_1, r_2 is made in the framework of Arrow's theory. Let us consider the following two scenarios.

Scenario 1. Several investors with different wealth face a given investment, which is the same for all of them. In this scenario, which we are not going to address in this work, the j^{th} ARA coefficient depends on the j^{th} investor's wealth W_j through Arrow's equation (Arrow, 1965).

$$r_j = (-1)u_j''(W_j)/u_j'(W_j), \quad W_j \geq 0 \tag{12}$$

where the first derivative $u_j' > 0$ and the second derivative $u_j'' < 0$. In the case of risk neutrality, $u_j'' = 0$ so that $r_j = 0$. It is usual assumed that r_j decreases with the increase of the investor's wealth (Copeland and Weston, 1988).

Scenario 2. A single investor faces several investments or goals. This is the scenario in this work. Then, Arrow's equation turns into

$$r_j = (-1)u_j''(R_j)/u_j'(R_j), \quad R_j \geq 0 \tag{13}$$

For the ease of notation, we here write R_j instead of $\hat{R}_j$ to denote random return. In this case, j=1,2 for Goals 1,2 respectively. Both derivatives are specified by making the return $R_j = \bar{R}_j$. Here, r_j increases with the increase of the $\bar{R}_j$ expected return, other things being equal.

Quadratic utility functions are the only functions verifying the previously mentioned property (Kallberg and Ziemba, 1983). With the purpose of eliciting the ARA coefficients we will here use the following quadratic utility function

$$u_j = 2b_jR_j - c_jR_j^2, \quad b_j, c_j > 0, \quad j = 1,2 \tag{14}$$

Equations (13) and (14) yield

$$r_j = \frac{1}{(b_j / c_j) - \bar{R}_j}, \quad j = 1,2 \tag{15}$$

By maximizing utility (14) we have

$$b_j - c_jR_j = 0 \Rightarrow R_j^* = \frac{b_j}{c_j}, \quad j = 1,2 \tag{16}$$

being R_j^* the return that maximizes the utility function in (14). From (15) and (16) we get

$$r_j = \frac{1}{(R_j^*) - \bar{R}_j}, \quad j = 1,2 \tag{17}$$

To elicit the ARA coefficients, the analyst should conduct a test through which the investor discloses his risk aversion for each goal. It is developed as follows.

(i) Test input. Concerning Goal 1, the test starts with a fictitious investment H_1 from an opportunity set, which is not characterized as an ethical set of assets. Investment H_1 has zero mean value and σ standard deviation. Concerning Goal 2, the test requires considering a fictitious investment H_2 from ethical assets. Investment H_2 also has zero mean value and σ standard deviation of the observed returns. Therefore, H_1 and H_2 have equal volatility. However, the investor's risk aversion can differ from one to another. From (17) we have

$$r_{H_j} = \frac{1}{\left(R_j^* - \bar{R}_{H_j}\right)} = \frac{1}{R_j^*}, \quad j = 1,2 \tag{18}$$

where r_{H_j} is the ARA coefficient for each H_j fictitious investment since mean value $\bar{R}_{H_j}$ is equal to zero.

(ii) Formulating the test. The analyst asks the investor: "If you really are an extremely social conscious investor, then your risk aversion for a sustainable investment such as H_2 will be relatively low, namely, lower than your risk aversion for H_1, which has the same expected return and risk as H_2 but it is not characterized as sustainable. Taking into account this, would you like to compare your risk aversion for H_1 to your risk aversion for H_2?".

(iii) Test output. Once the ration has been specified equation (18) yields

$$R_2^* = \left(r_{H_1} / r_{H_2}\right) R_1^* \tag{19}$$

(iv) ARA coefficients for Goal 1 and Goal 2. From equations (15) and (18) we have

$$r_1 R_1^* = \frac{1}{1 - \left(\bar{R}_1 / R_1^*\right)} \tag{20}$$

$$r_2 R_1^* = \frac{1}{\left(r_{H_1} / r_{H_2}\right) - \left(\bar{R}_2 / R_2^*\right)} \tag{21}$$

From equations (20) and (21), we have:

$$r_1 / r_2 \approx r_{H_1} / r_{H_2} \tag{22}$$

where ratio $r_{H_1} / r_{H_2} > 1$. Thus, the ARA coefficients are elicited in an approximate way.

Let us now try to interpret the previous results. Goals 1 and 2 lead to the following relationship (Pratt, 1964, Ballestero, 2001):

$$\max \left[E_{u_j}\left(\hat{R}_j\right)\right]_{norm} \approx \left[u_j\left(\bar{R}_j\right)\right]_{norm} - (1/2) r_j \sigma_j^2\left(\hat{R}_j\right), \quad j = 1,2 \tag{23}$$

where subscript "norm" means that the respective expression is normalized by the first derivative of the utility u_j specified at the $\bar{R}_j$ mean value.

Moreover,

$\sigma_j^2\left(\hat{R}_j\right) = X_j V_j X_j^T$ is the portfolio variance, which means risk for both goals the financial and the sustainability goal and X_j is the solution (vector of portfolio weights) to the following model:

$$\min X_j V_j X_j^T$$
$$s.t. \qquad \bar{R}_j \geq \begin{cases} g_0 & if \quad j=1 \\ e_0 & if \quad j=2 \end{cases} \tag{24}$$

together with the non-negativity conditions.

As the objective function (portfolio variance) is a measure of risk, Goal 1 and Goal 2 involve constrained risk minimization for the traditional (purely financial) investor and the extremely sustainable conscious investor, respectively.

Now, consider the so-called sustainable conscious investor in this paper. This investor has an intermediate profile between the traditional and the extremely sustainable conscious. Here, vector X is the solution of model (11). Notice that the objective function, namely,

$$\min X\left(r_1V_1 + r_2V_2\right)X^T = r_1XV_1X^T + r_2XV_2X^T \tag{25}$$

is a composite index of variability instead of a portfolio variance. Then, how to measure the financial risk that the sustainable conscious investor bears? This risk is given by

$$XV_1X^T \tag{26}$$

which is valid whatever the sustainability characterization of the portfolio. In fact, matrix V_1 includes the co-variances of all assets, whether the asset is sustainable or not.

Therefore, matrix V_1 is a financial matrix while matrix V_2 does not describe financial risk. Obviously, we generally have

$$XV_1X^T \neq X_jV_jX_j^T \quad (j=1) \tag{27}$$

because solution X is generally different from solution X_j, $j=1$.

Finally, to highlight the meaning of the model in terms of risk aversion, consider the above Pratt's relationship focusing on its negative term on the right-hand side. The greater the negative term (in absolute value), the smaller the expected utility on the left-hand side, other things being equal. This term is the product of two factors: portfolio variance, which is an observable risk measure and the r_j risk aversion coefficient, which describes the investor's perception of risk and his/her risk assessment from utility. The investor's expected utility suffers from the joint impact of both factors. We can plausible assume that the extremely sustainable conscious investors have less risk aversion for sustainable assets than the weakly sustainable ones, other things being equal. Indeed, if one is willing to invest in sustainable products, than one tends to close his eyes to the risk inherent in such investment.

As noted, defining Goal 1 and Goal 2 does not require assuming a special utility function. In contrast, the problem of eliciting risk aversion requires using a particular type of utility. The utility function chosen for this purpose in the previous section is valid whatever the sustainable profile of the investor. This is because even the extremely sustainable conscious investors are concerned about potential losses in their investments.

6. Conclusions

MV-SGP models provide satisficing solutions in the uncertainty case from the standard expected utility theory. In this paper, we have proposed a model that based on a MV-SGP approach provides satisficing portfolios. The obtained portfolios are obtained using a two-phase procedure.

In the first phase the opportunity set of assets (mutual funds) is obtained. This is done applying adequate financial and sustainable screens to the initial universe of assets and then ranking the assets using TOPSIS.

Application of financial and sustainable screens is based on the practice of a well-known mutual funds' rating agency, Morningstar and on its recently published sustainability rating for mutual funds.

TOPSIS is a well-known ranking method based on minimization of the distance of each decision alternative (mutual fund) to the positive ideal alternative and maximization of the distance of each alternative to the negative ideal alternative. In this paper, the ideal alternatives have been set as the funds obtaining the maximum and minimum rates, respectively.

In the second phase satisfying portfolios are obtained using a MV-SGP approach. In MV-SGP, the approach essentially consists of specifying the expected utility equation corresponding to each goal. The first goal reflects only the purely financial side of the target, while the second one reflects the sustainability side. Two approaches have been presented from Ballestero (2001) including the estimation of ARA coefficients, which are critical parameters in the MV-SGP achievement function.

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