

## Design of a continuous review stock policy

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**Abstract** In stock management system with inventory management policy (s,Q) sometimes managers find that is a spread between the level of service they have calculated and the real one. This is because there are exact and approximate ways of calculating CSL (cycle service level). The former are difficult to calculate (and not typically used in business), and the latter tend to neglect the deviations in the reorder point (also called undershoots) this results in differences mentioned above. This paper presents an structured review of the literature on the methodology to be followed in the design of inventory management policy (s,Q) when demand process is stationary with discrete probability function, independent and identically distributed, and replenishment period L constant and pinpoints the most significant research gaps and also proposes guidelines for the practical application of the most usual approximations. Models are considered for lost sales and backorders.

**Keywords:** CSL (Cycle Service Level), Management Policy (s,Q), undershoot.

### 1.1 Introduction

For design a continuous review policy first it is necessary to know nature of the demand, will be explained in section 1.2. The listed items will not be detail explained as it has been published in others papers. To explain two types of scenarios that can occur when there is demand but no product to serve, these are either made backorder or lost the sale, this is explained in section 1.3. In section 1.4. we explain the inventory management policy that focuses on this paper, this is the policy of continuous review (s,Q). Once we have selected management policy, the next step is to define the design criteria to determine policy parameters. The metric used in this article is the cycle service level (CSL). The different definitions for CSL are detailed in section 1.5.

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On section 1.6. based on previous section definitions we have made a summary table which lists various published methods and their characteristics. Finally we draw conclusions in section 1.7.

## **1.2 Nature of demand**

The first step for design an inventory management system is to study the item characteristics that the design is intended for. This analysis has two main purposes:

1. Identify the relative importance of item (Silver et al. 1998) and
2. To facilitate the selection on the best procedure for forecasting and inventory policy (Fogarty et al 1991).

For the first purpose the ABC Classification is used, it will be explained in section 1.2.1. For second purpose is used items categorize according to their demand pattern, that must be compared to probability distribution function to represent it.

### ***1.2.1 ABC classification***

ABC classification applied to inventory management is (Fogarty et al. 1991):

1. Classify each item based on their relative importance.
2. Establish a management approach consistent with importance degree according to items classification based on their relative importance.

ABC analysis identifies the most important items and classified as A type. The minor, are classified as C. Rest places in B type (Zipkin 2000). ABC classification does not limit use to these three categories. I.e., (Campbell 1975) use five different classes. And others authors limit classification to a maximum of six (Silver et al. 1998) and (Graham 1988). Traditional approach of ABC classification usually is made based on a single criterion, often this is demand value or demand volume (Teunter et al. 2010). However, a number of authors (Buzacott 1999), (Ramathan 2006), (Zhou and Fan 2007), (Ng 2007) and (Ding and Sun 2011) have considered use several criteria (such as supply security, obsolescence rate, delivery time, etc...). Based on above have been developed different multicriteria classifications. Any of the criteria annotated above serve to make an ABC classification.

### 1.3 Backorders versus lost sales

Two situations may occur in an inventory system when there is demand:

1. That there is enough stock in warehouse to satisfy completely, and
  2. That stock is not sufficient and therefore the inventory system is in stockout.
- An important system feature is what happens when this situation occurs. Basically, there are two extreme cases (Silver et al. 1998): (i) Backordering case: Demand that can not be served is deferred to following cycle and will be served as soon as system receives an order to supply large enough (Hadley and Whitin 1963); (ii) Lost sales case: Demand that can not be satisfied with available stock is lost and becomes lost sales. Generally not known and therefore, these lost sales are not recorded in the company (Thomopoulos 2007).

In classical inventory models, it is common to assume that excess demand is backorder. However, studies that analyze customer behavior in practice (Gruen et al. 2002) and (Verhoef and Sloot 2006) argue that unmet demand is more common than be lost. Inventory systems, including lost sales appear to be more difficult to analyze and resolve. Also, lost sales inventory systems require different replenishment policy to minimize the replacement cost compared to backorders systems (Bijvank and Vis 2011). Most of the available inventory theory do not talk about adjustments to be made for lost sales model see (Guide and Srivastava 1997), (Kennedy et al. 2002), (Silver 2008) and (Williams and Tokar 2008). The optimal policy for continuous review model with lost sales is not well known. Few authors explain it (Johansen and Thorstenson 1996). In real life, there are often situation that combine both scenarios. However, most inventory management models are developed for one of the two (Silver et al. 1998). From a mathematical point of view it is usually easier obtain the model if the demand can be differ (Silver 1981) and (Zipkin 2008), is why most literature focuses on this case.

### 1.4 Inventory Management policy (s,Q)

Selection of inventory policy depends on how often to check the inventory level (Cardós et al. 2009). Inventory management policies with random demand are divided into two main categories: periodical and continuous review. If the status of the inventory is permanently reviewed, we talk about continuous review policy. In the other case is periodical review policy. Most inventory management models are based on assumptions rather restrictive view (Silver et al. 1998) and (Axsäter 2000). For example, consider the demand for unit size and a normal distribution for the demand during the replenishment lead time. In most inventory management systems, these simplifications circumstances are allowed. Sometimes these simplifications fundamentally differ from the actual conditions.

The inventory management policy (s,Q) is known by the name order point-order quantity. Also known as reorder point system (Krajewski and Ritzman 2000). Where s is reorder point and Q quantity ordered. The model (s,Q) is an important model in literature production and management in operations research and in practice (Hadley and Whitin 1963), (Johnson and Montgomery 1974), (Silver and Peterson 1985) and (Nahmias 2008). In inventory management policy (s,Q) a fixed quantity Q is ordered whenever the inventory position reaches reorder point or falls below this (Silver et al. 1998). Order is received L periods later, L can be constant or variable. Are defined:

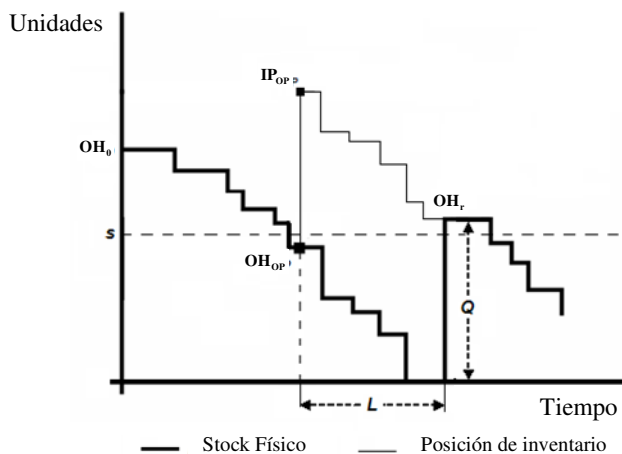
- Inventory Position (IP), measure the item's ability to satisfy future demand (Krajewski and Ritzman 2000). This includes scheduled receptions and available stock and would be subtracted backorders. This doesn't take into account the committed (Yeh et al. 1997), (Krajewski and Ritzman 2000) and (Rao 2003).

$$IP = OH + SR - BO \quad (1.1)$$

Where OH is on-hand, SR is scheduled receptions and BO are backorders.

On (Silver et al. 1998) in addition to subtract the commitment (C) :

$$IP = OH + SR - BO - C \quad (1.2)$$



**Fig. 1.1** Evolution of physical stock and the inventory position in a system (s,Q). Source: Own

Face with authors who say that periodic policies allow coordinate replenishment of various items, and cost savings that this implies (Sani and Kigsman 1997), (Eynan and Kropp 1998), (Chiang 2006) and (Chiang 2007) other authors such as (Yeh et al. 1997) assume that continuous review is needed to ensure an adequate level of service and (Rao 2003) says that if lead time is small, (s,Q) police is more efficient and can be manage with very little inventory, also says that in sporadic demand scenarios is better than periodic review.

## 1.5 Cycle Service Level as a design requirement

Once management policy is defined, we must establish design the criteria to determine the policy parameters. Basically there are two methods:

1. Which minimize cost, and
2. Which minimize inventory average to a certain service level.

In the first case, inventory policies should be considered different types of costs (holding inventory, order and stockout costs) (Schneider 1981). Some authors assume that shortages stock costs can be expressed analytically simple, focusing on cost minimization. In practice, these costs are difficult to establish and estimated, are discarded in favour of a focus on a predetermined service level view satisfaction (Cohen et al. 1988) and (Larsen and Thorstenson 2008), for this reason most commonly used design requirements relate to customer service. Service level is closely related to stock-outs or product lack when there is a request for it. A stock-out not only causes immediate profit loss by not serving an order, but also causes a loss of long-term benefit because it reduces the possibility of receiving new orders from that customer (Lejeune 2008). See (Anderson et al. 2006) for a study on the effects of stock-outs.

Most commonly used design requirements are those relate to customer service as cycle service level (CSL) o P1 (Vereecke and Verstraeten 1994); (Cardós et al. 2006);(Cardós and Babiloni 2009) and (Cardós and Babiloni 2011), and percentage of demand satisfied with physical stock, called fill rate (FR) o P2 (Dunsmuir and Snyder 1989), (Janssen et al. 1998), (Segerstedt 1994), (Snyder 1984), (Strijbosch et al. 2000); (Yeh et al. 1997). Present paper focuses on CSL.

In literature there are two definitions of CSL. First, hereinafter referred to classical, defines CSL as the probability of not incurring stock-outs during the replenishment cycle. This probability, also known as P1, is equivalent to the safety factor used to calculate k safety factor when demand is normally distributed (Silver et al. 1998). Therefore, CSL is the fraction of cycles in which a stockout doesn't occur. (Silver et al. 1998) defines stockout as the moment in which the available physical stock is zero. Therefore:

$$CSL = 1 - P(\text{stockout in a replenishment cycle}) \quad (1.3)$$

(Axsäter 2000) defines CSL as “probability of no stockout per order cycle”. According to (Cardós and Babiloni 2011), this definition and the corresponding expression should be improved as follow:

1. Demand fulfilment is not taken into account and
2. There are a number of situations where this definition is scarcely useful. I.e., under intermittent or slow-movements demand context the probability of no demand when physical stock is equal to zero is not negligible, so a stockout situation and demand fulfilment can be compatible.

(Chopra and Meindl 2001) proposes a convenient definition of CSL: fraction of replenishment cycles that end with all customer demand met for (Cardós et al. 2006) propose another CSL definition: “fraction of cycles in which having demand nonzero is been fully satisfied with physical stock”. When applied to continuous review policy, cycle demand is always positive so this definition can be simplified becoming expression proposed by (Chopra and Meindl 2001), see equation 1.4. Moreover this definition is useful not only from technical standpoint but also from management perspective.

$$CSL = P(\text{cycle demand} \leq \text{available stock} | \text{cycle demand} \geq 0) \quad (1.4)$$

(Chopra and Meindl 2001) say “...that stockout occurs in a cycle if demand during lead time is greather than reorder point” and also propose an estimation method:

$$CSL = P(\text{Demand during lead time of } L \text{ weeks} \leq s) \quad (1.5)$$

## 1.6 Methods for calculating CSL for inventory Management policy (s,Q)

In this section we review the existing calculation methods for CSL and collected in a summary table to find gaps in the research:

**Table 1.1** Review of methods for calculating CSL in (s,Q) inventory management policy.

Author (year)	Method Type	s vs Q	Ignores U <sup>1</sup>	Applicable with backorders and/or lost sales
(Chopra and Meindl 2001)	Approximate	For any s	Yes	Both
(Silver et al. 1998)	Approximate	For any s	Yes	Only lost sales
(Cardós et al. 2009)	Approximate	For any s	Yes	Only backorders
(Cardós and Babiloni 2008)	Exact	s < Q	No	Only lost sales
(Cardós et al. 2009)	Exact	s ≥ Q	No	Only lost sales
(Cardós et al. 2009)	Exact	For any s	No	Only backorders

<sup>1</sup>U is undershoot or deviation at the reorder point.

## 1.7 Conclusions

This paper has revealed some relevant research gaps that will be addressed in future research projects: (i) the need of better approximations for CSL estimation; (ii) all approximate methods despised undershoot; and (c) although we have not

exposed in the tail any exact calculation methods we should said that the exact methods requires a high computational effort.

In future research we will try to find a CSL approximate formula in the lost sales context with reduced deviation from the exact value.

This paper is part of a wider research project devoted to identify the most simple and effective stock policy to properly manage any particular demand pattern based on the characteristics of the demand itself.

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