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Sharing a polluted river through environmental taxes*

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Sharing a polluted river through environmental taxes *

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Abstract

We consider a river divided into n segments. There are n agents located along the river who generate residues. The river requires cleansing and it entails some cost. We propose several rules to distribute the total pollutant-cleaning cost among all the agents. For each rule we provide an axiomatic characterization using properties based in water taxes. Moreover, we prove that one of the rules coincides with the weighted Shapley value of a game associated with the problem.

JEL classification: C71; D61.

Keywords: cost sharing, pollutant-cleaning cost, water taxes.

1 Introduction

The aggravation of environmental contamination in the last years is an important reason for the countries to institute taxes on the emission of polluting substances into the different natural environments. In the particular case of Spain, sanitation (or water) charges constitute the most representative environmental tax as more than two thirds of the autonomous regions' governments use them. The sanitation charges are principally a tax instrument for funding public sewage treatment services (Gago et al., 2006).

From a theoretical point of view, Ni and Wang (2007) develop a model to study how to allocate the pollutant-cleaning costs among the agents that cause this pollution. They consider a river which is divided into n segments.

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In each segment there are agents who discharge pollutant substances of some kind into the river. The authorities could guarantee the cleansing of the water for public use. They prove that the *Upstream Equal Sharing* method is the only rule that satisfies *Efficiency*, *Additivity*, *Irrelevance of Upstream Costs* and *Upstream Symmetry* (which states that for any given downstream costs, all upstream polluters share them equally). However, many situations exist where the last axiom cannot be applicable.

In this paper we consider the same model as in Ni and Wang (2007) and we characterize the set of rules satisfying *Efficiency*, *Additivity* and *Irrelevance of Upstream Costs*. We then characterize two rules by adding two properties to these.

Sometimes, residues that firms dump into the river are biodegradable and therefore pollution disappears over time. Moreover, in many cases it is possible to know the biodegradation rate. The property of *Biodegradation rate* ensures that the taxes paid by the agents depend on this rate.

Another property is *Weighted Tax*. In many countries there are several alternatives in the design of water tax rates. In most cases a variable component exists which depends on different factors, such as the volume of water consumed, the pollution load, the population of the municipality, etc. (Gago et al., 2006; OECD, 2006). This idea is collected by the axiom of *Weighted Tax*.

Ni and Wang (2007) prove that the two methods they present coincide with the *Shapley value* (Shapley, 1953b) of two different games. We prove that the rule characterized by using *Weighted Tax* coincides with the *weighted Shapley value* (Shapley, 1953a) of a new game, where, the value of a coalition represents the pollutant-cleaning costs in the segments polluted only by agents who belong to this coalition.

The paper is organized as follows. In Section 2 we introduce the model. In Section 3 we characterize the family of rules satisfying three properties. In Section 4 we introduce new properties in this context and we present two rules and characterization results for them. Moreover, we prove that one of the rule coincides with the *weighted Shapley value* of a particular cooperative game.

2 The model

We will follow the model introduced by Ni and Wang (2007). Consider a river which is divided into n segments indexed in a given order $i = 1, 2, ..., n$ from upstream to downstream. There are n household-firms (agents) located along the river, each of them is located in one of these segments according to the above order. Each firm generates a certain amount of pollutants of some kind, which, all households try to avoid.

Our attempt is to find rules or methods to allocate the total cost of pollution cleaning among all the household-firm pairs that are responsible for the dumping.

Formally, let $\mathcal{N} = \{1, 2, ...\}$ be the set of all possible agents. Let $N \subset \mathcal{N}$ be a finite set of agents. Usually we take $N = \{1, ..., n\}$. Let $C = (c_1, ..., c_n) \in \mathbb{R}^n_+$

be the pollutant-cleaning cost vector, where c_i represents the cost incurred by agent i.

A *pollution cost sharing problem* is a pair (N, C). When N is fixed we simply denote as C the problem.

A *solution* to a problem (N, C) is a vector $x = (x_1, ..., x_n) \in \mathbb{R}^n_+$ such that $\sum_{i \in N} x_i = \sum_{i \in N} c_i$, where x_i represents the cost share assigned to agent i.

A *rule* (or *method*) is a mapping x that assigns to each problem (N, C) a solution $x(N, C)$.

A *transfer utility game*, *TU game*, is a pair (N, v) where $N \subset \mathcal{N}$ is finite and $v : 2^N \to \mathbb{R}$ satisfies $v(\emptyset) = 0$. Given $w \in \mathbb{R}^N_{++}$, we denote the *weighted Shapley value* (Shapley, 1953a) with weights given by w as $\phi^w_i(N, v)$.

3 Characterization

Ni and Wang (2007) characterize the *Upstream Equal Sharing* method with four axioms: *additivity*, *efficiency*, *independence of upstream costs* and *upstream symmetry*. The latter ensures that all the upstream agents have equal responsibilities for a given downstream pollution cost. However, situations exist where this axiom cannot be applicable. For instance, in the Spanish autonomous regions Valencia and Catalonia, the water taxes applicable to the households take into account the population of the municipality where the house is located. In these cases we cannot assume that all the upstream agents are symmetric in regards to the pollution caused.

In this section we characterize the set of rules satisfying the other three axioms. Before presenting the main result we introduce the properties:

Efficiency (Eff) $\sum_{i \in N} x_i = \sum_{i \in N} c_i$.

Additivity (Add) For any $C^1 = (c^1_1, ..., c^1_n) \in \mathbb{R}^n_+$, $C^2 = (c^2_1, ..., c^2_n) \in \mathbb{R}^n_+$ and $i \in N$, $x_i(C^1 + C^2) = x_i(C^1) + x_i(C^2)$.

Independence of Upstream Costs (IUC) Let $l \in N$ and $C, C' \in \mathbb{R}^n_+$ such that $c_i = c'_i$ for all $i > l$. Then, for all $i > l$, $x_i(C) = x_i(C')$.

Now we present the family of rules satisfying Add, Eff and IUC. These rules divide the cost of each segment j (c_j) among the agents responsible for it ($i \in N$ such that $i \leq j$) proportionally to a vector $p^j \in \mathbb{R}^n_+$. Namely,

Theorem 1 *A rule x satisfies Eff, Add and IUC if and only if for each $j = 1, ..., n$ there exists a weight system $\left(p^j_i\right)_{i \in N} \in \mathbb{R}^n_+$ such that $p^j_i = 0$ when $i > j$, $\sum_{i=1}^n p^j_i = 1$ and $x_i(C) = \sum_{j=1}^n p^j_i c_j$ for all $C \in \mathbb{R}^n_+$ and all $i \in N$.*

Proof. *See the Appendix.* ■

4 Other results

In this section we provide axiomatizations of two rules adding in Theorem 1 different properties based on possible and real taxes over pollution.

In many cases all the agents throw the same kind of residues into the water. Moreover, the residues are biodegradable and thus the pollution disappears over time; for instance: organic food waste, garden waste, forest residues, some industrial waste... In many occasions it is possible to know the biodegradation rate of the residues, say δ. If it happens, the cost that an agent pays for a polluted area should depend on this biodegradation rate. We introduce a new property following this idea:

Biodegradation Rate (BR) Given $j \in N$, for any $i \in N$ such that $i < j$, $x_i(0, ..., 0, c_j, 0, ...0) = \delta^{j-i} x_j(0, ..., 0, c_j, 0, ...0)$.

We assume that $0 \leq \delta \leq 1$. Notice that $\delta = 0$ means that the residue of agent i only affects its own area. In this case BR means that every agent pays the cost corresponding to its own area, namely $x_i(C) = c_i$ for all C and $i \in N$. Furthermore, $\delta = 1$ means that the residue is non-biodegradable. In this case BR coincides with Upstream Symmetry.

In the next theorem we study the effects of adding BR to the properties in Theorem 1.

Theorem 2 . *A rule x satisfies Add, Eff, IUC and BR if and only if for each $j = 1, ..., n$ there exists a weight system $\left(p_i^j\right)_{i \in N} \in \mathbb{R}_+^n$ such that $p_i^j = 0$ when $i > j$, $p_i^j = \delta^{k-i} p_k^j$ for any $i < k \leq j$, $\sum_{i=1}^n p_i^j = 1$ and $x_i(C) = \sum_{j=1}^n p_i^j c_j$ for all $C \in \mathbb{R}_+^n$ and all $i \in N$.*

Proof. *See the Appendix.* ■

In most autonomous regions of Spain there exists a difference between the rates applicable to domestic uses and those applicable to industrial ones. The autonomous regional governments of Aragon, Catalonia, Madrid, Galicia, Murcia, Navarre and La Rioja determine the base of the tax for industrial uses by estimating or directly measuring the pollution load (Gago et al., 2005). Further, as previously highlighted, in Valencia and Catalonia the rates applicable to domestic uses consider the population of the municipality. In all these situations the taxes can be modulated considering different factors, such as pollution load, population of the cities, monthly water consumption, etc. (See Gago et al., 2006). Other countries like Austria, Canada, France, Germany, Italy, Sweden, USA,... have also applied water taxes with similar features (OECD, 2006). These ideas are captured by the following axiom:

Weighted Tax (WT) Let $w = (w_i)_{i \in N} \in \mathbb{R}_+^N$. We say that x satisfies WT with respect to w if for any $i, j, k \in N$ such that $i < k \leq j$,

$$\frac{x_i(0, ..., 0, c_j, 0, ..., 0)}{x_k(0, ..., 0, c_j, 0, ..., 0)} = \frac{w_i}{w_k}.$$

This property states that the amount that each agent pays for a polluted area is given by some factor, which can be exogenous or endogenous.

WT generalizes *Upstream Symmetry* because when $w_i = w_j$ for all $i, j \in N$, both properties coincide.

In the next theorem we study the effects of adding *WT* to the properties in Theorem 1.

Theorem 3 . *A rule x satisfies Add, Eff, IUC and WT if and only if for each $j = 1, ..., n$ there exists a weight system $\left(p_i^j\right)_{i \in N} \in \mathbb{R}_+^n$ such that $p_i^j = 0$ when $i > j$, $p_i^j = \frac{w_i}{\sum_{l=1}^{j} w_l}$ for all $i \leq j$, $\sum_{i=1}^{n} p_i^j = 1$ and $x_i(C) = \sum_{j=1}^{n} p_i^j c_j$ for all $C \in \mathbb{R}_+^n$ and all $i \in N$.*

Proof. *See the Appendix.* ■

Ni and Wang (2007) prove that the solution they propose are related with some natural *TU* games they introduce. We now relate the solutions given by Theorem 3 with the weighted Shapley values of other *TU* game.

Given a pair (N, C) we define the *TU* game $\left(N, v^C\right)$ where $v^C(S) = \sum_{\{1,...,i\} \subset S} c_i$ for all $S \subset N$. Namely $v^C(S)$ represents the pollutant-cleaning costs in the segments polluted only by agents in S.

Theorem 4 . *Let x^w the solution given by Theorem 3. Then, x^w coincide with the weighted Shapley value of v^C, $\phi^w\left(N, v^C\right)$.*

Proof. *See the Appendix.* ■

The *Upstream Equal Sharing* method coincides with the solution given by Theorem 3 when $w_i = w_k$ for all $i, k \in N$. By Theorem 4 the *Upstream Equal Sharing* method also coincide with the Shapley value of the *TU* game v^C.

5 Appendix

We make a formal proof of the results stated in the paper.

5.1 Proof of Theorem 1.

Let x be a rule defined as in 1. We first prove that x satisfies Eff, Add and IUC:

x satisfies Eff:

$$\sum_{i=1}^{n} x_i(C) = \sum_{i=1}^{n} \sum_{j=1}^{n} p_i^j c_j = \sum_{j=1}^{n} c_j \left(\sum_{i=1}^{n} p_i^j \right) = \sum_{j=1}^{n} c_j. \blacksquare$$

x satisfies Add: Let C and $C' \in \mathbb{R}^n_+$ and $i \in N$. Thus,

$$\begin{aligned} x_i(C+C') &= \sum_{j=1}^{n} x_i^j(C+C') = \sum_{j=1}^{n} p_i^j (c+c')_j \\ &= \sum_{j=1}^{n} p_i^j \left(c_j + c_j'\right) = \sum_{j=1}^{n} p_i^j c_j + \sum_{j=1}^{n} p_i^j c_j' \\ &= x_i(C) + x_i(C').\blacksquare \end{aligned}$$

x satisfies IUC: Let $l \in N$ and $C, C' \in \mathbb{R}^n_+$ such that $c_i = c_i'$ for all $i > l$. Let $i \in N$, $i > l$. Since $p_i^j = 0$ when $i > j$,

$$\begin{aligned} x_i(C) &= \sum_{j=1}^{n} p_i^j c_j = \sum_{j=i}^{n} p_i^j c_j = \sum_{j=i}^{n} p_i^j c_j' \\ &= \sum_{j=1}^{n} p_i^j c_j' = x_i(C').\blacksquare \end{aligned}$$

We now prove the reciprocal. Assume that x is a solution satisfying Eff, Add and IUC. For each $j \in N$, let $1_j = (y_1, ..., y_n) \in \mathbb{R}^n_+$ be such that $y_j = 1$ and $y_i = 0$ when $i \neq j$. We define $p^j = x(1_j)$.

Let x^p be the rule induced by the weight system $\left\{p^j\right\}_{j \in N}$. We will prove that $x = x^p$ by several claims. The claims are proved following Bergantiños and Vidal-Puga (2004).

Claim 1 $\left\{p^j\right\}_{j \in N}$ is a weight system.

Proof of Claim 1. Since x satisfies Eff, $\sum_{i=1}^{n} x_i(1_j) = 1$. By definition of solution, $x_i(1_j) \in \mathbb{R}^n_+$. Let $i, j \in N$ such that $i > j$. We now prove that $p_i^j = 0$. Since x satisfies IUC, $x_i\left(1_j\right) = x_i\left(0, ..., 0\right)$. Since $x\left(0, ..., 0\right) \in \mathbb{R}^n_+$ and $\sum_{l=1}^{n} x_l\left(0, ..., 0\right) = 0$, $x_i\left(0, ..., 0\right) = 0$. $\blacksquare$

Claim 2 Let $c_j \in \mathbb{Q}_+$ (a non-negative rational number), then $x_i(0, ..., c_j, ..., 0) = c_j x_i(0, ..., 1, ..., 0)$.

Proof of Claim 2. Let $c_j = 1/q$, where $q \in \mathbb{N}$. By Add, $x_i(0, ..., 1, ..., 0) = \sum_{k=1}^{q} x_i(0, ..., \frac{1}{q}, ..., 0) = q x_i(0, ..., \frac{1}{q}, ..., 0)$. Thus,

$$x_i\left(0, ..., \frac{1}{q}, ..., 0\right) = \frac{x_i(0, ..., 1, ..., 0)}{q} = c_j x_i(0, ..., 1, ..., 0). \tag{1}$$

Let $c_j \in \mathbb{Q}_+$, say $c_j = \frac{p}{q}$. By Add,

$$x_i\left(0, ..., \frac{p}{q}, ..., 0\right) = p x_i\left(0, ..., \frac{1}{q}, ..., 0\right).$$

Then by (1),

$$x_i\left(0, ..., \frac{p}{q}, ..., 0\right) = \frac{p}{q} x_i(0, ..., 1, ..., 0).\blacksquare$$

Claim 3 Let $c_j \in \mathbb{R}_+ \backslash \mathbb{Q}_+$ (a non-negative irrational number), then $x_i(0,...,c_j,...,0) = c_j x_i(0,...,1,...,0)$.

Proof of Claim 3. Let $c_j \in \mathbb{R}_+ \backslash \mathbb{Q}_+$. Then, there exists $\{b_l\}_{l=1}^{\infty}$ such that $b_l \in \mathbb{Q}_+$, $b_l < c_j$ and $\lim_{l\to\infty} b_l = c_j$.

Let $l \in \mathbb{N}$. Since $x(0,...,c_j - b_l,...,0) \in \mathbb{R}_+^n$ and $\sum_{i\in N} x_i(0,...,c_j - b_l,...,0) = c_j - b_l$,

$$0 \leq x_i(0,...,c_j - b_l,...,0) \leq c_j - b_l.$$

By Add, $x_i(0,...,c_j,...,0) = x_i(0,...,c_j - b_l,...,0) + x_i(0,...,b_l,...,0)$. So,

$$0 \leq x_i(0,...,c_j,...,0) - x_i(0,...,b_l,...,0) \leq c_j - b_l.$$

Since $b_l \in \mathbb{Q}_+$, $x_i(0,...,b_l,...,0) = b_l x_i(0,...,1,...,0)$.Then,

$$0 \leq x_i(0,...,c_j,...,0) - b_l x_i(0,...,1,...,0) \leq c_j - b_l.$$

Thus,

$$0 \leq \lim_{l\to\infty} \left[x_i(0,...,c_j,...,0) - b_l x_i(0,...,1,...,0)\right] \leq \lim_{l\to\infty} \left[c_j - b_l\right].$$

So,

$$0 \leq x_i(0,...,c_j,...,0) - c_j x_i(0,...,1,...,0) \leq 0.$$

Therefore,

$$x_i(0,...,c_j,...,0) = c_j x_i(0,...,1,...,0).\blacksquare$$

Claim 4 Given $i \in N$ and $C \in \mathbb{R}_+^n$, $x_i(c_1,...,c_n) = \sum_{j=1}^n x_i(0,...,0,c_j,0,...,0)$.

Proof of Claim 4. It follows from the fact that x satisfies Add.$\blacksquare$

Since $x_i^p(c_1,...,c_n) = \sum_{j=1}^n p_i^j c_j$, and by Claims 2 and 3, $x_i(0,...,c_j,...,0) = c_j x_i(0,...,1,...,0) = c_j p_i^j$ for all $j \in N$ and all $c_j \in \mathbb{R}_+$, it is clear that $x = x^p$.$\blacksquare$

5.2 Proof of Theorem 2

We first prove that x satisfies BR. Let $i,j,k \in N$ such that $i < k \leq j$. Let $(0,....,c_j,...,0) \in \mathbb{R}_+^n$. Then,

$$\begin{aligned}
x_i(0...,c_j,...,0) &= \sum_{l=1}^n p_i^l c_l = p_i^j c_j = \delta^{j-i} p_j^j c_j \\
&= \delta^{k-i}\delta^{j-k} p_j^j c_j = \delta^{k-i} p_k^j c_j \\
&= \delta^{k-i} x_k(0...,c_j,...,0).\blacksquare
\end{aligned}$$

We now prove the reciprocal. Let x be a rule satisfying Add, Eff, IUC and BR. By Theorem 1 for each $j = 1,...,n$ there exists a weight system $\left(p_i^j\right)_{i\in N} \in \mathbb{R}_+^n$ such that $p_i^j = 0$ when $i > j$, $\sum\limits_{i=1}^n p_i^j = 1$ and $x_i(C) = \sum\limits_{j=1}^n p_i^j c_j$ for all $C \in \mathbb{R}_+^n$ and all $i \in N$. We now prove that $p_i^j = \delta^{k-i} p_k^j$ for any $i < k \leq j$.

Let $i,j,k \in N$ such that $i < k \leq j$. By the proof of Theorem 1, $p^j = x\left(1_j\right)$. Since x satisfies BR,

$$p_i^j = x_i\left(1_j\right) = \delta^{j-i} x_j\left(1_j\right) = \delta^{k-i}\delta^{j-k} x_j\left(1_j\right) = \delta^{k-i} x_k\left(1_j\right) = \delta^{k-i} p_k^j.\blacksquare$$

5.3 Proof of Theorem 3

We first prove that x satisfies WT. Let $i, j, k \in N$ such that $i < k \leq j$. Let $(0, ..., c_j, ..., 0) \in \mathbb{R}^n_+$. Then,

$$\begin{aligned} x_i(0..., c_j, ..., 0) &= \sum_{l=1}^{n} p_i^l c_l = p_i^j c_j = \frac{w_i}{\sum\limits_{l=1}^{j} w_l} c_j \\ &= \frac{w_i}{w_k} \frac{w_k}{\sum\limits_{l=1}^{j} w_l} c_j = \frac{w_i}{w_k} x_k(0..., c_j, ..., 0). \blacksquare \end{aligned}$$

We now prove the reciprocal. Let x be a rule satisfying Add, Eff, IUC and WT. By Theorem 1 for each $j = 1, ..., n$ there exists a weight system $\left(p_i^j\right)_{i \in N} \in \mathbb{R}^n_+$ such that $p_i^j = 0$ when $i > j$, $\sum\limits_{i=1}^{n} p_i^j = 1$ and $x_i(C) = \sum\limits_{j=1}^{n} p_i^j c_j$ for all $C \in \mathbb{R}^n_+$ and all $i \in N$. We now prove that $p_i^j = \frac{w_i}{\sum\limits_{l=1}^{j} w_l}$ for any $i < k \leq j$.

Let $i, j, k \in N$ such that $i < k \leq j$. By the proof of Theorem 1, $p^j = x\left(1_j\right)$. Since x satisfies Eff and WT,

$$\frac{1}{p_j^j} = \frac{\sum\limits_{l=1}^{j} p_l^j}{p_j^j} = \sum_{l=1}^{j} \frac{x_l\left(1_j\right)}{x_j\left(1_j\right)} = \sum_{l=1}^{j} \frac{w_l}{w_j} = \frac{\sum\limits_{l=1}^{j} w_l}{w_j}.$$

By WT,

$$p_i^j = x_i\left(1_j\right) = \frac{w_i}{w_j} x_j\left(1_j\right) = \frac{w_i}{w_j} p_j^j = \frac{w_i}{\sum\limits_{l=1}^{j} w_l}. \blacksquare$$

5.4 Proof of Theorem 4

Let $w = (w_i)_{i \in N} \in \mathbb{R}^N_+$. Let $\{u_S\}_{S \subset N}$ be a family of TU games such that $u_S\left(T\right) = 1$ if $S \cap T \neq \varnothing$ and $u_s\left(T\right) = 0$ otherwise. It is well known that $\{u_S\}_{S \subset N}$ is a basis for the set of all TU games. Kalai and Samet (1987) define the value ϕ^{w*} as the unique linear value satisfying that for each $S \subset N$, $\phi_i^{w*}\left(u_S\right) = \frac{w_i}{\sum_{k \in S} w_k}$ if $i \in S$ and $\phi_i^{w*}\left(u_S\right) = 0$ otherwise. Moreover, they prove that for each $w \in \mathbb{R}^N_+$ and each TU game v, $\phi^{w*}\left(v\right) = \phi^{w}\left(v^*\right)$ where $v^*\left(S\right) = v\left(N\right) - v\left(N \backslash S\right)$ for all $S \subset N$.

For each $j = 1, ..., n$, let $\left(N, v^j\right)$ be the TU game where for all $S \subset N$, $v^j\left(S\right) = c_j$ if $S \cap \{1, ..., j\} \neq \varnothing$ and $v^j\left(S\right) = 0$ otherwise. Notice that $v^j = c_j u_{\{1,...,j\}}$ for all $j \in N$.

Given $i \in N$,

$$\begin{aligned} x_i^w\left(C\right) &= \sum\nolimits_{j=1}^{n} p_i^j c_j = \sum\nolimits_{j=i}^{n} \frac{w_i}{\sum_{k=1}^{j} w_k} c_j = \sum\nolimits_{j=i}^{n} \phi_i^{w*}\left(v^j\right) \\ &= \sum\nolimits_{j=1}^{n} \phi_i^{w*}\left(v^j\right) = \sum\nolimits_{j=1}^{n} \phi_i^{w}\left(v^{j*}\right) = \phi_i^{w}\left(\sum\nolimits_{j=1}^{n} v^{j*}\right). \end{aligned}$$

Let $S \subset N$. Then, $v^{j*}(S) = v^j(N) - v^j(N\backslash S) = c_j - v^j(N\backslash S)$. Since $v^j(N\backslash S) = c_j$ when $N\backslash S \cap \{1,...,j\} \neq \varnothing$ and $v^j(N\backslash S) = 0$ when $N\backslash S \cap \{1,...,j\} = \varnothing$,

$$v^{j*}(S) = \begin{cases} c_j & \text{if } \{1,...,j\} \subset S \\ 0 & \text{otherwise.} \end{cases}.$$

Now it is trivial to prove that for all $S \subset N$, $v^C(S) = \sum_{j=1}^{n} v^{j*}(S)$. Hence, $x_i^w(C) = \phi_i^w\left(v^C\right)$.■

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