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ABSTRACT

We derive a parametric formula of the Watts' poverty index for the bivariate lognormal distribution of price indices and nominal living standards. This enables us to analyze the contributions of price and nominal living standard distributions to poverty, to estimate poverty when only means and variances are known. We also derive a statistical inference framework. Using data from peasants in Rwanda in four quarters, we show that poverty estimates based on scarce information are generally not significantly different from nonparametric estimates based on full survey information.

KEY WORDS: Measurement and analysis of poverty, Income distribution, Personal income distribution.

JEL CLASSIFICATION: I32, O15, D31.

1. Introduction

Differences in prices across households arise from several situations. Firstly, structural adjustment plans implemented in LDCs are often challenged as entailing poverty rises[1]. In particular, large movements of absolute and relative prices[2], which may be an important channel of changes in living standards. Secondly, geographical and seasonal differences in prices that households face is a typical feature of LDCs, generated by agricultural output fluctuations, imperfect markets, high transport and commercialisation costs, and information problems. The World Bank (1992) presents seasonal price ratios for twenty rural developing countries in the 1980s and five products (rice, maize, wheat, sorghum, millet). The ratios show a generally high sensitivity of agricultural prices to seasons[3]. These variations imply serious consequences for poor peasants who have often limited access to capital markets. In Africa, Baris and Couty (1981) suggest that the seasonal price fluctuations may worsen the social differenciation. Similar evidence

[1] The World Bank (1990), Bourguignon, De Melo, Morrisson (1991), Sahn and Sarris (1991).

[2] For example, Sahn, Dorosh and Youngs (1997)) argue that in Ghana the market liberalisation during the adjustment program of the end 1980s has lead to price decreases (or moderate increases) despite a devaluation of 100 percent. Between 1984 and 1990 the prices of major staple foods fell and the rate of decline was faster than in the 1970s and early 1980s. This was accompanied by substantial changes in relative prices since prices of imported goods sharply rised.

[3] That is also well known for industrial countries in general (Riley (1961)).

and concern have been raised for geographical differences in prices across households. As discussed by Sen (1981), particularly in periods of famines, differences in prices that households face can dramatically affect their entitlements relations and their capacity to acquire food.

In these situations, poverty measures may incorporate substantial errors caused by unaccounted large price differences across households or seasons (Jazairy, Alamgir and Panuccio (1992)). For example, Muller (1999a) shows for a large range of poverty measures that local and seasonal price differences have a statistically significant and large impact on poverty measurement in Rwanda. Then, understanding the contribution of the price distribution in poverty assessment, is crucial for welfare policies.

In practice, applied price indices are generally Laspeyres or Paasche price indices. Though, the role of price index dispersion in poverty estimation has not been studied from a theoretical point of view. Particularly, there is no explicit result about the price distribution contribution to poverty. The present paper attempts to fill this lacuna by using a bivariate distribution model.

Numerous authors[4], insist on the necessity of using axiomatically sound poverty measures for inferences on poverty. One of the main axiomatically sound poverty

[4] e.g. Atkinson (1987).

measure is the Watts' measure (Watts (1968)). We focus on this indicator and we study three different estimators of the Watts' measure that are based on different requirements of empirical information, and for which we derive the asymptotic distributions.

Are poverty estimators based on a bivariate lognormal model reliable? Can we usefully extrapolate poverty from the sole observation of empirical means and variances of nominal welfare and prices? The aim of this article is to answer these questions firstly by analyzing a distribution model for prices and living standards, secondly by deriving the asymptotic laws of poverty estimators for statistical inference, thirdly by estimating poverty using data from Rwanda.

The situation to which the analyst is confronted is often delicate in terms of available information. It happens that the researcher does not have access to data collected from household surveys. Sometimes, no survey has been carried out for the period and the region of interest. Sometimes, there has been a survey, but the collected information has not been transferred on magnetic support and only manual statistical treatment has been done (e.g. for historical data). Sometimes, the survey information has been entered on tapes or diskettes, but these supports have been lost, destroyed or are unusable. In some situations, nobody knows how to interpret the recorded information. In other cases the data is not legally or administratively available to researchers, or too costly to purchase. Finally, when the data for the observed sample can be obtained, it is often the case that no accurate information is available about the sampling scheme.

However, in many cases, simple statistics of the variables of interest have sometimes been published, in particular mean and standard deviations of living standards and price indices. Sometimes, poverty measures have been estimated and published. Unfortunately, even in these cases the calculated poverty statistics may not correspond to the specific needs of the researcher. Indeed, often the published poverty measures cannot be considered as axiomatically valid from the point of view of poverty theory[5].

Even when an axiomatically sound poverty measure is used (for example a Foster-Greer-Thorbecke measure with a severity parameter larger than two; some Clark-Hemming-Ulph-Chakravarty measures; Watts' measure), the estimation results can be very sensitive to the choice of the poverty line. Unfortunately, the method for calculating the poverty line is a very contentious subject[6], and poverty statistics are generally published only for one or two lines. Then, it is unlikely that the researcher would have chosen exactly the same poverty line and without

[5] See Foster, Greer and Thorbecke (1984) and Zheng (1997) for axiomatic analyses of poverty indicators.

[6] See Ravallion (1998) for a detailed discussion of methods of poverty line calculation.

access to household level data it is impossible for her to calculate new poverty measures with her poverty lines of interest.

We first define in section 2 the Watts' poverty measure. Under lognormality assumptions, we derive an expression of the Watts' measure in terms of the parameters of the joint distribution of price indices and nominal living standards. We derive in section 3, theoretical estimators of the distribution parameters and of the Watts' measure, as well as their asymptotic covariance matrices. In section 4, we describe the data used in the estimation and we test the distribution assumptions. We compare in section 5 estimates of Watts' measures calculated using our three estimators. Finally, section 6 concludes.

2. Watts' poverty index

2.1. The parametric formula

The living standard indicator for household i at period t is

$$y_{it} = \frac{c_{it}}{es_i \ I_{it}} = \frac{w_{it}}{I_{it}} \tag{2.1}$$

where c_{it} is the value of the consumption of household i at period t ; w_{it} is the standard of living of household i at date t; es_i is the equivalence scale of household i and I_{it} is the price index associated with household i and period t. We denote $w_{it} = c_{it}/es_i$, the non-deflated living standard indicator (nominal living standard). This variable is of primary empirical importance, since it corresponds to what can be obtained from most statistical reports of household surveys, therefore from official statistics and from many articles.

The Watts' poverty measure (Watts (1968)) is defined as

$$W = \int_0^z -\ln(y/z)\,d\mu(y) \tag{2.2}$$

where μ is the cumulative density function of living standards y, and z is the poverty line.

The Watts' measure satisfies the focus, monotonicity, subgroup consistency, transfer and transfer sensitivity axioms. Moreover, it is the only poverty measure that can be written as a difference in levels of a social welfare function and that satisfies monotonicity, continuity, decomposability and scale invariance (Zheng (1993)). Owing to its axiomatic properties, the Watts' measure is superior to the head-count index (P_0), or even the poverty gap index (P_1).

To separate the contributions of the distribution of price indices and the distribution of nominal living standards, we now rewrite eq. 2.2 in terms of the joint distribution of w and I, using the joint cumulative distribution function, F.

$$W = \int \int_{\Omega} - \ln((w/I)/z) \; dF(w, I) \tag{2.3}$$

where $\Omega = \{(w, I)| \; w > 0, I > 0, w/I < z\}$. Eq. (2.3) implies that the poverty line, z, is defined independently of the distributions of nominal living standards and price indices. The methods for calculating poverty lines are very varied (Ravallion (1998)), and the latter assumption may not always be satisfied. In that case, z should be replaced by an explicit function z(F) and complementary terms are to be added to the expressions obtained in this paper. Since no general result can be derived for these very varied specifications, we do not pursue this direction in this paper. In that sense, the poverty lines used in the application must be considered as fixed values given once for all.

We shall obtain explicit expressions of contributions of non-deflated living standards and prices by approximating F using a bivariate lognormal distribution. The choice of the lognormal distribution is supported by the fact that histograms of nominal living standards and price indices have unimodal asymmetrical and leptokurtic shapes, and the observations of these variables are always positive. The lognormal approximation has been frequently used in applied analysis of living standards (e.g. Alaiz and Victoria-Feser (1990), Slesnick (1993)). In some cases (e.g. van Praag, Hagenaars and van Eck (1983)) income data has been found consistent with the lognormal distribution. The assumption of lognormality of income has as well been exploited in theoretical economics (e.g. Hildenbrand (1998)). Log-wage or log-price equations have frequently been estimated, often implicitly relying on error terms related to normality assumptions, sometimes asymptotically. Eaton (1980), Deaton and Grimard (1992), for example, assume lognormality for price distributions. Other distribution models for living standards or incomes[7] or other distribution models for prices[8] can also be used to represent the data, but will not lead to an explicit expression for the Watts' measure. In particular, Singh-Maddala or generalised Gamma distributions are generally a better fit for incomes than the lognormal distribution. It may also be possible that the bivariate lognormal distribution for nominal living standards and price indices can be significantly dominated by another distribution family.

However, the reason why we adopt a lognormal specification is not that it corresponds to a close fit to the data, but rather because we search for a bivariate distribution model for nominal living standards and price indices, which has the

[7] Champernowne (1952), Salem and Mount (1974), Kloek and van Dijk (1978), Singh and Maddala (1976), Slottje (1984), Hirschberg and Slottje (1989).

[8] Creedy and Martin (1994).

well-behaved characteristics evoked above and which leads to a parametric expression of the poverty index. Here, the question of statistical fit is secondary in regard to the use of the distribution model as an analytical tool. The parametric approach has been shown to be useful in poverty measurement, as demonstrated for example in Cowell and Victoria-Feser (1996) for the treatment of data contamination. Nonetheless, even in the case of imperfect statistical fit, we would like to know if poverty estimates based on the distribution model are statistically close to the best poverty estimates.

The Watts' poverty measure is the only axiomatically sound poverty indicator for which a parametric formula can be derived under bivariate lognormal distribution of price index and nominal living standard[9]. However, the statistical methods that we shall implement in the following of this paper can easily be applied to the Head-Count index, the Gini coefficient and the variance of logarithms[10]. To

[9] Owing to the presence of integrals that cannot be solved explicitly, no explicit formula is possible for the other known axiomatically sound poverty measures such as Foster-Greer-Thorbecke poverty indices (P_a) and Clark-Hemming-Ulph-Chakravarty's measures with values of poverty severity parameters consistent with the transfer axiom.

[10] Head-Count index and Gini income inequality measures are explicitly known when y follows a lognormal distribution LN(m, σ^2) (for example in Hammer et al. (1997)). Their formula are respectively: $P_0 = \Phi\left[\frac{\ln z - \mu}{\sigma}\right]$ and $G = 2\Phi\left[\frac{\sigma}{\sqrt{2}}\right] - 1$.

Parametric formulae for a few inequality indicators under income Gamma distribution have also been proposed (McDonald and Jensen (1979)) or under income Pareto Distribution (Chipman (1974)).

shorten the exposition we focus from now exclusively on the Watts' index. We now present the expression of the Watts' measure under the lognormality assumption.

Proposition 2.1. *(Muller, 2006)*

If nominal living standards and price indices follow a bivariate lognormal distribution law, LN $\left[\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right]$*, then the Watts' index is equal to:*

$$\begin{aligned} W &= (\ln z - m_1 + m_2).\,\Phi\left(\frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right) \\ &\quad + \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}.\,\phi\left(\frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right) \end{aligned} \tag{2.4}$$

where ϕ and Φ are respectively the p.d.f. and c.d.f. of the standard normal distribution. The knowledge of $Z = \frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2+\sigma_2^2-2\rho\sigma_1\sigma_2}}$ and $S = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$ is sufficient to calculate W.

$$W = S.[Z.\Phi(Z) + \phi(Z)] = S.G(Z) \tag{2.5}$$

by definition of G.

Eq. 2.4 suggests that unless all price indices are very concentrated around 1, price indices should not be neglected in the estimation of the Watts' poverty measure. It is also clear that the parameters associated respectively with the distributions of w and I play similar roles. Note that m_1 and σ_1 (respectively m_2 and σ_2) are the mean and the standard deviation of the *logarithms* of nominal living standards (respectively of the logarithms of price indices). To shorten, we denote *variabilities* the standard deviation parameters σ_1 and σ_2.

Eq. 2.5 shows that the Watts' measure can be decomposed in terms of two sufficient statistics: S, which is the standard deviation of the logarithm of the real living standards, that we denote *global variability*; and Z, which is the *standardised logarithm of the poverty line* in real terms. S is equal to the variance of logarithms and can be considered as an inequality index, although Foster and Ok (1999) showed that it may seriously disagree with Lorenz rankings.

$\Phi(Z)$ is equal to the head-count index under the hypothesis of lognormality. Function G(Z) is a primitive function (with value $\frac{1}{\sqrt{2\pi}}$ at Z = 0) with respect to Z of the head-count index when σ is fixed. We denote G(Z) the *cumulating poverty incidence*. Consequently, the elasticity of W with respect to any independent variable is the sum of the elasticity of the global variability and the elasticity of the cumulative poverty incidence. The components of the gradient of

W with respect to parameters are $\frac{\partial W}{\partial m_1} = -\Phi(Z) < 0; \frac{\partial W}{\partial m_2} = \Phi(Z) > 0; \frac{\partial W}{\partial \sigma_1} = \frac{\sigma_1 - \rho\sigma_2}{S}\phi(Z); \frac{\partial W}{\partial \sigma_2} = \frac{\sigma_2 - \rho\sigma_1}{S}\phi(Z); \frac{\partial W}{\partial \rho} = \frac{-\sigma_1\sigma_2}{S}\phi(Z)$.

Some of the formulae of this paper, notably expression 2.4, can directly be obtained by considering the distribution of real living standards y. However, it is necessary to consider the joint bivariate distribution of (x, I) for two purposes:

1. Understanding the contribution of the distribution of each of these variables, notably when one is concerned by levels and variabilities of prices, or by the correlation parameter;

2. Estimating the poverty measure when only means and standard deviations are available. Indeed, the published statistics are generally in terms of nominal living standards (typically per capita consumption, per capita income or per adult-equivalent consumption). Very often, the mean and standard deviation of *real* living standards are not published and cannot be calculated from the same statistics for price indices and nominal living standards.

2.2. Comparison with other approaches under scarce information

What we need are only means and standard deviations of income (or living standard) and price indices (as we shall show later the correlation parameter ρ can be set to zero with still the formula providing good results because of the generally weak correlation of prices and nominal living standards). Although such information is not systematically available, it is published in a sufficient number of cases[11] to make our approach useful.

The main difficulty is generally the availability of standard deviations. When they have not been published, for example for incomes, they can often be approximated, either by using income subgroup information, or by using regional income information that is often available. In that case, the national standard deviations are likely to be underestimated since intra-regional dispersion is neglected. This may be a minor default when the differences across regions are large. A statistical model can also be used to reconstitute the share of the intra-regional dispersion. Although, such solution is not ideal, its imperfections are shared by the widely used methods based on extrapolation of subgroup incomes.

Furthermore, other sources of information can be used to calculate the standard deviations or the variability parameters, under lognormal assumption. First, there sometimes exist published estimates of the parameters of the lognormal distribution (e.g. Kloek and van Dijk (1978), McDonald and Ransom (1979)). Second, when a simple random sampling scheme has been used (e.g. République

[11] A few recent examples that come to our mind are: Muller (1989, 1999b), Sundrum (1990), Dercon and Krishnan (1994), Blundell and Preston (1998), Cavendish (1999).

Tunisienne (1997)), the standard deviation is equal to the square root of the sample size multiplied by the standard sampling error. Then, when sampling standard errors or confidence intervals are published, which should be a normal practice for surveys, it may be possible to calculate the needed standard deviations. Third, since parametric formulae for the Gini coefficient are known under lognormality, the value of these measures can be used to recover the variability parameters when means are known. Since published values of P_0 and Gini coefficients are widely available, our approach is likely to be applicable in many contexts.

Finally, the parametrised formulae for welfare indicators are generally provided without the associated elements for statistical inference, i.e. the estimator *and* the method for calculating confidence intervals. We stress in this paper the necessity of providing a complete framework of statistical inference so as to ensure a proper use of the parametrised formula for the Watts' index. The next sections deal with this problem.

3. Statistical Inference for the Watts' Measure

In this section, we first discuss some estimators of the parameters of the bivariate distribution. Then, we propose several estimators of the Watts' poverty measure. Finally, we propose a predictor of this measure when the price index distribution changes.

3.1. MLE

We can estimate the parameters of the joint distribution by using samples of price indices and living standards. This estimation is useful on several grounds. First, it informs about the shape of the considered distributions. Second, it quantifies the respective effects of both levels and variabilities of the logarithms of nominal living standards and of the logarithms of price indices. Finally, the estimates can be incorporated in an estimator of W. If the distributions of w and I are jointly lognormal, then the maximum likelihood estimators (MLE) of (m_i, σ_i), i = 1 and 2, and ρ, are consistent, asymptotically normal, efficient and invariant. under random sampling, they are: $\hat{m}_1 = \frac{1}{n}\Sigma_i \ln w_i$ and $\hat{m}_2 = \frac{1}{n}\Sigma_i \ln I_i$; $\hat{\sigma}_1^2 = \frac{1}{n}\Sigma_i (\ln w_i - \mathrm{m}_1)^2$ and $\hat{\sigma}_2^2 = \frac{1}{n}\Sigma_i (\ln I_i - \mathrm{m}_2)^2$; and $\hat{\rho} = \frac{\frac{1}{n}\Sigma_i (\ln w_i - \mathrm{m}_1).(\ln I_i - \mathrm{m}_2)}{\hat{\sigma}_1 \hat{\sigma}_2}$. Moreover, $\hat{m}_1$ and $\hat{m}_2$ are unbiased estimators.

We denote the covariance matrix of $(\hat{m}_1, \hat{m}_2, \hat{\sigma}_1, \hat{\sigma}_2, \hat{\rho})'$ as Σ_L. Owing to the invariance property of the MLE, the MLE of the Watts' measure can be defined using eq. 2.4 and substituting the MLEs for the distribution parameters.

Proposition 3.1. *(Muller, 2006) Under random sampling, the MLE of the Watts' measure under bivariate lognormality is:*

$$W1 = (\ln(z) - \hat{m}_1 + \hat{m}_2).\,\Phi\left(\frac{\ln z - \hat{m}_1 + \hat{m}_2}{\sqrt{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 - 2\hat{\rho}\hat{\sigma}_1\hat{\sigma}_2}}\right) + \sqrt{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 - 2\hat{\rho}\hat{\sigma}_1\hat{\sigma}_2}.\,\phi\left(\frac{\ln z - \hat{m}_1 + \hat{m}_2}{\sqrt{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 - 2\hat{\rho}\hat{\sigma}_1\hat{\sigma}_2}}\right) \quad (3.1)$$

The asymptotic variance of W1, which is asymptotically normal, is

V(W1) $= \nabla W1' \sum_L \nabla W1,$

where $\nabla W1$ *denotes the gradient vector of W1, calculated with respect to* $(\hat{m}_1,\, \hat{m}_2, \hat{\sigma}_1, \hat{\sigma}_2,\, \hat{\rho})$.

Proof: Direct application of the delta method.

The various asymptotic covariance matrices presented in this paper can be consistently estimated by replacing parameters m_i, σ_i^2 $(i = 1, 2)$ and ρ with consistent estimates, for example with the MLEs. Then, confidence regions of parameter estimates can be easily derived.

3.2. MME

We now propose to investigate the sole use of observed mean and standard-deviations of w and I to produce an estimator of poverty. This is possible because the means $(M_i, i = 1, 2)$ and the variances $(S_i^2, i = 1, 2)$ of w and I following jointly a bivariate lognormal law, can be *explicitly* calculated from the parameters of the distribution. We first define estimators $\tilde{\mathrm{m}}_i$ and $\tilde{\sigma}_i^2$ (i = 1,2) of the distribution parameters, using the method of moments. The MME of m_i and σ_i (i = 1, 2) are:

$$\tilde{\sigma}_i = \sqrt{\ln\left(1 + \frac{\left(\tilde{M}_i\right)^2}{\tilde{S}_i^2}\right)} \tag{3.2}$$

$$\tilde{m}_i = \ln(\tilde{M}_i) - \tilde{\sigma}_i^2/2 \tag{3.3}$$

where $\tilde{M}_1$ and $\tilde{S}_1^2$ are respectively the empirical mean and the empirical variance of the nominal living standards, and $\tilde{M}_2$ and $\tilde{S}_2^2$ are respectively the empirical mean and the empirical variance of the price indices.

Estimators ($\tilde{\mathrm{m}}_1$, $\tilde{\mathrm{m}}_2$, $\tilde{\sigma}_1^2$, $\tilde{\sigma}_2^2$) and ($\hat{\mathrm{m}}_1$, $\hat{\mathrm{m}}_2$, $\hat{\sigma}_1^2$, $\hat{\sigma}_2^2$) are both consistent, although not asymptotically equivalent. ($\hat{\mathrm{m}}_1$, $\hat{\mathrm{m}}_2$, $\hat{\sigma}_1^2$, $\hat{\sigma}_2^2$) is efficient when the actual distribution is lognormal, in contrast with ($\tilde{\mathrm{m}}_1$, $\tilde{\mathrm{m}}_2$, $\tilde{\sigma}_1^2$, $\tilde{\sigma}_2^2$) that is generally not efficient.

We now define a novel estimator W2 of the Watts' poverty measure by plugging the MMEs in eq. 2.4 under the hypothesis ($\rho = 0$). The hypothesis of non-correlation of the logvariables, which does not modify the derivation of the formula of $(\tilde{\mathrm{m}}_1, \tilde{\mathrm{m}}_2, \tilde{\sigma}_1^2, \tilde{\sigma}_2^2)$, is crucial. Indeed, firstly it may often correspond to a plausible situation. Secondly, it eliminates the need for estimates of ρ, which are typically not available in survey publications.

Definition 3.2.

$$\begin{aligned} W2 &= (\ln z - \tilde{m}_1 + \tilde{m}_2).\, \Phi\left(\frac{\ln z - \tilde{m}_1 + \tilde{m}_2}{\sqrt{\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2}}\right) \\ &\quad + \sqrt{\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2}.\, \phi\left(\frac{\ln z - \tilde{m}_1 + \tilde{m}_2}{\sqrt{\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2}}\right) \end{aligned} \tag{3.4}$$

Various combinations of parameters of the distribution are asymptotically Gaussian with the following covariance matrices, without or with the assumption that $\rho = 0$.

Proposition 3.3. *(Muller, 2006)*

The asymptotic covariance matrix of the MMEs $(\tilde{m}_j, \tilde{\sigma}_j)'$, $j = 1, 2$ *is*

$$\tilde{\Sigma}_j = [D_j' \Phi_j^{-1} D_j]^{-1}/N \tag{3.5}$$

$$\textit{where } D_j = \begin{bmatrix} -e^{(m_j+\sigma_j^2/2)} & -\sigma_j e^{(m_j+\sigma_j^2/2)} \\ -2e^{2m_j+2\sigma_j^2} & -4\sigma_j e^{2(m_j+\sigma_j^2)} \end{bmatrix} \textit{ and}$$

$\Phi_j, j = 1,2$ are 2×2 matrices with component at row k and column l ($k = 1,2; l = 1,2$):

$\Phi_{kl} = p\lim_{n \to +\infty} \left\{ \frac{1}{n} \sum_{i=1}^{n} f_{ik} f_{il} \right\}$ *can be estimated by* $\hat{\Phi}_{kl} = \frac{1}{n} \sum_{i=1}^{n} f_{ik} f_{il}$.

f_{ik} *denotes the* k^{th} *moment condition* f_k *(k= 1,2) for observation i.*

$$\begin{pmatrix} f_1 \equiv w_i - e^{m_1+\sigma_1^2/2} \\ f_2 \equiv w_i^2 - e^{2m_1+2\sigma_1^2} \end{pmatrix} \textit{ for } j = 1;$$

$$\begin{pmatrix} f_1 \equiv I_i - e^{m_2+\sigma_2^2/2} \\ f_2 \equiv I_i^2 - e^{2m_2+2\sigma_2^2} \end{pmatrix} \textit{ for } j = 2.$$

Proposition 3.4. *a) The asymptotic covariance matrix of*

$(\tilde{m}_1, \tilde{\sigma}_1, \tilde{m}_2, \tilde{\sigma}_2)'$ under the hypothesis $\rho = 0$ is

$$\varSigma_M = \begin{bmatrix} \tilde{\Sigma}_1 & 0 \\ 0 & \tilde{\Sigma}_2 \end{bmatrix}$$

b) The asymptotic covariance matrix of $(\hat{m}_1, \hat{\sigma}_1, \tilde{m}_2, \tilde{\sigma}_2)'$ is, under the hypothesis $\rho = 0$:

$$\Sigma_N = \begin{bmatrix} \frac{1}{N} IF_1^{-1} & 0 \\ 0 & \tilde{\Sigma}_2 \end{bmatrix},$$

where IF_1^{-1} is the inverse of the block of IF corresponding to $(\hat{m}_1,\ \hat{\sigma}_1)$. IF is the total information matrix for the whole sample of price indices and nominal living standards.

c) V(W2) = $\nabla W2' \Sigma_M \nabla W2$,

where $\nabla W2$ is deduced from the formula of the gradient of W2 calculated at the MMEs of the parameters.

3.3. The sampling estimator

When the survey and the sampling scheme description are available, the Watts' measure can be directly estimated using ratios of Horwitz-Thompson estimators (e.g. Gouriéroux (1981))[12]. These estimates are denoted W3 .

W3 = $\frac{\sum_{s=1}^{n} \frac{\ln(y_s/z) 1[y_s<z]}{\pi_s}}{\sum_{s=1}^{n} \frac{1}{\pi_s}}$ where π_s is the inclusion probability of household s (s = 1,...,n). In the applied section 5, we use an estimator for sampling standard

[12] Alternatively, if one considers that the proper level to conduct poverty analysis is the individual one, this formula is to be modified by weighting the expression at the numerator and denominator by the household size of household s at date t. This approach has been criticised on the ground that it corresponds to equal members' shares of household living standard or household income. In any case, it would not change the qualitative results of this paper.

errors that is inspired from the method of balanced repeated replications[13], which is adapted to the actual survey (see appendix). We investigate the empirical performance of the proposed estimators and simulators, using data from rural Rwanda in the next sections.

4. Data and Tests

4.1. The data

Rwanda is a small rural country in Central Africa. In 1983, it was one of the poorest country in the world, with per capita GNP of US $ 270 per annum. More than 95 percent of the population live in rural areas (Bureau National du Recensement (1984)).

Data for the estimation is taken from the Rwandan national budget-consumption survey, conducted by the Government of Rwanda and the French Cooperation and Development Ministry, in the rural part of the country from November 1982 to December 1983 (Ministère du Plan (1986a))[14]. 270 households were surveyed about their budget and their consumption. Because of missing values, 256 observations

[13] See Krewski and Rao (1981), Roy (1984) for discussions of the properties of this type of estimator.

[14] The main part of the collection has been designed with the help of INSEE (French National Statistical Institute).

are used in the estimation. The consumption indicators are of very high quality (see Muller (1999a) for details).

Agricultural year 1982-83 is a fairly normal year in terms of climatic fluctuations (Bulletin Climatique du Rwanda (1982, 1983, 1984)). The collection of the consumption data was organised in four rounds, corresponding to four quarters (A, B, C, D) of the agricultural year 1982-83[15].

Table 1 shows for the year and all quarters, the mean and the standard deviation of per capita consumption (deflated and non-deflated) and the Laspeyres price index.

Several studies of price surveys in Rwanda have shown the existence of substantial geographical and seasonal price dispersion[16]. We have computed elementary price indicators of the main categories of goods, for every quarter and every cluster of the sample. Muller (1999a) discusses the type and the sample of prices used. The deflation is based on Laspeyres price indices (I_{it}) for each household and each quarter, in which the basis is the annual national average consumption. All households in the same cluster are assumed to face the same prices. Thus, the price index simultaneously accounts for geographical and quarterly price dispersions.

[15] Round A: 01/11/1982 until 16/01/1983. Round B: 29/01/1983 until 01/05/1983. Round C: 08/05/1983 until 07/08/1983. Round D: 14/08/1983 until 13/11/1983.

[16] Niyonteze and Nsengiyumva (1986), O.S.C.E. (1987), Ministère du Plan (1986b), Muller (1988).

Table 1: Mean and standard deviation of living standards and price indices

	Annual	A	B	C	D
Deflated Per Capita Consumption	10613 (5428)	2750 (1701)	2702 (1620)	2850 (1968)	2310 (1511)
Price Index	1.0487 (0.063)	1.1087 (0.129)	0.9534 (0.101)	1.0476 (0.131)	1.0847 (0.097)
Non Deflated Per Capita Consumption	10905 (5355)	2995 (1826)	2539 (1475)	2902 (1834)	2468 (1524)

Standard deviations in parentheses. 256 observations.

4.2. Tests of distribution assumptions

We now turn to the tests of distribution assumptions. The lognormal assumption introduced in Section 2 is a convenient approximation that is probably rejected by statistical tests with some datasets. When the considered distributions are found close to lognormality, estimator W1 will perform well. However, we also want to explore its usefulness for distributions that are statistically distinguishable from lognormal distributions, and to compare there W1 and W3.

To implement this exploration, we conduct several tests of lognormality assumptions: Skewness-Kurtosis tests; Shapiro-Wilk tests (Shapiro and Wilk (1965), Royston (1982)); Shapiro-Francia tests (Shapiro and Francia (1972)); and Kolmogorov-Smirnov tests. All tests are done for the quarterly and the annual per capita nominal consumption distributions, and for the quarterly and the annual price index distributions. The corresponding P-values are shown in Table 2.

Skewness-Kurtosis tests yield results similar to those of other tests (except sometimes at 10 percent level for nominal living standards in period A and annually). Kolmogorov-Smirnov tests, which have low power, can reject neither the lognormality of price indices at 5 percent level at quarters A and D, nor the lognormality of living standards at quarters A, B, C. Shapiro-Wilk (W), and Shapiro-Francia (W'), tests yield very close results.

Table 2: P-values of lognormality tests

Test:	1	2	3
price index in A	0.0030	0.00008	0.00062
price index in B	0.0001	0.00000	0.00001
price index in C	0.0000	0.00000	0.00001
price index in D	0.0064	0.00016	0.00051
annual per capita consumption	0.0916	0.32117	0.20879
per capita consumption in A	0.0861	0.21209	0.11655
per capita consumption in B	0.0431	0.01719	0.00868
per capita consumption in C	0.5249	0.84615	0.52032
per capita consumption in D	0.0000	0.0000	0.00001

Test:	4	5	6	7
price index in A	0.364	0.0644	0.06862	0.10313
price index in B	0.000	0.0207	0.00029	0.00113
price index in C	0.011	0.0129	0.00073	0.00768
price index in D	0.155	0.1431	0.06970	0.12212
per capita consumption in A	0.816			
per capita consumption in B	0.163			
per capita consumption in C	0.934			
per capita consumption in D	0.028			

Tests:

1: 256 households. Skewness-Kurtosis of the variable in logarithm

2: 256 households. Shapiro-Wilk W of the variable in logarithm

3: 256 households. Shapiro-Francia W' of the variable in logarithm

4: 256 households. Kolmogorov-Smirnov of the variable in logarithm for N($\hat{m}$,$\hat{\sigma}^2$)

5: 90 clusters. Skewness-Kurtosis of the variable in logarithm

6: 90 clusters. Shapiro-Wilk W of the variable in logarithm

7: 90 clusters. Shapiro-Francia W' of the variable in logarithm

We prefer the W' test because of its better statistical properties in this context. Its P-values imply to reject the lognormality of the price index distributions. One might object that this result could be attributed to the clustering of observations since all households in the same cluster are attributed the same value of the price index. However, even when considering only one observation at the cluster level, the lognormality of price indices is as well rejected by the W' test, except in quarters A and B and at 10 percent level only. Moreover, these latter outcomes may be obtained only because of small sample size.

In contrast, the lognormality of the living standard distribution is not rejected by the W' test at 5 percent level, at quarters A and C, or for the year. It is rejected at other quarters. Using different equivalent scales does not change the test results. It is interesting to note that we dispose of a benchmark data that does not overly determine the results of estimator comparisons with a too good fit to lognormality.

Muller (1999b) shows the results of χ^2 tests of independence between nominal living standards and price indices that indicate at every quarter that the independence between price indices and nominal living standards cannot be rejected. Here, $\rho = 0$ would be an acceptable hypothesis.

On the whole, the preliminary tests support the adopted distribution assumptions. We are now ready to present the estimation results.

5. Estimations

In this section we compare the performances of our three poverty estimators, W1, W2, W3. Finally, we discuss the availability of the information necessary for the W1 and W2 calculations.

Muller (1999d) shows for six poverty lines estimates of the three estimators W1, W2, W3, together with the sampling standard errors of W3 and the standard errors of W1 and W2, for all quarters and the whole year. Table 3 summarizes this information by presenting the means over the six poverty lines for these statistics. The poverty measured using any of our indicators is unambiguously higher in quarter D, after the dry season, than in other quarters and lower in quarter B, after the bean harvests.

5.1. Comparison of Estimators

We now compare the quarterly and annual estimates obtained with the three estimators W1, W2, W3 by examining the means over the six poverty lines of their relative variations that are shown in Table 3 as well as the means of their respective standard errors. Muller (1999d) shows detailed results for all lines. It is

Table 3: Effects of level and variability of the logarithm of price indices

	A	B	C	D	Y
r_1	0.764 (0.0244)	1.182 (0.0268)	0.903 (0.0108)	0.844 (0.0164)	0.829 (0.0238)
r_2	0.940 (0.0177)	0.943 (0.0184)	0.935 (0.0186)	0.982 (0.00550)	0.968 (0.0119)
r_3	0.710 (0.0376)	1.121 (0.00527)	0.841 (0.0272)	0.828 (0.0211)	0.800 (0.0334)

The first number in each cell is the mean over 6 poverty lines. The number in parentheses is the standard deviation over the six lines.

r_1 is the ratio of the Watts' index under ($m_2 = 0$), over the Watts index without restriction;

r_2 is the ratio of the Watts' index under ($\sigma_2 = 0$), over the Watts index without restriction;

r_3 is the ratio of the Watts' index under ($m_2 = 0$ and $\sigma_2 = 0$), over the Watts index without restriction.

Table 4 : Watts' Poverty indices

	$W3$	$W1$	$W2$	$(W3-W1)/W1$	$(W2-W1)/W1$
A	0.1105	0.1159	0.1095	-0.0476	-0.0616
	(0.0160)	(0.0142)	(0.0503)	(0.00890)#	(0.0170)#
B	0.0841	0.0905	0.1023	-0.1811	0.1476
	(0.0152)	(0.0113)	(0.0134)	(0.265)#	(0.0434)#
C	0.1091	0.1166	0.1111	-0.0690	-0.0525
	(0.0126)	(0.0143)	(0.0180)	(0.0124)#	(0.0143)#
D	0.1861	0.2035	0.1717	-0.0923	-0.1683
	(0.0358)	(0.0201)	(0.0229)	(0.0242)#	(0.0464)#
Y	0.0562	0.1445	0.07432	-0.1363	0.2331
	(0.00554)	(0.00841)	(0.0118)	(0.0634)#	(0.0805)#

The first number in each cell is the mean of W over the six poverty lines. The mean over the six lines of standard errors is in parentheses. # denotes the standard deviation over the six lines.

A, B, C, D denote the results of each of the quarters. Y denotes the results based on annual per capita consumption to estimate chronic annual poverty. In the latter case, the poverty lines are multiplied by four.

important to understand that these estimators rely on very different information requirements. The estimation of W3 and its standard error requires the observation of the survey sample and an accurate knowledge of the sampling scheme (not only the sampling weights). The estimation of W1 requires the observation of the survey sample, without knowledge of the sampling scheme. The estimation of W2 requires only the knowledge of the mean and the standard deviation for the nominal living standard and the price index.

The estimates produced by using W1 and W3 are generally very close for all lines and all quarters, and never significantly different at the 5 percent level when the standard errors of W3 are considered for the test[17]. This is as well the case when the standard errors of W1, $\hat{\sigma}_{W1}$, are used for the comparison. The absolute deviation between W1 and W3 is larger in quarter D at the time of the annual poverty crisis (-12.0 percent through -5.4 percent for different poverty lines), although it is still non significant. Even if in other quarters too, the absolute deviation between W1 and W3 varies with the poverty line, W1 always slightly

[17]Of course, and this is also true for comparisons of W3 and W2, or of W1 and W2, we could combine the standard errors associated to both estimators in the comparison. Because the standard errors of W1 and W2 are subject to the lognormality hypothesis, which is not the case for the standard error of W3, we prefer to fix the value of one estimate and check if the sampling estimate is significantly different. This means in practice that we check that the numbers obtained with W1 and W2 belong to the confidence interval of our best estimate, W3.

overestimates poverty when compared with W3. This is consistent with a lower tail of the distribution of real living standards that is thicker than that of a lognormal distribution.

The comparison of W1's and W3's estimates shows that the distribution model provides a reasonable approximation of poverty even if the lognormality of distributions is rejected in several periods. These results validate the model as an analytical tool and simulation device. However, the use of W3 is limited by the availability of the MLEs of parameters, which are not typically published. Naturally, when a complete household survey dataset is available, it would generally be inefficient to first estimate the functional form of the distribution and then use the estimated parameters to calculate poverty measures.

Let us now turn to W2 estimates. The main interest of W2 is that when household level data are hard to come by, or too costly to treat, using W2 could make approximate measurement practicable. However, this is relevant only if we can exhibit actual cases where the approximation is of sufficient quality. We first compare with W1's estimates. Using W2 underestimates poverty in quarters A, C, D and the year, and overestimates poverty in quarter B. These underestimations and overestimations are sometimes substantial[18]. The lower the poverty line, the

[18]-23.4 through -11.4 percent in quarter D following the poverty line; -4.2 through

greater the relative absolute deviation. The differences between W1 or W2 arise from the gap between MLEs and MMEs of the distribution parameters and from the assumption of a null correlation for W2.

More interesting for our purpose are the deviations between W3 and W2, i.e. between respectively the best estimator and the estimator based on scarce information. At the 5 % level, W1 appears to be in the confidence interval of W3 for all quarters and all poverty lines, but not when considering annual consumption for the calculation of poverty estimates. This latter result is related to consumption smoothing occurring when using annual consumption instead of quarterly consumption. However, at the 1 % level, W1 is in the confidence interval of W3 even when using the annual per capita consumption and with all poverty lines.

The fact that differences between W2 and W3 are generally non significant indicates that there exist realistic situations for which the model can be used for poverty predictions when only empirical means and standard deviations of price indices and nominal living standards are known. Naturally, nothing ensures that the same differences would be systematically non significant for all data sets. On the contrary, that is certainly not the case for some surveys based on very large

-8.6 percent in quarter A; 9.8 through 21.0 percent in B; -3.6 through -7.3 percent in C; 13.31 through 34.1 percent for the year.

sample for which the high accuracy of the estimator W3 would make it preferable to W2. However, because there are many situations of scarce data where W3 and W1 are impossible to calculate, and since W2 has been shown to perform well for a national survey in a very poor country, we believe that it can be useful to poverty study.

The relative variations between W2 or W3, and W1, provide indications about the extent of the approximation involved in the model. Clearly, using W1 or W2 instead of W3 changes the estimated poverty, but the estimates from the three estimators remain close enough to provide meaningful information. This is particularly the case when one keeps in mind the size of errors that can occur when using non axiomatically sound indicators, when non deflating, when using a wrong poverty line. For example, our own experience of development practice has shown us that the number of the poor in some countries can be increased by several times by using administrativelly chosen poverty lines rather instead of poverty lines anchored on nutritional minima.

Finally, the distance of W2 and W1 from W3 is generally larger for quarters in which the lognormality of nominal living standards has been rejected (B and D). This is consistent with the restrictions imposed by the model, although the result was not obvious a priori since the lognormality of price indices had been rejected at all quarters.

5.2. The necessary information

When direct sampling estimates of poverty are not available, it seems to us that there is little to loose by using W2. When sampling estimates of poverty are available, but based on non axiomatically sound indicators or on inappropriate poverty line, or without proper deflation, using W2 would provide a useful control for fragile inferences.

The availability of mean and standard deviation of both nominal living standard and price index is crucial for the use of W2. As we said above, this information is sometimes present in publications or can be easily reconstructed. Moreover, limited data treatment by statistical institutes should often be sufficient to generate these statistics. In rare cases, it is even possible to know the MLEs of the distribution parameters. Then, W1 can be used for poverty measurement.

The model used for W2 is also of interest for an univariate approach. Indeed, in some cases, mean and standard deviation of *real living standards* can be found. The formula of W can then be easily contracted with m = m_1 - m_2 and $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$, by assuming that the real living standards follow an univariate lognormal law L(m, σ). Muller (1999c) discusses additional axiomatic properties of the Watts' measure in this case. The methods used in this paper can therefore be readily adapted. Another context where one may want to use this univariate approach is when there is no information at all about prices and that one is ready, or constrained, to neglect completely the price index dispersion in poverty measurement.

Finally, rarer information is necessary for the calculation of the asymptotic covariance matrices for W1 and W2. However, under the assumption $\rho = 0$, which may be frequently satisfied in practice, simplifications occur and only estimators of σ_1 and σ_2 are necessary beyond the sample moments of variables. Again, the MMEs given by eq. 3.2 can be used. *Therefore, in favorable cases, while still with scarce information, not only an axiomatic sound poverty measure can be estimated, but also the accuracy of its estimation, as long as one accepts the lognormality approximation.*

6. Conclusion

Available statistical information for poverty analysis is often quite limited when it is impossible to access data collected from household survey. Nonetheless, in such situations, the mean and standard deviation of living standards and price indices of the variables of interest are sometimes published. In some cases, poverty measures have been estimated, but the calculated poverty statistics do not correspond to the researcher's requirements.

Moreover, large geographical and temporal differences in prices exist in agricultural developing countries because of market imperfections and high output seasonality. They may also occur during structural adjustment periods, or because of weather, economic or political shocks that are frequent in LDCs. Unfortunately, price differences across households and periods seriously affect poverty measurement.

We propose in this article a simultaneous solution to the data availability problem and the price dispersion problem. Using a bivariate lognormal model of the distributions of price indices and nominal living standards, we derive a parametric formula of the Watts' poverty measure and its associated statistical inference framework.

We show that this formula can be useful for three tasks, which we illustrate using national data from Rwanda, a very poor African country:

1. Analyzing the interactions of price indices and nominal living standards for poverty analysis.

2. Deriving estimates of an axiomatically sound poverty measure (and of its

accuracy), which accounts for price dispersion and for any desired poverty line, from the sole knowledge of the mean and standard deviation of price indices and nominal living standards.

APPENDICES:

Proof of Proposition 3.6:

Under $\rho = 0$, the two distributions of w and I are independent since those of $\ln w$ and $\ln I$ are. Then, $\mathrm{cov}(\tilde{\mathrm{m}}_1, \tilde{m}_2) = \mathrm{cov}(\hat{\mathrm{m}}_1, \tilde{\sigma}_2) = \mathrm{cov}(\hat{\sigma}_1, \tilde{m}_2) = \mathrm{cov}(\hat{\sigma}_1, \tilde{\sigma}_2)$ $= 0$.

For one observation, the block $\mathrm{IF}_1^{-1} = \begin{bmatrix} \sigma_1^2(1-\rho^2) & 0 \\ 0 & \frac{\rho^4.(\sigma_1-\sigma_2)^2+4\sigma_1^2\sigma_2^2+\rho^2\left(4\sigma_1^2\sigma_2^2-2\sigma_1^2-2\sigma_2^2\right)}{\sigma_1^4.(2\sigma_2^2+\rho^2.(1-\rho^2).(-1+\sigma_2^2))} \end{bmatrix}$ since IF is diagonal for $\rho = 0$.

The variance of W2 is obtained by application of the delta method. []

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