A discusión

POVERTY SIMULATION AND PRICE CHANGES*

Christophe Muller

WP-AD 2006-13

Correspondence to: Christophe Muller. Departamento de Fundamentos del Análisis Económico. Universidad de Alicante, Campus de San Vicente, 03080 Alicante, Spain. E-mail: cmuller@merlin.fae.ua.es

IVIE working papers offer in advance the results of economic research under way in order to encourage a discussion process before sending them to scientific journals for their final publication.

Christophe Muller

ABSTRACT

Spatial price dispersion varies because of climatic fluctuations, market imperfections, economic growth or economic policies. These variations are often neglected in poverty studies.

In this paper, we propose a simple simulation formula to assess the effect on poverty of a change in the spatial mean or spatial variance of price indices without having to model each household situation. This approach constitutes a convenient first step of the analysis of the impact of change in price distributions before more sophisticated investigation of causality structures and household heterogeneity.

1.   Introduction

Variations in prices are fundamental for poverty analysis, notably in LDCs (Less Developed Countries) where most of the poor live and where price dispersion can be large. Indeed, differences and changes in prices of the goods affect the purchasing power of the poor. This paper proposes a new and simple statistical approach to investigate how changes in price distribution affect poverty.

Price variations in developing countries stem from several sources. On one hand, structural adjustment plans or other economic policy measures are accompanied by large temporal and geographical movements of absolute and relative prices, which may substantially affect the distribution of real living standards. For example, Alderman and Shively (1996) and Sahn, Dorosh and Younger (1997) show that in Ghana that the market liberalisation during the adjustment program of the late 1980's has led to a price decrease (or moderate increase) despite a devaluation of 100 percent. In this country, between 1984 and 1990, the prices of major staple foods fell and the rate of decline was faster than in the 1970's and early 1980's. This was accompanied by substantial changes in relative prices. Adjustment plans have raised a misgiving of poverty[1] rise and the impact of these plans on poverty has been the object of many studies[2]. However, although most authors mention the importance of movements in aggregate relative prices on the wake of adjustment, they do not consider how these plans affect the spatial distribution of prices. This may be important when large differences exist in price levels that households, particularly poor households, face in different regions.

Many other economic policies are designed to change prices[3]. Some

---

[1] The World Bank (1990, 2000), Bourguignon, De Melo, Morrisson (1991), Sahn and Sarris (1991).

[2] e.g. Kanbur (1987), Demery and Squire (1990), The World Bank (1990), Bourguignon, de Melo and Morrisson (1991), Duncan and Howell (1992), Schneider (1992), Sahn and Sarris (1991), Balisacan (1995), Sahn, Dorosh and Younger (1997).

[3] See for example Muellbauer (1974a, 1974b), Besley and Kanbur (1988),

policies aim at improving or stabilising the prices of inputs mainly bought by the poor. Other policies attempt to improve or stabilise the prices of the outputs produced intensively with the inputs used by the poor. All these economic measures are likely to affect the spatial structure of prices. Badiane and Shively find reductions in the local prices and local price variance following the economic reforms during 1983 in Ghana. Similarly, we shall examine the changes in level and spatial variance of price indices.

Some authors have analysed how price-changing policies affect poverty. For example, Ravallion and van de Walle study how food pricing reforms affect poverty in Indonesia. They simulate a hypothetical reform of rice pricing and find that the poverty orderings critically depend on the choice of poverty lines. Then, we shall use several poverty lines.

On the other hand, geographical and seasonal differences in prices that households face are common in LDCs, and typically generated by agricultural output fluctuations, imperfect markets, high transport and commercialisation costs, and information problems. Evidence of regional differences in price levels have been found in many LDCs[4]. This is significant because, as discussed by Sen (1981), differences in prices that households face can dramatically affect their capacity to acquire food.

Evidence also exists of large intertemporal price movements. The World Bank (1992) shows seasonal price ratios for twenty rural developing countries in the 1980's and five products (rice, maize, wheat, sorghum and millet). The ratios exhibit a generally high sensitivity of agricultural relative prices across seasons. Such variations may severely harm poor peasants who have often limited access to capital markets. In Africa, Baris and Couty (1981) suggest that the seasonal

---

Pinstrup-Andersen (1988), Bevan, Collier and Gunning (1990), Ravallion and van de Walle (1991), Duncan and Jones (1993), Koen and Phillips (1993), Lipton and Ravallion (1993), Alderman and Shively (1996), Bardhan (1996), Binswanger and Deininger (1997), Badiane and Shively (1998).

[4]e.g., Ravallion and van de Walle (1991b), Kanbur (1987), Badiane and Shively (1998), Muller (2002).

price fluctuations may make the social differentiation worse. Barrett and Dorosh (1996) fin dsubstantial farmer's welfare impact of changing rice prices in Madagascar. Similar evidence and concern have been raised for geographical differences in prices across households or for industrialised countries (Riley, 1961). Consequently, it is important to consider local price indices rather than national or regional inflation indicators, and to account for the seasonal variability in prices.

Price distributions can be summarised in a first approach by the mean and variance of price indices across households. Then, changes in price distributions can be approximated by changes in statistics of the price index distribution. The latter statistics are easier to gather or to conjecture than a whole description of the joint distribution of living standards and all prices. In this paper, we develop a simple prediction formula based on the changes in these two statistics to assess the effect on poverty of the changes in the price distribution without having to model the situation of every individual household. The approach in this paper does not incorporate agents' responses and general equilibrium effects, which are discussed in Bourguignon et al. (2002). What we provide is a tool to produce preliminary estimates without entering in the modelling of a full economy.

We discuss in Section 2 the Watts poverty measure on which our analysis is based. In Section 3, we present an estimator of the effect of price changes on the Watts poverty measure under lognormality. We present in Section 4 an application to seasonal price variations in Rwanda. Finally, Section 5 concludes the paper. The proofs are presented in the Appendix.

2.   The Watts poverty measure

In this section, we discuss the living standard indicator, then the Watts poverty measure on which the analysis is based and finally a parametric formula of this poverty measure under bivariate lognormality of nominal living standards and price indices. The living standard indicator for household i at period t is defined as $y_{it} = \frac{c_{it}}{es_i \, P_{it}} =$

$\frac{w_{it}}{P_{it}}$, where c_{it} is the value of the consumption of household i at period t, es_i is the equivalence scale of household i and P_{it} is the price index associated with household i and period t. We label $w_{it} = c_{it}/es_i$, the non-deflated living standard indicator (nominal living standard).

Atkinson (1987) and Ravallion (1994), among others, insist on using axiomatically sound poverty measures for inferences on poverty. One of the main axiomatically sound poverty measures is the Watts measure (Watts, 1968, Zheng, 1993). We focus on this indicator because of its attractive axiomatic properties and of the existence of a convenient parametric formula under lognormality. However, the methods of the paper can also be extented to the cases of the Head-Count Index and the Gini coefficient of inequality, and to many other welfare indicators under different distribution assumptions. The Watts poverty measure is defined as

$$W = \int_0^z -\ln(y/z)\, d\mu(y), \tag{1}$$

where μ is the cumulative distribution function of living standards y, and z is the poverty line. The Watts measure satisfies the monotonicity, sub-group consistency, transfer and transfer sensitivity axioms[5].

Eq. 1 supposes that the poverty line, z, is defined independently of the distributions of real living standards. Many methods exist for calculating poverty lines (Ravallion, 1998) and this assumption may not always be satisfied. In such a case, z becomes an explicit function $z(\mu)$ and complementary terms must be added to the formulae in this

---

[5] Focus axiom: The proverty index $P(y, z)$ on independent of the income distribution above z. Monotonicity: $P(y, z)$ is increasing if one poor person has a drop in income. Transfer: $P(y, z)$ increases if income is transferred from a poor person to someone richer. Transfer-sensitivity: The increase in $P(y, z)$ in the previous Transfer axiom is inversely related to the income level of the donator. Sub-group consistency: if an income distribution is partitioned in two sub-groups y' and y'', then an increase in $P(y'', z)$ with $P(y', z)$ constant, increases $P(y, z)$.

paper. We do not pursue this approach to avoid mixing too many different issues.

We can obtain an explicit parametric expression of the Watts measure by approximating the joint distribution of (w, I) with a bivariate lognormal distribution. This will help us to separate these contributions of the distributions for non-deflated living standards and prices. The choice of the lognormal distribution is supported by the fact that histograms of nominal living standards and price indices have unimodal asymmetrical and leptokurtic shapes. Moreover, these variables are always positive.

The lognormal approximation has been frequently used in applied analysis of living standards[6]. Log-wage or log-price equations have also been estimated, often implicitly relying on error terms that satisfy normality assumptions. Eaton (1980), Deaton and Grimard (1992), assume for example, lognormality for price distributions. Other distribution models for living standards or incomes[7] or models for prices[8] could also be used to represent the data, but they would not lead to a parametric expression for the Watts measure.

Finally, the reason for a lognormal specification in this paper is not its good fit with the data, but rather the availability of a convenient parametric expression of the Watts measure. The parametric approach has been shown to be useful in poverty measurement, as demonstrated in the example of Cowell and Victoria-Feser (1996) for the treatment of data contamination. In this paper, we use it to calculate the impact of price changes.

As we showed in Muller (2001), the Watts poverty measure is the only axiomatically sound and subgroup-consistent poverty indicator used by empirical economists for which a parametric formula can be derived under bivariate lognormal distribution of price index and nom-

---

[6]See van Praag, Hagenaars and van Eck (1983), Alaiz and Victoria-Feser (1996), Slesnick (1993).

[7]Salem and Mount (1974), Kloek and van Dijk (1978), Singh and Maddala (1976), Hirschberg and Slottje (1989).

[8]Creedy and Martin (1994).

inal living standard. However, the statistical methods we implement in this paper can easily be applied to the Head-Count index or the Gini coefficient of inequality. To shorten the exposition and because the Head-Count index has mediocre theoretical properties, we focus on the Watts measure of which we now present the formula under lognormality.

Proposition 1 *(Muller, 2001)*

If the nominal living standards w and the price indices P follow a bivariate lognormal distribution law, $LN\left[\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right]$, then the Watts measure is equal to:

$$
\begin{aligned}
W &= (\ln z - m_1 + m_2)\,\Phi\left(\frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right) \\
&\quad + \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}\,\phi\left(\frac{\ln z - m_1 + m_2}{\sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}}\right),
\end{aligned}
$$

where ϕ and Φ are respectively the p.d.f. and c.d.f. of the standard normal distribution.

Now we are ready to explain how to obtain simulations of poverty in this framework.

3.   A Simulator for the Watts Measure

In this section, we first present the parameters' estimators of the bivariate distribution. Then, we propose a simulator of the Watts poverty measure when the distribution of price indices changes.

3.1. MLE

We first recall a few well-known formulae that will be used for our estimators of W. The parameters of the joint distribution can be estimated from joint samples of price indices and living standards. All parameters, except the correlation coefficient ρ, can also be estimated from separate information sources about prices and incomes.

Proposition 2

If the distributions of w and P are jointly lognormal, then the maximum likelihood estimators (MLE) of (m_i, σ_i), i = 1 and 2, and ρ, are consistent, asymptotically normal, efficient and invariant. They are:

$$\hat{m}_1 = \frac{1}{n}\Sigma_i \ln(w_i) \, and \, \hat{m}_2 = \frac{1}{n}\Sigma_i \ln(P_i);$$

$$\hat{\sigma}_1^2 = \frac{1}{n}\Sigma_i \left(\ln(w_i) - m_1\right)^2 and \, \hat{\sigma}_2^2 = \frac{1}{n}\Sigma_i \left(\ln(P_i) - m_2\right)^2;$$

$$\hat{\rho} = \frac{\frac{1}{n}\Sigma_i \left(\ln(w_i) - m_1\right).(\ln(P_i) - m_2)}{\hat{\sigma}_1 \, \hat{\sigma}_2}, \, where \, n \, is \, the \, sample \, size.$$

The Fisher information matrix associated with $(\hat{m}_1, \hat{m}_2, \hat{\sigma}_1, \hat{\sigma}_2, \hat{\rho})$ is given in the Appendix.

3.2. MME

We now use the mean and variance of P to produce alternative estimators of the parameters m_2 and σ_2. The mean (M_2) and the

variance (S_2^2) of P following a bivariate lognormal law can be *explicitly* calculated from the parameters of the distribution. Then, these formulae can be used to define method-of-moment estimators (MME) of m_2 and σ_2 as follows.

Definition 1 *The MME of m_2 and σ_2 are:*

$$\tilde{\sigma}_2(\tilde{M}_2, \tilde{S}_2) = \sqrt{\ln\left(1 + \frac{\tilde{M}_2}{\tilde{S}_2^2}\right)} \tag{2}$$

$$\tilde{m}_2(\tilde{M}_2, \tilde{S}_2) = \ln(\tilde{M}_2) - \tilde{\sigma}_2^2/2 \tag{3}$$

where $\tilde{M}_2$ and $\tilde{S}_2^2$ are respectively the empirical means and the empirical variance of the price indices.

Proof: The characteristic function of the normal law $\mu \equiv N(m, \sigma^2)$ is the following (Johnson and Kotz, 1973).

$$G(t) = \int e^{i.(ut)} d\mu(u) = e^{itm - \frac{1}{2}t^2\sigma^2} \tag{4}$$

Equation 4 gives for $t = -i.r$, $u = \ln(v)$: $\int v^r.dLN(v) = e^{rm + \frac{1}{2}r^2\sigma^2}$. In particular, using specific values of r we have for $X = y$ following a joint lognormal distribution:

$$EX = e^{m + \sigma^2/2} \tag{5}$$

$$E(X^2) = e^{2m + 2\sigma^2} \tag{6}$$

$$VX = E(X^2) - (EX)^2 = e^{2m + \sigma^2}(e^{\sigma^2} - 1) \tag{7}$$

Equations. 5 and 7 enable us to define the MME $(\tilde{m}_2, \tilde{\sigma}_2)'$ because in the bivariate lognormal distribution the corresponding moments are separable and can be deducted from the marginal distribution of the price indices. QED.

Proposition 3 *Estimators $\tilde{m}_2$ and $\tilde{\sigma}_2^2$ are consistent, although generally not efficient. Their asymptotic covariance matrix is presented in the next proposition.*

The asymptotic covariance matrix of the MMEs $(\hat{m}_2, \tilde{\sigma}_2)'$ is

$$\Sigma_1 = [D'\,\Phi^{-1}D]^{-1}/n \tag{8}$$

where n is the sample size, $D = \begin{bmatrix} -e^{(m_2+\sigma_2^2/2)} & -\sigma_2 e^{(m_2+\sigma_2^2/2)} \\ -2e^{2m_2+2\sigma_2^2} & -4\sigma_2 e^{2(m_2+\sigma_2^2)} \end{bmatrix}$

and where $\Phi_{kj} = p\lim\limits_{n\to+\infty} \left\{ \frac{1}{n}\sum_{i=1}^n f_{ik}f_{ij} \right\}, (i,j = 1,2)$ can be estimated by using $\hat{\Phi}_{kj} = \frac{1}{n}\sum_{i=1}^n f_{ik}.f_{ij}\ (k,j = 1,2)$, for each observation i, the two moments are: $f_1 = P_i - e^{m_2+\sigma_2^2/2}$ and $f_2 = P_i^2 - e^{2m_2+2\sigma_2^2}$.

Finally, we derive the formula of the asymptotic covariance matrix of $(\hat{m}_1, \hat{\sigma}_1, \tilde{m}_2, \tilde{\sigma}_2)$ that we will use for the standard error of the simulator of W.

Proposition 4 *Let IF be the total information matrix for the joint sample of price indices and nominal living standards. Under the hypothesis $\rho = 0$, the asymptotic covariance matrix of $(\hat{m}_1, \hat{\sigma}_1, \tilde{m}_2, \tilde{\sigma}_2)'$ is:*

$$\Sigma_2 = \begin{bmatrix} \frac{1}{n}IF_1^{-1} & 0 \\ 0 & \Sigma_1 \end{bmatrix},$$

where IF_1^{-1} is the inverse of the block of IF corresponding to $(\hat{m}_1, \hat{\sigma}_1)$.

$$IF_1^{-1} = \begin{bmatrix} \sigma_1^2(1-\rho^2) & 0 \\ 0 & \frac{\rho^4(\sigma_1-\sigma_2)^2+4\sigma_1^2\sigma_2^2+\rho^2\left(4\sigma_1^2\sigma_2^2-2\sigma_1^2-2\sigma_2^2\right)}{\sigma_1^4(2\sigma_2^2+\rho^2(1-\rho^2)(-1+\sigma_2^2))} \end{bmatrix}$$

3.3. A simulator of the effects of price changes on poverty

We now propose a simulator of the Watts poverty measure, WS, which describes the impact of changes in the mean and in the variance of the price index distribution on poverty. These changes are indexed by two new parameters related to natural intuitions: θ_1, θ_2. By definition, θ_1 is the proportional change in the mean price index, M_2, and θ_2 is the proportional change in the variance of price indices, S_2^2. When $\theta_1 = \theta_2 = 1$, the price index distribution stays the same. This approach is possible because we use Watts poverty formula in which the estimators of the mean and variance of the price indices are explicitly incorporated. The formula of the simulator is the following:

Definition 2

$$\begin{aligned} WS \; = \; & (\ln z - \hat{m}_1 + \tilde{m}_2(\theta_1 M_2, \theta_2 S_2^2)). \\ & \Phi\left(\frac{\ln z - \hat{m}_1 + \tilde{m}_2(\theta_1 M_2, \theta_2 S_2^2)}{\sqrt{\hat{\sigma}_1^2 + \tilde{\sigma}_2^2(\theta_1 M_2, \theta_2 S_2^2) - 2\hat{\rho}\hat{\sigma}_1\tilde{\sigma}_2(\theta_1 M_2, \theta_2 S_2^2)}}\right) \\ & + \sqrt{\hat{\sigma}_1^2 + \tilde{\sigma}_2^2(\theta_1 M_2, \theta_2 S_2^2) - 2\hat{\rho}\hat{\sigma}_1\tilde{\sigma}_2(\theta_1 M_2, \theta_2 S_2^2)} \\ & .\phi\left(\frac{\ln z - \hat{m}_1 + \tilde{m}_2(\theta_1 M_2, \theta_2 S_2^2)}{\sqrt{\hat{\sigma}_1^2 + \tilde{\sigma}_2^2(\theta_1 M_2, \theta_2 S_2^2) - 2\hat{\rho}\hat{\sigma}_1\tilde{\sigma}_2(\theta_1 M_2, \theta_2 S_2^2)}}\right) \end{aligned} \quad (9)$$

where θ_1 and θ_2 are simulation parameters describing changes in

the price distribution. $\tilde{m}_2(.,.)$ and $\tilde{\sigma}_2(.,.)$ denotes the functions defined in equations 2 and 3.

Note that this formula incorporates the estimator of ρ. However, this estimator can only be used when prices and living standards are simultaneously observed for the same sample. This is not the case if price information does not come from the household consumption survey. Since the situation of null or weak correlation is not unrealistic at least in some contexts (for example in Muller, 2001), it is interesting to consider the same predictor in which $\hat{\rho}$ has been replaced by zero. The hypothesis that nominal living standards and price indices are not correlated is often necessary because there is little hope of recovering the correlation parameter from published information on prices and living standards. In the case $\rho = 0$, the asymptotic covariance matrix of WS can be related to the asymptotic covariance matrix Σ_2, as the following proposition shows.

Proposition 5

Under the hypothesis $\rho = 0$, $V(WS) = G.\Sigma_2.G'$ where

$$G = \left[\frac{\partial W}{\partial(m_1\,\sigma_1\,m_2\,\sigma_2)'}\right] \cdot \left[\frac{\partial(m_1\,\sigma_1\;m_2(\theta_1'M_2,\theta_2'S_2^2)\;\sigma_2(\theta_1'M_2,\theta_2'S_2^2))'}{\partial(m_1\,\sigma_1\,\theta_1'M_2\,\theta_2'S_2^2)}\right] \cdot$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \theta_1 & 0 \\ 0 & 0 & 0 & \theta_2 \end{bmatrix} \cdot \left[\frac{\partial(m_1\,\sigma_1\,m_2\,\sigma_2)'}{\partial(m_1\,\sigma_1\,M_2\,S_2^2)}\right]^{-1},$$

$$where \; \left[\frac{\partial(m_1\,\sigma_1\,m_2\,\sigma_2)'}{\partial(m_1\,\sigma_1\,M_2\,S_2^2)}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\partial m_2}{\partial M_2} & \frac{\partial m_2}{\partial S_2^2} \\ 0 & 0 & \frac{\partial \sigma_2}{\partial M_2} & \frac{\partial \sigma_2}{\partial S_2^2} \end{bmatrix},$$

$$\frac{\partial m_2}{\partial M_2} = \frac{1}{M_2}\left[1 + \frac{S_2^2}{M_2^2 + S_2^2}\right], \quad \frac{\partial m_2}{\partial S_2^2} = -\frac{1}{2\left(M_2^2 + S_2^2\right)},$$

$$\frac{\partial \sigma_2}{\partial M_2} = -\frac{1}{\sqrt{\ln\left(1 + \frac{S_2^2}{M_2^2}\right)}}\frac{1}{\left(M_2^2 + S_2^2\right)}\frac{S_2^2}{M_2}, \quad and \quad \frac{\partial \sigma_2}{\partial S_2^2} = \frac{1}{2\sqrt{\ln\left(1 + \frac{S_2^2}{M_2^2}\right)}}\frac{1}{\left(M_2^2 + S_2^2\right)}.$$

The asymptotic covariance matrice Σ_2 can be consistently estimated by replacing parameters m_i, σ_i^2 $(i = 1, 2)$ and ρ with consistent estimates. Each Jacobian matrix can also be estimated consistently by using consistent estimators of parameters. When the hypothesis $\rho = 0$ is rejected, no convenient close form is available for the formula of the asymptotic covariance matrix of WS. We do not provide the complicated general formula of $V(WS)$ in that case, because the coefficient ρ is not likely to be estimable without data on prices and incomes from the same survey. We now discuss the data used for a brief application of the above formulae.

4. Application to seasonal price variations in Rwanda

The data for the estimation is extracted from the Rwandan national budget-consumption survey, conducted by the Government of Rwanda and the French Cooperation and Development Ministry, in the rural part of the country from November 1982 to December 1983 (Ministère du Plan, 1986a)[9]. In 1983, Rwanda had a per capita GNP of US $ 270 per annum, which is a very low level. More than 95 percent of the population lived in rural areas (Bureau National du Recensement, 1984). 270 households were surveyed about their consumption. Due to missing values, 265 observations are used for the estimations.

The collection of the consumption data was organised in four rounds, corresponding to four quarters (A, B, C, D) of the agricultural year 1982-83: Round A: 01/11/1982 until 16/01/1983; Round B: 29/01/1983 until 01/05/1983; Round C: 08/05/1983 until 07/08/1983; Round D:

---

[9]The main part of the collection has been designed with the help of INSEE (French National Statistical Institute).

14/08/1983 until 13/11/1983. Agricultural year 1982-83 was a normal climatic year (Bulletin Climatique du Rwanda, 1982, 1983, 1984).

Several studies of price surveys in Rwanda have established the existence of substantial geographical and seasonal price dispersion[10]. We have calculated price indicators of the main categories of goods, for every quarter and every cluster of the sample. We use a Laspeyres price index specific to each household and each quarter, in which the basis is the annual national average consumption. Thus, the price index simultaneously accounts for geographical and quarterly price dispersions.

For the poverty estimator, we use poverty lines located in relevant parts of the distribution. z_A is four times the minimum over the four quarters of the first quintiles of quarterly living standards. z_B is the sum of the first quintiles of the four quarterly living standards. z_C is the first quintile of annual living standards. z_D, z_E, z_F, have respectively the same definitions calculated from second quintiles. These definitions correspond to the annual poverty lines. The quarterly poverty lines, used for estimating the quarterly poverty indicators, are the annual poverty lines divided by four.

In the simulations, we do not incorporate the responses of households to changes in prices they face, nor the shift of the equilibrium of the economy caused by price shocks. An approach followed by Ravallion and van de Valle (1991) to account for household responses, is to estimate equivalent income functions using a demand model and to simulate the new value of each household's equivalent income after the price changes. Here, we focus on the very short term effects, neglecting household responses.

We now examine the consequences on poverty of non-marginal seasonal shocks. These shocks are interesting because they are related to the typical seasonal price fluctuations that Rwandan households face each year. Naturally, different years may have different patterns of price fluctuations, creating price shocks households cannot anticipate.

---

[10]Niyonteze and Nsengiyumva (1986), O.S.C.E. (1987), Ministère du Plan (1986b), Muller (1988).

We shall describe the seasonal shocks in prices by different values of mean and variance of the distribution of price indices. However, these parameters will always be defined in reference with the observed quarterly distributions of price indices.

All the parameters from the joint distribution can be estimated from the data for each quarter, and we shall consider them as known. We are interested in changes in the level M_2 and variance S_2^2 of prices. From the estimated mean and standard deviation of price indices at every quarter, we calculate the largest absolute deviation between two successive quarters for the mean and for the standard-deviation. That is, we consider additional increments in mean and variance of price indices comparable to the largest increment observed for two successive quarters. These numbers are used as a benchmark for the simulation of price changes that are therefore analog to seasonal changes. The calculation gives approximately a 15 percent increase for M_2; and 60 percent for S_2^2. Consequently and to keep this illustration short, we only examine two simulations: M_2 changed into 1.15 M_2 ($\theta_1 = 1.15$); and S_2^2 changed into 1.6 S_2^2 ($\theta_2 = 1.6$).

The poverty estimates resulting from the simulations based on equation 9 are shown in Table 1, along with the relative variations of W, and their standard errors. χ^2 tests of independence show the hypothesis ($\rho = 0$) is not rejected in our data. Then, we can use Proposition 7 for the calculation of the standard errors.

The simulation results reveal that substantial poverty changes occur at every quarter owing to the price shocks represented by a 15 percent increase in mean of price indices. In contrast, a 60 percent increase in variance of price indices moderately changes poverty as described by the Watts measure. Then, the studied seasonal price distribution shocks on poverty in Rwanda can be considered as mostly aggregate shocks, consequences of the change in the general level of prices in the country. It seems that very large shocks in variances of price indices, as compared to typical seasonal shocks, would be required to seriously modify the poverty measured with the Watts measure in Rwanda. This suggests stressing studies of seasonal price

shocks on the aggregate price shocks before entering into spatial and product decompositions. Of course, policy in this domain for Rwanda should be first guided by a sound understanding of the macroeconomic aspects and by the size of the seasonal fluctuations of the aggregate price index.

The magnitude of the poverty change is generally a decreasing function of the poverty line. The fact that the choice of the poverty line can substantially affect the relative variation in poverty shows the importance of considering a broad range of lines in such an analysis.

5. Conclusion

What is the impact on poverty of a change in the distribution of prices across households? This question is important because its answer may determine the design of policy responses to exogenous price shocks affecting household welfare.

To assess this impact, we propose a simple statistical tool based on the mean and the variance of spatial price indices that does not require the modelling of the situation for every individual household. The proposed simulator permits a first approach to the question of interest by determining if level and dispersion of prices[11] are likely to influence poverty substantially.

In a short illustration for Rwanda, we find that seasonal variations in prices have an effect on poverty mostly through the change in the mean of spatial price indices and very little through the change in the variance in the spatial price indices. This suggests the need to develop modelling approaches that first emphasize the role of aggregate price shocks.

Additional empirical elements would be necessary to determine what changes in the price index mean and its variance are associated with different policies. Once in possession of such characteristics of

---

[11]These price statistics may be associated for example with climatic fluctuations, market imperfections, economic growth or economic policies

price shocks, the proposed simulator can be used to investigate these policies.

One could argue that all the simulations could be done more efficiently by using non-parametric methods based on joint observation of all prices and living standards for a representative sample of households (as in Barrett and Dorosh, 1996). This is partly true when such data are available. However, such availability is not a frequent occurrence and only mean and variance of price indices can generally be found at best.

There are several reasons for adopting a parametric approach of the problem. First, using a distribution model enables the researcher to complement missing information about prices and living standards. Much can be done from the sole knowledge of the values of means and variances of price indices and living standards, and by deriving from them the parameters of the joint distribution. Second, the parametric approach enables us to exhibit meaningful parameters that are easy to grasp: the mean and the variance. Not only can the main characteristics of the distribution of price indices be expressed in terms of the values of these parameters, but the definition of the simulations of interest can also be designed naturally in reference to the mean and variance of the price index. This makes a simulation easy to implement, and its results easier to communicate. Third, anchoring the approach on a well-known distribution shape enables the researcher to mobilise information about changes of moments of the price or living standard distributions. For example, we have shown how the largest seasonal gap in aggregate price means (or variance of price indices) could be naturally incorporated in the analysis. Such a task would be much less obvious in a non-parametric setting. Finally, there are examples of a fruitful use of parametric distribution modelling in welfare analysis, e.g. in Slottje (1987) who uses a multivariate distribution model to enter explanatory variables of the distribution through parameters. A similar approach could be implemented with our model,

at the cost of complicating it. However, non-parametric approaches become necessary when one wants to accurately understand the involved mechanisms at household level.

Table 1: Simulations

1.15 M_2

Lines	$z_F >$	$z_E >$	$z_D >$	$z_C >$	$z_B >$	z_A
	0.2515	0.2149	0.1661	0.1535	0.1012	0.0822
A	[34.6]	[36.7]	[40.1]	[41.1]	[46.4]	[48.9]
	(0.0233)	(0.0211)	(0.0178)	(0.0169)	(0.0126)	(0.0108)
	0.2201	0.1841	0.1372	0.1252	0.0773	0.0606
B	[41.4]	[44.5]	[49.5]	[51.0]	[59.0]	[63.0]
	(0.0210)	(0.0187)	(0.0153)	(0.0143)	(0.0100)	(0.0083)
	0.2512	0.2154	0.1676	0.1552	0.1036	0.0847
C	[34.8]	[36.9]	[40.3]	[41.4]	[46.8]	[49.4]
	(0.0234)	(0.0212)	(0.0179)	(0.0170)	(0.0127)	(0.0110)
	0.3844	0.3397	0.2777	0.2611	0.1889	0.1608
D	[27.8]	[29.5]	[32.3]	[33.2]	[37.7]	[39.9]
	(0.0302)	(0.0281)	(0.0248)	(0.0239)	(0.0192)	(0.0172)

1.60 S_2^2

Lines	$z_F >$	$z_E >$	$z_D >$	$z_C >$	$z_B >$	z_A
	0.1865	0.1569	0.1185	0.1088	0.0693	0.0553
A	[−00.21]	[−00.16]	[−00.058]	[−00.02]	[00.167]	[00.27]
	(0.0162)	(0.0145)	(0.0120)	(0.0113)	(0.0082)	(0.0070)
	0.1569	0.1287	0.0931	0.0842	0.0497	0.0382
B	[00.80]	[01.03]	[01.43]	[01.56]	[02.29]	[02.69]
	(0.0137)	(0.0120)	(0.0096)	(0.0089)	(0.0060)	(0.0049)
	0.1884	0.1594	0.1215	0.1118	0.0724	0.0584
C	[01.07]	[01.32]	[01.7]	[01.18]	[02.65]	[03.06]
	(0.0163)	(0.0146)	(0.0121)	(0.0114)	(0.0083)	(0.0071)
	0.3028	0.2644	0.2122	0.1983	0.1395	0.1171
D	[00.68]	[00.83]	[01.09]	[01.17]	[01.64]	[01.89]
	(0.0223)	(0.0205)	(0.0178)	(0.0170)	(0.0133)	(0.0117)

The tables of simulations correspond successively to the following changes in parameters: 1.15 M_2 instead of M_2; 1.60 S_2^2 instead of S_2^2. The first line of each cell for a given poverty line and a given quarter (A through D) shows the simulated Watts poverty measure. The second line, in brackets,

shows the percentage of variation when compared to the Watts poverty measure estimated with the initial distribution parameters. The third line, in parentheses, shows the estimated standard error of the predictor WS, $\hat{\sigma}_{WS}$.

BIBLIOGRAPHY

ALDERMAN, H. and G. SHIVELY, "Economic Reform and Food Prices: Evidence from Markets in Ghana," *World Development*, vol. 24, n. 3, 521-534, 1996.

ALAIZ, M.P. and M.-P. VICTORIA-FESER, "Modelling Income Distribution in Spain: A Robust Parametric Approach," Distributional Analysis research Programme, Working Paper no 20, LSE-STICERD, June 1996.

ATKINSON, A.B., "On the Measurement of Poverty," *Econometrica*, vol. 55, n. 4, 749-764, July 1987.

BADIANE, O. and G.E. SHIVELY, "Spatial integration, transport costs, and the response of local prices to policy changes in Ghana," *Journal of Development Economics*, vol. 56, 411-431, 1998.

BALISACAN, A.M., "Anatomy of Poverty during Adjustment: The Case of the Philippines," *Economic Development and Cultural Change*, 33-62, 1995.

BARDHAN, P., "Efficiency, Equity and Poverty Alleviation: Policy Issues in Less Developed Countries," *The Economic Journal*, 106, 2349-1356, September 1996.

BARIS, P. And P. COUTY, "Prix, marchés et circuit commerciaux africains," *AMIRA* No 35, December 1981.

BARRETT, C.B. and P.A DOROSH, "Farmer's Welfare and Changing Food Prices: Nonparametric Evidence from Rice in Madagascar," *American Journal of Agricultural Economy*, 656-669, August 1996.

BESLEY, T. and R. KANBUR, "Food Subsidies and Poverty Alleviation," *The Economic Journal*, 98, 701-719, September 1988.

BEVAN, D., P. COLLIER and J.W. GUNNING, "Price Controls and the Transition to Market Clearing: Theory and an Application to Tanzania," Applied Economics Discussion Paper Series, Institute of Economics and Statistics, Oxford, May 1990.

BINSWANGER, H.P. and K. DEININGER, "Explaining Agricultural and Agrarian Policies in Developing Countries," *Journal of Economic Literature*, Vol. XXXV, 1958-2005, December 1997.

BOURGUIGNON, F., de MELO, J. and C. MORRISSON, "Poverty and Income Distribution During Adjustment: Issues and Evidence from

the OECD Project," *World Development*, vol. 19, No. 11, 1485-1508, 1991.

BOURGUIGNON, F., L. PEREIRA AND N. STERN, "Evaluating the Poverty Impact of Economic Policies: Some Analytical Challenges", mimeo The World Bank, March 2002.

BULLETIN CLIMATIQUE DU RWANDA, "*Climatologie et Précipitations,*" Kigali, 1982, 1983, 1984.

BUREAU NATIONAL DU RECENSEMENT, "*Recensement de la Population du Rwanda, 1978. Tome 1: Analyse,*" Kigali, Rwanda, 1984.

COWELL, F.A. and M.-P., VICTORIA-FESER, "Poverty Measurement with Contaminated Data: A Robust Approach," *European Economic Review*, 40, 1761-1771, 1996.

CREEDY, J. and V.L. MARTIN, "A Model of the Distribution of Prices," *Oxford Bulletin of Economics and Statistics*, 56, I, 1994.

DAVIDSON, R. and J. McKINNON, "Estimation and Inference in Econometrics," Oxford University Press, 1993.

DEATON, A. and F. GRIMARD, "Demand Analysis for Tax Reform in Pakistan," Working Paper Princeton University research program in Development Studies, January 1992.

DEMERY, L. and L. SQUIRE, "Macroeconomic Adjustment and Poverty in Africa: An Emerging Picture," *The World Bank Research Observer*, Vol. 11, No. 1, 39-59, February 1996.

DUNCAN, A and J. HOWELL, "*Structural Adjustment & the African Farmer,*" Oversee Development Institute, London, 1992.

DUNCAN, A and S. JONES, "Agricultural Marketing and Pricing Reform: A Review of Experience," *World Development*, Vol. 21, No. 9, 1495-1514, 1993.

EATON, J., "Price Variability, Utility and Savings," *Review of Economic Studies*, XLVII, 513-520, 1980.

HIRSCHBERG, J.G. and D.J. SLOTTJE, "Remembrance of Things Past the Distribution of Earnings across Occupations and the Kappa Criterion," *Journal of Econometrics*, 42, 121-130, 1989.

JOHNSON, N.L. and S. KOTZ, "*Distributions in Statistics: Continuous Multivariate Distributions,*" John Wiley & Sons, Inc., 1973.

KANBUR, S.M., "Structural Adjustment, Macroeconomic Adjustment and Poverty: A Methodology for Analysis," *World Development*, 1515-1526, 1987.

KLOEK, T. and H. K. VAN DIJK, 1978, "Efficient Estimation of Income Distribution Parameters," *Journal of Econometrics*, 8, 61-74.

KOEN, V. and S. PHILLIPS, "Behaviour of Prices, Household Incomes, and Consumption During the First Year," I.M.F. Occasional Paper 104, June 1993.

LIPTON, M. and M. RAVALLION, "Poverty and Policy," chapter 42, in J.BEHRMAN and T.N. SRINIVASAN, *Handbook of Development Economics*, vol. 3, 1993, North Holland.

MINISTERE DU PLAN, "*Méthodologie de la Collecte et de l'échantillonnage de l'Enquête Nationale sur le Budget et la Consommation 1982-83 en Milieu Rural*," Kigali, 1986.

MINISTERE DU PLAN, "*Enquête des Prix PCI au Rwanda*," Kigali, 1986.

MUELLBAUER, J., "Prices and Inequality: The United Kingdom Experience," *The Economic Journal*, 32-55, March 1974a.

MUELLBAUER, J., "Inequality Measures, Prices and Household Composition," *Review of Economic Studies*, 493-502, 1974b.

MULLER, C., "*E.N.B.C.: Relevés de prix en milieu rural*," Ministère du Plan, Kigali, Rwanda, 1988.

MULLER, C., "Prices and Living Standards. Evidence for Rwanda," *Journal of Development Economics*, Vol. 68, 187-203, 2002.

MULLER, C., "The Properties of the Watts Poverty Index under Lognormality," *Economics Bulletin*, , Vol. 9, No.1, pp1-9, August 2001.

NIYONTEZE, F. And A. NSENGIYUMVA, "*Dépouillement et présentation des résultats de l'enquête sur marchés Projet Kigali-Est*," Projet Kigali-Est, Juin 1986.

O.S.C.E., "Note statistique rapide sur la comparaison des niveaux des prix du Rwanda," Bruxelles, 1987.

PINSTRUP-ANDERSEN, P., "Food prices and the poor in developing countries," *European Journal of Agricultural Economics*, 12, 069-085, 1988.

RAVALLION, M., "*Poverty Comparisons. A Guide to Concepts and*

Methods," Harwood Academic Publishers, London, 1994.

RAVALLION, M., "Poverty Lines in Theory and Practice," Living Standards Measurement Study Working Paper No. 133, July 1998.

RAVALLION, M. and D. VAN DE WALLE, "The Impact on Poverty of Food Pricing Reforms: A Welfare Analysis for Indonesia," *Journal of Policy Modeling*, Vol. 13, 1991a.

RAVALLION, M. and D. VAN DE WALLE, "Urban-Rural Cost-of-Living Differentials in a Developing Economy," *Journal of Urban Economics*, 29, 113-127, 1991b.

RILEY, H.E., "Some aspects of Seasonality in the Consumer Price Index," *American Statistical Association Journal*, March 1961.

SAHN, D.E. and A. SARRIS, "Structural Adjustment and the Welfare of Rural Smallholders: A Comparative Analysis from Sub-Saharan Africa," *The World Bank Economic Review*, Vol. 1, No. 2, 1991.

SAHN, D.E., P.A. DOROSH, S.D. YOUNGER, "*Structural Adjustment Reconsidered*," Cambridge University Press, 1997.

SALEM, A. B. Z. and T. D. MOUNT, "A Convenient Descriptive Model of Income Distribution: The Gamma Density," *Econometrica*, Vol. 42, No. 6, November 1974.

SCHNEIDER, H., "*Adjustment and Equity in Cote d'Ivoire*," Development Centre Studies, OECD, Paris, 1992.

SEN, A., "*Poverty and Famines*," Clarendon Press, Oxford, 1981.

SINGH, S.K. and G.S. MADDALA, "A Function for Size Distribution of Incomes," *Econometrica*, Vol. 44, No. 5, 1976.

SLESNICK, D.T., 1993, "Gaining Ground: Poverty in the Postwar United States," *Journal of Political Economy*, vol. 101, No 1.

SLOTTJE, D.J., "Relative Price Changes and Inequality in the Size Distribution of Various Components of Income," Journal of Business and Economic Statistics, Vol. 5, No. 1, 19-26, January 1987.

VAN PRAAG, B., A. HAGENAARS and W. VAN ECK, 1983, "The Influence of Classification and Observation Errors on the Measurement of Income Inequality," *Econometrica*, Vol. 51, No. 4, July.

WATTS, H.W., "An Economic Definition of Poverty" in D.P. Moynihan (ed.), "*On Understanding Poverty*," 316-329, Basic Book, New York, 1968.

THE WORLD BANK, "*Making Adjustment Work for the Poor. A Framework for Policy Reform in Africa,*" The World Bank, Washington, 1990.

THE WORLD BANK, "*World Development Report 1990. Poverty,*" Oxford University Press, 1990.

THE WORLD BANK, "*World Development Report 1992,*" Oxford University Press, 1992.

THE WORLD BANK, "*World Development Report 2000/2001. Attacking Poverty,*" Oxford University Press, 2000.

ZHENG, B., "An axiomatic characterization of the Watts poverty index," *Economic Letters,* 42, 81-86, 1993.

Appendix:

The average Fisher information matrix of $(\hat{m}_1, \hat{m}_2, \hat{\sigma}_1, \hat{\sigma}_2, \hat{\rho})$ is:

$$I\bar{F} = \begin{bmatrix} \frac{1}{\sigma_1^2(1-\rho^2)} & \frac{-\rho}{\sigma_1\sigma_2(1-\rho^2)} & 0 & 0 & 0 \\ \frac{-\rho}{\sigma_1\sigma_2(1-\rho^2)} & \frac{1}{\sigma_2^2(1-\rho^2)} & 0 & 0 & 0 \\ 0 & 0 & \frac{(2-\rho^2)}{\sigma_1^2(1-\rho^2)} & \frac{-\rho^2}{\sigma_1\sigma_2(1-\rho^2)} & \frac{\rho}{\sigma_1^2(1-\rho^2)} \\ 0 & 0 & \frac{-\rho^2}{\sigma_1\sigma_2(1-\rho^2)} & \frac{(2-\rho^2)}{\sigma_2^2(1-\rho^2)} & \frac{\rho}{\sigma_2^2(1-\rho^2)} \\ 0 & 0 & \frac{\rho}{\sigma_1^2(1-\rho^2)} & \frac{\rho}{\sigma_2^2(1-\rho^2)} & \frac{(1+\rho^2)}{(1-\rho^2)^2} \end{bmatrix}$$

Proof of Proposition 3:

The vector of moments to consider for price index observation i is:

$$\left(f_1 \equiv P_i - e^{m_2+\sigma_2^2/2}, f_2 \equiv P_i^2 - e^{2m_2+2\sigma_2^2} \right)'$$

The associated moment conditions are $P_i - e^{m_2+\sigma_2^2/2} = 0$, $P_i^2 - e^{2m_2+2\sigma_2^2} = 0$.

Let be $F_i(\theta) = (f_1, f_2)'$. Here, f_{ik} denotes the kth moment condition f_k for observation $i, k = 1, 2$.

It is clear that $E[F_i] = 0$. This defines the 'estimating equations' for the MMEs. The formula in the proposition provides the asymptotic covariance matrix of the MMEs (see Davidson and McKinnon, 1993). Matrices D and Φ have been calculated in the considered case by using the estimating equations.

Proof of Proposition 4: Direct consequences of Propositions 2, 3 and the non-correlation of $(\hat{m}_1, \hat{\sigma}_1)$ and $(\tilde{m}_2, \tilde{\sigma}_2)$. Under $\rho = 0$, the two distributions of w and P are independent since those of $\ln w$ and $\ln P$ are independent. Then, $cov(\tilde{m}_1, \tilde{m}_2) = cov(\hat{m}_1, \tilde{\sigma}_2) = cov(\hat{\sigma}_1, \tilde{m}_2) = cov(\hat{\sigma}_1, \tilde{\sigma}_2) = 0$. IF_1^{-1} is the inverse of the relevant block of IF because the blocks on the second diagonal of IF are null. QED.

Proof of Proposition 5: The mapping g transforming $(\hat{m}_1,\ \hat{\sigma}_1,\ \tilde{m}_2,\tilde{\sigma}_2)$, whose covariance matrix, Σ_2, is known, into WS can be decomposed into

$$(\hat{m}_1,\ \hat{\sigma}_1,\ \tilde{m}_2,\tilde{\sigma}_2) \overset{g_1}{\mapsto} (\hat{m}_1,\ \hat{\sigma}_1,\ \tilde{M}_2,\tilde{S}_2^2) \overset{g_2}{\mapsto} (\hat{m}_1,\ \hat{\sigma}_1,\ \theta_1\tilde{M}_2,\theta_2\tilde{S}_2^2)$$

$$\overset{g_3}{\mapsto} (\hat{m}_1,\ \hat{\sigma}_1,\ \tilde{m}_2(\theta_1\tilde{M}_2,\theta_2\tilde{S}_2^2),\ \tilde{\sigma}_2(\theta_1\tilde{M}_2,\theta_2\tilde{S}_2^2)) \overset{g_4}{\mapsto} WS\ .$$

Then, by using the delta method, the asymptotic covariance matrix of WS is $(Jg).\Sigma_2.(Jg)'$ where $g = g_4\, o\, g_3\, o\, g_2\, o\, g_1$, and J denotes the operator Jacobian matrix. This yields by using the chain rule:

$$V(WS) = Jg_4\, Jg_3\, Jg_2\, Jg_1.\Sigma_2.(Jg_1)'(Jg_2)'(Jg_3)'(Jg_4)',$$ which gives the result. QED.