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A virtual reality web environment create tovisualize Archimedean Polyhedra and their Catalan duals

Creación de un entorno web de realidad virtual para visualizar Poliedros Arquimedianos y sus Duales Catalanes

Paulo Henrique Siqueira[1, *]

1- Departamento de Expresión Gráficam, Universidad Federal de Paraná. Curitiba, Brasil.

* E-mail: paulohs@ufpr.br

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Abstract

This paper shows the use of web resources to create environments for visualizing Archimedean polyhedra and the respective Catalan duals. These environments were created using Virtual Reality (VR) resources, which allow the visitor to manipulate the polyhedra and compare the polyhedra and their duals. Geometric transformations of translation and rotation were used to build the virtual rooms, using HTML page hierarchies structures, and inserting each polyhedron in its respective virtual room. The resources presented in this work can be used in the classroom to visualize polyhedra with immersive goggles and even in Augmented Reality (AR) using smartphones and tablets. Other studies that can be developed with the polyhedra modeled in this article are areas, volumes, plane sections and the Euler relation. This article shows the possibility of creating teaching materials with a simple, free technology that makes a great contribution to improving the teaching of Geometry, as well as other areas that use graphic representations of 3D objects.

Keywords: Virtual Reality; Graphical visualization; Archimedean polyhedra; Catalan polyhedra; Duality.

Resumen

Este artículo muestra el uso de recursos web para crear entornos para visualizar los poliedros de Arquímedes y los respectivos duales de Catalan. Estos entornos fueron creados utilizando recursos de Realidad Virtual (VR), que permiten al visitante manipular los poliedros y compararlos y sus duales. Para la construcción de las salas virtuales se utilizaron transformaciones geométricas de traslación y rotación, utilizando estructuras de jerarquías de páginas HTML, e insertando cada poliedro en su respectiva sala virtual. Los recursos presentados en este trabajo se pueden utilizar en el aula para visualizar poliedros con gafas inmersivas e incluso en Realidad Aumentada (RA) utilizando smartphones y tablets. Otros estudios que se pueden desarrollar con los poliedros modelados en este artículo son áreas, volúmenes, secciones planas y la relación de Euler. Este artículo muestra la posibilidad de crear materiales didácticos con una tecnología sencilla y gratuita que hace un gran aporte a la mejora de la enseñanza de la Geometría, así como de otras áreas que utilizan representaciones gráficas de objetos 3D.

Palabras claves: Realidad Virtual; Visualización gráfica; poliedros de Arquímedes; poliedros de Catalan; Dualidad.

Introduction

The topics presented in some disciplines that involve three-dimensional concepts can be understood, by students, more efficiently by using auxiliary resources. The concrete materials, made with 3D printers, can be used in Mathematics classes [1], Biology [2], or disciplines with content that involve development and spatial skills [3]. The web environments creation or applications for teaching can collaborate in the teaching of Biology [4] and Physics [5, 6] and has been used as an attractive alternative to aid student learning.

Modeling objects using virtual technologies can also help in learning content that involves concepts in three dimensions (3D). Virtual Reality (VR) allows the creation of an immersive and interactive environment with the objects manipulation through controls and immersive goggles [7].

Environments developed in VR can help with the visualization of physical, biological or chemical phenomena [8], simulations of training situations[9], educational games [10], Medicine [11], Nursing [12], construction simulations [13] and other areas related to education [14, 15, 16].

Learning content involving polyhedra in disciplines such as Technical Drawing, Euclidean Geometry or Descriptive Geometry almost always requires auxiliary materials, such as planned polyhedra, printed in 3D, or assembled with alternative materials [17]. Content that involves calculating areas and volumes, visualizing faces and edges, or exploring the properties of solids, can be explored with manipulable materials or modeled in virtual environments.

The virtual environments programmed in VR can complement the use of traditional courseware in teaching polyhedra, as students can interact and visualize solids and their properties in a more effective and meaningful way. VR can help students interact with representations of modeled polyhedra, facilitating visualization and understanding of object properties.

This article presents web resources for creating immersive environments that enable the manipulation of Archimedes and Catalan polyhedra with VR and Augmented Reality (AR) technologies. When viewing polyhedra using AR, links to pages developed in VR are available to visitors. A DNA ribbon was modeled in the virtual environments, establishing connections between the elements of the polyhedra and their respective duals.

On the page programmed in AR, students view and manipulate the polyhedra from different points of view and access the pages programmed in VR to manipulate the representations of solids with mobile devices, computers or they can even immerse themselves in the scene using VR goggles.

Modeling of Archimedean Polyhedra

A convex Archimedean polyhedron is formed by regular polygons and each vertex is the extremity of the same number of edges. There are 13 Archimedean polyhedra, 7 of which are obtained through truncation of Plato's Solids. The vertices of these polyhedra are combinations of two or more different regular polygons [18, 19]. Polyhedra can be represented using the coordinates of their vertices or through the constructions of polygons that represent the solids faces.

In [20] the codes used to represent each Archimedes polyhedron are presented using the symmetries and angles between edges and faces that define these solids. The modeling of Archimedes' polyhedra for creating the virtual rooms in this article is the same as that presented in the work presented in [20]. Figure 1 shows the Snub Cube and Rombicosidodecahedron polyhedra modeled with the VR resources available for HTML programming with scripts and commands of A-Frame [21].

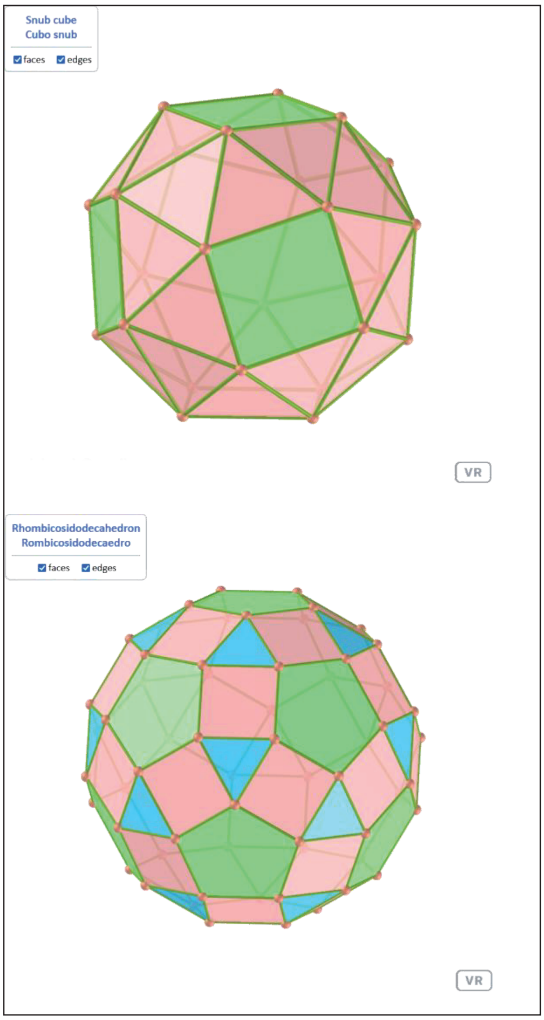

Figure 1: Modeling of Snub Cube and Rombicosidodecahedron polyhedra.

The modeled polyhedra can be accessed individually, and visitors can manipulate and visualize these solids in a variety of ways on any device. In this form of visualization, students can choose to visualize only the faces or the edges of each polyhedron. The captions present the solids names in English and Portuguese. Models of Archimedes' polyhedra, virtual rooms and QR codes for viewing each polyhedron using AR technology are available on the page:

https://paulohscwb.github.io/polyhedra/archimedes/

Duality of Polyhedra

The duality concept in Geometry is associated with a secondary structure of a polyhedron. According to Wenninger [22], this structure is defined by the one-to-one correspondence between the vertices of one polyhedron with the faces of the other polyhedron. The edges between the pairs of vertices of a polyhedron have a one-to-one correspondence with the edges between the pairs of faces of the dual polyhedron.

Consider Archimedes' Rhombicuboctahedron, which has 8 triangular faces, 18 square faces, 48 edges and 24 vertices (Figure 2). The correspondence between each face of this polyhedron with the vertices of the dual determines a solid with 26 vertices and 24 faces called Deltoidal Icositetrahedron. This is one of the Catalan polyhedra [18, 23]. The edges of the dual polyhedron are orthogonal to the edges of the primal polyhedron, and the dual of the dual is always the primal polyhedron.

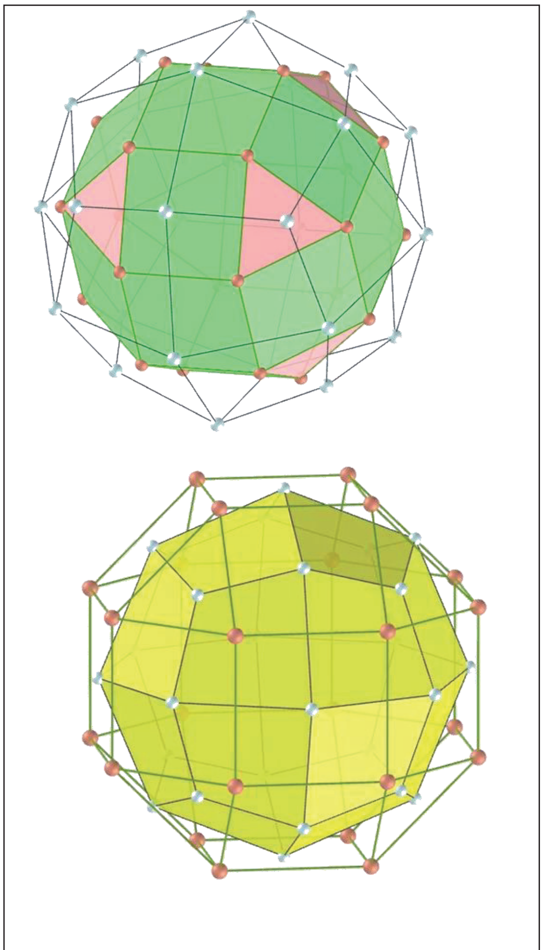

Figure 2: Correspondence between edges, faces and vertices of Rhombicuboctahedron and Deltoidal Icositetrahedron.

The faces of Catalan polyhedra are not regular: they are isosceles triangles, rhombuses, deltoids (quadrilaterals with two disjoint pairs of congruent adjacent sides), symmetric irregular pentagons and acute scalene triangles. Therefore, the modeling of these polyhedra can be done using the vertices coordinates, defining which vertices form each solid's face. This information is inserted into *obj* format files [24], which work in a very intuitive way.

For example, a quadrilateral that has vertices 1, 2, 3, and 4 and a triangle that has vertices 3, 4, and 5 are defined in an *obj* file (Figure 3). Faces are defined with *f* commands, which have sequences of vertices defined in ordered*v* commands with 3D coordinates. Colors and textures can be defined in the materials file with *mtl* extension and the *usemtl* command.

The *obj* files with the edges are created in the same way, using the line commands *l*. The HTML programming codes with the A-Frame resources of each polyhedron are made by inserting the links of the files that contain the polyhedra modeled in *obj* format files.

```
v 0 0 0
v 0 0 1.2
v 0 1.3 1.2
v 0 1.3 0
v 2.5 1.3 0
usemtl Cor1
f 1 2 3 4
usemtl Cor2
f 3 4 5
```

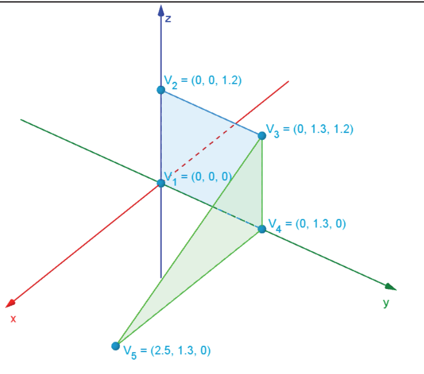

Figure 3: Excerpt from a file code in obj format that defines a quadrilateral and a triangle.

Figure 4 shows the code for modeling a Triakis Tetrahedron in VR. Between lines 10 and 12, the links to the files containing the polyhedron and edges are defined. In lines 16 and 17, the polyhedra modeled in the scene are inserted. Finally, between lines 18 and 25, the *a-sphere* commands are used to represent the 8 vertices of the Triakis Tetrahedron. The other Catalan polyhedra are defined using the same HTML file type.

```
1  <!DOCTYPE html>
2  <html>
3   <head>
4    <script src="https://aframe.io/releases/1.3.0/aframe.min.js"></script>
5   </head>
6   <body>
7   <a-scene>
8   <a-entity camera></a-entity>
9   <a-assets>
10   <a-asset-item id="poliedro" src="catalan/TriakisTetrahedron.obj"></a-asset-item>
11   <a-asset-item id="linhas" src="catalan/TriakisTetrahedronL.obj"></a-asset-item>
12   <a-asset-item id="material" src="catalan/solidos.mtl"></a-asset-item>
13  </a-assets>
14  <a-sky color="aliceblue"></a-sky>
15  <a-entity scale="3,3,3" position="0,0,0">
16   <a-obj-model src="#poliedro" mtl="#material"></a-obj-model>
17   <a-obj-model src="#linhas" mtl="#material"></a-obj-model>
18   <a-sphere position="1.06066,-1.06066,-1.06066" radius="0.05"></a-sphere>
19   <a-sphere position="-1.06066,-1.06066,1.06066" radius="0.05"></a-sphere>
20   <a-sphere position="-1.06066,1.06066,-1.06066" radius="0.05"></a-sphere>
21   <a-sphere position="0.63639,-0.63639,0.6363" radius="0.05"></a-sphere>
22   <a-sphere position="0.63639,0.63639,-0.63639" radius="0.05"></a-sphere>
23   <a-sphere position="-0.63639,0.63639,0.63639" radius="0.05"></a-sphere>
24   <a-sphere position="-0.63639,-0.63639,-0.6363" radius="0.05"></a-sphere>
25   <a-sphere position="1.06066,1.06066,1.06066" radius="0.05"></a-sphere>
26  </a-entity>
27  </a-scene>
28  </body>
29 </html>
```

Figure 4: Code for modeling a Triakis Tetrahedron in VR using A-Frame resources.

Figure 5 presents the modeling of the Catalan polyhedra Pentagonal Icositetrahedron and Deltoidal Hexecontahedron with *obj* files, which represent the duals of the Archimedes Cube Snub and Rhombicosidodecahedron polyhedra shown in Figure 1.

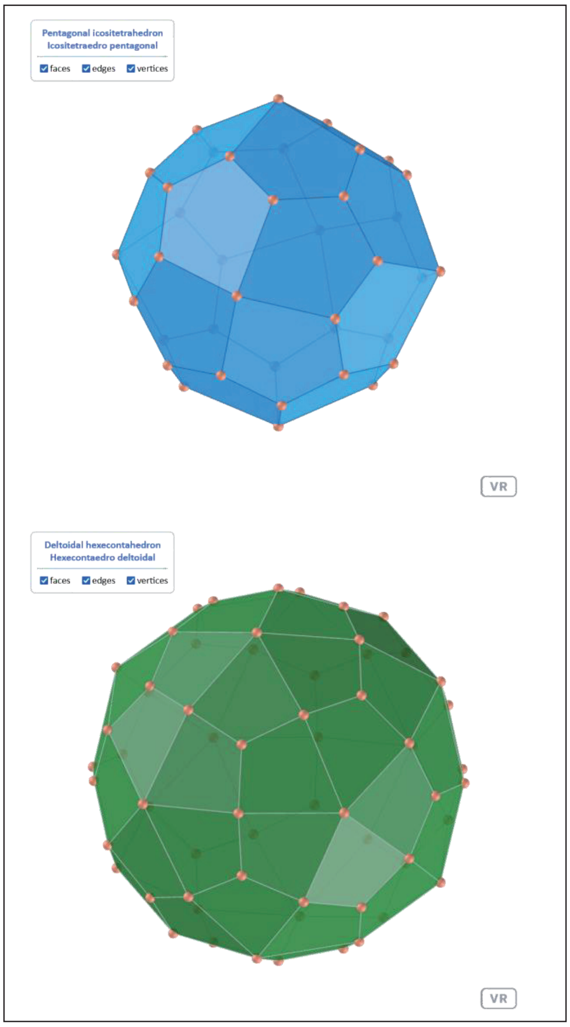

Figure 5: Modeling of the Pentagonal Icositetrahedron and Deltoidal Hexecontahedron

Catalan polyhedra models, virtual rooms and QR codes for viewing each polyhedron using AR technology are available on the page:

https://paulohscwb.github.io/polyhedra/catalan/

Immersive rooms with modeled polyhedra

The modeled polyhedra by Archimedes and Catalan were inserted into VR immersion rooms, which can be accessed using any device. Support tables for the polyhedra, equirectangular background photos and a projection screen with the properties of some solids were inserted into the programmed environments.

The A-Frame's gravity and shadow effects properties were programmed into the virtual rooms, with the aim of improving the feeling of immersion. Figure 6 shows an overview of the environment of a virtual room with Archimedes' solids, without the equirectangular image in the background.

The polyhedra models are supported on the tables, with labels containing the respective names in English and Portuguese of each solid. Figure 7 shows one of the tables with Archimedes' polyhedra, presenting the background image in an equirectangular format [25].

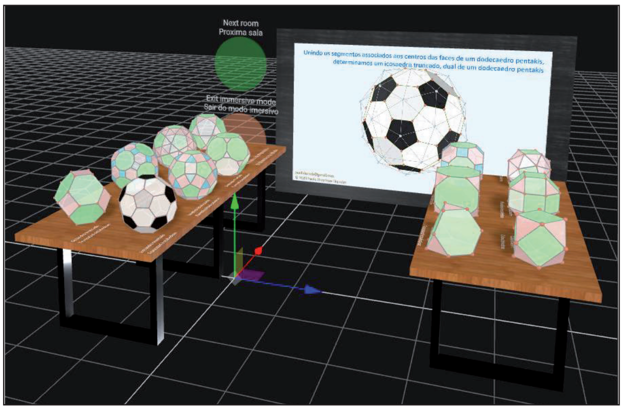

Figure 6: Overview of a virtual room with Archimedes' polyhedra.

Figure 7: Detail of the virtual room with one of the tables containing 7 Archimedes polyhedra.

Using controls from immersive goggles, the click of a mouse or the touch of a smartphone or tablet, the visitor can move the polyhedra, change virtual rooms and leave the immersive environment. Figure 8 shows one of the tables with Catalan polyhedra, with the background image in equirectangular format [26]. Figure 9 shows the use of Virtual Reality goggles manipulation controls in virtual rooms with Archimedes and Catalan polyhedra.

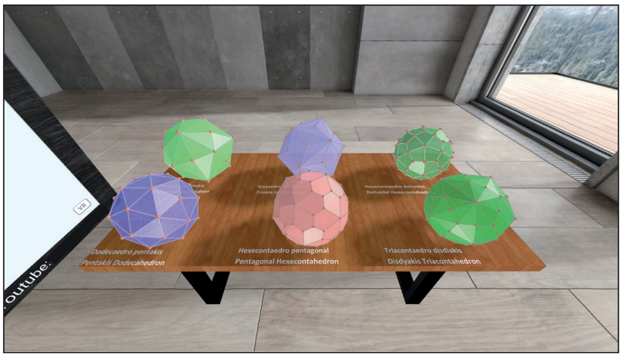

Figure 8: Detail of the virtual room with one of the tables containing 6 Catalan polyhedra.

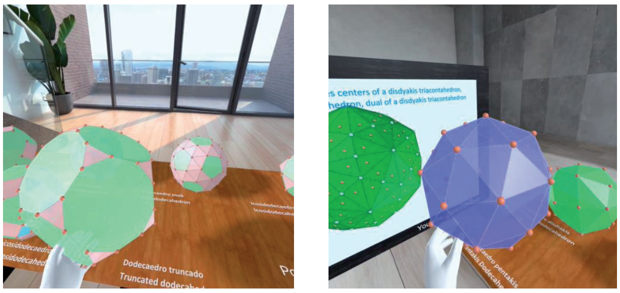

Figure 9: Details of the immersive rooms with polyhedron manipulations using VR goggles controls.

Connections between the polyhedra elements and their duals

The relations between elements of Archimedean polyhedra elements and their respective Catalan duals, can be understood more efficiently when connections are established between these solids. In this article, a DNA ribbon serve as a basis for visualizing the connections between the elements of dual polyhedra.

The construction of a DNA ribbon can be done using two cylindrical helices symmetric about the z axis. Consider the helix radius r = 3, and the first pair with an Archimedes polyhedron at point A and its respective Catalan dual in the symmetric position A' (Figure 10).

Consider that each pair of polyhedra is located with a rotation of 45º, about the z axis in relation to the previous pair. Therefore, the second pair of polyhedra has the same x and y coordinates, which can be found using the Pythagorean theorem in the right triangle with hypotenuse r and equal legs:

$$x^2 + x^2 = r^2 \Rightarrow 2x^2 = 3^2 \Rightarrow x^2 = 4,5 \Rightarrow x \cong 2,12.$$

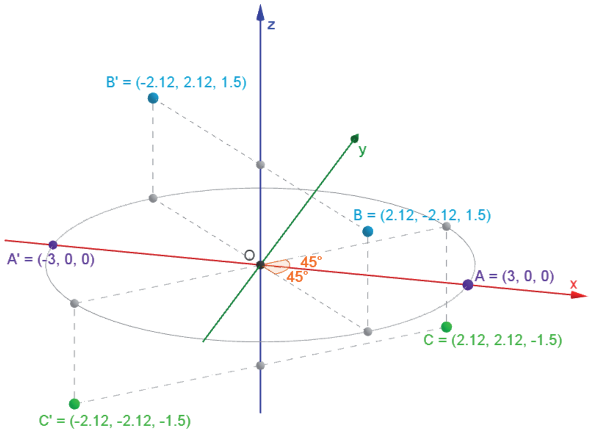

Figure 10: Construction of a DNA ribbon with two cylindrical hélices.

Using the vertical distance z = 1.5, we have the second pair of duals at points B and B'. The third pair of polyhedra has the coordinate z = -1.5 and is located in the symmetric positions of B and B' about the x axis. The other 10 pairs of polyhedra can be positioned following the same reasoning. To improve the visualization of this DNA ribbon in the virtual room, a rotation of the helices with an angle of 90º around the y axis is considered.

Modeling the DNA ribbon allows visitors to visualize the overlapping vertices of a polyhedron, which determine the centers of the faces of their respective dual. Figure 11 shows the modeling of this DNA ribbon with two cylindrical helices.

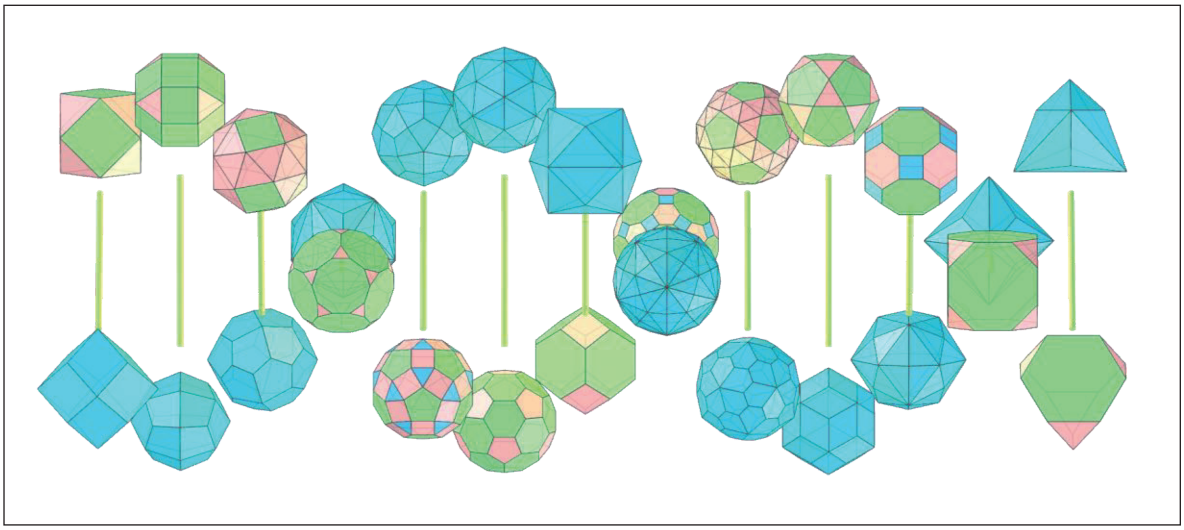

Figure 11: Modeling of the DNA ribbon with the connections between the Archimedes and Catalan dual polyhedra.

In the same way presented in the polyhedra viewed individually, visitors can choose to view only the edges or only the faces of the polyhedra, in addition to manipulating the visualization of the DNA ribbon. The spheres that represent the vertices of the polyhedra were omitted in the cylindrical helices, to make the processing of the graphical representation faster.

An immersive virtual room was created to visualize these connections between the dual polyhedra of Archimedes and Catalan. Figure 12 shows the detail with the table, the DNA ribbon and the equirectangular image [27] in the created virtual environment.

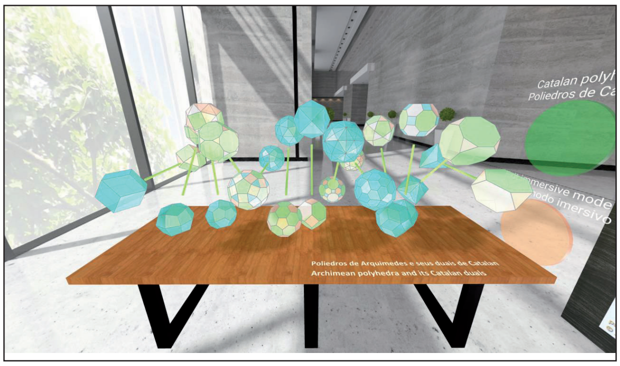

Figure 12: Virtual room with the DNA ribbon showing the connections between the Archimedes and Catalan dual polyhedra.

In addition to virtual rooms and individual polyhedron visualization capabilities, Augmented Reality technology can be used to visualize each polyhedron and the DNA ribbon with the connections between the dual polyhedra.

The programming codes for using AR are practically the same as those shown in Figure 4, with the indication of the QR code marker for each polyhedron model [20]. Figure 13 shows using the AR feature to visualize the DNA ribbon presented in this section.

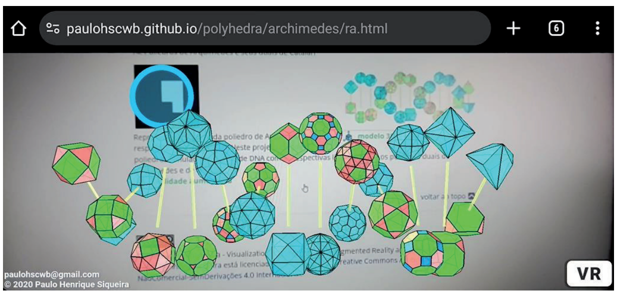

Figure 13: Visualization of the DNA ribbon using the Augmented Reality feature.

Conclusions

This article shows the use of web pages to visualize Archimedes and Catalan polyhedra using Virtual Reality and Augmented Reality technologies. By viewing printed markers, students can view solids in AR on any device with a webcam and internet access, with links to the VR views.

The polyhedra modeling and the virtual rooms creation showed in this article use the hierarchical structures of web page programming with A-Frame scripts, facilitating the insertion of several polyhedra on the same page. The result shows that it is a useful tool for use in the classroom, as it allows students to view and manipulate graphic representations of polyhedra on their devices or to use VR goggles for complete immersion in virtual rooms.

The programmed environments can be explored in Geometry classes, helping to understand the polyhedra elements and properties or topics such as calculating areas and volumes, Euler's relation, plane sections or simply visualizing each modeled solid. The DNA ribbon modeled in this article allows visualization of the connections existing between the elements of Archimedean polyhedra and their respective Catalan duals.

All elements of the polyhedra can be viewed in VR and AR and visitors can move the camera around the scene to find the best views of the solids with A-frame developed tools to orbit the camera around the objects.

The web page programming tools shown in this article are simple and intuitive, and can be used in classrooms with printed materials or with immersive goggles. Students access the page programmed in AR, view the solids with their respective printed markers and can interact with the polyhedra programmed in VR. In this way, students can explore the geometric concepts involved in a more efficient and dynamic way.

Some advantages of creating AR and VR environments as web pages for use in the classroom are practicality, low cost, excellent performance, simple programming and operation on all types of devices with internet access. Another advantage of this tool is the almost immediate loading of pages, as they are programmed in HTML with references from VR libraries developed with the JavaScript language.

The visitors do not need to download applications and several markers can be used on the same HTML page, which allows the creation of teaching materials with different themes programmed in AR and VR. This tool can be used in other disciplines, such as Statistics, Biology, Differential and Integral Calculus, Physics, Geography, Chemistry, Engineering and other areas that use 3D graphical representations.

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