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Probabilistic Weibull reliability of a shaft design subjected to bending and torsion stress

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Abstract
The circular shaft serves as the axis of rotation for the components. It is subjected to flexion and tearing, indicating that fatigue is the mode of failure. The range of stresses resulting from the mean and alternating loads determines the occurrence of fatigue failure. The deterministic fatigue analysis, calculated using the stress average obtained from SN curves, can only represent the mean life. This is because the stress range is not a single number, and therefore it cannot provide the reliability level for the stress. The study employs the Weibull distribution to estimate loads and parameters for a probabilistic shaft design under bending and torsion. The minimum strength is assessed using corresponding stress analysis to determine the reliability index for the designed shaft.

Keywords: probabilistic design; Weibull distribution; reliability; stress-strength.

Confiabilidad probabilística Weibull en el diseño de un eje sometido a esfuerzos de flexión y torsión

Resumen
El eje circular sirve de eje de rotación de los componentes. Está sometido a flexión y desgarro, lo que indica que el modo de fallo es la fatiga. El rango de tensiones resultante de las cargas medias y alternas determina la aparición del fallo por fatiga. El análisis determinista de la fatiga, calculado a partir de la tensión media obtenida de las curvas SN, sólo puede representar la vida media. Esto se debe a que el rango de tensiones no es un número único y, por lo tanto, no puede proporcionar el nivel de fiabilidad de la tensión. El estudio emplea la distribución de Weibull para estimar las cargas y los parámetros de un diseño probabilístico de un eje sometido a flexión y torsión. La resistencia mínima se evalúa mediante el correspondiente análisis de tensiones para determinar el índice de fiabilidad del eje diseñado.

Palabras clave: diseño probabilístico; distribución de Weibull; fiabilidad; tensión-resistencia.

1 Introduction

A shaft is a spinning, often circular cross-section that is used in a variety of applications to convey power and rotational motion [1]. The main goal of a shaft design is to make them safe and to continuously enhance this goal, engineers focus on increasing their dependability and efficiency [2]. The shaft design is considered to be a typical mechanical engineering issue in which the effect of the uncertainty, i.e., environment or usage circumstances and material strength qualities, is to try to reduce this uncertainty by applying safety factors [3].

In general, a shaft failure results from bending stress perpendicular to the shaft axis, and the shaft fracture resulting from torsional stress most frequently disposed at a 45° angle to the shaft axis [4], i.e. the stresses acting on the shaft is the σ_X, σ_y and τ_{xy} [5]. Bending and torsion imply that the failure mode of shafts is by fatigue. Then, when shafts are exposed to alternating strains over extended periods, fatigue develops.

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To estimate shaft fatigue time, statistical models are crucial since the fatigue behavior of shafts under cyclic stress is random [6]. The Weibull distribution, on the other hand, is the distribution that, using the weakest link concept, could be one of the best predictors of fatigue life [7]. The Weibull distribution is a natural distribution for forecasting lifespan as a result. Moreover, as the Weibull model for fatigue estimate effectively predicts life, it should be utilized to forecast the fatigue SN reliability R(t) [8]. The Weibull distribution has the property of lognormal distribution. This shows that the shape parameters have not changed even though the position and scale parameters have changed [9]. i.e. $X_i \sim W(\beta, \eta, \gamma) \rightarrow \min(X_1, X_2, \ldots X_m) \sim W(\beta, \eta, \gamma_m^{-1/\beta})$. In that case, at least one data set is also Weibull distributed. Based on these facts, we present a probabilistic Weibull model analysis of shaft designs under bending and torsional loading to predict the lifetime SN number [10]. The Weibull distribution withstand stress analysis is then used to calculate the reliability R(t) of the shaft under variable stress [11].

2 Fatigue

Fatigue occurs when a material is subjected to cyclic loads below the threshold at which failure can occur [12]. The cumulative and irreversible nature of damage caused by fatigue processes and the fact that failures often occur suddenly make it difficult to detect incremental changes in material behavior over time [13]. The material behavior under this type of load is different from that under static load materials that can withstand large static loads and can fail at lower loads when repeated many times. Fatigue failure is caused by repeated loading. It is reported that 90% of all devastation is caused by it. On the other hand, fatigue is affected by three basic factors [14].

The three most important factors that affect fatigue are: many cycles, a wide range of applied loads, and high maximum stresses (bending and torsion). There is no universal theory to explain how materials behave under fatigue and cyclic loading, making material fatigue and damaging formation a complex process [15]. By measuring the fatigue life of a component under a given load cycle sequence, fatigue analysis aims to predict fatigue life in actual operation. Fatigue prediction techniques often fall into two categories [16]. The first focuses on predicting crack initiation using a combination of damage as a function of component stress [17]. The second approach is based on the mechanism of continuous fatigue life by calculating damage cycles [18]. According to the deterministic point of view, there are three main ways used in design to identify a cyclically loaded component that would fail over time [19]. In general, the life forecast of sensitive to fatigue parts is based on the safe-life approach [20]. These three methods are linear elastic fracture mechanics, stress-life method, and strain-life method. The purpose of these techniques is to predict N, cycles to failure at a given stress level [21]. This error cycle is categorized as follows: $1\ N \leq 10^3$ the cycles as low-cycle fatigue, whereas high-cycle fatigue is $N > 10^3$ cycles [22].

This deterministic method of fatigue calculations is used by most contemporary fatigue standards (ASTM E606/E606M). Due to the use of deterministic algorithms, fatigue estimation always produces the same result given an input [23]. The random nature of fatigue and deterministic fatigue techniques that use characteristic values and high safety factors to explain are fraught with uncertainty [24]. As the issue of fatigue assessment is so complicated and has not yet been properly resolved, probabilistic approaches are required for accurate damage prediction, constructive design, and fatigue risk analysis [25]. Therefore, it is essential to evaluate and predict the fatigue life of elements using mathematical and probabilistic models [1]. Therefore, according to the weakest-link principle, the survival probability of a uniformly loaded volume $V(0)$ is given by all probabilities $m = \frac{V_0}{dV}$ volume elements survive [26].

$$Q_s = 1 - p = \prod_{i=1}^{m}(1 - dP) = (1 - \lambda(s)dV)^m \quad (1)$$

And introducing $V_0 = mdV$ in eq. (2)

$$Q_s = \left(1 - \frac{\lambda(S)V_0}{m}\right)^{\frac{m}{\lambda(s)dV}\lambda(s)dV} \quad (2)$$

As both m and dV rise indefinitely, Eq. (3) transforms into an exponential function, as shown by

$$Q_s = e^{-(\lambda(s)V_0)} \quad (3)$$

2.1 *Fatigue Failure Mode for Shafts*

A rotating part that conveys power or motion is called a shaft. Typically, its cross-section is round. It provides an axis of rotation or oscillation for a variety of parts, such as cranks, pulleys, and gears. Shafts are designed to transmit motion, so they must be subjected to bending, axial rotation, and torsional loads to transmit motion [27]. Torque is transferred from input to output by torsional stress. The shaft component transmits torque xy. At stable operating levels, the torque is often fairly constant and the outer surface experiences a maximum shear stress due to torsion [28]. A given load element alternates between compression and tension with each rotation of the shaft, so that a constant bending moment applied to the rotating shaft produces a reversible moment in rotation. For deterministic methods of fatigue analysis, the axial stress is lower than the bending stress [29]. Thus, a shaft's failure mechanism is due to fatigue when it is subject to alternating loads over time. The cumulative damage from each cycle approaches a tipping point, leading to failure [30]. As a result, unlike most failure types, fatigue causes failure at loads much below the maximum value. As a result, fatigue is probabilistic rather than deterministic. The ASTM acknowledges that the Weibull distribution is the function that may estimate the life of fatigue [31]. Hence, the mean stress and the alternating stress may be determined using Mohr circle based on the bending stress x, the axial stress y, and the torsion stress xy data. As a result, according to, the values of the Weibull distribution may be approximated using the principal stresses σ_1 and σ_2 [32].

The foundation of the Mohr circle and the formulation of the maximum stress estimation are described in the next section.

2.2 Generalities of Mohr Circle

The Mohr circle, which was found by Culmann (1866) and explored in detail by Mohr (1882), is a graphical representation of the aforementioned stress connections [33].

$$Cos^2\theta = \frac{1+\cos 2\theta}{2} \tag{4}$$

$$Sen^2\theta = \frac{1-\cos 2\theta}{2} \tag{5}$$

Think about the stressed-out plane condition, where only σ_x, σ_y and τ_{xy} act. Assume that we are aware of these coordinate stresses. Finding the stress state in rotating configurations is the goal in this situation. i.e., determine the major stressors σ_1, σ_2 [34]. Let be θ the angle where the main stresses are operating between the original coordinate system and the rotated system e.g. σ_1, σ_2 and τ_{max} [35]. Based on the normal stresses σ_x, σ_y and τ_{xy}, the major stressors are predicted [36].

The trigonometric double angle is used to mathematically justify the Mohr circle method. i.e.

$$\sigma_x{'} = \sigma_x \cos^2\theta + \sigma_y sen^2\theta + 2\tau_{xy} sin\theta cos\theta \, 0$$

$$\sigma_y{'} = \sigma_x \, \mathrm{sen}^2\theta + \sigma_y \cos^2\theta - \tau_{xy} sin\theta cos\theta$$

$$\tau_{xy} = (\sigma_y - \sigma_x) sin\theta \, cos\,\theta + \tau_{xy}(cos^2\theta - sen^2\theta) \tag{6}$$

The Mohr circle is a tool that makes it easier to see how rotating the axis affects the stresses and the second-rank tensor [37]. A rotation does not occur for a $xy = 0$, which implies that the stresses produced in the normal plane are acting equally in the normal plane [38]. Nevertheless, as the main stresses are occurring on the rotational plane if $xy \neq 0$, then the Weibull distribution should be utilized rather than the normal distribution [39].

The necessary equations for the estimation of the main stresses as well as the angle of action are presented below.

Maximum stress

$$\sigma_1 = \frac{\sigma_X + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_X + \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{7}$$

Minimum stress

$$\sigma_2 = \frac{\sigma_X + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_X + \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{8}$$

Mohr circle center

$$\mu = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2} \tag{9}$$

Maximum shear stress (ratio)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_X + \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2} \tag{10}$$

Principal angle

$$\theta_p = \frac{\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)}{2} \tag{11}$$

Shear angle θ_s

$$\theta_s = \frac{\tan^{-1}\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)}{2} \tag{12}$$

Thus, depending on the strength of the material, the maximum shear stress theory and the deformation energy theory provide a safe state S y and the safety factor S f.

$$DE = \sqrt{\sigma_x^2 - \sigma_x\sigma_Y + \sigma_y^2} < S_y/S_f \tag{13}$$

And the maximum stress is.

$$\sigma_{max} = S_y/S_f \tag{14}$$

The parameters of the Weibull distribution can be obtained entirely from the primary stresses using the Mohr circle-Weibull relation [40]. However, this fatigue failure prediction method does not consider stress as a random variable [41].

3 Weibull Distribution

In this section, the Weibull model used in fatigue analysis is presented along with the theoretical basis and characterization definitions. Due to its versatility, the Weibull distribution is often used in reliability and service life studies [42]. Depending on the value of the Weibull parameter, the Weibull distribution can be used to describe different life behaviors. The values of the shape and scale parameters have a strong influence on the distribution characteristics [43]. The probability density function of the Weibull distribution is given by.

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} exp^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{15}$$

The Weibull distribution cumulative and reliability functions are

$$F(t) = 1 - exp^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{16}$$

$$R(t) = exp^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{17}$$

The linear form of Eq. (17) is given by.

$$\ln\left(-\ln\left(1 - F(t_i)\right)\right) = -\beta\ln\eta + \beta\ln t_i =$$

$$Y_i = \beta[-\ln(\eta) + \ln(t_i)] \tag{18}$$

Or equivalently is given by

$$Y_i = b_0 + \beta X_i \tag{19}$$

Where $Y_i = \ln(-\ln(1 - F(t_i))) = -\beta \ln\ (\eta)$, and $x_i = \ln(t_i)$, $F(t_i)$. The median rank is given by.

$$F(t_i) = \frac{1 - 0.3}{n + 0.4} \tag{20}$$

The sample n is determined by the reliability R(t).

$$n = \frac{-1}{\ln(R(t))} \tag{21}$$

The mean μ_y and the standard deviation σ_y of the vector Y are

$$\mu_y = \sum_{i=1}^{n} \frac{Y_i}{n} \tag{22}$$

$$\sigma_y = \sqrt{\sum_{i=1}^{n} \frac{Y_i}{n}\left(\frac{Y_i - \mu_y}{n - 1}\right)^2} \tag{23}$$

4 Weibull Distribution Properties

Concerning to location and scale transformation the Weibull distribution is stable, which is the first of two crucial Weibull distribution characteristics for fatigue lifetime prediction.

$$X \sim W(\beta, \eta, \gamma) \leftrightarrow \frac{X - a}{b} \sim W \frac{\gamma - a}{b}, \frac{\eta}{b}, \beta \tag{24}$$

This implies that while the shape parameters are kept unchanged, the transformation constants a and b are given new scale and position parameters in relation to their former values. In other words, assuming the scale and position parameters stay the same, the converted variable is likewise a Weibull random variable. Their resilience to the most straightforward random variable manipulations, $X_i = 12, \ldots, m$, demonstrates their independence and uniform distribution.

$$X_i \sim W(\beta, \eta, \gamma) \rightarrow \min(X_1, X_2, \ldots, X_m) \sim W(\gamma, \eta_m^{-\frac{1}{\beta}}, \beta) \tag{25}$$

This shows that a random variable's distribution is the same as a collection of independently distributed random variables if it is a Weibull variable. There is a minimum with a collection of identical random values X_i through X_m that are independently distributed and have a common $Cdf\ \ F(x)$.

$$F_{min}(x) = 1 - [1 - F(x)]^m \tag{26}$$

$$\begin{aligned} F_{min}(x) &= 1 - \left\{1 - \left(1 - exp^{\left[-\left(\frac{x-\lambda}{\gamma}\right)^{\beta}\right]}\right)\right\}^m \\ &= 1 - exp^{\left[-\left(\frac{x-\lambda}{\gamma m^{-\frac{1}{\beta}}}\right)^{\beta}\right]} \end{aligned} \tag{27}$$

5 Fundaments of the Fatigue Weibull Model

The Weibull model is used for fatigue life of constant cycle components based on the following principles:

According to the weakest link theory, the fatigue of a longitudinal member is equal to the minimum fatigue of its components.

$$F_{min}(x) = F_{nl}(x) = 1 - [1 - F_l(x)]^n \tag{28}$$

The second fundament is stability, which means that the selected distribution family must hold for different lengths, it is to say, a parametric family of $cdfs$ is used to represent the cdf for fatigue of a longitudinal element of length l, according to Eq. (18) the element must be

$$\begin{aligned} &F\big(x; \beta(nl), \eta(nl), \gamma(nl)\big) = \\ &1 - [1 - (29)\, F\left(\big(x; \beta(nl), \eta(nl), \gamma(nl)\big)\right]^n \end{aligned} \tag{29}$$

The Weibull distribution was the only distribution that could satisfy this functional equation.

A third concept, known as limit behavior, is that in extreme cases where the component size of an element tends to zero or the number of components tends to infinity, the distribution function is asymptotic and the distribution in the independent case are the Weibull and Gumbel distributions.

$$\begin{aligned} \text{Lim}\, f(x) &= \lim\, [\beta, \eta, \gamma(1 \\ &\quad - exp^{\eta x})^{\beta-1} exp\{\beta, \eta \\ &\quad - \gamma(exp^{\eta x} - 1)^{\beta}] \\ &= \beta, \eta, \gamma * 1 \end{aligned} \tag{30}$$

$$\begin{aligned} \text{Lim}\, f(x) &= \lim_{0\to\infty} [\beta, \eta, \gamma(1 - e^{\eta x})^{\beta-1} e\{\beta, \eta \\ &\quad - \gamma(e^{\eta x} - 1)^{\beta}] \\ &= \beta, \eta, \gamma * 1^{\beta-1} * 0 \end{aligned} \tag{31}$$

The approach to fully estimate the Weibull distribution parameters directly from the primary stresses [39].

6 Estimation of Weibull Parameters by Method

Step 1:
Choose reliability level.

Step 2:

Using the $R(t)$ indices from step 1, compute the corresponding n values in equation (20). As a result, use the F_{ti} values from equation (20) and the mean and standard values. to calculate the appropriate element Y_i. Deviation from equations (22) and (23).

Step 3:

Perform a stress analysis to determine the maximum and minimum primary stress and calculate the ratio. $\sigma_1;\sigma_2$.

Step 4.
By utilizing the Weibull distribution's initial shape parameter value, $\ln(t_{io})=\frac{Y_i}{\beta}$ it is

$$\beta=\frac{-4\mu_y}{\ln\left(\frac{\sigma_1}{\sigma_2}\right)} \tag{32}$$

Estimate standardized logarithms ($\ln(t_{oi})$) elements, and calculate the initial μ_{og}, μ_o and the $g(x)$ values are given by.

$$\mu_{og}=\sum_{i=1}^{n}\frac{\ln\,(t_{oi})}{n}=\frac{\mu_y}{\beta} \tag{33}$$

$$\mu_o=\sum_{i=1}^{n}\frac{t_{oi}}{n} \tag{34}$$

$$g(x)=\sum_{i=1}^{n}\frac{Y_i}{n}=\exp(\mu_{og}) \tag{35}$$

Step 5.
Determine the corresponding β value given by

$$\beta=\frac{-4\mu_y}{\ln\left(\frac{\sigma_1}{\sigma_2}\right)*(0.9947)} \tag{36}$$

Step 6.
With the principal stresses σ_1 and σ_2, estimate the mean μ. Then, based on the Mohr circle estimation μ and the estimated t_{oi} mean value μ_o of step 5, determine the corresponding scale η parameter of the Weibull distribution as

$$\eta=\frac{\mu}{\mu_o} \tag{37}$$

The β value of step 5 and the η value of step 6 corresponds to the Weibull parameters that represents the observed stresses σ_1 and σ_2 values.

Finally, given that stress behavior is typically viewed as falling below a log-normal distribution provided by

$$f(t)=\frac{1}{t\sigma_g\sqrt{2\pi}}\exp\left\{-\left(\frac{ln\,(t_i-\mu_g}{2\sigma_g}\right)^2\right\} \tag{38}$$

With log-mean μ_g and log-standard deviation σ_g parameters, thereby because both μ_g and σ_g [44] the log-normal distribution's parameter connection in terms of β and η is given by

$$\beta=\frac{\sigma_y}{\sigma_g} \tag{39}$$

$$\eta=\exp\left\{\mu_g-\frac{\mu_y}{\beta}\right\} \tag{40}$$

Where μ_y and σ_y from Equation (22) mean and standard deviation, respectively (23). Then the negative μ_g and the standard deviation is given by

$$\sigma_g=\frac{\sigma_y}{\beta} \tag{41}$$

$$\mu_g=\ln(\eta)+\frac{\mu_y}{\beta} \tag{42}$$

In the next part, the approach is now applied, and the results are compared to those of the deterministic method.

7 Weibull Shaft Lifetime Application

The used data to run the numerical application are an alternator rotor is supported by the shaft in Fig. 1. It is constructed from AISI 4340 steel that has been heating treated, ground, and given a Brinell hardness range of 323–370. (R, 35 to 40). The shaft will experience an axial force from the rotor of up to $130\ MPa$ and a maximum bending stress of $607\ MPa$. The predicted dependability is $R(t)=95\%$, and the shaft will rotate at a speed of $12000\ rpm$. Tensile yield, the properties of the material, correspond to a material with a 277 Brinell hardness, Sy, and ultimate strength, Sut, with $Sy=113Mpa$ and $Sut=128Mp$, respectively (see Figs. 1 and 2) [45].

This suggests that if the system is in equilibrium, the total sum of all forces and moments must equal zero.

8 Parameters of the Design Shaft

Step 1. The desired reliability index according to the data is 95%, however, to have an integer n value the reliability level to use will be equal to $R(t)=0.9535$.

Step 2. By using Eq. (31) the n value is estimated as follows:

$$n=\frac{-1}{\ln(0.9535)}=21$$

Then, using Eq. (31) according to the n value estimated, the corresponding $F(ti)$ elements, and Yi elements, in the same way, the μ_y and σ_y values are given in Table 1.

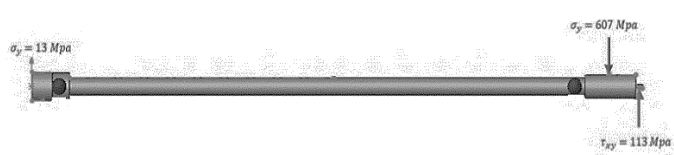

Figure 1. Shaft
Source: The authors

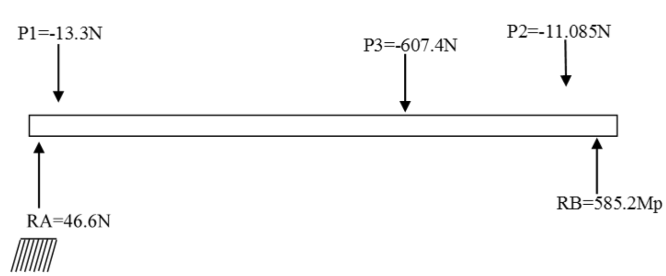

Figure 2. Free-Body diagram
Source: The authors

Table 1.
Weibull $\beta = 3.710802$ and $\eta = 409.156$
Example of a table.

n	$F(t_i)$	Y_i	$\ln(t_{oi})$	t_{oi}	n	$F(t_i)$	Y_i		$\ln(t_{oi})$		t_{oi}
1	0.03271	-3.403483	-1.7148667	0.1799877	12	0.546729	-0.234122		-0.117964		0.888728
2	0.079439	-2.491662	-1.2554397	0.2849505	13	0.593458	-0.105285		-0.0530486		0.948334
3	0.126168	-2.003463	-1.0094577	0.3644166	14	0.640187	0.021928		0.0110488		1.01111
4	0.172897	-1.661646	-0.8372308	0.4329077	15	0.686916	0.149526		0.0753395		1.0782502
5	0.219626	-1.394398	-0.7025764	0.4953075	16	0.733645	0.279845		0.1410017		1.1514266
6	0.266355	-1.172054	-0.5905467	0.5540243	17	0.780374	0.415962		0.2095851		1.2331664
7	0.313084	-0.979381	-0.4934674	0.6105058	18	0.827103	0.562502		0.2834202		1.3276629
8	0.359813	-0.807447	-0.4068375	0.6657524	19	0.873832	0.727616		0.3666139		1.4428407
9	0.406542	-0.650492	-0.3277546	0.7205398	20	0.920561	0.929311		0.4682391		1.5971792
10	0.453271	-0.504509	-0.2542	0.7755367	21	0.96729	1.22966		0.6195719		1.8581324
11	0.5	-0.366513	-0.1846699	0.8313787		μ_y=	-0.545624	μ_{oi}=	-0.274916	μ_o=	0.878673
						σ_y=	1.1751169	σ_{oi}=	0.5920901	σ_o=	0.4454875

Source: The authors

Determinant of the stresses is given by.

$$Det = 78910 - 12769 = 66141$$

corresponding geometric mean is

$$g(x) = \sqrt{66141} = 257.178926\ Mpa.$$

the arithmetic mean is given by

$$\mu = \frac{607 + 130}{2} = 368.5\ Mpa.$$

The eigenvalues are given by $\lambda = \mu \pm \sqrt{\mu^2 - g(x)^2}$

$$\sigma_1 = \mu + \sqrt{368.5Mpa^2 - 257.1789Mpa^2} = 345.6443Mpa$$

$$\sigma_2 = \mu - \sqrt{368.5Mpa^2 - 257.1789Mpa^2} = 191.3557Mpa.$$

The angle acting on this system is given by Eq. (25), this is equivalent to

$$\theta = \tan^{-1}\left(\frac{257.178926Mpa}{345.6443Mpa}\right) = 36.6513352$$

In such a way the principal stresses σ_1 and σ_2 are equal to *345.6443Mpa, and 191.3557Mpa,* respectively.

Determine the value of β of the Weibull distribution as

$$\beta = \frac{-4(-0.54562)}{\ln\left(\frac{345.6443Mpa}{191.3557Mpa}\right) * (0.9947)} = 3.71080171$$

The scale value η of the Weibull distribution is determined as

$$\eta = \frac{368.5Mpa}{0.900634} = 409.1562166$$

The parameter that represents the Weibull distribution family is given by $W \sim (\beta = 3.71080717, \eta = 409.1562166)$. Then, these Weibull parameters are the stress distribution parameters (see Table 1).

9 Estimation of strength parameters and its corresponding reliability

The basis of the investigation is the initial stress intensity ratio from the Weibull analysis. This gives the minimum material strength and maximum principal stress at which failure occurs. After that, the procedure will be explained.

Step 1.
Determine the principal stresses.
In this case the principal stresses are $\sigma_1 = 345.6443Mpa.$ and $\sigma_2 = 191.3557Mpa.$

Step 2.
Used the estimated Weibull parameters.

$$\eta = 409.1562166 \quad \text{and} \quad \beta = 3.71080171$$

Step 3.
Estimate the value of g_x

$$g_x = \sqrt{345.6443 * 191.3557} = 257.17894$$

Step 4.
Select the reliability level and using the corresponding β value estimate the values of t_{oi}.
If the desired reliability is 95% and $\beta = 3.71080171$ then.

$$t_{oi} = (-\ln(R(t))^{\frac{1}{\beta}} \tag{43}$$

$$t_{oi} = (-\ln(0.95)^{\frac{1}{3.71}} = 0.44906317$$

Step 5.
Estimate the minimum strength.

$$Strength_{min} = \eta * t_{oi} \tag{44}$$

$$Strength_{min} = 409 * 0.4490 = 183.7368904$$

Step 6.
Estimate the minimum strength average.

$$Strength_{min} = \mu_{min} - \sigma_{min} \quad (45)$$

$$\mu_{min} = Strength_{min}/0.90 \quad (46)$$

$$\mu_{min} = \frac{183.7368904}{0.90} = 204.1521004$$

$$Strength_{min} = 409 * 0.4490 = 183.7368904$$

Step 7.
Determine t_o

$$t_o = \sqrt{\sigma_2/\sigma_1} \quad (47)$$

$$t_o = \sqrt{191.3557/345.644} = 0.744056972$$

Step 8.
Estimate $Strength_{min}$ with

$$Strength_{min} = 1/\eta^{1/\beta} \quad (48)$$

$$Strength_{min} = 1/409.156216^{1/3.71} = 0.19769094$$

Step 9.
Determine the scale parameter as

$$\eta = \frac{Strength_{min}}{((-\ln(R(t))^{\frac{1}{\beta}})} \quad (49)$$

$$\eta = \frac{183.7368904}{((-\ln(0.95)^{\frac{1}{3.71}})} = 409.1560003$$

Step 10.
Estimate the corresponding design reliability using equation 17.

$$Strength_{min} = \eta * t_{oi} \text{ and } strength_{max} = \eta/t_{oi} \quad (50)$$

$$Strength_{min} = 409.1562166 * 0.744056972 = 304.4351375$$

$$R(t) = exp^{-\left(\frac{409.156}{304.435}\right)^{3.71}} = 0.949944802$$

As can be seen, the reliability of the design for the diameter and the used material corresponds to 95% and the estimated Weibull family for both stress and resistance represents the fatigue behavior completely.

10 Conclusion

By predicting the primary stresses and the lowest and average loads, the probabilistic design for the analysis of the chosen element can be estimated, in this case, an axis, and first, evaluate the validity of the design for the stress and load. It is also feasible to identify the associated loads for the resistance and the accompanying Weibull parameters using the estimated Weibull distribution parameters. This allows for the calculation of the dependability of the resistance, which accurately depicts the resistance distribution. It is feasible to construct appropriate decision-making for the type of stress and resistance by determining the related distributions, as shown in this study.

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