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Power Losses Reduction in Three-Phase Unbalanced Distribution Networks Using a Convex Optimization Model in the Complex Domain

Reducción de pérdidas de potencia en sistemas de distribución trifásicos desbalanceados usando un modelo de optimización convexa en el dominio complejo

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Abstract

This article presents a solution methodology to minimize power losses in three-phase unbalanced distribution networks. This approach involved an efficient complex-domain model that is categorized under mixed-integer convex optimization. The methodology employed consisted of efficient load rotation at each constant power node via a three-phase rotation matrix that allows defining each load connection to minimize the expected power imbalance at the terminals of the substation, as well as the total grid power losses, and improve voltage profile performance at each system phase. The load imbalance, expressed as a percentage, can be defined as a function of the active, reactive, or apparent power. In addition, considering the complex-domain representation of three-phase electrical networks under steady-state conditions, a mixed-integer convex model was formulated to reduce the power imbalances. With the purpose of determining the initial and final power losses of these distribution systems, the successive approximations method was employed to address the three-phase power flow problem. As a result, numerical validations in the IEEE 25-bus system and a 35-node three-phase feeder showed that the final active power losses vary depending on the objective function analyzed. Therefore, for the test feeders studied, it is necessary to evaluate each objective function, with the aim of finding the one that yields the best numerical results. Power losses reductions of about 3.8056 % and 6.8652 % were obtained for both test feeders via the proposed optimization methodology. All numerical validations were performed in the Julia programming environment, using the JuMP optimization tool and the HiGHS solver.

Keywords

Power losses reduction, mixed-integer convex formulation, power imbalances, three-phase power flow solution, numerical analysis, complex domain optimization model.

Resumen

En este artículo se presenta una metodología para la minimización de las pérdidas de potencia activa en sistemas de distribución trifásicos desbalanceados. Este enfoque utilizó un modelo eficiente en el dominio complejo que pertenece a la categoría de optimización convexa de enteros mixtos. La metodología empleada consistió en la rotación de las cargas en cada nodo de potencia constante a través de una matriz trifásica de rotación que permitió definir cada conexión de carga. Lo anterior con el objetivo de minimizar los desbalances de potencia en los terminales de la subestación y las pérdidas de potencia totales, y mejorar los perfiles de tensión en cada una de las fases del sistema. El desbalance de cargas, expresado como un porcentaje, puede definirse en función de las potencia activa, reactiva o aparente. Además, se formuló un modelo entero mixto convexo con el propósito de minimizar los desbalances de potencia, considerando la representación en el dominio complejo de las redes eléctricas trifásicas en condiciones estacionarias. En aras de determinar las pérdidas de potencia iniciales y finales de estas redes, se empleó el método de aproximaciones sucesivas para resolver el problema de flujo de potencia trifásico. Como resultado, las validaciones numéricas realizadas en el sistema IEEE de 25 nodos y en una red trifásica de 35 nodos mostraron que las pérdidas finales de potencia activa varían dependiendo de la función objetivo analizada. Por lo tanto, para los alimentadores de prueba estudiados, es necesario evaluar cada función objetivo, en aras de encontrar la que produzca los mejores resultados numéricos. Se obtuvieron reducciones de 3.8056 % y 6.8652 % en las pérdidas de potencia para los dos sistemas de prueba mediante la metodología de optimización propuesta. Todas las validaciones numéricas se realizaron en el entorno de programación Julia, utilizando la herramienta de optimización JuMP y el solucionador HiGHS.

Palabras clave

Reducción de pérdidas de potencia, formulación convexa de enteros mixtos, desbalance de potencias, solución de flujo de potencia trifásico, análisis numérico, modelo de optimización en el dominio complejo.

1. INTRODUCTION

Electrical distribution networks are the most extensive systems in any country's general electrical grid [1]. These systems are responsible for providing electricity to all energy users in medium- and low-voltage level applications while ensuring reliability, continuity, security, and efficiency [2], [3]. The general structure of electrical distribution networks has three phases, *i.e.*, three conductors – phases a, b, and c – are used to efficiently distribute energy. Under balanced operating conditions, these networks are very efficient, as they ensure the lowest possible energy losses [4], [5]. Nevertheless, the real nature of electrical networks is unbalanced, given that (i) the distribution transformers that interface energy users and the grid have single-, two-, and three-phase connections [6], [7]; (ii) due to the length of the feeders, grid construction does not apply the concept of *transposition*, which is typically used in transmission networks [8]; and (iii) the electrical behavior of each node's users is not balanced, which implies that different current levels are requested per phase [9].

The above can cause unbalanced distribution networks to experience higher levels of energy losses and deteriorated voltage profiles in comparison with their balanced equivalents [10].

To efficiently deal with the issue of unbalanced operation in three-phase asymmetric distribution networks, the specialized literature has proposed multiple strategies, some of which are currently known as *phase-balancing* or *load redistribution problems*. Some of the classical and recent solution methodologies for this problem are presented below.

The authors of [10] developed a mixed-integer convex model to minimize power losses in three-phase distribution networks, using a quadratic formulation. This optimization model is based on the concept of *electrical momentum*, which allows the resistive effect of all distribution branches to be included in the value of the expected objective function. Numerical results in the 8-, 15-, and 25-bus grids demonstrated the effectiveness of the proposed optimization model when compared to heuristic algorithms such as a genetic algorithm (GA) or the vortex search algorithm (VSA).

In [11] an optimization model based on the general active power imbalance, using a real variable domain representation of redistributed unbalanced loads in three-phase networks is presented. The general mixed-integer convex structure of this model ensures solution repeatability. Numerical results in the 8-, 15-, and 25-bus feeders demonstrated the effectiveness of this proposal in comparison with metaheuristic optimizers such as the sine-cosine algorithm (SCA) and the black hole optimizer (BHO).

The study by [8] proposed a specialized GA to reduce active power losses in three-phase asymmetric distribution networks. The main advantage of this approach is that it incorporates single-, two-, and three-phase loads connected to the grid. Numerical results in a 19- and a 37-node grid demonstrated this proposal's ability to provide a set of feasible solutions to reduce power losses, thus providing more alternatives for the physical implementation of phase-balancing plans.

In [12], the VSA was implemented with a discrete codification to minimize the active power losses of a three-phase unbalanced distribution network. A master-slave solution methodology involving the combination of the VSA and the three-phase power flow method based on the iterative backward/forward approach yielded effective results in 8-, 25-, and 37-bus grids when compared to a GA. In addition, the results obtained with the VSA were improved by applying the discrete version of the crow search algorithm.

Additional works on optimal phase-balancing in three-phase distribution networks employ particle swarm optimization (PSO) [9], [13], the heuristic algorithm based on pole measurement [7], [14], and GAs based on group theory [6], among others.

Considering the above, the contributions of this research are presented below.

- A new formulation of the efficient nodal load redistribution problem for a three-phase distribution network using a mixed-integer convex model defined in the complex domain.
- The evaluation of three possible objective functions, demonstrating that different energy losses levels can be obtained in an unbalanced distribution network.
- The application of the successive approximations power flow method for three-phase unbalanced distribution networks using the nodal admittance matrix and its upper-diagonal lower decomposition.

It should be noted that the main goal of this research is to minimize the expected grid power losses via load redistribution at all nodes, which also allows for the improvement of voltage profiles in each phase of the distribution network. In addition, within the scope of our work, it is considered that (i) peak-load data have been provided by the distribution company and all users have the same energy consumption profile, *i.e.*, the electrical grid feeds only one type of user (residential, commercial, or industrial); (ii) the electrical infrastructure is three-phase and the impedance couplings between each pair of phases are negligible; and (iii) all the loads can be independently rotated in positive and negative sequences, *i.e.*, the system does not contain any electrical machines that are directly connected to the grid or all of them are integrated via speed variators.

For the three-phase power flow implementation via the successive approximation's method, it is assumed that all the loads are solidly grounded and have a star connection. In addition, the system is three-phase and three-wire, *i.e.*, no neutral cables have been installed [12].

The remainder of this research document is structured as follows. Section II presents the general power flow formulation for electrical networks, which uses the nodal admittance representation, an essential tool for determining the initial and final power losses of the grid as a function of the load connection. Section III describes the proposed mixed-integer convex formulation for the efficient redistribution of the nodal load connections while considering three objective functions. Section IV shows the general implementation of the solution methodology and its integration with the successive approximations power flow formulation. Section V describes the main characteristics of the three-phase version of the 35-bus network, which was used as a test system. Section VI shows all the computational validations carried out, and, finally, Section VII provides the main concluding remarks of this research.

2. METHODOLOGY

This section presents the proposed solution methodology to reduce power losses and improve voltage profiles in three-phase unbalanced distribution networks. Firstly, the three-phase power flow problem is formulated and solved via the successive approximation's method. Secondly, the mixed integer convex programming model (MICP) is formulated, which includes three different objective functions.

2.1 The three-phase power flow problem

Under steady-state conditions, an electrical distribution network with a single- or three-phase representation can be expressed via a set of nonlinear non-convex equations regarding the apparent power equilibrium per node (i.e., the combination of Kirchhoff's laws and Tellegen's second theorem) [15]. This representation is possible since it is assumed that the electrical grid is fed using balanced voltage profiles following a sinusoidal behavior with constant amplitude and frequency at the terminals of the substation. The general power flow problem in three-phase networks can be formulated using (1).

$$-\mathbb{I}_{d3\varphi} = \mathbb{Y}_{ds3\varphi}\mathbb{V}_{s3\varphi} + \mathbb{Y}_{dd3\varphi}\mathbb{V}_{d3\varphi} \quad (1)$$

Here, $\mathbb{I}_{d3\varphi} \in \mathcal{C}^{3(n-1)\times 1}$ is the vector of the demanded currents, defined in the complex domain for a network of n nodes where the substation's injected current is removed; $\mathbb{V}_{d3\varphi} \in \mathcal{C}^{3(n-1)\times 1}$ denotes the vector of complex voltages in all the demand nodes, which are the unknown variables of interest; $\mathbb{V}_{s3\varphi} \in \mathcal{C}^{3\times 1}$ is a complex vector that defines the voltage output at the substation terminals, *i.e.*, a perfectly known vector; $\mathbb{Y}_{ds3\varphi}\mathcal{C}^{3(n-1)\times 3}$ is a submatrix stemming from the nodal admittance matrix that associates the substation bus with the remaining demand nodes; and $\mathbb{Y}_{dd3\varphi} \in \mathcal{C}^{3(n-1)\times 3(n-1)}$ is a square matrix defined in the complex domain that is always invertible when the system is radially connected (*i.e.*, there are no isolated nodes). Note that the symbol $\mathcal{C}$ is used to represent the set of complex numbers.

Equation (1) looks like a set of linear equations due to its structure. However, note that the demanded currents are hyperbolic functions of the voltage profiles and constant power consumptions of the nodes, along with their connection type (*i.e.*, Wye or Triangle) [12].

To illustrate the possible load connections of a particular node k node in a three-phase network, consider the Wye and Triangle connections presented in Figures 1 and 2.

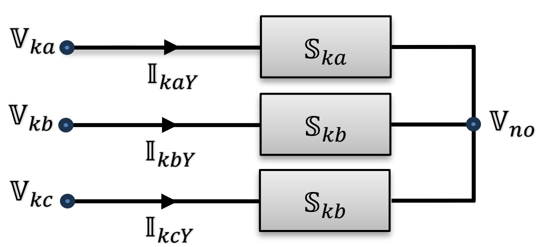

Figure 1. Schematic connection of a wye load on bus k. Source: authors.

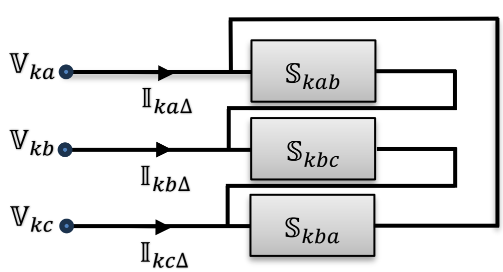

Figure 2. Schematic connection of a triangle load on bus k. Source: Authors.

In Figure 1, the common point to which all the loads are connected is assumed to be solidly grounded, *i.e.*, without a differential potential with respect to the neutral point at the terminals of the substation (this implies that $\mathbb{V}_{no} = 0$). Thus, each phase current can be calculated as defined from (2) to (4).

$$\mathbb{I}_{kaY} = \left(\frac{\mathbb{S}_{ka}}{\mathbb{V}_{ka}}\right)^{*} \quad \{\forall k \in \mathcal{D}\} \tag{2}$$

$$\mathbb{I}_{kbY} = \left(\frac{\mathbb{S}_{kb}}{\mathbb{V}_{kb}}\right)^{*} \quad \{\forall k \in \mathcal{D}\} \tag{3}$$

$$\mathbb{I}_{kcY} = \left(\frac{\mathbb{S}_{kc}}{\mathbb{V}_{kc}}\right)^{*} \quad \{\forall k \in \mathcal{D}\} \tag{4}$$

Where $\mathbb{I}_{kaY}$, $\mathbb{I}_{kbY}$, and $\mathbb{I}_{kcY}$ represent the net current absorption for a Wye load connected at bus k; $\mathbb{V}_{ka}$, $\mathbb{V}_{kb}$, and $\mathbb{V}_{kc}$ represent the complex line-to-ground voltage variables associated with node k; $\mathbb{S}_{ka}$, $\mathbb{S}_{kb}$, and $\mathbb{S}_{kc}$ denote the complex power consumption per phase at bus k; and $\mathcal{D}$ is defined as the set containing all the load nodes.

According to Figure 2, the calculation of each current is the sum of two components, as each phase has two loads connected to it. Equations (5)-(7) present the current flows of the triangle loads.

$$\mathbb{I}_{ka\Delta} = \left(\frac{\mathbb{S}_{kab}}{\mathbb{V}_{ka} - \mathbb{V}_{kb}}\right)^{*} - \left(\frac{\mathbb{S}_{kca}}{\mathbb{V}_{kc} - \mathbb{V}_{ka}}\right)^{*} \{\forall k \in \mathcal{D}\} \tag{5}$$

$$\mathbb{I}_{kb\Delta} = \left(\frac{\mathbb{S}_{kbc}}{\mathbb{V}_{kb} - \mathbb{V}_{kc}}\right)^{*} - \left(\frac{\mathbb{S}_{kab}}{\mathbb{V}_{ka} - \mathbb{V}_{kb}}\right)^{*} \{\forall k \in \mathcal{D}\} \tag{6}$$

$$\mathbb{I}_{kc\Delta} = \left(\frac{\mathbb{S}_{kca}}{\mathbb{V}_{kc} - \mathbb{V}_{ka}}\right)^{*} - \left(\frac{\mathbb{S}_{kbc}}{\mathbb{V}_{kb} - \mathbb{V}_{kc}}\right)^{*} \{\forall k \in \mathcal{D}\} \tag{7}$$

Where $\mathbb{I}_{ka\Delta}$, $\mathbb{I}_{kb\Delta}$, and $\mathbb{I}_{kc\Delta}$ represent the net current absorption for a triangle load connected at bus k; and $\mathbb{S}_{kab}$, $\mathbb{S}_{kbc}$, and $\mathbb{S}_{kca}$ denote the complex power consumption between each pair of phases on bus k.

Considering the definitions of the Wye and Triangle current consumptions per node, it is convenient to obtain a general formula to calculate each three-phase current at node k. Thus, the Wye currents in (2)-(4) can be compacted using (8).

$$\mathbb{I}_{k3\varphi Y} = diag^{-1}\left(\mathbb{V}_{k3\varphi}^{*}\right)\mathbb{S}_{k3\varphi}^{*}, \{\forall k \in \mathcal{D}\} \tag{8}$$

Where $\mathbb{V}_{k3\varphi}^{*} = [\mathbb{V}_{ka}^{*} \quad \mathbb{V}_{kb}^{*} \quad \mathbb{V}_{kc}^{*}]^{T}$ and $\mathbb{S}_{k3\varphi}^{*} = [\mathbb{S}_{ka}^{*} \quad \mathbb{S}_{kb}^{*} \quad \mathbb{S}_{kc}^{*}]^{T} = [\mathbb{S}_{kab}^{*} \quad \mathbb{S}_{kbc}^{*} \quad \mathbb{S}_{kca}^{*}]^{T}$. In addition, $diag(v)$ is a matricial operation that transforms the vector v into a diagonal matrix.

The Triangle load connections defined in (5)-(7) can be generalized using (9).

$$\mathbb{I}_{k3\varphi\Delta} = diag^{-1}\left(M\mathbb{V}_{k3\varphi}^{*}\right)\mathbb{S}_{k3\varphi}^{*} + diag^{-1}\left(M^{T}\mathbb{V}_{k3\varphi}^{*}\right)H\mathbb{S}_{k3\varphi}^{*}\{\forall k \in \mathcal{D}\} \tag{9}$$

Where

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} H = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Finally, considering the possibility of having a node k with loads in a Wye or a Triangle connection, the demanded current at this node can be generally calculated as defined in (10).

$$\mathbb{I}_{kd3\varphi} = \mathbb{I}_{k3\varphi Y} + \mathbb{I}_{k3\varphi\Delta}, \quad \{\forall k \in \mathcal{D}\} \tag{10}$$

Now, by analyzing (1) and (10), it can be stated that a nonlinear relationship between voltage and current makes it necessary to apply an iterative solution method in order to find the numerical solution to the power flow problem in three-phase unbalanced networks. In this context, the iterative solution to this problem, which is defined in (1), requires the following formula:

$$\mathbb{V}_{d3\varphi}^{t+1} = \mathbb{Y}_{dd3\varphi}^{-1} \left(\mathbb{Y}_{ds3\varphi}\mathbb{V}_{s3\varphi} + \mathbb{I}_{d3\varphi}^{t}\right) \tag{11}$$

Where t is the iteration counter, and each component of $\mathbb{I}_{d3\varphi}^{t}$ is calculated using (10). Note that (11) is known in the literature as the *successive approximation power flow method* [12]. The iterative process starts when $t = 0$, setting each component of the vector $\mathbb{V}_{d3\varphi}^{0}$ as $\mathbb{V}_{k3\varphi}^{0} = \mathbb{V}_{s3\varphi}$, for $k = 2,3, \dots, n$, with $\mathbb{V}_{s3\varphi}$ defined as follows (12):

$$\mathbb{V}_{s3\varphi} = \begin{bmatrix} 1 \\ e^{-i\frac{2\pi}{3}} \\ e^{i\frac{2\pi}{3}} \end{bmatrix} V_{nom} \tag{12}$$

Where V_{nom} represents the nominal operating voltage of the three-phase network under analysis. The iterative process with (11) is carried out until the convergence criterion is met. This criterion is defined in (13).

$$max\left\{\left|\left|\mathbb{V}_{d3\varphi}^{t+1}\right| - \left|\mathbb{V}_{d3\varphi}^{t}\right|\right|\right\} \leq \varepsilon \tag{13}$$

Where ε is the acceptable tolerance, typically set between 1×10^{-6} and 1×10^{-10}.
To evaluate the total power losses level of the distribution network, (14) is used.

$$\mathbb{S}_{loss} = \mathbb{V}_{3\varphi}^{T}\left(\mathbb{Y}_{bus3\varphi}\mathbb{V}_{3\varphi}\right)^{*} \tag{14}$$

Where $\mathbb{V}_{3\varphi} = [\mathbb{V}_{s3\varphi} \quad ; \quad \mathbb{V}_{d3\varphi}]^{T}$ represents the vector containing all the voltages of the network, including that of the substation.

The flow diagram in Figure 3 summarizes the general solution of the three-phase power flow problem in unbalanced distribution networks via the successive approximations power flow method.

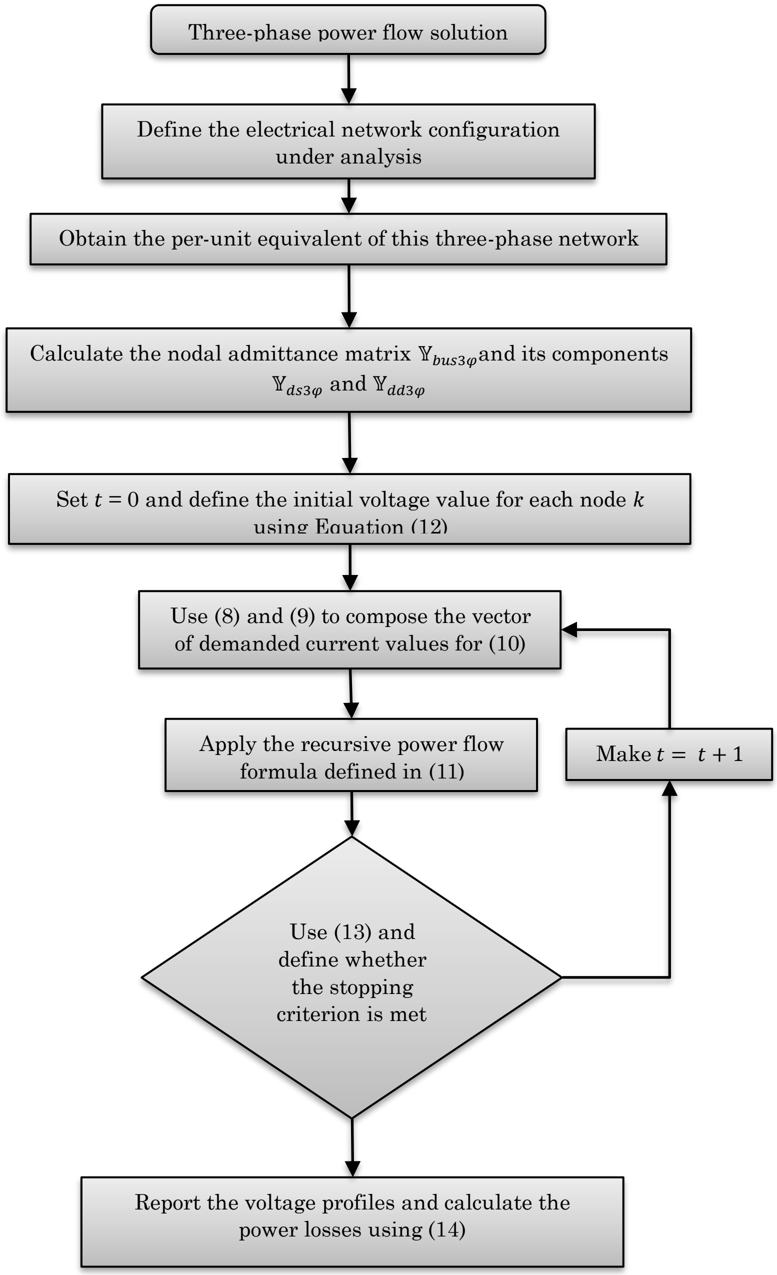

Figure 3. General implementation of the power flow model in three-phase networks using the successive approximations method. Source: Authors.

2.2 Optimal load redistribution modeling

In reducing the power losses of three-phase distribution networks, the nodal connection of each load type plays a fundamental role, as all the voltages in the demand nodes are a function of the demanded current in (9) and (10), which also defines the expected power losses level expressed in (15).

To formulate the efficient load redistribution model for the terminals of the substation, consider the following information for a 4-node grid (Table 1).

Table 1. 4-node example for load redistribution at the substation terminals. Source: Authors.

Node	$\mathbb{S}_{da}$	$\mathbb{S}_{db}$	$\mathbb{S}_{dc}$
1	$800 + j500$	$700 + j400$	$750 + j750$
2	$0 + j0$	$850 + j300$	$400 + j600$
3	$500 + j250$	$0 + j0$	$0 + j0$
4	$750 + j450$	$950 + j650$	$0 + j600$
Total	$2050 + j1200$	$2500 + j1350$	$1150 + j1950$

Regarding the total active and reactive power consumption per phase (Table 1), note that the total active and reactive power values are unbalanced, *i.e.,* these values are different from each other. Therefore, the ideal active and reactive power consumption per phase can be defined as follows:

$$\mathbb{S}_p = \frac{1}{3}\left(\sum_{f \epsilon F} \mathbb{S}_f\right) = \frac{\mathbb{S}_a + \mathbb{S}_b + \mathbb{S}_c}{3} \tag{15}$$

Which, for the example in Table 1, takes a value of $\mathbb{S}_p = 1900 + j1500$. Here, F is the set containing all the phases, *i.e.*, $F = \{a, b, c\}$.

Regarding $\mathbb{S}_p$ as the expected load value per phase, three possible load redistribution scenarios can be analyzed, which are defined in (16)-(18).

$$U_{p\,\%} = \frac{100}{3\mathrm{Re}\{\mathbb{S}_p\}} \sum_{f \in F} \left|\mathrm{Re}\{\mathbb{S}_f\} - \mathrm{Re}\{\mathbb{S}_p\}\right| \tag{16}$$

$$U_{q\,\%} = \frac{100}{3\mathrm{Im}\{\mathbb{S}_p\}} \sum_{f \in F} \left|\mathrm{Im}\{\mathbb{S}_f\} - \mathrm{Im}\{\mathbb{S}_p\}\right| \tag{17}$$

$$U_{s\,\%} = \frac{1}{2}\left(U_{p\,\%} + U_{q\,\%}\right) \tag{18}$$

Where $U_{p\,\%}$, $U_{q\,\%}$, and $U_{s\,\%}$ denote the percent of active, reactive, or average apparent power losses at the substation terminals, which should ideally be zero, as is the case for three-phase balanced networks. The main characteristic of these objective functions is that all of them are convex, implying that an optimal value can be found if and only if all the constraints are convex or mixed-integer convex.

In three-phase networks, the phase-balancing problem implies that only a few load movements per node are admissible. Table 2 presents the load connection possibilities per node.

Each of the six possible load connections in Table 2 can be represented (19)-(20) with a binary matrix per node x_{kfg}, where the subscript k means the nodal connection, f represents the initial load connection, and g denotes the final load connection. In addition, as observed in Table 2, this binary variable only admits one position filled by one in each row and column, which yields the following set of linear-integer constraints.

Table 2. Load connection possibilities per node. Source: Adapted from [11].

Connection	Phases	Variable
1	ABC	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2	CAB	$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
3	BCA	$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
4	ACB	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$
5	BAC	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
6	CBA	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$\sum_{f\in F} x_{kfg} = 1, \quad \{k \in N \wedge \ g \in F\} \tag{19}$$

$$\sum_{g\in F} x_{kfg} = 1, \quad \{k \in N \wedge \ f \in F\} \tag{20}$$

Where N represents the set containing all the nodes of the network. In addition, according to Equations (16)-(18), the total apparent power demand per phase at the terminals of the substation (*i.e.*, $\mathbb{S}_f$) can be defined using (21).

$$\mathbb{S}_f = \sum_{k\in N}\sum_{g\in F} x_{kfg}\, \mathbb{S}_{kf}, \quad \{f \in F\} \tag{21}$$

Where $\mathbb{S}_{kf}$ represents the initial load connection at each node. The optimization model defined from (16) to (21) has a convex structure of mixed integers, which implies that an optimal solution can be reached using the branch and bound method combined with the modified simplex optimization algorithm [16].

Figure 4 illustrates the general implementation of the proposed load redistribution model and its connection with the power flow problem presented in Figure 3.

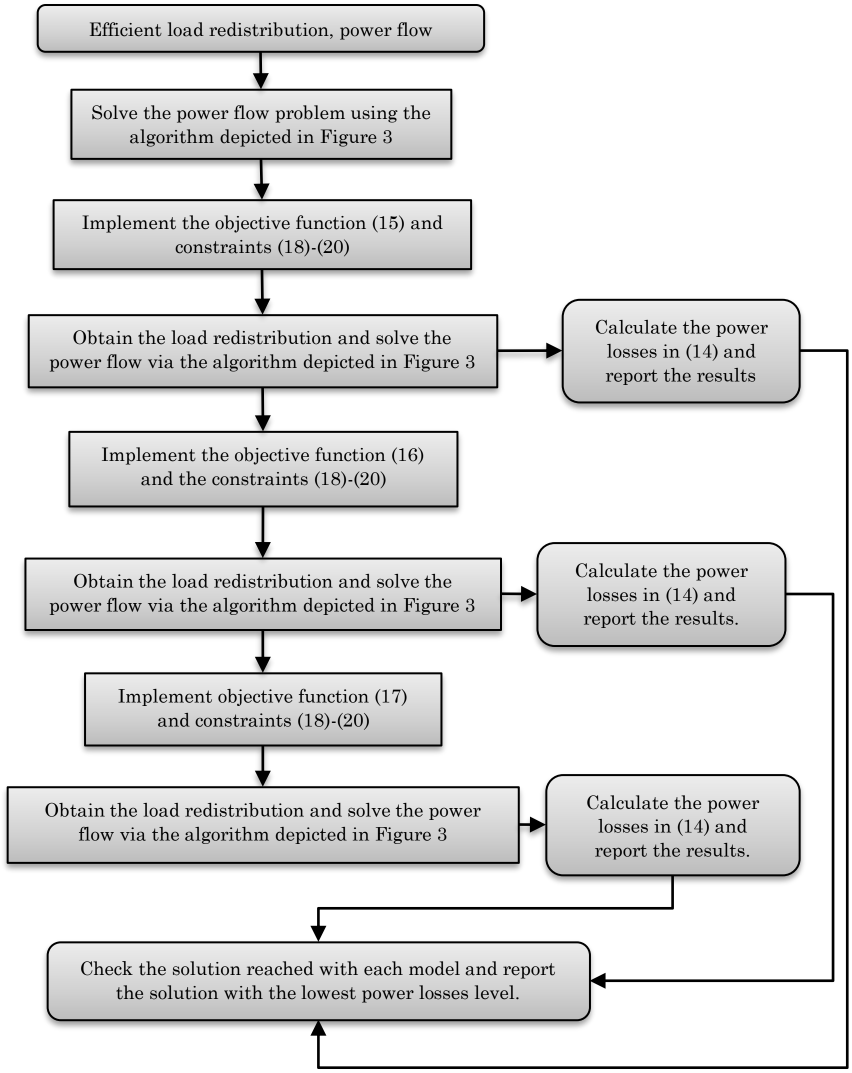

Figure 4. General implementation of optimal load redistribution at the substation terminals in order to minimize grid power losses. Source: Authors.

3. RESULTS AND DISCUSSION

This section describes the main numerical results obtained and discusses the application of the load balancing methodology to three-phase asymmetric networks, which combines a MICP approach with an efficient power flow solution. The first part of this section describes the main characteristics of the 35-bus system, and the final part focuses on the numerical validations, analyses, and discussions.

3.1 Three-phase 35 bus system

To demonstrate the applicability of the proposed methodology to reduce power losses in unbalanced three-phase distribution networks, which employ a hybrid optimization approach based on the power flow formulation and the mixed integer convex model (15)-(20), the three-phase equivalent of a single-phase 35-bus network is used. Figure 5 presents the nodal interconnections of this grid.

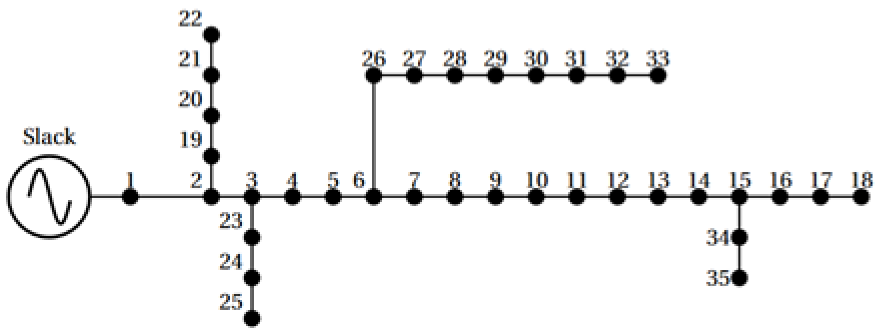

Figure 5. Nodal connections of the 35-bus system. Source: Authors.

Note that this is a three-phase network that works with a line-to-neutral voltage of 15 kV at the terminals of the substation. In addition, Table 3 presents the per-phase parametric data of the 35-bus grid.

Table 3. Per-phase parametric information of the three-phase 35-bus system. Source: Authors.

k	m	R_{km} (Ω)	X_{km} (Ω)	k	m	R_{km} (Ω)	X_{km} (Ω)
1	2	0.0922	0.0477	2	19	0.1640	0.1565
2	3	0.4930	0.2511	19	20	1.5042	1.3554
3	4	0.3660	0.1864	20	21	0.4095	0.4784
4	5	0.3811	0.1941	21	22	0.7089	0.9373
5	6	0.8190	0.7070	3	23	0.4512	0.3083
6	7	0.1872	0.6188	23	24	0.8980	0.7091
7	8	1.7114	1.2351	24	25	0.8960	0.7011
8	9	1.0300	0.7400	6	26	0.2030	0.1034
9	10	1.0400	0.7400	26	27	0.2842	0.1447
10	11	0.1966	0.0650	27	28	1.0590	0.9337
11	12	0.3744	0.1238	28	29	0.8042	0.7006
12	13	1.4680	1.1550	29	30	0.5075	0.2585
13	14	0.5416	0.7129	30	31	0.9744	0.9630
14	15	0.5910	0.5260	31	32	0.3105	0.3619
15	16	0.7463	0.5450	32	33	0.3410	0.5302
16	17	1.2860	1.7210	15	34	0.2819	0.4012
17	18	0.7320	0.5740	34	35	0.1958	0.2714

The information presented above (22) shows that, for a particular distribution line, the three-phase impedance matrix takes the following structure:

$$\mathbb{z}_{km} = \begin{bmatrix} R_{km} + jX_{km} & 0 & 0 \\ 0 & R_{km} + jX_{km} & 0 \\ 0 & 0 & R_{km} + jX_{km} \end{bmatrix} \quad (22)$$

Table 4 presents the per-phase complex power consumption of the 35-bus system.

Table 4. Load information for each phase in the 35-bus grid. Source: Authors.

Node	$\mathbb{S}_{da}$ (kVA)	$\mathbb{S}_{db}$ (kVA)	$\mathbb{S}_{dc}$ (kVA)
1	$0+j0$	$0+j0$	$0+j0$
2	$100+j50$	$100+j60$	$50+j50$
3	$50+j0$	$70+j40$	$50+j40$
4	$120+j75$	$100+j80$	$150+j90$
5	$60+j20$	$60+j30$	$30+j30$
6	$400+j180$	$0+j0$	$300+j150$
7	$200+j150$	$110+j70$	$100+j100$
8	$200+j0$	$100+j100$	$150+j150$
9	$120+j75$	$0+j0$	$0+j0$
10	$0+j0$	$600+j400$	$0+j0$
11	$130+j100$	$0+j0$	$0+j0$
12	$125+j75$	$60+j35$	$155+j100$
13	$60+j110$	$60+j35$	$60+j35$
14	$120+j80$	$190+j80$	$0-j400$
15	$60+j10$	$0+j0$	$0+j0$
16	$60+j20$	$110+j80$	$60+j20$
17	$60+j20$	$150+j95$	$0+j0$
18	$90+j40$	$100+j0$	$90+j40$
19	$300\,j150$	$0+j0$	$90+j40$
20	$210+j50$	$85+j40$	$70+j75$
21	$90+j40$	$110+j40$	$110+j20$
22	$300+j400$	$0+j0$	$90+j40$
23	$90+j50$	$70+j0$	$0+j0$
24	$300+j200$	$0-j600$	$250+j100$
25	$120+j75$	$0+j0$	$150+j100$
26	$60+j25$	$80+j25$	$0+j0$
27	$210+j145$	$80+j25$	$0+j0$
28	$60+j20$	$48+j24$	$60+j20$
29	$120+j70$	$185+j75$	$220+j90$
30	$200-j500$	$0+j400$	$300+j350$
31	$150+j70$	$120+j90$	$150+j70$
32	$210+j100$	$120+j35$	$180+j50$
33	$60+j40$	$100+j350$	$0+j0$
34	$0+j0$	$0+j0$	$60+j10$
35	$0+j0$	$60+j450$	$0+j0$
Total	$4435+j1940$	$2868+j2059$	$2925+j1370$

3.2 Numerical validations

For the computational implementation or our proposal, the Julia programming environment (version 1.9.2) [17] was employed. The simulations were carried out on a 64-bit version of Microsoft Windows 10 Single Language, on a computer with an AMD Ryzen 7 3700 2.3 GHz processor and 16.0 GB RAM. The solution to the MICP model was obtained by means of the JuMP optimization environment and the HiGHS solver [16], [18].

3.2.1 Benchmark case

To determine the effectiveness of the proposed complex-domain MICP model in reducing the power losses of the 35-bus system, the objective functions defined from (15) to (17) were evaluated. This, in order to determine the active, reactive, and apparent power imbalances of the reference case, *i.e.*, the information presented in Table 4. The following results were obtained (note that $\mathbb{S}_p = 3409.3333 + j1789.6667$ kVA):

- The total active power imbalance (*i.e.*, $U_{p\%}$), as defined by (15), was about 20.0561 %.
- The total reactive power imbalance (*i.e.*, $U_{q\%}$), as defined by (16), was about 15.6630 %.
- The total apparent power imbalance (*i.e.*, $U_{s\%}$), as defined by (17), was about 17.8445 %.

In addition, the initial power losses were calculated using (14) after solving the power flow problem via the successive approximations method described in Figure 3. The power losses for this system were $\mathbb{S}_{loss} = 473.1181 + j337.3563$ kVA. However, in electrical engineering applications, active power losses constitute the main interest, *i.e.*, the real part of $\mathbb{S}_{loss}$ ($P_{loss} = \text{Re}\{\mathbb{S}_{loss}\} = 473.1181$ kW).

3.2.2 Results after solving the proposed MICP model

To reduce power losses in the studied three-phase 35-bus grid, the optimization model defined from (15) to (20) was solved while considering a single-objective analysis, *i.e.*, each objective function (15)-(17) was optimized independently. Table 5 reports the general percent of active, reactive, and average apparent power imbalances before and after solving the optimization model for each of the performance indicators.

Table 5. Numerical results before and after optimizing the load connections. Source: Authors.

Objective function	Before optimization (%)	Active power losses (kW)	After optimization (%)	Active power losses (kW)
$\boldsymbol{U_{p\,\%}}$	20.0561	473.1181	0.02607	450.8602
$\boldsymbol{U_{q\,\%}}$	15.6330		0.02483	445.2722
$\boldsymbol{U_{s\,\%}}$	17.8445		0.02545	440.6374

The numerical results in Table 5 show that:

- All the objective functions can be minimized to values between 0.0245 and 0.0265 %, which means that the proposed optimization model (15)-(20) is efficient.
- In all cases, minimizing the indices $U_{p\,\%}$, $U_{q\,\%}$, and $U_{s\,\%}$ allows for reducing the expected power losses value with respect to the initial case. The power losses are reduced by about 4.7045 % when the active power imbalance is minimized. This reduction is about 5.8856 % for the reactive power imbalance and 6.8652 % for the average apparent power imbalance.

It is worth mentioning that, for this system, the best result regarding active power losses minimization (Table 5) was obtained by reducing the average apparent power imbalance. Nevertheless, this result cannot be generalized to any three-phase unbalanced distribution network due to the nonlinear relation between power losses and voltage profiles, in addition to the fact that the optimization model (15)-(20) is a mixed-integer linear approximation of the optimal load-balancing problem, and it assumes that there are ideal voltages and that all the loads can be moved to the substation terminals. In other words, each objective function must be evaluated to select the alternative that allows for higher reductions in total power losses in a given grid.

3.2.3 Voltage profile behavior

To demonstrate the positive effect of power losses minimization on the electrical performance of the studied three-phase distribution network with respect to power savings, the behavior of the per-phase voltage profile is shown depicted in Figures 6-8.

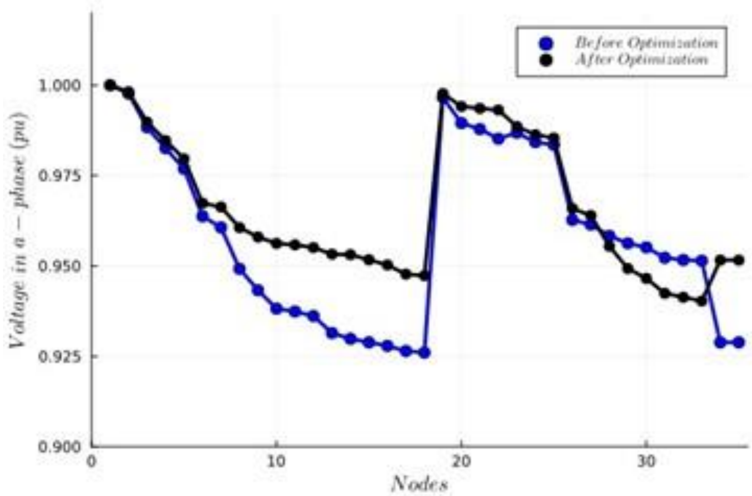

Figure 6. Per-node voltage profile behavior in the a-phase before and after implementing the load redistribution optimization approach. Source: Authors.

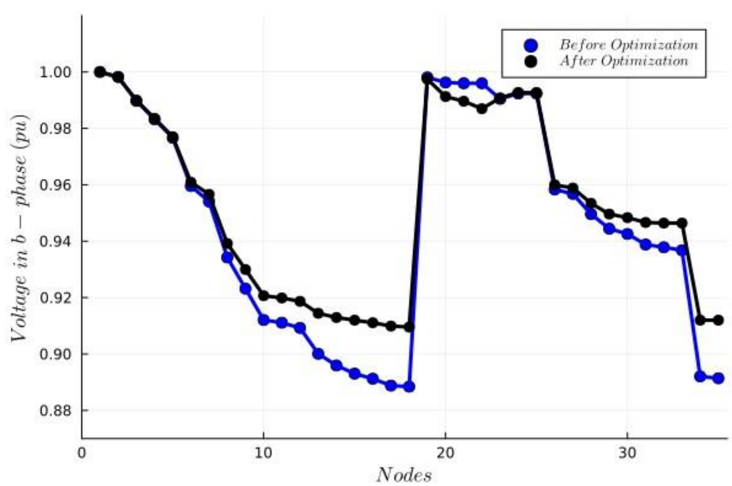

Figure 7. Behavior of the voltage profile per node in the b-phase before and after implementing the load redistribution optimization approach. Source: Authors.

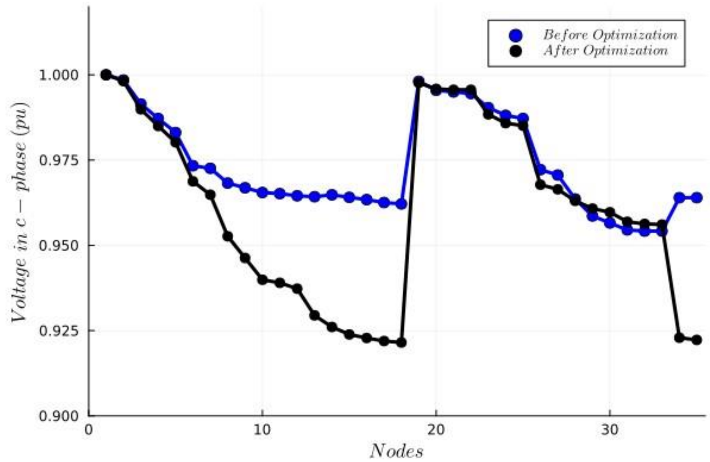

Figure 8. Per-node voltage profile behavior in the *c*-phase before and after implementing the load redistribution optimization approach. Source: Authors.

The main results observed in these figures are the following:

- The voltage profile behaviors for phases a and b show that minimizing the total grid power losses allows improving the voltage magnitude of most nodes (Figures 6 and 7), whereas the voltage profile of phase c deteriorated with respect to the benchmark case (Figure 8). However, this is an expected result, as phases a and b were initially overloaded when compared to phase c (Table 3), which implies that, after load redistribution, part of the loads present in phases a and b were transferred to phase c.
- The overall voltage regulation of the 35-bus grid improved after load redistribution using the proposed optimization model. Note that the worst voltage magnitude occurred in phase *b* for the benchmark case, with a magnitude of 0.8895 pu at node 18, which defines an overall general regulation value of about 11.05 %. Nevertheless, after implementing the optimization procedure, the worst voltage profile occurred at node 18 in phase *b* with a value of 0.9150 pu, *i.e.*, the voltage regulation was about 8.95 % in this context. This is a significant result, as regulatory policies for distribution companies typically impose a voltage regulation value of about 10 % in medium-voltage applications. This criterion is met in this research.

3.2.4 Comparative analysis

To demonstrate the effectiveness of the optimization model defined from (15) to (20) with respect to the reduction of expected power losses in an unbalanced distribution network by minimizing the total equivalent load imbalance at the substation terminals, the IEEE 25-grid reported by [10] and [11] was considered. Table 6 reports the general percent active, reactive, and average apparent power imbalances before and after solving the optimization model for each one of the performance indicators.

Table 6. Numerical results before and after optimizing the load connections. Source: Authors.

Objective function	Before optimization (%)	Active power losses (kW)	After optimization (%)	Active power losses (kW)
$\boldsymbol{U_{p\,\%}}$	16.6012		0	72.5775
$\boldsymbol{U_{q\,\%}}$	13.9165	75.4207	0	72.7812
$\boldsymbol{U_{s\,\%}}$	15.2588		0.00873	72.5505

The numerical results in Table 6 show that:

- All the objective functions can be minimized to values between 0 and 0.0088%, which means that the proposed optimization model is efficient regarding the improvement of the power imbalances at the terminals of the substation.
- In all cases, minimizing the indices $U_{p\%}$, $U_{q\%}$, and $U_{s\%}$ allows for reducing the expected power losses value with respect to the initial case. The power losses are reduced by about 3.7698 % when the active power imbalance is minimized. This reduction is about 3.4997 % for the reactive power imbalance and 3.8056 % for the average apparent power imbalance. These results confirm that, for IEEE 25-bus grid, the most effective approach involves minimization of average apparent power imbalance, as defined in (17).
- The proposed approach reports a final value of about 72.5505 kW regarding the expected grid power losses when the average power imbalance is minimized. In contrast, the mixed-integer quadratic convex presented by [11] reports a final value of about 72.2816 kW, which implies a difference of about 0.2689 kW (0.3565%) between both methodologies. These results confirm that our complex-domain proposal provides an adequate approximation of the solution reported with the MIQC approach in [11], with the main advantage that the proposed MICP model is simpler, since it does not require information about the branches in the optimization stage.
- A comparative analysis with metaheuristic optimizers was performed, which included PSO, Chu & Beasley GA, and BHO. When the optimization model (15)-(20) is solved with these methods and their solution is evaluated via the successive approximations power flow approach in order to determine the final power losses values, the standard deviations are different from zero, *i.e.*, the mean value differs from the minimum value after multiple executions (these variations were between 0.50 and 1.75 %), implying that the metaheuristic optimizers cannot converge to the same numerical solution, which is always ensured with the proposed MICP approach.

4. CONCLUSIONS

This research presented a new optimization methodology based on a mixed-integer convex formulation in the complex domain to redistribute the constant power load consumption of three-phase unbalanced distribution networks, with the aim of minimizing the grid power imbalances at the terminals of the substation, reducing the expected grid power losses, and improving the voltage profile performance at each phase of the system. This solution methodology is based on two stages. The first stage solves the proposed MICP model while separately considering three objective functions, *i.e.*, the active, reactive, and average apparent power imbalances. The second stage solves the three-phase power flow problem in order to identify which objective function yields lower power losses compared to the benchmark case.

Numerical results in the 35-bus grid show that power losses can be reduced by 4.7045-6.8652 % using the proposed methodology, depending on the objective function analyzed.

The most important result of this research has to do with the combination of a MICP formulation in the complex domain (as a relaxation of the optimal phase-balancing problem in three-phase distribution grids) and an efficient power flow method based on successive approximations, with the aim of reducing the power losses of unbalanced distribution networks after implementing the load connection plans provided by the solution of the MICP model. Note that the solutions found are optimal for each objective function under analysis, since the objective function is convex and the solution space is mixed-integer convex, which implies that combining the branch and cut method with an interior-point optimizer is possible and can ensure a globally optimum.

It is essential to note that the proposed approach requires evaluating all objective functions (*i.e.*, the active, reactive, and apparent power imbalances) in order to determine the best alternative for reducing the total power losses of a specific three-phase grid. This is an important element of our proposal; due to the MICP method's relaxation of the power balance constraints by the MICP method, the solution obtained is optimal, but it can correspond to an approximate solution in the case of the full power flow model.

A possible limitation of our contribution is that it neglects the effect of the distribution branch impedances on the calculation of power losses, since the MICP approach only focuses on balancing loads at the terminals of the substation. However, when the power flow problem is solved with the successive approximations method, the final power losses are adequate with regard to the expected reduction values, even though they correspond to an approximation of the exact optimization model.

As future work, (i) different objective function structures could be evaluated in the three-phase grid model, such as quadratic functions and norms, in combination with convex approximation; and (ii) metaheuristic optimization techniques could be applied to define the best load combination per node in order to minimize power losses via a master-slave optimization methodology. In this methodology, a metaheuristic approach could be assigned to define the load connection per node in the master stage, and the successive approximations power flow method could be used in the slave stage to guide the exploration and exploitation stages.

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CONFLICTS OF INTEREST OF THE AUTHORS

The authors declare no conflict of interest.

CONTRIBUTION OF AUTHORS

O. D. Montoya: Conceptualization, Methodology, Research, Review, and Editing. C. A. Ramírez-Vanegas: Methodology, Research, Review, and Editing. J. R. González-Granada: Methodology, Research, Review and Editing. All authors have read and agreed to the published version of the manuscript.

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