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OPTIMAL ORDERING POLICY FOR THE TIME DEPENDENT DETERIORATION WITH ASSOCIATED SALVAGE VALUE WHEN DELAY IN PAYMENTS IS PERMISSIBLE

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RESUMEN:
En este artículo de investigación, se ha hecho un esfuerzo para desarrollar el modelo matemático, cuando las unidades en el inventario están sujetas al deterioro con el paso del tiempo, y el proveedor ofrece a su distribuidor el período de crédito para liquidar la cuenta de la adquisición de las unidades. El valor residual se asocia al deterioro de las unidades. El objetivo es reducir al mínimo el minorista total del costo de inventario. Se propone un algoritmo para encontrar la política óptima de pedidos. Un ejemplo numérico es la posibilidad de estudiar el efecto del período permisible de crédito y el deterioro de la variable de decisión y el costo total del minorista.

ABSTRACT :
In this research article, an attempt is made to develop mathematical model when units in inventory are subject to deterioration with time, and the supplier offers his retailer the credit period to settle the account of the procurement units. The salvage value is associated to the deterioration units. The objective is to minimize the retailer's total inventory cost. An algorithm is proposed to find optimal ordering policy. A numerical example is given to study the effect of allowable credit period and deterioration on decision variable and the total cost of the retailer.

KEY WORDS : Economic order quantity (EOQ), allowable credit period, time–dependent deterioration, salvage value.

MSC 90B05

1. INTRODUCTION

Trade credit is the most effective tool of the supplier to encourage retailer to buy more and to attract more retailers. Goyal (1985) derived an economic order quantity inventory model when the supplier offers a credit period to settle the retailer's account. Shah et al. (1988) extended Goyal's model by allowing shortages. Mandal and Phaujdar (1989) developed a mathematical model including interest earned from the sales revenue on the stock remaining beyond the settlement period. Shah (1993) studied an an inventory model for constant deterioration of units in an inventory under the scenario of permissible a credit period. Jamal et al. (1997) developed an inventory model to allow for shortages under the permissible delay in payments. Shah (1993) analyzed an inventory model when the supplier offers a credit period to settle the retailer's account by considering stochastic demand. Shah (1997) derived a probabilistic order – level system with lead – time when delay in payments is permissible. Jamal et al. (2000) formulated a mathematical model when retailer can settle the account either at the end of the credit period or later incurring interest charges on the un-paid balance for the over-due period.

Arcelus et al. (2003) formulated a mathematical model to maximize the retailer's profit when the supplier offers a credit period and /or price discount on the purchase of regular order when units in an

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inventory are subject to a constant deterioration. Related articles are Haley and Higgins (1973), Kingsman (1983), Chapman at al.(1984), Dallenbach (1986, 1988), Ward and Chapman (1987), Chapman and Ward (1988), Raafat (1991), Shah (1993), Jaggi and Aggrawal (1994), Aggrawal and Jaggi (1995), Shinn et al. (1996), Shah and Shah (1998), Chung (1998), Chu et al. (1998), Shah and Shah (2000), Chung (2000), Goyal and Giri (2001), Chung et al. (2001), Sarker et al. (2001), Teng (2002), Shinn et al. (2003), Salameh et al. (2003), Shah et al. (2004), Shah (2004), Lokhandwala et al. (2005), Shah and Trivedi (2005), Chung and Liao (2006), Shah (2006), Yang and Wee (2006), Liao (2007) .

In this article, an attempt is made to develop a mathematical model when the units in an inventory are subject to deterioration with respect to time and the supplier offers a credit period to the retailer to settle the account. The salvage value is associated to the deteriorated units. The objective is to minimize retailer's total cost. An algorithm is given to explore the computational flow. The effects of deterioration rates of units salvage value and a credit period on objective function and decision variable is validated using numerical example.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions are used to develop the aforesaid model:

- The system deals with a single item.
- The demand rate of *R*-units per time unit is deterministic and constant.
- The replenishment rate is infinite.
- The lead time is zero and shortages are not allowed.
- The deterioration rate of units in an inventory follows the Weibull distribution function given by

$$\theta(t) = \alpha\beta t^{\beta-1}, 0 \le t \le T \tag{2.1}$$

- where α $(0 \le \alpha < 1)$ denotes a scale parameter, β $(\beta \ge 1)$ denotes a shape parameter and t $(t > 0)$ is the time to deterioration.
- The deteriorated units can neither be repaired nor be replaced during the cycle time.
- During the fixed a credit period; *M*, the unit cost of the generated sales revenue is deposited in an interest bearing account. The difference between sales price and unit cost is retained by the system to meet the day–to–day expenses of the system. At the end of the credit period the account is settled and interest charges are payable on the un-paid account.
- The salvage value, γC ($0 \le \gamma < 1$) is associated to deteriorated units during the cycle time. Here *C* is the purchase cost of an item.

The following notations are used in developing the model:

R : demand rate per unit of time.
C : purchase cost per unit.
P : unit selling price (P > C).
h : inventory holding cost per unit per time unit excluding interest charges.
A : ordering cost per order.
I_e : interest earned per unit per annum.
I_c : interest charged per unit in stock per annum by the supplier to the retailer. $(I_c > I_e)$
T : cycle time. (decision variable)
M : allowable credit period offered by the supplier to the retailer for settling the accounts.

3. A MATHEMATICAL MODEL

Let $Q(t)$ be the on-hand inventory at any instant of time t ($0 \le t \le T$) of a cycle. The depletion of units in an inventory is due to the demand and the deterioration of units. The instantaneous state of $Q(t)$ at any instant of time is described by the differential equation

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R, \qquad 0 \le t \le T \tag{3.1}$$

with the initial condition *Q(0)* = Q and the boundary condition *Q(T)* = 0, where $\theta(t)$ is given by the equation (2.1).

Taking series expansion and ignoring second and higher power of α (assuming α to be very small), the solution of the differential equation (3.1) using the boundary condition *Q(T)* = 0 is given by

$$Q(t) = R\left[T - t + \frac{\alpha T}{\beta+1}\left(T^{\beta} - (1+\beta)t^{\beta}\right) + \frac{\alpha\beta t^{\beta+1}}{\beta+1}\right] \tag{3.2}$$

Using *Q(0)* = Q, we get procurement quantity as

$$Q = R\left[T + \frac{\alpha T^{\beta+1}}{\beta+1}\right] \tag{3.3}$$

The number of units that deteriorate; *D(T)* during one cycle is given by

$$D(T) = Q - RT = \frac{\alpha R T^{\beta+1}}{\beta+1} \tag{3.4}$$

Hence, the cost due to deterioration (CD) is

$$CD = \frac{\alpha C R T^{\beta+1}}{\beta+1} \tag{3.5}$$

and salvage value of deteriorated units; SV is

$$SV = \frac{\alpha\gamma\ C R T^{\beta+1}}{\beta+1} \tag{3.6}$$

The inventory holding cost; IHC during the cycle is

$$IHC = h\int_0^T Q(t)dt = hR\left[\frac{T^2}{2} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right] \tag{3.7}$$

and the ordering cost; OC per order is

$$OC = A \tag{3.8}$$

About the interest charged and the interest earned the following two cases arise:

Case 1 : $M \le T$ *(figure. 1)*

Case 2 : $M > T$ *(figure 2)*

Next we compute the interest charged and the interest earned in both cases:

Case 1: $M \le T$ i.e. the offered credit period is less than or equal to the cycle time.

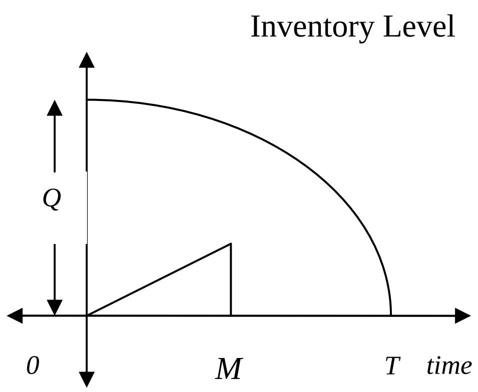

Figure 3.1 $M \le T$

The retailer can sale units during *[0, M]* at a sale price; *P* per unit which he can put at an interest rate I_e per unit per annum in an interest bearing account. So the total interest earned during *[0, M]* is:

$$IE_1 = PI_e \int_0^M Rt\,dt = \frac{PI_e RM^2}{2} \tag{3.9}$$

During *[M, T]*, the supplier will charge the interest to the retailer on the remaining stock at the rate I_c per unit per annum. Hence, total interest charges payable by the retailer during *[M, T]* is

$$IC_1 = CI_c \int_M^T Q(t)\,dt$$

$$= CI_c R\left[\frac{(T-M)^2}{2} - \frac{\alpha MT}{\beta+1}\left(T^{\beta} - M^{\beta}\right) + \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left(T^{\beta+2} - M^{\beta+2}\right)\right] \tag{3.10}$$

The total cost *($K_1(T)$)* per time unit is

$$K_1(T) = \frac{1}{T}\left[OC + IHC + CD + IC_1 - IE_1 - SV\right] \tag{3.11}$$

The necessary condition for $K_1(T)$ to be minimum is given in

$$\frac{\partial K_1(T)}{\partial T} = -\frac{1}{T^2}(\chi_1 + \chi_2 + \chi_3 + \chi_4) + \frac{1}{T}\left(\psi_1 + \zeta\left(\vartheta_1 + \vartheta_2\right)\right) = 0$$

where

$$\chi_1 = hR\left(\frac{T^2}{2} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right) + \frac{C\alpha RT^{\beta+1}}{\beta+1}(1-\gamma) + A$$

$$\chi_2 = CI_c R\left(\frac{T^2}{2} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right)$$

$$\chi_3 = -CI_c\frac{pI_e RM^2}{2}$$

$$\chi_4 = CI_c R\left(TM - \frac{M^2}{2} + \frac{\alpha T(T^{\beta}M + \frac{(-\beta-1)M^{\beta+1}}{\beta+1})}{\beta+1} + \frac{\alpha\beta M^{\beta+2}}{(\beta+1)(\beta+2)} \right)$$

$$\psi_1 = hR\left(T + \frac{\alpha T\left(T^{\beta}\beta + T^{\beta} + \frac{(-\beta-1)T^{\beta+1}}{T} \right)}{\beta+1} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)T} \right)$$

$$\zeta = \frac{C\alpha R T^{\beta+1}}{T}(1-\gamma)$$

$$\vartheta_1 = CI_c R\left(T + \frac{\alpha T\left(T^{\beta}\beta + T^{\beta} + \frac{(-\beta-1)T^{\beta+1}}{T} \right)}{\beta+1} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)T} \right)$$

$$\vartheta_2 = -R\left(M + \frac{\alpha\left(T^{\beta}M + \frac{(-\beta-1)M^{\beta+1}}{(\beta+1)} \right)}{\beta+1} + \frac{\alpha T^{\beta}\beta M}{\beta+1} \right)$$

and solve it for *T* by a mathematical software. For obtained *T*, $K_1(T)$ is minimum only if

$\frac{\partial^2 K_1(T)}{\partial T^2} > 0$ for all T

Case 2: $T > M$ i.e. the offered credit period by the supplier to the retailer for settling the account is greater than cycle time.

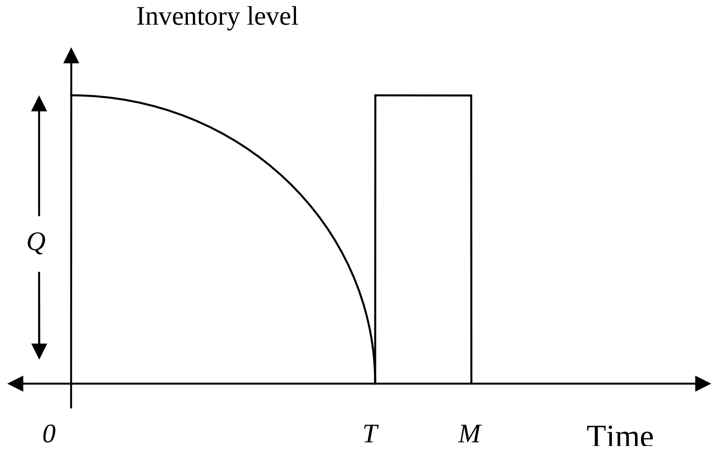

Figure 3.2 $M > T$

Here, the interest charges is zero i.e.

$$IC_2 = 0 \tag{3.12}$$

and the interest earned; IE_2 is

$$IE_2 = PI_e \left[\int_0^T Rtdt + RT\left(M - T\right) \right] = PI_e R\left(M - \frac{T}{2}\right) \tag{3.13}$$

Thus, total cost $(K_2(T))$ per time unit is

$$K_2(T) = \frac{1}{T}\left[OC + IHC + CD + IC_2 - IE_2 - SV\right] \tag{3.14}$$

For the optimal value T, solve

$$\frac{\partial K_2(T)}{\partial T} = -\frac{1}{T^2}\left\{ hR\left(\frac{T^2}{2} + \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right) + \frac{C\alpha RT^{\beta+1}}{\beta+1}(1-\gamma) + A - PI_e R(M - \frac{T}{2}) \right\}$$

$$+\frac{1}{T}\left\{ \begin{array}{l} hR\left(T + \alpha T\left(\dfrac{T^{\beta}\beta + T^{\beta} + \dfrac{(-\beta-1)T^{\beta+1}}{T}}{\beta+1} \right) + \dfrac{\alpha\beta T^{\beta+2}}{(\beta+1)T} \right) + \dfrac{C\alpha RT^{\beta+1}}{T}(1-\gamma) \\ + \dfrac{PI_e R}{2} \end{array} \right\} = 0$$

which minimizes $K_2(T)$ only if $\frac{\partial^2 K_2(T)}{\partial T^2} > 0$ for all T.

For $T = M$, we have

$$K_1(M) = K_2(M) = hR\left[\frac{M}{2} + \frac{\alpha\beta T^{\beta+1}}{(\beta+1)(\beta+2)}\right] + \frac{C(1-\gamma)\alpha RM^{\beta}}{\beta+1} + \frac{A}{T} - \frac{PI_e RM}{2} \tag{3.15}$$

4. A COMPUTATIONAL ALGORITHM

To find the optimal cycle time and hence the total cost, we follow the decision policy as shown in the flow chart in Figure 4.1 in the Appendix.

5. ANALYTICAL RESULTS

Proposition 5.1 The total cost is an increasing function of the deterioration rate (α).
Proof:

$$\frac{\partial K_1(T)}{\partial \alpha} > 0$$

where

$$\frac{\partial K_1(T)}{\partial \alpha}=\frac{1}{T}\left(\begin{array}{l} hR\left(\frac{\beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right)+(1-\gamma)\frac{CRT^{\beta+1}}{\beta+1} \\ +CI_c\left(\begin{array}{l} R\left(\frac{\beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right) \\ -R\left(\begin{array}{l} T\left(\frac{T^{\beta}M+\frac{(-\beta-1)M^{\beta+1}}{\beta+1}}{\beta+1}\right) \\ +\frac{\beta M^{\beta+2}}{(\beta+1)(\beta+2)} \end{array}\right) \end{array}\right) \end{array}\right)>0 \text{ for all } \alpha$$

$$\frac{\partial K_2(T)}{\partial \alpha}=\frac{\frac{hR\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{CRT^{\beta+1}}{\beta+1}-\frac{C\gamma RT^{\beta+1}}{\beta+1}}{T}>0 \text{ for all } \alpha .$$

Proposition 5.2 The total cost is a decreasing function of the allowable delay period *M*.

Proof:

$$\frac{\partial K_1(T)}{\partial M}=-CI_C R\left(\frac{T-M+\frac{\alpha T\left(T^{\beta}+\frac{(-\beta-1)M^{\beta+1}}{M}\right)}{(\beta+1)}+\frac{\alpha\beta M^{\beta+2}}{(\beta+1)M}-PI_e RM}{T}\right)<0$$

for all *M*

$$\frac{\partial K_2(T)}{\partial M}=-\frac{PI_e R}{T}<0 \qquad \text{for all } M$$

Proposition 5.3 The total cost is a decreasing function of the salvage parameter γ.

Proof:

$$\frac{\partial K_1(T)}{\partial \gamma}=\frac{\partial K_2(T)}{\partial \gamma}=-\frac{C\alpha RT^{\beta+1}}{\beta+1}<0 \qquad \text{for } 0<\gamma<1$$

In the next section, we consider a numerical example to validate the analytical results.

6. NUMERICAL EXAMPLE

In this section, we illustrate the aforesaid mathematical model using a numerical example.

Consider an inventory system with following parametric values in proper units:
[A, C, h, P, α *,* β *,* γ *, R,* I_c*,* I_e*, M]*
= [250, 50, 5, 75, 0.02, 1.5, 0.1, 1000, 18%, 14%, 15/365]

Then Case 1 is the optimal decision policy. The optimum cycle time T = 0.1853 years < 0.0411 = M and minimum total cost of an inventory system is $K_1(T)$ = $ 2298.74 and $\frac{\partial^2 K_1(T)}{\partial T^2}$ = 1.5728 > 0.

Next, we study the variations of the delay period; *M*, deterioration rate; α , shape parameter; β and salvage value; γ on decision variable and objective function.

Table 1: Effect of *M* and α when $\beta = 1.5,\ \gamma = 0..1$

α	M	15 days	20 days	25 days	30 days
0.02	T	0.1853	0.1850	0.1845	0.1840
	TC	2298.74	2170.06	2040.81	1909.08
0.04	T	0.1824	0.1820	0.1816	0.1810
	TC	2327.60	2198.79	2069.38	1937.48
0.06	T	0.1796	0.1770	0.1780	0.1783
	TC	2355.79	2227.14	2097.29	1965.51
0.08	T	0.1770	0.1767	0.1763	0.1758
	TC	2383.34	2254.26	2124.57	1992.35

Table 2: Effect of *M* and β when $\alpha = 0.02$, $\gamma = 0..1$

β	M	15 days	20 days	25 days	30 days
1.5	T	0.1854	0.1850	0.1847	0.1840
	TC	2298.74	2170.06	2075.17	1909.08
1.7	T	0.1861	0.1858	0.1854	0.1847
	TC	2288.84	2160.20	2065.34	1899.30
1.9	T	0.1867	0.1864	0.1860	0.1853
	TC	2282.32	2153.71	2058.87	1892.88
2.1	T	0.1871	0.1868	0.1865	0.1858
	TC	2278.00	2149.41	2057.59	1888.63

Table 3: Effect of *M* and γ when $\alpha = 0.02$, $\beta = 1.5$

γ	M	15 days	20 days	25 days	30 days
0.1	T	0.1854	0.1850	0.1847	0.1840
	TC	2298.74	2170.06	2075.17	1909.08
0.3	T	0.1860	0.1857	0.1853	0.1846
	TC	2292.34	2163.68	2068.80	1902.75
0.5	T	0.1867	0.1863	0.1860	0.1852
	TC	2285.90	2157.26	2062.41	1896.39
0.7	T	0.1874	0.1870	0.1867	0.1860
	TC	2279.43	2150.81	2055.97	1890.00

It is observed that the buyer's total cost decreases with increases in the delay period for a fixed deterioration rate. This is because the buyer can earn interest by generating more revenue from the sold items. The deterioration of the units increases the buyer's total inventory cost. Increase in the shape parameter, i.e. increase in the deterioration rate with time, increases the cycle time of the buyer and decreases the total cost of an inventory system. The buyer can reduce his total cost by incorporating salvage value to the deteriorated units instead of considering it to be a total loss.

7. CONCLUSIONS

The economic order quantity for time dependent deteriorated units with associated salvage value when the supplier offers a credit period to the retailer to settle the account is analyzed in this study. The model is sensitive to deterioration rate 'α' and a credit period 'M'. The retailer can keep an eye for low deterioration rate to reduce his total inventory cost. To reduce his total inventory cost, he can buy deteriorated units at a lower price and sell it off at the earliest.

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REFERENCES

[1] AGGARWAL, S. P. and JAGGI, C. K., (1995): Ordering policies of deteriorating items under permissible delay in payments. **Journal of the Operational Research Society,** 46, 658 – 662.

[2] ARCELUS, F. J., SHAH, NITA. H. and SRINIVASAN, G., (2003): Retailer's pricing credit and inventory policies for deteriorating items in response to temporary price discount credit incentives. **International Journal of Production Economics**, 81 – 82, 153 – 162.

[3] Chapman, C. B., Ward, S. C., Cooper, D. F. and Page, M. J., (1984): Credit policy and Inventory Control. **Journal of the Operational Research Society**, 35, 1055 – 1065
.
[4] CHAPMAN, C. B. and WARD, S. C., (1988): Inventory control and trade credit –A future Reply. **Journal of Operational Research Society**, 39, 219 – 220.

[5] CHU, P., CHUNG, K. J. and LAN, S. P., (1998): Economic order quantity of deteriorating items under permissible delay in payments. **Computers and Operational Research** , 25, 817 – 824.

[6] CHUNG, K. J., (1998): A theorem on the deterioration of economic order quantity under conditions of permissible delay in payments. **Computers and Operations Research**, 25, 49 – 52.

[7] CHUNG, K. J. (2000): The inventory replenishment policy for deteriorating items under permissible delay in payments. **OPSEARCH**, 37, 267 – 281.

[8] CHUNG, K. J. and LIAO, J. J. (2006) : The optimal ordering policy in a DCF analysis for deteriorating items when trade the credit depends on the order quantity. **International Journal of Production Economics**, 100(1), 116 – 130.

[9] CHUNG, H. J., HUNG, C. H. and DYE, C. F., (2001): An inventory model deteriorating items with linear trend demand under condition of permissible delay in payments. **Production and Planning and Control**, 12, 274 – 282.

[10] DAELLENBACH, H. G., (1986): Inventory Control and trade credit. **Journal of the Operational Research Society**, 37, 525 – 528.

[11] DAELLENBACH, H. G., (1988): Inventory Control and trade credit – a rejoinder. **Journal of the Operational Research Society**, 39, 218 – 219.

[12] GOYAL, S. K., (1985): Economic order quantity under conditions of permissible delay in payments. **Journal of the Operational Research Society**, 36, 335 – 338.

[13] GOYAL, S. K. and GIRI, B. C., (2001): Recent trends in modeling of deteriorating inventory. **European Journal of Operational Research**, 134, 1 – 16.

[14] HALLEY, C. G. and HIGGINS, R. C., (1973): Inventory policy and trade credit financing. **Management Science**, 20, 464 – 471.

[15] JAGGI, C. K. and AGGARWAL, S. P., (1994): Credit financing in economic ordering policies of deteriorating items**. International Journal of Production Economics**, 34, 151 – 155.

[16] JAMAL, A. M. M., SARKER, B. R. and WANG, S., (1997): An ordering policy for deteriorating items with allowable shortages and permissible delay in payments. **Journal of the Operational Research Society**, 48, 826 – 833.

[17] , A. M. M., SARKER, B. R. and WANG, S., (2000): Optimal payment time for a retailer under permitted delay of payments by the wholesaler. **International Journal of Production Economics**, 66, 59 – 66.

[18] KINGSMAN, B. G., (1983): The effect of payment rules on ordering and stockholding in purchasing. ***Journal of the Operational Research Society***, 34, 1085 – 1098.

[19] LIAO, J. J., (2007): On an EOQ model for deteriorating items under permissible delay in Payments. Applied Mathematical Modeling, 31(3), 393 – 403.

[20] LOKHANDWALA, K., SHAH, NITA H. and SHAH, Y. K., (2005): Optimal ordering policies under conditions of extended payment privileges for deteriorating items. ***Revista Investigacion Operacional***, 26(3), 1 – 8.

[21] MANDAL, B. N. and PHAUJDAR, S., (1989): Some EOQ models under permissible delay in payments. **International Journal of Management and Systems**, 5, 99 – 108.

[22] RAAFAT, F., (1991): Survey of literature on continuously deteriorating inventory model. **Journal of the Operational Research Society**, 42, 27 – 37.

[23] SALAMEH, M. K., ABBOUD, N. E., E1 – KSSAR, A. N.and GHATTAS, R. E., (2003): Continuous review inventory model with delay in payments. **International Journal of Production Economics**, 85, 91 – 95.

[24] SARKER, B. R., JAMAL, A. M. M. and WANG, S., (2001): Optimal payment time under permissible delay foe production with deterioration. **Production Planning and Control**, 11, 380 – 390.

[25] SHAH, NITA H., (1993): A lot – size model for exponentially decaying inventory when delay in payments is permissible. **Cahiers du centre D' Etudes de Recherche Operationnell Operations Research, and Applied Mathemati**k, 35, 1 – 9.

[26] SHAH, NITA H., (1993): Probabilistic order level system when delay in payments is permissible. **Journal of the Korean Operations Research and Management Science**, 18, 175 – 183.

[27] SHAH, NITA H., (1997): Probabilistic order – level system with lead – time when delay in payments is permissible. **TOP**, 5, 297 – 305.

[28] SHAH, NITA H., (2004): Probabilistic order – level system when items in inventory deteriorate and delay in payments is permissible. **Asia - Pacific Journal of Operational Research**, 21, 319 – 331.

[29 SHAH, NITA H., (2006): Inventory model for deteriorating items and time value of money for a finite time horizon under the permissible delay in payments. **International Journal of System Sciences**, 37, 9 -15.

[30] SHAH, NITA H. and SHAH, Y. K., (1998): A discrete – in – time probabilistic scheduling model for deteriorating items under conditions of permissible delay in payments. **International Journal of Systems Sciences**, 29(2), 121 – 125.

[31] SHAH, NITA H. and SHAH, Y. K (2000): Literature survey on inventory model for deteriorating items**. Economic Annals**, 44, 221 – 237.

[32] SHAH, V. R., PATEL, H. C. and SHAH. Y. K., (1988): Economic ordering quantity when delay in payments of orders and shortages are permitted. **Gujarat Statistical Review**, 15, 51 – 56.

[33] SHAH, NITA H., SHAH, B. J. and SHAH, Y. K., (2004): An order level lot – size model with stock dependent demand for time dependent deterioration under permissible delay in payments. **Far East Journal of Theoretical Statistics**, 12, 201 – 220.

[34] SHAH, NITA H. and TRIVEDI, C. J., (2005): An EOQ model for deteriorating items under permissible delay in payments when supply is random. **Revista Investigación Operacional**, 26 157–168.

[35] SHINN, S. W. AND HWANG, H. P and SUNG, S., (1996): Joint price and lot – size determination under conditions of permissible delay in payments and quantity discounts for freight cost. **European Journal of Operational Research**, 91, 528 – 542.

[36] SHINN, S. W. and HWANG, H., (2003): Retailer's pricing and lot – sizing policy for exponentially deteriorating products under the conditions of permissible delay in payments. **Computers and Industrial Engineering,** 24(6), 539 – 547.

[37] TENG, J. T., (2002): On the economic order quantity under conditions of permissible delay in payments. **Journal of the Operational Research**, 53, 915 – 918.

[38] WARD, S. C. and CHAPMAN, C. B., (1987): Inventory control and trade credit –a reply to Daellenbach. **Journal of Operational the Research Society**, 32, 1081 – 1084.

[39] YANG, P. C. and WEE, H. M., (2006): A collaborative inventory system with delay in payment for deteriorating items. **Mathematical and Computer Modelling**, 43, 209 – 221.

APPENDIX

Figure 4.1 Flow – chart for optimal decision policy.