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Teaching Building Structures Graphical Methods with GeoGebra

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HIGHLIGHTS

- Teaching-learning process of Building Structures
- Analysis of structures with graphical methods
- Parameterized structure analysis with GeoGebra
- Lateral Soil Pressure with Geogebra, generic problem

ABSTRACT

Graphic methods can be used when teaching Building Structures to solve some problems such as analysis of cables and arches through funiculars tracing; analysis of trusses with the Cremona method; graphically obtaining the lateral earth pressure of the ground on a wall and many other problems. This article presents some tools to solve these problems using GeoGebra, a free interactive mathematical software for education created by Markus Hohenwarter. The usefulness of Geogebra in teaching Building Structures will be presented through specifically created resources. These are new resources with approaches and utilities that commercial software does not have either. For each problem studied, the theory on which it is based is shown, the general description of the parameters used in Geogebra, and the application that has been created. In addition, the learning objectives to be achieved and the results docents will be shown.

Keywords: *Geogebra, Building structures, Funicular, Arches, Trusses, Lateral Soil Pressure*

1. INTRODUCCIÓN

Geogebra is a free interactive mathematical software for education created by Markus Hohenwarter from his thesis [1]. It is commonly used to solve problems of mathematics [2] [3] [4] [5] [6] [7]as geometry, algebra, statistics and calculus but it can be used in other areas as physics [8] [9] [10]of different educational levels, including engineering [11] [12], with resources applicable to civil engineering structures [13]. To a lesser extent there are resources in architecture for generating 3D shapes [14]. For building structures there are hardly any publications on their use, Pérez applies it to the optimization of precast concrete [15] and Escalada applies it to the study of an arcade of the Mosque of Cordoba [16].

Graphical methods for structural analysis have been used since ancient times for numerous stability problems. [17] [16]. Giving way from the second half of the 20th century to analysis with computer programs based on matrix methods and finite elements finite elements method (FEM).

In this article, graphical methods are joined in a wide variety of problems, with automated and parameterized analysis thanks to Geogebra software. Its implementation will be seen in the teaching of several subjects of Building Structures in the Degree of Fundamentals of Architecture (GFA for its acronym in Spanish).

2. METHODOLOGY

A series of resources have been created with Geogebra; they are used in Building Structures subjects at the ETSAM of the Technical University of Madrid (UPM for its acronym in Spanish). These resources are integrated into the remote teaching platform Moodle, accessible by registered students. The resources are available to practice and learn, and this activities can be evaluated as shown below.

As a sample of the multitude of problems that can be solved using graphical methods, equilibrium calculations will be studied, such as:

a) analysis of cables and arches through funiculars and antifunicular tracing. In both case, the internal forces will be balanced with the external force polygon;

b) analysis of trusses with the Cremona method, where the loads are balanced node by node with the requests of the bars that concur in it;

c) obtaining lateral earth pressure on a wall by graphic methods, where the thrust is balanced by the reaction between the wall and the ground and the friction force between the ground that slides and the ground that remains.

For each problem studied are shown: theoretical basis, the general description of the parameters used in Geogebra, and the application that has been created. In addition, the learning objectives to be achieved and the teaching results in terms of abilities through the exercises solved by the students also will be shown.

2.1. Geogebra

Geogebra has a graphical interface that allows the representation in 2D and 3D of functions and geometry. It can be a useful tool to solve structural problems. Two graphical views can be displayed simultaneously with different information (its application will be shown later), in addition to other views that are not used in the resources created, such as symbolic calculation, 3D graphic view, spreadsheet and probability calculations.

3. CABLES

Cables and arches are structures where geometry is decisive in their structural behavior, the former through tension, the latter through compression and both cases can be solved with graphic methods with the tools offered by Geogebra.

3.1. Theoretical basis and parameters.

In a cable there can only be tension stresses along its axis and when loaded it automatically adopts the shape of the most effective structure according to the load system, the funicular polygon. There are many resources for tracing funiculars in the Geogebra community e.g. [18]. The solution is obtained by solving the balance of each force with two other adjacent ones in funicular. The forces can be in any direction, as in many physical problems; in Building Structures the predominant situation is the gravitational action, hence the acting forces, are vertical and downwards.

The first step to trace the funicular is to draw the loads scaled; a point is chosen (O, named pole), whose position is arbitrary and external to the loads; the force polygon is closed joining the extremes of the loads with the pole. These two vectors (polar rays) represent the value and direction of the forces that balance the loads and form the polygon of forces (**Fig. 1**). Tracing parallels to the different polar rays forming polygons with apices at the line of actions of the loads the funicular is formed (**Fig. 1**, left). The rise and thrust (H) of the funicular changes as the pole separates from the loads (the shorter the distance, the smaller the thrust, and the higher the rise). Therefore, there is no single solution.

3.1. Application.

This resource for funicular polygons is publicly available through Geogebra, it can be used for first-year students of Structures and as a means for students in the last years of the Degree to remember the layout of the funicular for later use, as will be seen in the following apart or as antifunicular.

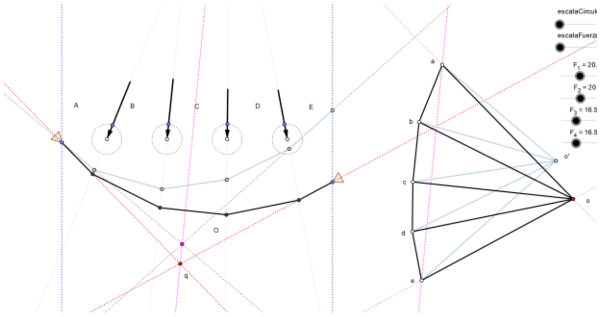

Fig. 1*: Funicular polygons passing through two know points [17]*

3.2. Teaching objectives and results.

funiculars are studied in the first courses of Building Structures [19], as well as the procedures to make them pass through two or three specific points. The typical case of the funicular going through three points will be used in three-hinged arches. The educational objective of the funiculars is not only to determine the form that a thread takes in a specific load system, but also the relationship it has with the bending moment diagram. On a beam, the funicular tracing of the loads coincides with the moment diagram. To know the value of the moment at each point is enough to multiply the thrust by the distance between the funicular and the line that joins the supports. In frames, the moment diagram is obtained by multiplying thrust by the distance between it and the directrix of the bars, measured perpendicularly.

The use of antifuniculars in advanced Structures courses allows students to dare to design structures with greater formal freedom, something that they would not do otherwise. As an example (**Fig. 2**) in the Project of Structures subject of the 5th year of the Degree in Fundamentals of Architecture, a team of students can use them for the project and analysis of a long-span roof in the form of an arch made with steel bars as a curved truss.

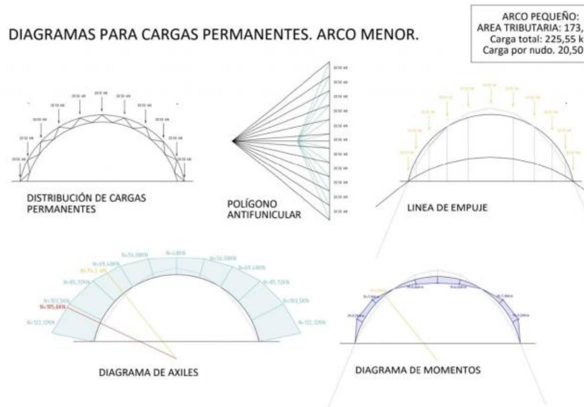

Fig. 2: *Work carried out by María García Piñeira, Zaid Aissaoui and Adriana Fernández López for the Structures Project subject in the 2020-2021 academic year*

The advantage of studying funiculars with GeoGebra is that it can be parameterized with variable load values and variable pole position so that it is easy to see how these parameters influence the shape of the funicular and cable stresses. In addition, it is sometimes difficult to make the student who even comes with a good mathematical and physical background see that a problem can have more than one solution. The layout of a funicular has infinite solutions, only if you want it to pass through three certain points the solution is unique, but those three points can be chosen by the structural designer based on their interests to achieve a specific structural behavior.

4. MASONRY ARCHES AND VAULTS

4.1. Theoretical basis.

Arches and masonry vaults are common in historical structures. For centuries, they have been built taking just balance into account. The only physical variable is the self-weight, which in a homogeneous material is proportional to the volume, and therefore the problem can be solved with only geometry and graphic methods.

The materials used have high compressive strength, so they usually easily comply with the strength condition. Nevertheless, their resistance to tension is very small and therefore they crack (in case of disaggregated materials such as adobe or rammed earth, etc., the lack of cohesion can cause the structure to fail) or they detach (with bonded materials such as brick or the masonry). In the case of disaggregated materials, this appearance of cracking can lead to deterioration of the structure and its subsequent failure. In materials bonded with different loading situations, these fissures can be opened or closed. To avoid failure, the appearance of tensions must be avoided or reduced.

Cracks in masonry structures allow the arch to adapt to the strains. This is only possible to understand within the framework of the plastic analysis of structures. If there is a line of thrusts (trajectory of forces) within the arch, it will not fail. The safety of the arch, therefore, depends on its geometry [20].

This makes it an easily solvable problem with graphic methods and therefore with Geogebra. It is about tracing the antifunicular, also known as the trajectory of the loads or thrust line. If a thrust line that passes through the interior of the masonry can be traced, (middle third in disaggregated or center-half in bonded materials), tensions are avoided, and the problem is solved. In addition, this line of thrusts must not have an excessive inclination relative to the contact surface between pieces (less than the angle of friction so that it does not slide), this requirement is resolved with the stereotomy of the pieces. This methods was common until early XX century. It began to fall into disuse later with the Cross method and practically disappeared with the use of computers to calculate structures

The typical thrust line of three-hinged arches can be easily drawn. Because they are made of elastic materials and their building process prevents, in most cases, hyperstatism, it is usually considered that the joints are in the arch springer and, by symmetry, in its keystone. If the arch is symmetrical, one half must be in equilibrium with the other and therefore the thrust line of the half arch can be drawn.

If a symmetric arch is subjected to gravitational load (**Fig. 3**) with Geogebra the thrust line of the half-arch loads can be traced. For it, the arch or vault is divided into strips that coincide with the voussoirs. The -weight of each piece and the filling or load on it are determined. Following the steps of the funicular construction, the polygon of forces (right) is drawn, arranging all loads as vectors, consecutively. By symmetry, this path must be horizontal in the keystone and therefore the pole, O, is chosen in the horizontal of the upper end of the loads. The polar rays that connect each end of the loads with the pole are traced. The trajectory of the loads on the arch (left) is obtained by drawing the parallel to the polar rays consecutively on the arch, between the lines of action of the sliding vector representing the load, starting with the keystone.

This way, loads trajectory is obtained on the arch. The value of the compression stresses is represented to scale in the polygon of forces, (the length of polar rays). Each load is balanced with the two contiguous forces in the polygon. Each inner force can be decomposed at each arch section, in two components: a parallel one (normal stress) and a perpendicular (tangential stress) one. To prevent the voussoirs from slipping, the tangential component must be less than the frictional force (proportional to the axial component and the tangent of the friction angle of the material), which implies an adequate stereotomy of the arch, with the joints orthogonal to the thrust line.

4.2. Application.

It is a resource that, due to its simplicity, GFA final year students create with GeoGebra in the Building Structures Intensification course. (**Fig. 3**).

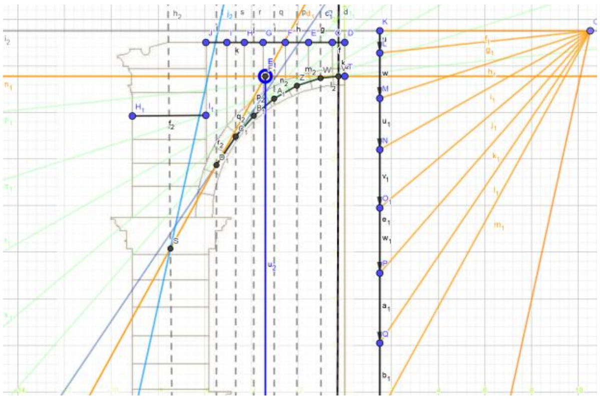

Fig. 3: *Anti-funicular form load trajectory in a masonry arch (left), and forces polygon (right) by Enrique Martínez-Gaya for the subject Intensification of Structures*

4.3. Objectives and docent results.

In addition to learning the theory of Arches out of the box, the goal of doing so with GeoGebra is the ability to do parametric computation.

Again, there is no single solution, although in this case solutions are more bounded than in the previous one. There are many ways to balance loads and thus many different lines of thrust. With the usual procedure, until now, the student traced a single load trajectory of the case presented as an exercise and it was more difficult for him to understand that this solution is only one of the many possible solutions; It was less intuitive.

The tracing of the trajectories of the loads on the section of an arch is done in a parametric way, with the pole and starting point of the funicular as variables. Hence, the students starting from the initial data and varying the positions of the variable points, can draw several thrust lines balanced with the loads. Some of them will satisfy the stability condition, passing through the interior of the section, and others will not, being outside of it. Those that meet the stability condition will be solutions to the problem.

Therefore, the first teaching objective is for the students to verify that there are multiple possible equilibrium situations. In this way they can play with the tool, the learning is more intuitive and the learning process in the classroom or outside is energized.

A second objective is for the student to understand that, unlike current concrete or steel structures, in which it is easy to distinguish what is load (for example, the enclosures) and what is structural (beams, joists, supports, etc..), in historic masonry structures both the seams and the arch are both load-bearing and resistant structures.

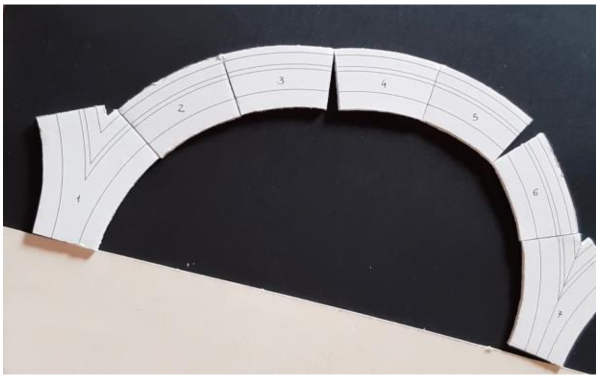

Fig. 4: *Model of masonry arch subjected to gravitational load and horizontal action.*

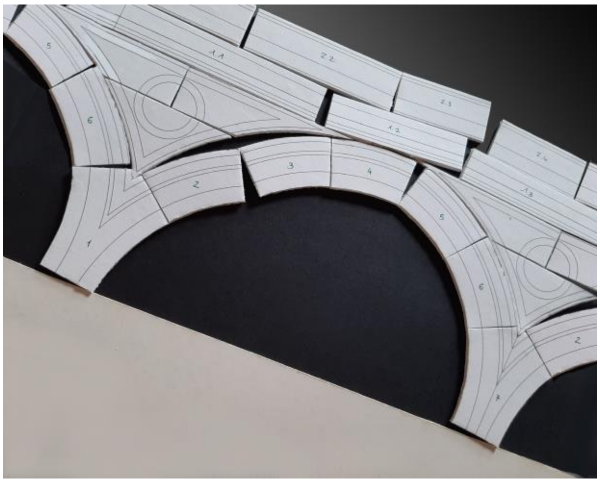

Fig. 5: Model of masonry arch and spandrel subjected to gravitational load and horizontal action.

In a complementary activity to this class, students can handle the physical model (**Fig. 4** y **Fig. 5**) of an arch with which they verify in a practical way how it behaves. Subsequently, the students are presented with an exercise in which they must draw the line of thrust of an archway with its spandrel subjected to gravitational actions.

The results obtained by students using this tool are satisfactory. At a general level, it can be affirmed that most of the students have understood the behavior at a global level.

5. TRUSSES ANALYSIS

5.1. Theoretical basis.

They are analyzed by the Cremona method, also called Cremona-Maxwell [21]. It is a graphic method where loads are balanced with inner forces in the directions of the bars. It was widely used when the balance of the forces that concur in a node was drawn by hand and to reduce the error when moving the forces in the drawing. Drawing each force only once reduced errors. Nowadays, thanks to CAD software that error is no longer possible, so the method is very useful when creating this Geogebra resource.

Tracing the Cremona polygon is similar to tracing the funicular with the difference that the direction of the internal force is determined by the direction of the bar and therefore there is only one solution. The concatenated tracing of the balance of all nodes, consecutively gives the Cremona polygon (**Fig. 6**, left).

5.2. Application.

This resource is aimed at students in their final year of the GFA, in the Structures Project subject. The students already know how to analyze and calculate trusses and it is intended that they design roof structures with great span.

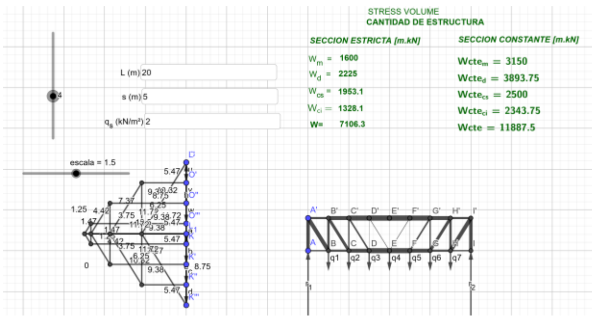

Fig. 6: Analysis of isostatic trusses for long-span roofs.

The application allows the analysis of isostatic trusses with variable data such as span, load per unit area, separation between trusses, to calculate a roof structure. The application allows the student to modify the geometry of the truss with the theoretical criteria taught in the theory of the subject, considering the cost expressed as the sum of the products of the length of the bars times their effort, see **Fig. 6** under the topic "cantidad de estructura".

To find the most efficient way (from the point of view of cost) the student can do a little research trying out different possible solutions. This formal exploration can be done with the geogebra resource , varying the position of the nodes or the edge at a global level. This rapid exploration of different alternatives allows the analysis to be obtained immediately and the students can see how the design and configuration decisions of the structure influence the final requests of the bars and therefore their dimensioning.

The analysis of the truss is shown in the thickness of the bars. The grater it is, the higher the internal forces. The Cremona polygon can also be consulted to obtain the number value of the axial stress.

5.3. Objectives and docent results.

The advantage over a conventional analysis program is that in it the structure must be fully defined to proceed with the analysis, not only the position of nodes and members but also their pre-dimensioning. Conventional use programs need this pre-dimensioning because they use a matrix method or FEM and must know the stiffness of the bars (depending on their length and section) to proceed with the analysis. These are more general methods useful for solving all types of structures at a professional level. In the learning stage, this makes the student believe that it is always necessary to start from a previous dimension of the bars. This is not always the case and in isostatic structures, it is not necessary. As it is only a problem of balance, to know the internal force of the bars it is only necessary to establish the balance node by node with the loads, reactions, and internal force of the bars previously analyzed, as is done in the created resource.

The student can easily try different alternatives and can look for other designs, an option that the usual structure calculation programs do not offer.

In **Fig. 7** is shown part of the research on the form of a student of the 2019-2020 academic year.

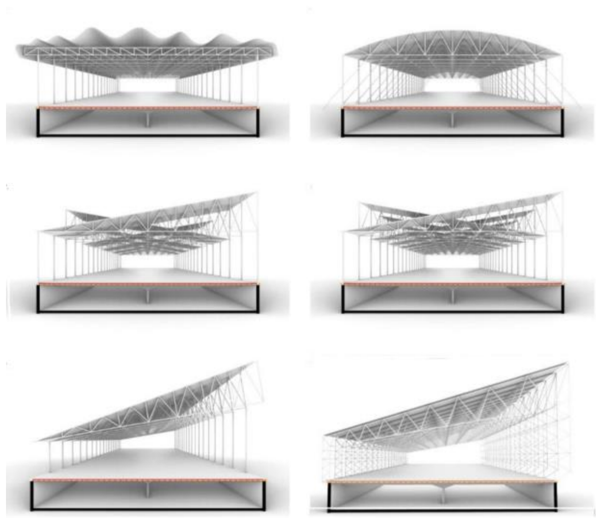

Fig. 7: *Study of different solutions to cover a space with a steel structure, image of Iñigo Clemente, Structures Project student of the 2019-2020 academic year*

6. LATERAL SOIL PRESSURE

6.1. Theoretical basis.

Soil lateral pressure failures are due to the sliding movement of one wedge of terrain over another.

Coulomb [22]proposes the first approach to the structural behavior of soil modeling considering its specific weight and its internal friction angle. He defined the concept of lateral earth pressure and deduced its value without needing to know the stresses in the soil retained by the structure.

He represents soil failure by a surface along which one wedge of soil slides. The lateral earth force is the largest reaction of all possible wedges. The worst surface is curved, but it would lead to great variability and would make the solution unattainable. Hence, the problem is simplified analyzing flat surfaces of contact between wedges. This approximation has given good enough results throughout history since they yield a value very close to the maximum force.

Once this force is known, the retaining structure can be designed.

Years later Rankine [23]obtained a specific formula for the trivial problem of a homogeneous terrain, without inclination, (α= 0), and without friction between soil and wall, that is the one that usually appears in soil mechanics handbooks.

6.2. Description of parameters

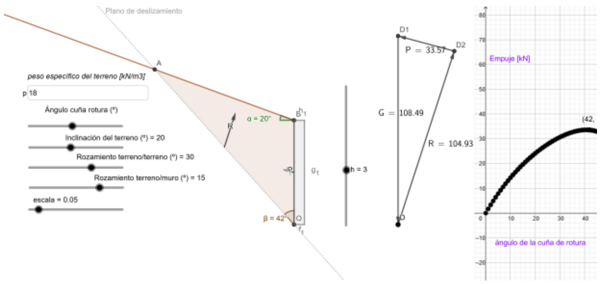

Fig. 8: Calculation of active lateral earth pressure on a retaining wall

Soil's Internal friction angle is usually around 30°. The friction angle between soil and wall is smaller, usually 20°. The critical wedge is around 30°. The case studied considers homogeneous soils, whose solution is trivial. The initial data can be changed with the sliders (**Fig. 8** on the left).

Calculations are simple: a polygon (triangle) of forces is built with the wedge weight, G, the reaction on the sliding plane, R, and the reaction on the wall, E, which is the lateral earth force. The problem is studied in section, so the sliding planes OA are straight (Fig.7). For a wall of height H, the position of point B is determined. All OAB wedges are studied, where O and B are fixed points. The line OA turns around point O forming the angle β with the intrados of the wall.

Using a slider for angle β, the problem can be studied for a given set of initial data. This is what is called Cullmann's graphic construction.

In the second graphic view, to the right of **Fig. 8**, the thrust for a certain value of the angle β is shown. The maximum value can be easily found and for what angle it is produced.

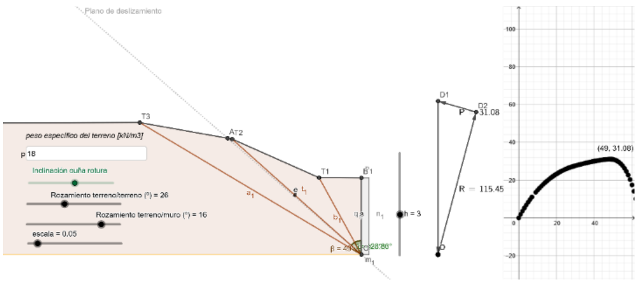

Fig. 9: Calculation of active lateral earth pressure for an irregular soil profile.

The process can be repeated for irregular soil profiles (**Fig. 9**). The additional factor added in this case is that the terrain is divided into different triangular wedges and a conditional formula on the position of point A, relative to the points T1, T2 and T3, is needed to calculate the weight. The position of points T1, T2 and T3 are determined by the user manually. Again in the graphic to the right the value of the lateral force of the wedge of soil against the wall is represented, and the maximum for which the wall should be designed. This problem does not have a specific formulation in Soil Mechanics handbooks. With this procedure other elements could be added, such as nearby constructions.

6.3. Objectives and docent results.

This tool is intended to help the students understand that the value of the active lateral force of the soil on a wall is not the result of a formula but depends on the equilibrium of the critical wedge when the sliding failure happens. We also want to highlight the application field of the current formula so that it cannot be used in other cases. Instead, the Coulomb method has a general application field. It covers all cases and can be represented with the GeoGebra tool adding other new parameters acting on the sliding wedge. The value of the lateral earth force depends on the geometric data of each case.

The student develops the ability to solve simple problems such as the following [24]:

Calculation of the overturning and sliding stability with sufficient safety of an open field retaining wall with mass concrete (**Fig. 10**)

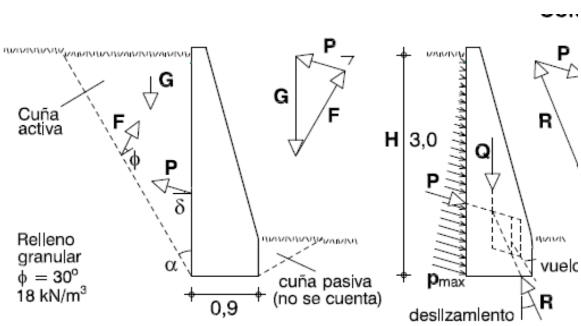

***Fig. 10:** Analysis of the overturning and sliding stability of a mass retaining wall in the open field.*

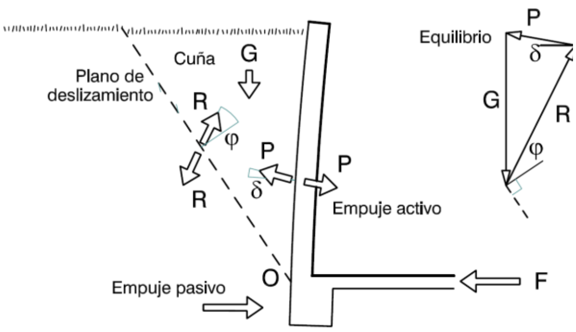

***Fig. 11:** Determination of the thrust of a homogeneous and horizontal ground on an English backfill.*

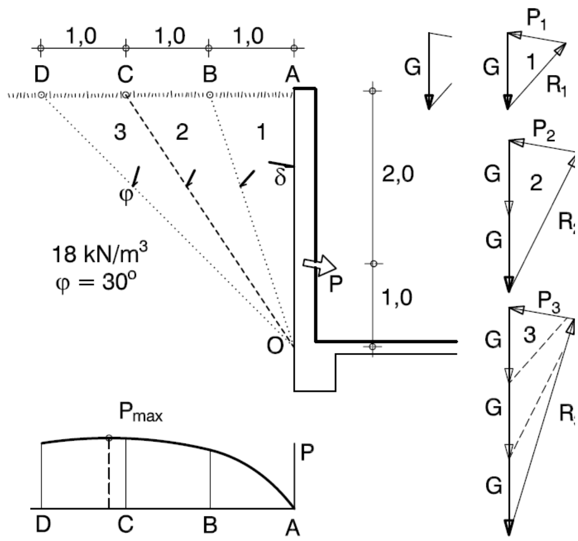

***Fig. 12:** Determination of the thrust of a homogeneous and horizontal ground on a basement wall.*

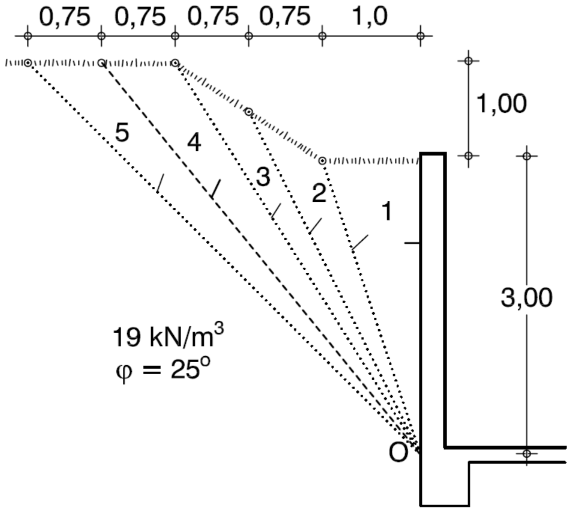

***Fig. 13:** Determination of the thrust of a homogeneous terrain with an irregular backfill.*

Calculation of the pressure produced by the ground in an English courtyard wall with known ground data (**Fig. 11**)

Like the previous one, the student develops the ability to calculate the thrust produced by the ground on a basement wall, knowing the parameters of the ground (**Fig. 12**).

These problems can be solved graphically or with Rankine's formula. The difference between both methods lies in the fact that graphically the student understands that the thrust is the greatest force that the sliding wedge exerts on the wall and from there develops the ability to solve different exercises that can only be solved graphically, such as the following:

Calculation of the thrust produced by the terrain with an irregular backfill layout on a wall is a problem that has no formula, and therefore the only way to solve it is with graphic procedures such as the one used with GeoGebra (**Fig. 13**).

Other problem without a formula that can be solved by graphical methods is the wall retaining a backfill with a slope equal to its internal friction angle, it can also be solved with the resource created, the student is asked to solve it graphically and then self-check his solution with Geogebra and see how the thrust varies with the variation of the ground angle. The objective of this exercise is for the student to see that the wedges are of indefinitely increasing size, but the thrust has a finite value (**Fig. 14**).

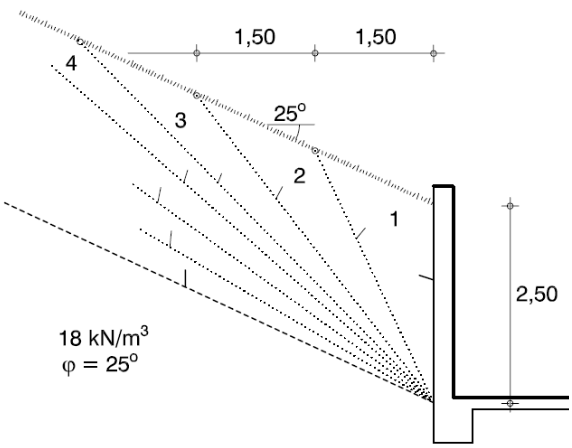

Fig. 14: *Determination of the thrust of homogeneous inclined terrain with a constant slope that coincides with the internal friction angle of the soil.*

The students use the resource varying the properties of the terrain and its shape, obtaining the value of the thrust and how it varies depending on the angle of the wedge, being able to obtain the worst, which is the one with which the wall is dimensioned and assembled.

The aim is for the student to develop the ability to calculate wall thrusts in very diverse but common configurations in the field of building structures, including those that cannot be solved with the usual formulae, such as determining the actions to calculate the stresses on the wall of heel hold (**Fig. 15**), the study of the ground thrust must include the fact that the BED wedge moves in solidarity with the wall. Again, there is no formulation for this problem.

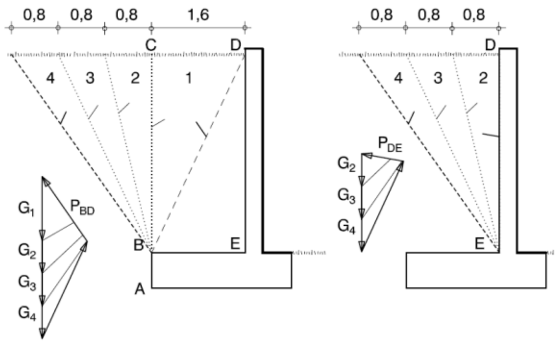

Fig. 15: *Determine the earth pressure on a retaining wall with heel.*

This resource is used in the subject Structural Types of the Master's Degree in Building Structures with satisfactory results. Proof of this is also the messages in the subject's forum, the students express how they have come to understand that the active thrust of terrain against a wall is not the result of applying a formula (which is only available for one case, that of horizontal and homogeneous terrain) but its physical meaning and later they demonstrate the ability to apply it through the proposed exercises.

7. LEARNING OUTCOMES

The questions raised here are in all cases equilibrium problems.

These tools are used in the usual teaching in different subjects of Building Structures. The learning results have been exposed in each of the sections below the GeoGebra resource used.

Globally, a transversal learning of the use of all these resources is that the structures can not only be solved by calculating them numerically, but also graphically, drawing solutions. In building, part of the architectural project is projected by drawing and graphic tools can be introduced from the first phases of the structural design.

They can also be used to draw moment diagrams and estimate the behavior of the structure at a global level, focused on concrete or steel frames. Or draw the thrust line of masonry structures. Therefore, the parameterized plotting resources created with Geogebra are very useful.

Its usefulness has also been shown in the formal search for solutions with the analysis of trusses and the disassociation that exists between the analysis and the pre-dimensioning of the structure. Again, it demonstrates the usefulness of parameterized graphical computations with Geogebra.

Finally, soil lateral pressure on a wall is a problem of balance. Its graphical resolution allows us to deal with all kinds of problems, many of which are not formulated today.

8. CONCLUSIONS

Initially, Geogebra was used as a tool for teaching mathematics subjects, and little by little it is making its way into other fields. Resources are emerging to support teaching in civil engineering structures, a few applicable to build. This article has shown several resources for teaching in this field of building structures by solving structural equilibrium problems, such as masonry arch thrust line, truss analysis, and wall thrust calculation.

This demonstrates the usefulness of Geogebra teaching of building structures. Its main advantage over other graphical tools is the possibility of programming the resolution based on the main parameters of each problem, the student can easily try different alternatives and learn by interacting with the tools.

The tools created allow facing problems that the usual structure calculation programs do not offer or automatically solving other problems that currently have no formulation, such as soil lateral pressure on walls.

These Resources reinforce the existing ones, facilitate learning when there is no single solution to the problem, or it can only be obtained graphically. It energizes the classroom that invites students to participate, it motivates them because they can play with the tools exploring different structural behavior.

9. ACKNOWLEDGMENTS

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