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Mathematical system based on taper functions for distribution by structural product of culms in three giant bamboo taxa

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Abstract

Aim of study: To generate a mathematical system to distribute structural products of bamboo culms.

Area of study: Northeastern region of the state of Puebla, Mexico.

Materials and methods: Eighty-seven culms of *Bambusa oldhamii* Munro, *Guadua aculeata* Rupr. and *Guadua angustifolia* Kunth were collected in Puebla, Mexico. Four taper functions were evaluated, the one with the best predictive capacity was fitted to model the diameter over and under wall together with a wall thickness model. The fitting strategy consisted of a system of additive equations using Weighted-Nonlinear Seemingly Unrelated Regression (WNSUR) procedure with autocorrelation correction, in combination with the Dummy Variable technique.

Main results: The Fang & Bailey case 1-a model was selected to describe the diameter over and under wall; the Cao and Papper model was used to model the wall thickness. The R^2_{adj} of the system fitted were 0.977, 0.944 and 0.918, and RMSE values 0.186 cm, 0.200 cm and 0.134 cm, for diameter over wall, diameter under wall, and wall thickness, respectively. *G. angustifolia* had the greatest taper and wall thickness, followed by *G. aculeata*. The highest proportion of primary product was presented by *G. angustifolia*. The system generated had parameters specific for each bamboo taxon.

Research highlights: The diameter profile of bamboo culms can be modeled by taper functions. A mathematical system for distribution by structural product type was developed consisting of: (1) a taper model for the diameter over wall and, (2) a function to estimate the commercial height..

Additional key words: *Guadua aculeata*; *Guadua angustifolia*; *Bambusa oldhamii*; diameter profile; primary product; round material; use in construction.

Abbreviation used: AIC (Akaike information criterion); CAR (continuous autoregressive error); dow (diameters over wall); Dow (diameter over wall at breast height); dowmin (minimum commercial diameter over wall at commercial height h); duw (diameters under wall); Duw (diameter under wall at breast height); DW (Durbin-Watson statistic); $\bar{E}$ (bias); FIML (full information maximum likelihood); h (height at different diameters over wall); H (total culm height); hst (stump height); R^2_{adj} (adjusted coefficient of determination); RMSE (root mean square error); v (volume); WNSUR (weighted-nonlinear seemingly unrelated regression); wt (wall thickness).

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Competing interests: The authors declare that they have no conflict of interest in carrying out the research work from which they derived the data used.

Introduction

Bamboo is widely distributed in tropical, subtropical, and some temperate zones of the planet (Du et al., 2018). This forest resource has attracted interest due to several ecological benefits such as its high potential for carbon sequestration, erosion control, water harvesting, and biodiversity conservation (Ceccon & Gómez-Ruiz, 2019). Its wide distribution, rapid growth rate, and excellent mechanical properties of its culms make bamboo a suitable material for multiple uses, among which construction stands out (Liese et al., 2015). The structural use of bamboo in its original form as it is found naturally in *ready-to-use* mode makes it a low-cost raw material in civil engineering works. In this sense, Trujillo (2016) indicated that external diameter and wall thickness are the most important dimensional properties for the use of bamboo culms in structural design, while the taper is the most relevant geometric property for the same purpose, so its study and analysis by taxon is essential.

Although more than 1,650 bamboo species have been recorded on the planet (Ruiz-Sánchez et al., 2021), not all of them can be used as a structural material in construction, so interest is focused on some genera that reach the largest dimensions in height and wall thickness, such as *Arundanaria, Bambusa, Cephalostachyum, Dendrocalamus, Gigantochloa, Melocanna, Phyllostachys, Schizostachyum, Guadua* and *Chusqea* (Correal, 2020). In Mexico, bamboo is naturally distributed in three main regions: (1) the western region, which includes the states of Michoacán, Colima, and Jalisco; (2) the southern region, which only includes the state of Chiapas; and (3) the central region, which includes the states of Puebla and Veracruz (Cedeño-Valdiviezo & Irogoyen-Castillo, 2011). It is possible to find natural stands of *Guadua aculeata* Rupr. bamboo, a native Mexican species that due to the commercial dimensions it reaches and its physical-mechanical properties, are frequently used in housing construction, mainly in the Totonaca culture region (Hernández & Torres, 2020). Important commercial plantations of the exotic bamboos *Guadua angustifolia* Kunth and *Bambusa oldhamii* Munro have also been established (Muñoz-Flores et al., 2021).

The species *B. oldhamii*, *G. aculeata* and *G. angustifolia* can reach heights of over 20 m and diameters over wall at breast height less than 20 cm in the case of the genus *Guadua*. Due to these characteristics, these taxa are labeled as giant bamboo (Toledo-Bruno et al., 2017). Their culms are used in rural constructions, furniture manufacturing, and handicrafts, and as tutors for tomato and vine crops (Camarillo-Cuenca et al., 2020). Recently, a large part of bamboo production has been absorbed by the construction industry, since in the last decade large tourism projects have been developed in which bamboo is considered the main material and structural element (Lorenzo et al., 2017). This allows for resilient constructions that are in harmony with the environment, while they function as carbon stores where this will remain sequestered for many years before being released into the atmosphere (Yadav & Mathur, 2021), an aspect that can contribute to reducing the phenomenon of global warming and climate change.

Despite the aforementioned importance, studies aimed at modeling the taper, predicting the external diameter, the wall thickness and therefore internal diameter, and estimating the distribution by type of product in bamboo culms are scarce both abroad and in Mexico. For this reason, there is a need to develop quantitative tools in the form of specific mathematical systems to determine the distribution of structural products for bamboo stands in both natural forests and commercial plantations. To address this issue, the approach used in timber-yielding tree species can be used, which consists of modeling the diameter profile using taper functions, whose mathematical expressions simulate a progressive thinning from the base to the tip of the stem (Ulak et al., 2022).

Strictly speaking, bamboos are not trees, but the shape of the culm is similar to that of a stem as its diameter also decreases as height increases and its value is zero when full height is reached (Inoue et al., 2021), so its shape could be adequately represented by taper models. The advantage of using this type of model is that an expression can be derived that allows estimating the commercial height when a given commercial diameter over wall of interest is predefined at the tip of the culm and vice versa (Tamarit-Urias et al., 2014). Concerning this topic, Zhang et al. (2021), and Ulak et al. (2022) concentrate different types of taper models to describe the stem profile of tree species, from which also expressions of total and commercial volume can be derived, making it possible to develop fully compatible volume estimation systems. Among the wide variety of taper models available, the segmented type such as that of Fang et al. (2000) and those of variable exponent such as those of Kozak (1997) are the most used in tree species.

The former is relevant in the case of bamboo harvesting because, unlike trees, where what is marketed is their volume, bamboo culms are marketed in linear meters mainly in the construction market and industry, where a minimum commercial diameter over wall at the tip is commonly required in purchase-sale operations. The main aim of this study was to generate a mathematical system to perform the distribution by structural product of bamboo culms. The specific ones were: (1) to evaluate the quality of fit of four taper models to describe the diameter profile over and under wall, (2) from the model with the best predictive capacity derivate an expression that estimates the commercial height, and 3) a function to modeling the wall thickness in the bamboo species *B. oldhamii, G. aculeata*, and *G. angustifolia* from commercial plantations of the first and third species, while the second corresponds to natural stands in the northeast of the state of Puebla, Mexico. The fundamental assumption that was taken as a basis was that at least one taper model can be applied to describe the diameter over wall profile of culms of the aforementioned species with a low estimation error.

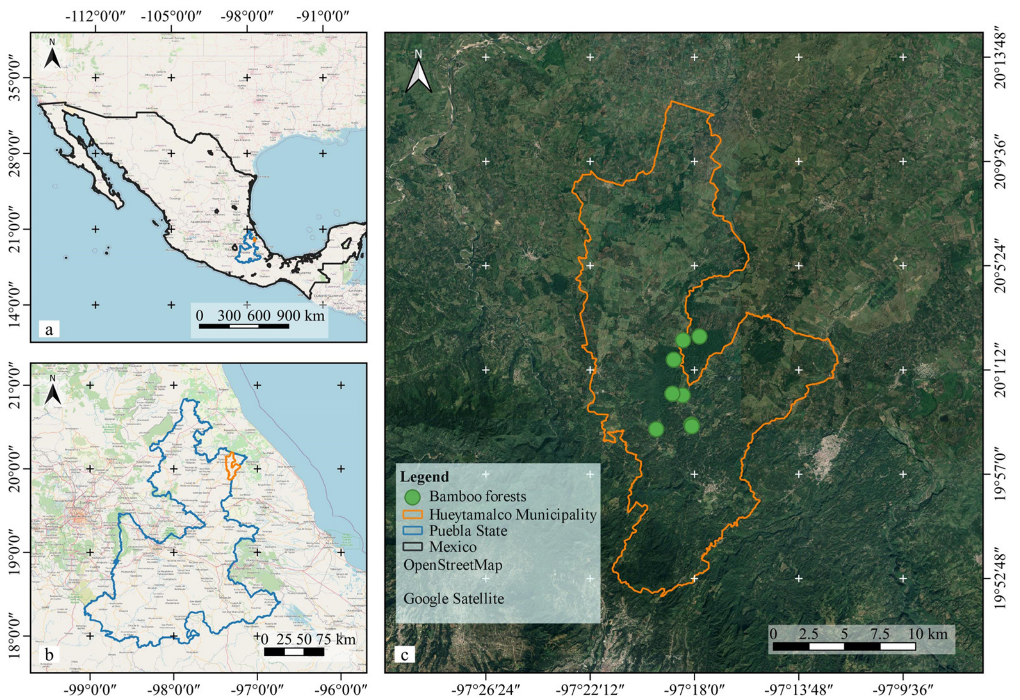

Figure 1. Geographical distribution of *Guadua aculeata* stands and commercial forest plantations of *Bambusa oldhamii* and *Guadua angustifolia*: a) Mexico, b) Puebla state, and c) study area.

Material and methods

Study area

The study was conducted in natural bamboo forests of *G. aculeata*, as well as in commercial plantations of *G. angustifolia* and *B. oldhamii* in the northeast of the state of Puebla, Mexico (Fig. 1), located between the Sierra Madre Oriental and the plains of "Tuxpan-Nautla" hydrological region No. 27. According to García (2004), the climate is of the Af(c) subtropical humid semi-warm type with year-round rainfall, a mean annual temperature of 21 °C and mean annual precipitation of 3,153 mm. The vegetation of the region is fragmented, as the continuous areas of primary vegetation are minimal, containing species such as *Brosimum alicastrum* Swarts., *Croton draco* Schltdl., *Matudaea trinervi* Lundell, *Cymbopetalum bailonii* N.A. Murray, *Guatteria amplifolia* Triana & Planch., *Alchornea latifolia* Sw., and *Dussia mexicana* (Standl.) Harms, as well as to a lesser extent *Cedrela odorata* L. and some species of *Quercus* (Ordóñez-Prado et al., 2022).

Data acquisition

Based on the culm maturity criteria by cohort referred to by Nath et al. (2018), we collected 87 mature bamboo culms aged 3 to 4 years, 30 of *B. oldhamii*, 27 of *G. aculeata*, and 30 of *G. angustifolia*. The sample captured the variability of sizes and shapes of commercial bamboos in the study area, with healthy specimens and free of physical or mechanical damage. Through destructive sampling, each selected culm was felled and sectioned. The following taper variables were measured in each specimen: total culm height (H); diameter over wall at breast height (Dow) obtained at a height of 1.3 m; stump height (hst) expressed in meters, defined as the distance between ground level and the upper part of the first culm node at which the felling cut is made; the height (h) at different diameters over wall (dow) along the culm until reaching the tip (dow=0). The culms of the bamboo species studied have the particularity of being hollow (the interior of the culm presents a space, except in the part of the nodes); for this reason, the diameter under wall at breast height (Duw) was measured, as well as the different diameters under wall (duw) at the same heights at which dow is recorded. This allowed the wall thickness (wt) to be calculated. Table 1 presents the basic descriptive statistics of the variables analyzed by species.

Data analysis

In total, 877 data pairs for dow – h and the same amount for duw – h, and wt – h were obtained. In a previous analysis, the database was used to fit 10 classic and current taper models reported in the specialized literature (McTague & Weiskittel, 2021), of which four were shown to be plausible candidates to describe the observed diameter over wall profile of bamboo culms (Table 2 and Fig. 2). These models have been frequently used to model the stem profile

Table 1. Basic descriptive statistics of the variables measured in the culms by species.

Variable[1]	N	Minimum	Maximum	Mean	SD[2]	CV[3]	Variance
			Bambusa oldhamii				
H (m)	30	7.12	23.71	18.54	5.46	29.46	29.84
Dow (cm)		4.30	12.40	9.07	2.38	26.24	5.68
Duw (cm)		2.30	9.35	6.68	2.08	31.14	4.31
hst (m)		0.02	0.37	0.19	0.08	41.01	0.006
wt (cm)		1.00	2.53	1.83	0.45	24.44	0.20
dow (cm)	285	1.10	12.80	6.79	3.04	44.77	9.25
duw (cm)		0.65	10.50	5.17	2.47	47.78	6.10
			Guadua aculeata				
H (m)	27	7.17	25.53	18.29	5.37	29.34	28.80
Dow (cm)		3.50	13.60	9.43	3.41	36.16	11.60
Duw (cm)		1.85	9.90	6.10	2.82	46.23	7.98
hst (m)		0.08	0.26	0.16	0.06	39.57	0.004
wt (cm)		0.70	3.50	1.93	0.64	33.36	0.41
dow (cm)	266	0.55	13.90	6.25	3.84	61.44	14.74
duw (cm)		0.30	10.70	4.27	2.93	68.62	8.56
			Guadua angustifolia				
H (m)	30	6.87	22.8	16.47	3.97	24.09	15.73
Dow (cm)		3.85	13.45	8.66	2.94	33.95	8.62
Duw (cm)		1.80	9.10	5.64	2.10	37.23	4.40
hst (m)		0.06	0.26	0.14	0.05	33.99	0.002
wt (cm)		0.88	3.50	2.17	0.74	34.17	0.55
dow (cm)	326	1.00	13.50	6.39	3.26	51.02	10.66
duw (cm)		0.40	10.40	4.63	2.43	52.48	5.93

[1] H = total culm height. Dow = diameter over wall at breast height (DBH). Duw = diameter under wall DBH. hst = stump height. wt = wall thickness. dow = diameter over wall at height h from the stump. duw = diameter under wall at height h from the stump. [2] SD = standard deviation. [3] CV = coefficient of variation.

of tree species (Diéguez-Aranda et al., 2006). They meet the constraint that when dow = 0, then H = h, a condition that allows predicting the dow at any commercial culm height required and vice versa (Tamarit-Urias et al., 2014; Hernández-Ramos et al., 2017). This analysis is essential to identify the taper model with the highest predictive capacity to describe the diameter over wall profile of the culms of the three bamboo species.

In the previous analysis referred to, different functions were also evaluated to predict wall thickness, of which the one corresponding to Cao & Pepper (1986) was the one selected as plausible in the present study (M5, Table 2). This function was successfully used by Zhang & Jiang (2015), and Stängle & Dormann (2018) because it has logical characteristics since wt is expressed as a function of dow and the variables h and H.

In this first phase, the estimation of the parameters of the taper models for the three species together was carried out, using the full information maximum likelihood (FIML) technique with the Model procedure of the SAS/ETS® 9.3 statistical package (SAS Institute Inc., 2011). The FIML technique minimizes the joint form errors that occur in the taper along the culm, maximizes the significance of the coefficients and assumes that the error is independent of a normal distribution (SAS Institute Inc., 2011).

The quality of fit of the analyzed models was evaluated by comparing the values of the following statistics: the highest adjusted coefficient of determination (R^2_{adj}), the lowest root mean square error (RMSE), the lowest bias ($\bar{E}$) and the lowest value in the Akaike information criterion (AIC), the latter being the better the smaller its value. For the selection of the best model, the rating criterion referred to by Tewari & Singh (2018) was applied, which consisted of ranking the values by statistic for each model. A value of 1 was assigned to the model whose statistic was the best, 2 to the second best, and so on for the rest of the values. The sum of the values formed the overall score of each model, and the best model was identified as the one that presented the lowest overall score.

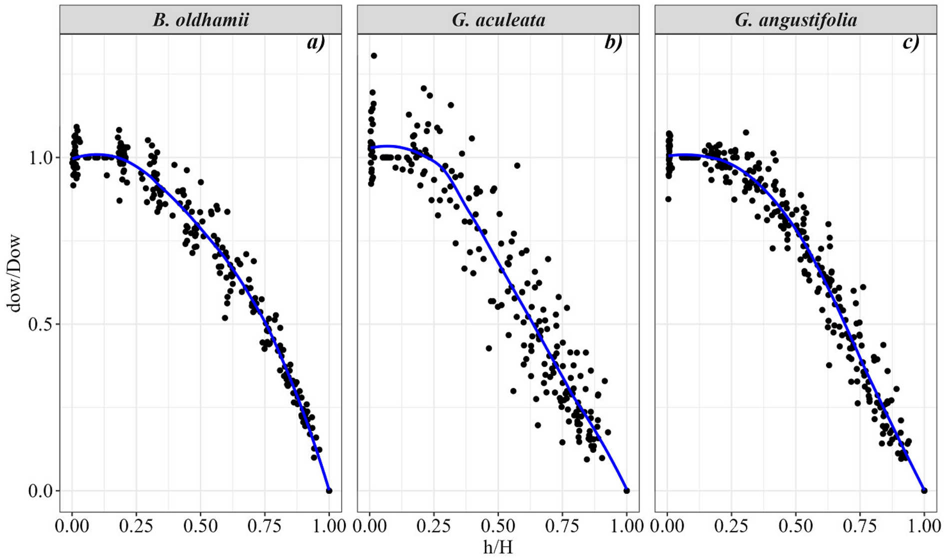

Figure 2. Relative diameter (dow/Dow) vs relative height (h/H) of the bamboo species analyzed.

$$R^2_{adj} = 1 - \left[\frac{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{(n-p)}}{\frac{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2}{(n-1)}} \right] \quad (1)$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{(n-p)}} \quad (2)$$

$$\bar{E} = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)}{n} \quad (3)$$

$$\text{AIC} = 2p + n \, ln \left(\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n} \right) \quad (4)$$

where $y_i, \hat{y}_i, \bar{y}$ are the observed, predicted and, average values, respectively; n is the number of observations; and p is the number of model parameters.

In the second phase, the best taper model was fitted to the diameter over and under wall data together with the wall thickness function (M5, Table 2). Based on Xu et al. (2021), a system of additive equations was formed, which was fitted using Weighted-Nonlinear Seemingly Unrelated Regression (WNSUR) procedure. This technique minimizes errors, maximizes the significance of the coefficients, and assumes that the error is independent of a normal distribution (Vonderach et al., 2018).

Given that the sample size per species was the minimum acceptable, to optimize the degrees of freedom of the error, based on Montgomery & Runger (2018), the dummy variables technique referred to in Rodríguez et al. (2015) was applied. This technique only requires fitting a single regression model, because the data is grouped by classes of interest. In this way, based on Barrio-Anta et al. (2006), each of the parameters of the system equations was expanded by including a parameter with an additive effect in conjunction with a dummy variable. The significance test of the parameters associated with each dummy variable was analyzed, with which it was detected whether each species requires specific values in each of the system parameters. The general structure of the expansion was:

$$\boldsymbol{\theta} + \gamma_i \text{I, with i = 1, … n}$$

where $\boldsymbol{\theta}$ is each parameter of the system equations; γ_i are the associated parameters to differentiate between species; I is a dummy variable whose value is 1 for a particular species and 0 for a different one; n is the number of species. The number of dummy variables is n-1.

During the simultaneous and compatible fitting of the system of additive equations based on Xu et al. (2020; 2021), different combinations of the functions (dow=duw+wt, duw=dow-wt, and wt=dow-duw) were tested to achieve the additivity property of the system. The option that provided superior goodness-of-fit statistics was selected, as well as the best logical graphic behavior in the dependent variables.

In this second phase, the correction of the autocorrelation problem was performed by incorporating a continuous autoregressive error (CAR) type error structure in which autoregressive terms of different order were tested (Montgomery & Runger, 2018); such a structure was defined with Eq. 5. In this case, the factor that reproduced the best homoscedastic pattern of the residuals vs. the predicted ones was selected. The autocorrelation correction was verified using the Durbin-Watson (DW) statistic, it is sought that the value of this statistic is close to 2 (Montgomery & Runger, 2018).

Table 2. Taper models of describing the diameter over wall profile of the culm.

Source	Model expression	Model
Fang & Bailey case 1-a (1999)	$\mathrm{dow} = \sqrt{\frac{\alpha_0}{\beta_0} \mathrm{Dow}^{\alpha_1} \mathrm{H}^{\alpha_2 - k/\beta_0} (\mathrm{H}-\mathrm{h})^{(k-\beta 0)/\beta 0}}$	1
	Solved to h	
	$\mathrm{h} = \mathrm{H} - \left(\frac{\mathrm{dow}^2}{\frac{\alpha_0}{\beta_0 \mathrm{Dow}^{\alpha_1} \mathrm{H}^{\alpha_2 - k/\beta_0}}} \right)^{\frac{\beta_0}{k-\beta_0}}$	
Lenhart & Clutter (1971)	$\mathrm{dow} = \left[\mathrm{Dow} \left(\frac{\mathrm{H}-\mathrm{h}}{\mathrm{H}-1.3} \right) \right]^{\frac{1}{\beta_1 - 2}}$	2
	Solved to h	
	$\mathrm{h} = \mathrm{H} - \frac{\mathrm{dow}^{\beta_1 - 2} (\mathrm{H}-1.3)}{\mathrm{Dow}}$	
Demaerschalk (1972)	$\mathrm{dow} = \mathrm{Dow} \sqrt{\beta_0 \left(\frac{\mathrm{H}-\mathrm{h}}{\mathrm{H}} \right)^{\beta_1}}$	3
	Solved to h	
	$\mathrm{h} = \mathrm{H} - \left(\frac{\mathrm{dow}^2}{\beta_0 \mathrm{Dow}^2} \right)^{\frac{1}{\beta_1}} \mathrm{H}$	
Kozak et al. (1969)	$\mathrm{dow} = \mathrm{Dow} \sqrt{\beta_0 \left(\frac{\mathrm{h}}{\mathrm{H}} - 1 \right) + \beta_1 \left(\left(\frac{\mathrm{h}}{\mathrm{H}} \right)^2 - 1 \right)}$	4
	Solved to h	
	$\mathrm{h} = \frac{\mathrm{H} \left(\sqrt{\mathrm{Dow}^2 \beta_0^2 + 4\beta_1 (\mathrm{Dow}^2 \beta_0 + \beta_1 \mathrm{Dow}^2 + \mathrm{dow}^2)} - \mathrm{Dow} \beta_0 \right)}{2\beta_1 \mathrm{Dow}}$	
Cao & Pepper (1986)	$\mathrm{wt} = \mathrm{dow} \left(\delta_1 + \delta_2 \frac{\mathrm{h}}{\mathrm{H}} + \delta_3 \left(\frac{\mathrm{h}}{\mathrm{H}} \right)^2 + \delta_4 \mathrm{H} \right)$	5

dow = diameter over wall (cm) at height *h* (m) from the stump. Dow = diameter over wall at breast height (m). H = total culm height (m). k = π/40000. h = commercial height at a minimum diameter over wall given at the tip of the culm (dow_{min}). wt = culm wall thickness. α_i, β_i, δ_i = parameters to estimate.

$$e_{ij} = \sum_{k=1}^{k=x} I_k \rho_k^{t_{ij} - t_{ij-k}} e_{ij-k} + \varepsilon_{ij} \quad (5)$$

where e_{ij} is the *j*-th ordinary residual at the i-th culm, e_{ij-k} is the *j*-th ordinary residual at the *i*-*k*-th culm, $I_{k=1}$ for $j > k$ and is zero for $j \leq k$, ρ_k is the autoregressive parameter of order *k* to be estimated, t_{ij}-t_{ij-k} is the distance in meters separating the *j*-th from the *j*-*k*-th observation within each culm with $t_{ij} > t_{ij-k}$, ε_{ij} is the error term under the independence condition.

The heteroscedasticity problem was also corrected employing power function to weight the variance of the residuals (Montgomery & Runger, 2018), for which different weighting factors were tested from combinations of the independent variables Dow and H (Dow, Dow^2, $\mathrm{Dow}^2\mathrm{H}$, H, H^2, $\mathrm{Dow}^{0.5}$) (Picard et al., 2012). In this case, the factor that reproduced the best homoscedastic pattern of the residuals vs. the predicted ones was selected. In this phase, to achieve a dynamic update of the residuals, the fit of the system of equations was performed simultaneously with both the autoregressive structure and the power function using the Model procedure of the SAS/ETS® 9.3 statistical package (SAS Institute Inc., 2011). The "*ggplot2*" library 3.3.5 of the R® statistical software 4.1.1 (R Development Core Team, 2021) was used to prepare the graphics.

The mathematical system to perform the distribution by type of structural product was generated from the best

Table 3. Goodness-of-fit statistics and the overall score of the taper models were evaluated to describe the diameter profile of bamboo culms.

Model	R^2_{adj}	RMSE	$(\bar{E})$	AIC	Overall score
1	0.9514	0.8263	-0.0252	-330.7	7
2	0.8684	1.3599	0.5804	540.2	12
3	0.9506	0.8335	0.0032	-317.6	10
4	0.9432	0.8935	-0.0176	-195.6	13

R^2_{adj} = adjusted coefficient of determination. RMSE = root mean square error. $(\bar{E})$ = bias. AIC = Akaike information criterion.

taper model, from which the equation that estimate the commercial height was derived when a minimum commercial diameter over wall (dow_{min}) is specified at the tip of the culm and vice versa. Two types of products were defined: primary and secondary. Products were basically considered primary and thus suitable for structural use in construction (as columns, beams, and so-called *morillos*, defined as pieces with dimensions smaller than those of the beams used in rural ceilings) when the $dow_{min} \geq 6$ cm, whereas products were considered as secondary when the $dow_{min} < 6$ cm. This latter group includes uses such as *tutors* in agricultural crops, stakes, poles, canes, fishing rods, chips, cellulosic material, etc. The dimensions in dow_{min} referred to were determined from a documentary review and interviews with both bamboo producers and professionals in bamboo construction. The mathematical system to be generated can be used to reproduce a table with distribution by type of structural product.

Results

The values of the goodness-of-fit statistics of the fitted taper models explained from 86.84% to 95.14% of the observed variability of the taper of the bamboo taxa studied; the estimation error ranged from 0.8263 cm to 1.3599 cm (Table 3). The rating system implemented revealed that model 1 presented the best predictive capacity; in particular, it had the highest R^2_{adj} value of 95.14%, as well as the lowest RMSE value of 0.8263 cm. Model 3 ranked second in plausibility. Such evidence led to selecting model 1 as the appropriate one to fit the data of over and under wall taper of the bamboo culms for the species studied. Model 1 has the additional advantage that the parameters α_0, α_1, and α_2 correspond to the total culm volume (v) model known as the Schumacher-Hall ($v=\alpha_0 Dow^{\alpha_1} H^{\alpha_2}$) (Fang & Bailey, 1999). Therefore, this model can additionally be useful to estimate the respective apparent volume and infer other relevant variables such as biomass and carbon content. In contrast, model 4 obtained the highest score, which makes it the least appropriate to describe the diameter profiles of the studied taxa.

The system of additive equations fitted through the WNSUR method and under the dummy variable technique showed non-significance in some parameters, so it is inferred that these are common, and therefore it is feasible to use the same values to describe profile culm. It was determined that the best combination of the system that achieves the additivity property, provides superior goodness-of-fit statistics, and has the best logical graphic behavior is defined as duw=dow-wt. Table 4 shows the statistics of the fitting, e.g., for dow, the model explained on average 97.7% (R^2_{adj}) of the observed variability with an estimation error < 0.186 cm (RMSE), which confirms that model 1 is suitable for describing the over wall taper of culms at the bamboo taxon level. In the adjustment of the final structure of additive equations system with indicator variables, all estimated parameters were highly significant ($p<0.05$), highlighting the robustness of the model to predict this response variable (Table 5).

The assumption made that at least one taper model is applicable to describe the culm diameter profile of the studied species was confirmed. Based on bias values, model 1 tends to slightly underestimate the over wall taper in all three species. As for the modeling of wt using Eq. 1, on average an explanation of variability of 91.8% (R^2_{adj})

Table 4. Fit statistics for model 1 and 5 of the bamboo species studied when applying the WNSUR procedure, autocorrelation correction and, variable dummy technique.

Model	R^2_{adj}	RMSE	$(\bar{E})$	AIC	DW
dow	0.977	0.186	0.020	-3235.288	1.237
duw	0.944	0.220	0.004	-2896.627	1.609
wt	0.918	0.134	0.016	-3859.416	1.960

R^2_{adj} = adjusted coefficient of determination. RMSE = root mean square error. $(\bar{E})$ = bias. DW = Durbin-Watson statistic. AIC = Akaike information criterion.

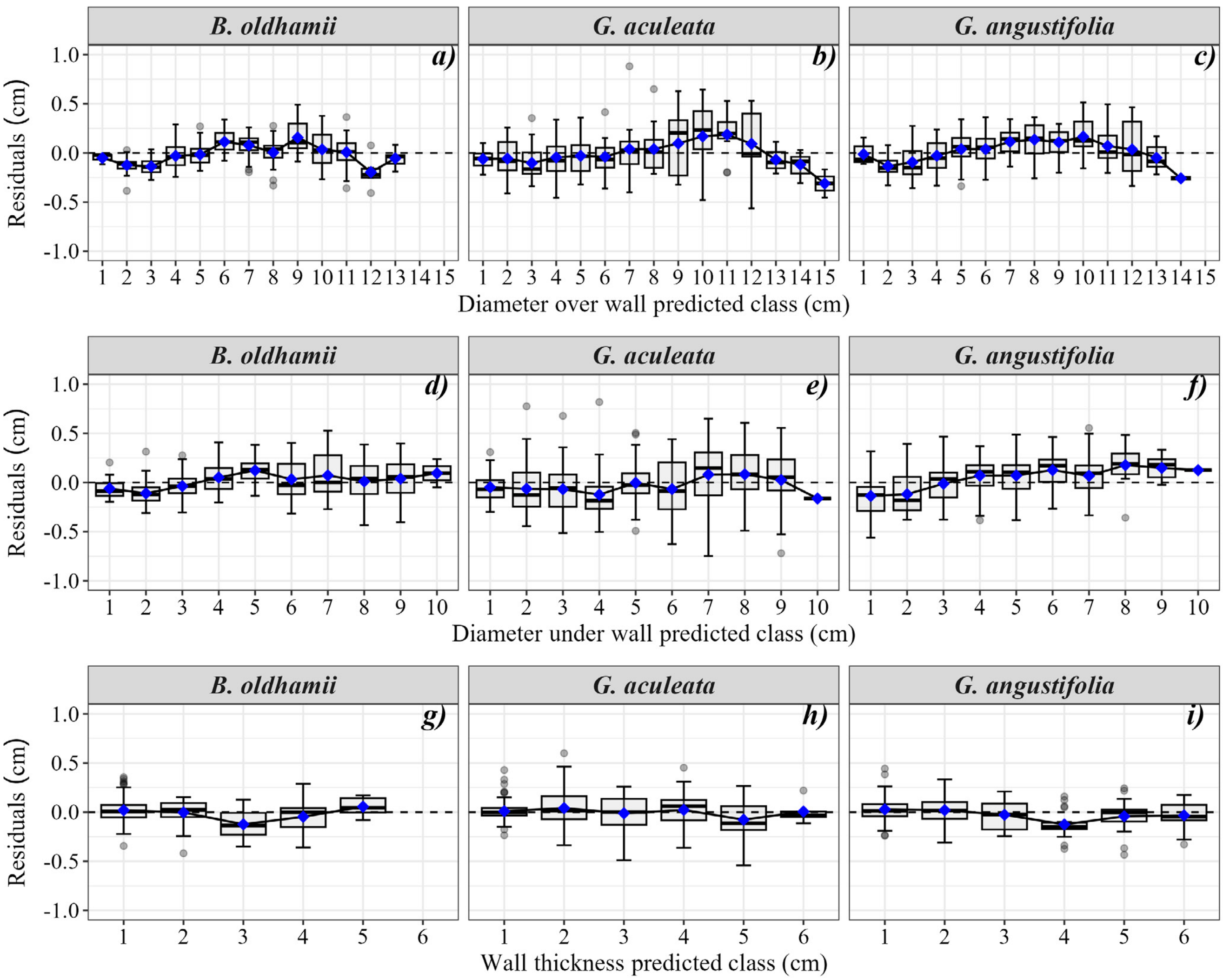

Figure 3. Graphical behavior of the residuals vs. predicted ones for the bamboo species studied.

was obtained with an estimation error < 0.134 cm (RMSE) (Table 4).

Regarding the values of the DW statistic, the autocorrelation correction with the CAR(2) structure minimized the effect of correlated errors, although to a lesser extent for dow which presented the lowest value (Table 4). The correction of heteroscedasticity for dow, wt, and duw determined the weighting factor that provided the best graphical behavior of the residuals vs. the predicted ones was the one corresponding to the variable $Dow^{0.5}$, the residuals

Table 5. Specific parameters by species obtained from the fitting of the system of additive equations.

Parameters	*Bambusa oldhamii*		*Guadua aculeata*		*Guadua angustifolia*	
	Estimate	SE	Estimate	SE	Estimate	SE
α_0	0.000056	0.000005	0.000056	0.000005	0.000056	0.000005
α_1	2.021715	0.070790	1.977602	0.061200	1.977602	0.061200
α_2	0.877979	0.068500	0.877979	0.068500	0.877979	0.068500
β_0	0.000038	0.000001	0.000031	0.000001	0.000034	0.000000
δ_1	0.571589	0.017200	0.678089	0.010400	0.678089	0.010400
δ_2	-1.204867	0.087100	-1.191966	0.090100	-1.822340	0.037000
δ_3	1.297631	0.131000	1.508161	0.137600	2.208541	0.056800
δ_4	-0.008110	0.000543	-0.012070	0.000862	-0.008110	0.000543

α_i and β_i parameters related to dow and duw. δ_i parameters related to wt. SE: standard error.

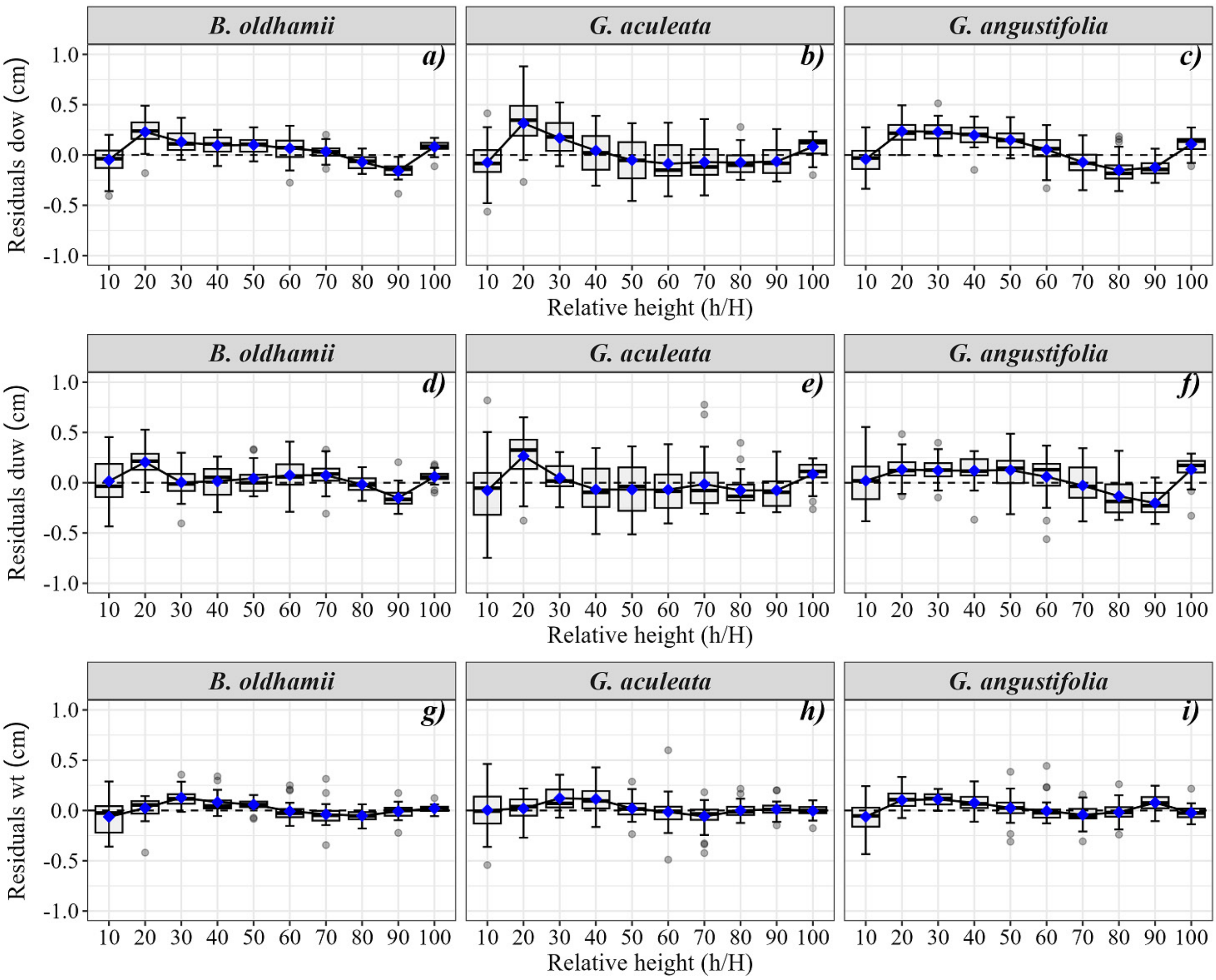

Figure 4. Graphical behavior of the error along different relative culm height for the bamboo species studied.

compared to those predicted for dow, wt, and duw by taxon show a pattern with a random distribution trend (Fig. 3).

The estimated dow presented errors smaller than ±5 mm by diameter class for bamboo species; for the dow_{min} of commercial interest, the error was less than ±2 mm (Figs. 4a-c). The errors in the estimation of dow as a function of h presented acceptable behavior along the culm, the errors remained within ±5 mm (Figs. 4d-f). This indicates that the model accurately represents the culm profile of the species and the proposed dow_{min} for the distribution of structural products (Fig. 5).

The estimation of dow at different heights with model 1, and modeling duw presents a logical behavior because they reproduce profiles similar to those observed in the taxa studied (Fig. 5). In this sense, the system of additive equations is consistent because a decrease in the diameter over and under wall is maintained, especially in the upper two thirds of the height (Fig. 5). The same situation occurs for wall thickness prediction. Additionally, the additivity property is fulfil led, where the duw results from the difference between the dow and the wt. It is also guaranteed that the predicted duw will never be greater than the dow. In addition, dow, and duw faithfully adhere to the different growth habit of each species since the dimensions and level of conicity vary by taxon. In this analysis, *G. angustifolia* is comparatively more conical (Fig. 5c), followed by *G. aculeata*, which implies that they have a larger base that contributes to both having greater stability. *B. oldhamii* presented a more homogeneous taper compared to the other two species, which makes it special for obtaining products of better decorative and artistic quality. It was found that the equation for duw tends to produce slightly overestimated values for *G. angustifolia* and *G. aculeata*.

Equation solved to h from model 1 was useful for estimating the commercial height at a minimum diameter over wall (dow_{min}) given at the tip of the culm (Table 2), which in turn is important to be able to carry out the distribution of culm length by product type and to derive the respective table with the classification of the commercial use of culms by product.

When a $dow_{min} \geq 6$ cm was set for the main use of the culm in structures for the construction industry and the rest

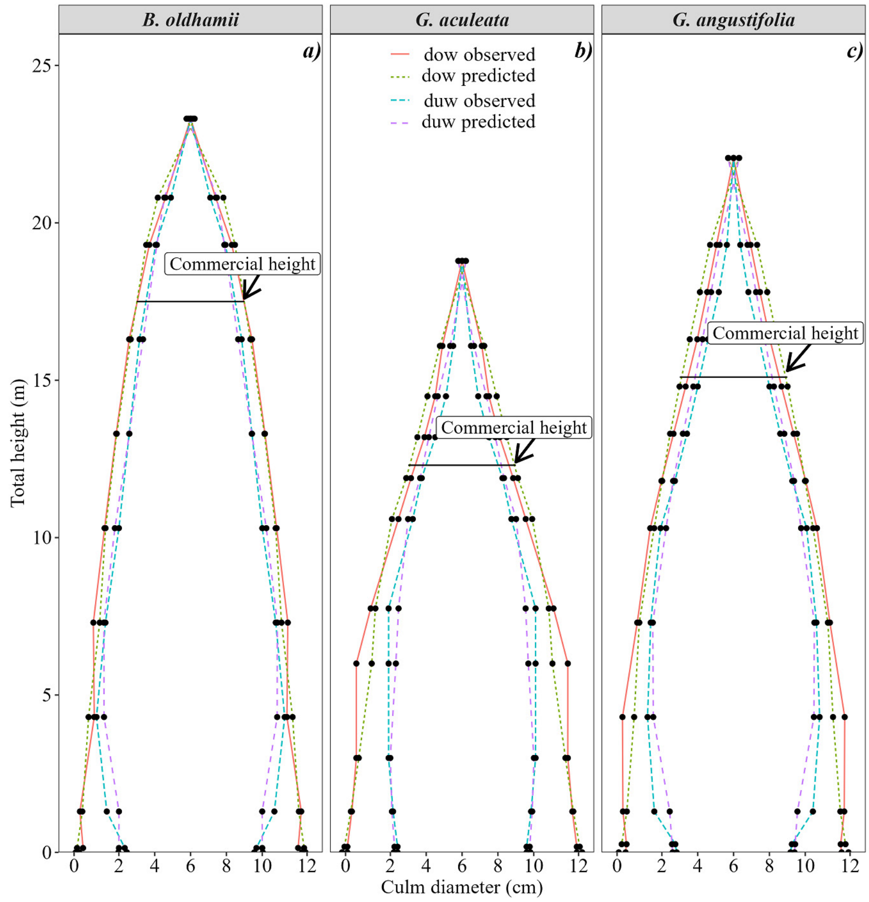

Figure 5. Observed and predicted diameter profile over (dow) and under (duw) wall describing the proposed system of additive equations, as well as estimation of commercial height when dow_{min} = 6 cm for culms with Dow = 12.35 cm per bamboo species.

of the culm diameter (dow_{min} < 6 cm) for other uses, slight differences were obtained in the estimated culm heights (lengths) for the species studied, which is indicative that the taper varies among taxa (Fig. 5).

The culm length distribution system by type of product can be adapted to other types of commercial uses and destinations by defining the minimum commercial diameter over wall of interest at the tip of the culm and then estimating the corresponding commercial height or length. The system allows knowing the linear meters (culm length) for some logical combination of Dow and H. The primary product with $dow_{min} \geq 6$ cm that was destined for structural use can be in the form of beams, columns, or *morillos*. The fact that *G. angustifolia* and *B. oldhamii* are the species that provide the most primary product highlights their potential for the establishment of commercial forest plantations.

Discussion

Of the taper models used, the Fang & Bailey (1999) one presented the best behavior for predicting the culm diameter profile of the three bamboo taxa studied. This result contrasts with that of Sanquetta et al. (2015), who determined that the description of such a profile in culms of *B. oldhamii* and *Bambusa vulgaris* Schrad. *ex* J. C. Wendl. can be done with a polynomial taper model with variable exponent, where *B. oldhamii* comparatively presented the best graphical behavior of the residuals vs. the predicted ones; likewise, they report that another taper model that was plausible for the same purpose was a fifth-order polynomial one. In the present study, model 3 of Demaerschalk (1972) was also plausible (Table 3).

The three bamboo species studied shared the parameters α_0 y α_2, in the model used to describe the dow and duw, while *G. aculeata* and *G. angustifolia* also shared α_1. As for the wt modeling the two *Guadua* species shared δ_1, while *B. oldhamii* shared δ_4 with *G. angustifolia* (Table 5).

In general, the values of the DW statistic were higher for wt than for duw; wt had the value closest to 2, which shows that the autocorrelation correction was adequate. In all variables, the value of this statistic was greater than 1.609, except for dow, which was 1.237. However, when the autoregressive error structure was not introduced, this statistic in no case exceeded the value of 0.95, from which it can be inferred that the autocorrelation could only be partially removed. It is important to point out that when higher order structures were tested, there were convergence problems in the fit of dow, wt and duw, in addition to non-significance in some parameters. In contrast, Rodríguez & Molina (2003) and Diéguez-Aranda et al. (2006) achieved a better autocorrelation correction in taper models for clones of *Populus* × *euramericana* and *Pinus sylvestris*, respectively.

Although the autocorrelation correction achieved in the present study was partial, this fact does not limit the practical applicability of the models, according to Hernández-Pérez et al. (2013), since it does not affect the prediction of the taper because the values of its parameters maintain logical consistency (Fig. 5). With regard to heteroscedasticity, an adequate correction was achieved by introducing weighting factors for dow, wt and duw, since a distribution pattern of the residuals was achieved that tends to be homoscedastic; however, in the diameter interval of 5 to 13 cm, *G. aculeata* had a wider dispersion in dow (Fig. 3b), which may be because the culms came from natural stands that have had minimal forestry management.

Although the diameter profile in culms has been little studied utilizing taper models, the satisfactory result obtained regarding the improvements in the statistical properties of model 1 (high significance of parameters, autocorrelation correction, and heteroscedasticity) confirms and ratifies the assumption that such models can be applied to bamboo taxa with low estimation errors. In Mexico, model 1 has also been successfully fitted in studies with coniferous and broadleaf tree species, such as that of Hernández-Ramos et al. (2017) for commercial plantations of *Eucalyptus urophylla* in which that model explained 96% of the observed variability, while Torres-Ávila et al. (2020) for hybrids between *Pinus caribaea* and *Pinus elliotti* found that the same model explained 97% of the observed variability. In this sense, in the present study, such a model was consistent because it explained at least 95% of the variability; together with the values obtained in the RMSE and bias statistics, it is inferred that model 1 can offer a reliable distribution of structural products of culms. Nevertheless, it was discovered that the basal part of the bamboo culms does not adhere to a neiloide-type dendrometric body, which translates into a partial deficiency of model 1. In this sense, Inoue et al. (2021) reported that the taper in bamboo culms can be determined by the so-called "Taper Index based on Form-Factor" that relates culm surface area with the form-factor corresponding to the apparent volume of the culm. Additionally, the difference calculated between the observed and estimated basal diameter is of the order of millimeters; therefore, the minimal differences presented did not affect the estimate of h. Besides, the most relevant aspect was to estimate the commercial height (h) as well as the commercial diameter over wall at the tip of the culm, whose mathematical expressions will make up the mathematical system of product distribution.

The prediction of dow, duw and dow_{min} is exemplified in Fig. 5, which was built with data from bamboo culms with similar Dow between taxa. In this graphical representation of the variables, the diameter of the culm at a height of 1.3 m (Dow) was used. The graphical analysis showed that the taper of *B. oldhamii* was smaller and different from that of the other two species (Fig. 5), which means that the decrease in diameter with respect to height was more gradual compared to the other taxa, whose decreases were more pronounced (abrupt). This aspect implies that this species, from the point of view of its taper, could have a greater potential for use for structural purposes. This dissimilarity may be due in part to the morphology of the clump and growth habit of the species, where *B. oldhamii* presents closed pachymorphic growth, which makes the culms emerge very close to each other, generating culms that are more homogeneous; in contrast, the genus *Guadua* presents open pachymorphic growth, where the shoots emerge with comparatively greater separation and therefore have more growing space for access to moisture, nutrients, and sunlight (Aguirre-Cadena et al., 2018).

Estimation of the wall thickness is key in the culms bamboo, which Trujillo (2016) states is the second most important dimensional property to take into account when the culms are to be used in structural design and construction. In this sense, *B. oldhamii* had the smallest thickness (Table 1), while *G. angustifolia* and *G. aculeata* had the greatest thickness (Figs. 3b and 3c), so the latter are promising sources because according to Hernández-Santiago & Torres-Hoyos (2020), the culms with $dow_{min} \geq 6$ cm are products with structural quality that meet the technical specifications in the construction market. It was also determined that wall thickness in *B. oldhamii* represented 25% of dow and in *G. angustifolia* and *G. aculeata* 32%. Additionally, based on Sanquetta et al. (2015), from the dow and duw equations, further studies could be aimed at estimating the timber volume that corresponds exclusively to the volume of the culm wall, as an important factor in estimating biomass and fiber production.

Future studies on bamboo culm taper should be aimed at overcoming limitations, such as: (1) using a larger sample size, besides incorporating covariates and the mixed effects model technique, which will allow fitting a system of additive equations separately for each bamboo taxon; and (2) the technical aspect that the evaluated taper models do not adequately predict the basal part of the culms because it does not adhere to the neiloide-type dendrometric body. In this sense, the need to generate specific taper models for giant bamboo species that consistently estimate the basal part is highlighted. Based on McTague & Weiskittel (2021), for the basal part of the culms of the taxa studied, it is necessary to explore other types of functions that are capable of appropriately describing the observed shape, such as a segmented function of increasing type or through some spline function.

In summary, this study showed that taper models can be used to describe the diameter profile over and under wall of mature and commercial culms of *B. oldhamii, G. aculeata* and *G. angustifolia*. Based on the selection of a taper model with high predictive capacity and a wall thickness function, a system of additive equations was formed, and a mathematical system was generated to carry out the distribution by structural product of culms of the three species in natural stands and in commercial plantations in northeast Puebla, Mexico. Comparatively, *G. angustifolia* and *G. aculeata* are more tapered and have a greater wall thickness, and *B. oldhamii* provides the greatest amount of primary product. The system will provide support in decision-making to generate strategies aimed at optimizing the production and distribution of products with predefined diameter over wall dimensions and determining the lengths in linear meters of the culm sections to be used for structural purposes in construction. The fitting strategy that was applied using WNSUR with autocorrelation correction in combination with the Dummy Variables technique is robust, modern, and up-to-date. It also fulfills the additivity property, so it can be applied to other giant bamboo species in other regions of the world.

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