ARTÍCULOS / ARTICLES

THE MEDIEVAL LATIN RECEPTION OF THE PSEUDO-ARISTOTELIAN *ON INDIVISIBLE LINES*: REASSESSING THE STATE OF THE ART

LA RECEPCIÓN LATINA MEDIEVAL DEL PSEUDOARISTOTÉLICO *SOBRE LAS LÍNEAS INDIVISIBLES*: REEVALUACIÓN DEL ESTADO DE LA CUESTIÓN

Clelia Crialesi

KU Leuven

Abstract

This article deals with the first Latin reception of Pseudo-Aristotle's *On Indivisible Lines* and its impact on the medieval debate about the continuum. Robert Grosseteste's and Albert the Great's references to this pseudo-Aristotelian text show that it could be regarded as a source for where to find information about the indivisibilist tenet, as well as an expansion of Aristotle's anti-atomistic critiques scattered throughout his authentic works. The use of *On Indivisible Lines* made by Henry of Harclay and Adam of Wodeham confirms this trend: the reading of this text could be twofold according to the tenet defended. While Henry argues against Pseudo-Aristotle to defend indivisibilism, Adam expands on pseudo-Aristotelian arguments to show the incongruities implied by indivisibilism.

Keywords

Pseudo-Aristotle; continuum; Robert Grosseteste; Albert the Great; Henry of Harcaly; Adam of Wodeham

Resumen

Este artículo trata de la primera recepción latina de *Sobre las líneas indivisibles* de Pseudo-Aristóteles y de su impacto en el debate medieval sobre el continuo. La referencia de Robert Grosseteste y Alberto Magno a este texto pone en evidencia que esta obra pseudoaristotélica podría ser tomada tanto como fuente para encontrar información sobre el principio de indivisibilidad como ampliación de las críticas antiatomistas de Aristóteles dispersas en sus obras auténticas. El uso de *Sobre las líneas indivisibles* que hacen Enrique de Harclay y Adán de Wodeham confirma que la lectura de este texto podría ser doble según el principio defendido: mientras que

Enrique toma el argumento pseudoaristotélico de la división de las líneas en partes iguales y lo replica, Adán amplía este argumento mostrando las incongruencias que implican sus adversarios indivisibilistas.

Palabras clave

Pseudo-Aristóteles; continuo; Robert Grosseteste; Alberto Magno; Enrique de Harclay; Adán de Wodeham

1. Introduction

With the rediscovery of Aristotle's natural works in the late 12th-century Latinate world, a new debate over the material substances arose, aiming to determine the nature of extension. This debate, which gained momentum especially in the 13th and 14th centuries, was marked by two opposing stances: 'divisibilism', which claims that everything that is extended – be it a geometric figure or a body – is indefinitely divisible; and 'indivisibilism', which maintains that extended things are grounded on extensionless elements, i.e., atoms that cannot be divided.[1] These positions were held by divisibilists and indivisibilists, respectively, the former espousing Aristotle's account of the continuum, the latter rejecting it. The textual starting point for both parties was Aristotle's *Physics*, for in books 4 and 6, Aristotle considers the continuous nature of the corporeal universe, refuting the positions of the atomists Leucippus and Democritus. Arguing for and against indivisibilism on the basis of this text was not a matter of doxography. Rather, it had many implications in the field of natural philosophy. For instance, the assumption that indivisibles are constitutive parts of the continuum of magnitude, motion, and time means holding that the whole world is made up of discrete quantities. Moreover, in the case of local motion, this issue led to question about both the continuity of the magnitude over which the physical body moves and the continuity of the time in which the motion takes place.[2]

A distinctive feature of the debate over the continuum in natural bodies between Aristotelians and atomists consists in mathematical arguments, and more precisely in arguments rooted in Euclidean geometry, raised to defend the one tenet or the other. However, it was especially those seeking to prove the existence of the indivisibles to be

[1] For an extensive analysis of this medieval dispute about the continuum as well as for the bibliography dedicated to it, we can refer to *Atomism in Late Medieval Philosophy and Theology*, edited by C. Grellard and A. Robert (Leiden and Boston: Brill, 2009); and Anneliese Maier, *Die Vorläufer Galileis im 14. Jahrhundert. Studien zur Naturphilosophie der Spätscholastik* (Rome: Edizioni di storia e letteratura, 1949), especially the chapter "Kontinuum, minima naturalia und aktuell Unendliches", 155-215.

[2] Aristotle, *Physics*, VI, 231b20-232a22.

false who recurred to Euclidean axioms, definitions, and proofs, willing to show the absurd conclusions deriving from such an assumption. These geometrical arguments drew interest and acquired philosophical prestige especially after Duns Scotus employed them in his *Ordinatio* – that is, the commentary on Peter Lombard's *Sentences* collecting his lectures at Oxford – to criticize indivisibilism by unveiling the contradictions between indivisibilism and geometry.[3]

From today's perspective, one can rightly hold that Aristotelian philosophy contributed only indirectly to the medieval debate on atomism by providing geometrical explanations, as the genuine core of the Aristotelian writings did not contemplate mathematics. However, medieval thinkers engaging with atomism could refer to other texts such as Al-Ghazali's *Summa theoricae philosophiae* (relying on Avicenna's *Danesh-Nameh-Alai*) and the pseudo-Aristotelian *De lineis indivisibilibus*. As for the former, we know to what extent it influenced some 13th century divisibilist thinkers. However, the latter's impact on the medieval debate about the continuum still needs to be fully appreciated. Indeed, what we know about this text and its reception in the Latin West is still very limited, as we still lack not only a critical edition but also a systematic study assessing its philosophical relevance within the medieval discussions.[4] After a survey on the contents of this pseudo-Aristotelian text, I will explore its earliest reception in the Latinate world.

2. *On Indivisible Lines*

On Indivisible Lines (i.e., *De lineis indivisibilibus*, henceforth DLI) is the Latin version of the original Greek Περὶ ἀτόμων γραμμῶν.[5] This brief Greek text was written by a member of the Academy – possibly Theophrastus – espousing Aristotle's philosophical

[3] See Cecilia Trifogli, "Duns Scotus and the Medieval Debate about the Continuum", *Medioevo* 29 (2004): 233-266; and Jean-Luc Solère, "Scotus Geometres. The Longevity of Duns Scotus's Geometric Arguments Against Indivisibilism", in *Scotism Through the Centuries: Proceedings of 'The Quadruple Congress' on John Duns Scotus*, edited by M. Dreyer, E. Mehl and M. Vollet (Munich: Franciscan Institute Publications 2013), 139-154.

[4] I am currently working on the critical edition of the Latin text for the *Aristoteles Latinus* project. https://hiw.kuleuven.be/dwmc/research/al

[5] The most recent and accurate study of the Greek text is Cédric Hugonnet, "Édition, traduction et commentaire du Περὶ ἀτόμων γραμμῶν du Pseudo-Aristote" (Université d'Aix-Marseille: unpublished PhD thesis, 2014). We refer to this work for a thorough analysis of the transmission of this text within the Greek manuscript tradition. Many other studies, translations into different languages, and critical editions of the *Περὶ ἀτόμων γραμμῶν* are available: Pseudo-Aristotle, *De lineis insecabilibus*, translated by H. H. Joachim, in *The Works of Aristotle*, edited by W. D. Ross (Oxford: Clarendon, 1913); Pseudo-Aristotele, *De lineis insecabilibus*, edited and translated by M. Timpanaro Cardini (Milan and Varese: Istituto Editoriale Cisalpino, 1970); Pseudo-Aristotle, *Problèmes mécaniques. Des lignes insécables*, translated by M. Federspiel and M. Decorps-Foulquier (Paris: Les Belles Lettres, 2017).

system.[6] The author argues against the theory of the indivisible lines that traces back to the Platonists of the Old Academy – possibly Xenocrates and Speusippus – who elaborated it to reject the Eleatic stances against plurality and movement. The theory criticized in DLI postulates the existence of atomic quantities, i.e., the indivisible lines, that structure both geometrical and physical objects. Writing at the time of Euclid's redaction of the *Elements* (between 339 and 300 B.C.E.), the pseudo-Aristotelian author rejects such theory by means of a series of arguments taken from the field of geometry, kinematics, and logic, showing the incongruous consequences arising from defending a primary and indivisible unit of linear measurement, such as indivisible lines.

The overall structure of DLI can be divided into six sections according to the specific issues addressed:

1. Five arguments in favor of the theory of indivisible lines;
2. Refutation of the five arguments;
3. Mathematical arguments against the theory of indivisible lines;
4. Definition of the line and its properties;
5. Definition of the point, also with regard to the line;
6. Logical arguments against the line containing points.

I will not explore the depths of all the contents of DLI. Rather, I will briefly focus on two arguments that a provisional analysis of the reception of this text has shown to be the starting point for further developments in the debate on atomism in the Middle Ages. In defending that a line segment cannot be composed of points nor of indivisible lines, the pseudo-Aristotelian author argues that, if we assume that points constitute lines, then we should also draw that all things be dissolved into points. And since solids are made of surfaces and surfaces of lines, then points would therefore be like 'elements' of bodies:

> Moreover, all things would be divided and dissolved into the point, and the point <would be> a part of the body, if a body <consists of> surfaces, and a surface of lines. But if the elements are those things from which each thing comes in the first place, then points would be the elements of bodies. Thus, elements would be identical with regard to the name and the species. From what has been said, therefore, it is clear why the line is not made of points.[7]

[6] Diogenes Laertius includes this text among Theophrastus' works (see *Vitae*, V, 42, 17), followed by Simplicius, *In Aristotelis quattuor libros de caelo commentaria*, III, 1, edited by J. L. Heiberg, *Commentaria in Aristotelem Graecam* 7 (Berlin: Reimer, 1894), 566; and Iohannis Philoponus, *In Aristotelis libros de generatione et corruptione commentaria*, I, 2, edited by H. Vitelli, Commentaria in Aristotelem Graeca 14.2 (Berlin: Reimer, 1897), 34.

[7] Milan, Biblioteca Ambrosiana, E 71 Sup., f. 157vb: "Amplius dividentur omnia et resolve<n>tur in punctum, et punctus pars corporis, si quidem corpus quidem planum, planum

Two remarks can be made about this passage: one concerns the theory criticized and another one concerns the critique itself. On the one hand, the theory criticized seems to support a reduction from bodies to gradually simpler geometrical elements, passing through surfaces and lines, and eventually arriving to points. In other words, this theory relies on the belief that every magnitude – hence also bodies – is ultimately composed of points or indivisible entities. On the other hand, Pseudo-Aristotle openly criticizes the conception of points as constituents of lines, by showing the objectionable implication that bodies also consist of points. Therefore, if points were seen as 'elements' of bodies, all elements would not be distinguished by name (i.e., they would be *univoca*) nor according to species, which is absurd.

The other argument I would like to consider is that of the division of a line segment into halves. It presupposes that indivisibles are distributed within a line segment in the same way numbers are within a series (of the kind $x_1 + x_2 + x_3 + ...$): for instance, three geometric points are arranged within a line like the numbers 1, 2, and 3 are arranged within the series of natural numbers. This conception of the extension of continuous magnitudes, such as lines, is based on an arithmetical model usually applied to discrete quantities, such as natural numbers. From this perspective, one could think of a line having *n* points and thus ascribe the specific arithmetical properties of evenness and oddness to these points. The argument runs as follow:

> Moreover, if every line, except the indivisible one, can be divided both into equal and into unequal parts – and not only that line which is composed of three or any other odd number of indivisibles – how will the indivisible line be indivisible? And the same happens also if a line <is divided> into two halves, for every <line made of> an odd number of indivisibles <can be divided into two halves> – and if not every line, only that which is made of an even number of indivisibles, is divisible into halves. But if whatever line is divisible into halves, then also the indivisible line will be divisible, when a line composed of an even number of indivisibles will be divided into unequal parts.[8]

autem ex lineis. Si autem ex hiis qui primum insunt singula sunt, elementa erunt haec, puncta utique erunt elementa corporum. Quare univoca elementa nec altera specie. Manifestum igitur ex dictis quoniam non est linea ex punctis». Cf. Hugonnet, "Περὶ ἀτόμων γραμμῶν du Pseudo-Aristote", 289-290 (972a 6-13). Milan, Biblioteca Ambrosiana, E 71 Sup. is the only manuscript that ascribes the Latin translation of the Περὶ ἀτόμων γραμμῶν to Robert Grosseteste. This is the main reason why I refer to this manuscript. A study of the manuscript tradition of the Latin translation of *On Indivisble Lines* is in progress.

[8] Milan, Biblioteca Ambrosiana, E 71 Sup., f. 157ra: "Amplius, <si omnis> linea de indivisibili et aequaliter et inaequaliter dividitur, et non ex tribus indivisibilibus et totaliter superfluis, quare indivisibilis indivisibilis? Similiter autem et si in duas partes <dividitur>, omnis enim quae ex superfluis. Si autem in duas partes quidem non omnis dividitur, sed quae ex perfectis. In duas partes autem divisas et quaecumque possible dividere, dividetur et sic indivisibilis. Cum autem ex impartibilibus in inaequalia dividetur". See Hugonnet, "Περὶ ἀτόμων γραμμῶν du Pseudo-Aristote", 240-241 (970a 26-33).

If it is true that every line composed of indivisibles can be divided into either equal or unequal parts, we cannot maintain that the indivisibles are truly such. We can divide into equal parts a line which is made of an odd number of indivisibles as well as we can divide into unequal parts a line which is made of an even number of indivisibles. However, in so doing, a remainder is produced, and we will have to admit that it is divisible on account of the premise of this reasoning. For instance, let a line be composed of 5 points and divided into halves; the product of this division will be 2 points and 2 points, plus 1 point as remainder. But since every line can indeed be divided into two – no matter how many indivisibles it is composed of – it will follow that the remaining single point will have to be divided as well. Thus, indivisibles are truly divisible, and the hypothesis of the existence of indivisibles is confuted.

As we will see in the next section, in the 13th century Robert Grosseteste and Albert the Great refer to the first argument, and more precisely to the theory of bodies being reduced to geometrical entities, yet from different perspectives. Furthermore, mention of the critique carried out in the same argument is made in the 14th century by Adam of Wodeham, who, together with Henry of Harclay – who had different purposes himself – engages with the division of line segments into equal and unequal parts.

3. The earliest Latin reception: Robert Grosseteste and Albert the Great

All these arguments raised against the theory of indivisible lines are consistent with Aristotle's unfavorable references to this theory (and atomism in general) in his *Topics* (IV, 1), *Physics* (III, 6), *Metaphysics* (I, 9), and *De caelo* (III, 1).[9] This consistency led almost all medieval thinkers to ascribe this text to Aristotle himself. An exception is Thomas Aquinas. In his commentary on Aristotle's *De caelo*, he argues that bodies are not produced by surfaces and refers to DLI, "in which it is shown that there are no indivisible lines, and which is ascribed to Theophrastus by some".[10] Thomas Aquinas states this likely on the basis of Simplicius' commentary on Aristotle's *De caelo*, available in William of Moerbeke's Latin translation.[11] Nevertheless, this remark does not appear

[9] A significant survey of the philosophical contents of the *Περὶ ἀτόμων γραμμῶν* with respect to Aristotle's critics of ancient atomism is due to Pieter Sjoerd Hasper, "Aristotle's Diagnosis of Atomism", *Apeiron* 39 (2006): 121-155.

[10] Thomas Aquinas, *In libros Aristotelis de caelo et mundo*, III, 1, 3, *In libros Aristotelis de caelo et mundo*, Leonine Commission (Rome: Typographia Polyglotta S. C. de Propaganda Fide 1886), 235-236: "Et de hoc dicit esse prius consideratum *in sermonibus de motu*, idest in VI *Physic.*, ubi probatum est quod lineae non sunt indivisibiles, neque ex indivisibilibus compositae. Invenitur autem quidam alius libellus, in quo probatur quod non sunt lineae indivisibiles: quem quidam dicunt esse Theophrasti". This was pointed out in Steven J. Williams, "Defining the Corpus Aristotelicum: Scholastic Awareness of Aristotelian Spuria in the High Middle Ages", *Journal of the Warburg and Courtauld Institutes* 58 (1995): 29-51, esp. 44.

[11] See Simplicius, *Commentaria in quatuor libros De coelo Aristotelis*, III, 1 (Venice: apud Hieronimum Scotum 1563), 198b: "in libro de indivisibilibus lineis quem quidam Theophrasto

in Robert Grosseteste's translation of Simplicius' commentary. Beside Aquinas' doubts, the attribution of DLI to Aristotle remained unquestioned throughout the Middle Ages. The authority of Aristotle undoubtedly contributed to its remarkable success in the Latin West. As a consequence, DLI is still preserved in seventy-four *codices* disseminated in libraries across Europe.[12] A preliminary exam of the manuscript tradition has shown that it is preserved mainly in anthologies of Aristotelian works, both authentic and spurious, such as: *Physics*, *De generatione et corruptione*, *De caelo*, *De sensu et sensato*, *Physiognomica*, *De sompno et vigilia*, *De memoria et reminiscentia*, *De longitudine et brevitate vitae*.

So far, scholarship credited the 13th-century polymath Robert Grosseteste (ca. 1170 – 1253) to be responsible for the first Latin translation of the text. Grosseteste played a pivotal role in delivering both Aristotle's works and Patristic literature to the Latinate world. He translated and commented on the *Nichomachean Ethics*, parts of the *De caelo* as well as on Simplicius' *Commentary*, the corpus of the Pseudo-Dionysius, works by John of Damascus, and the Greek lexicon *Suda*. In line with the tradition of the medieval translators of Aristotelian texts, his style of translation is characterized by a word-for-word approach, which is the outcome of a precise policy aiming at mirroring the Greek 'in all its irreducible differences and foreignness', as James McEvoy described it.[13]

The attribution of the Latin translation of DLI to Grosseteste was originally advanced in 1931 by Vincenzo Ussani.[14] This information was then reported – unaltered and with no other additions – in subsequent studies consecrated to Grosseteste's writings and translations as well as to the dissemination of Aristotelian works in the Middle Ages, by Harrison Thomson, Ezio Franceschini, and Auguste Mansion.[15]

tribuunt". The unpublished critical edition by B. Märien, prepared under the supervision of F. Bossier, can be consulted on the Aristoteles Latinus Database.

[12] As for the Greek manuscript tradition, only twenty-seven manuscripts are extant, the oldest dating from the end of the 13th century. See Hugonnet, "Περὶ ἀτόμων γραμμῶν du Pseudo-Aristote", 70.

[13] James McEvoy, *Robert Grosseteste* (Oxford: Oxford University Press, 2000), 117. See also Chalres Burnett, "Translating from Arabic into Latin in the Middle Ages: Theory, Practice, and Criticism", in *Éditer, traduire, interpreter: essais de methodologie philosophique*, edited by S. G. Lofts, P. W. Rosemann *et al.* (Louvain-la-Neuve: Éditions de l'Institut supérieur de philosophie, 1997), 55-78; Anna C. Dionisotti, "On the Greek Studies of Robert Grosseteste", in *The Uses of Greek and Latin: Historical Essays*, edited by A. C. Dionisotti, A. Grafton and J. Kraye (London: The Warburg Institut, 1988), 19-39; Henry P. F. Mercken, "Robert Grosseteste's Method of Translating: A Medieval Word Processing Program?", in *Tradition et traduction. Les textes philosophiques et scientifiques grecs au Moyen Âge Latin. Hommage à Fernand Bossier*, edited by R. Beyers, J. Brams *et al.* (Leuven: Leuven University Press, 1999), 323-337.

[14] S. Harrison Thomson reports this reference: Ussani, *Relazione dell'AA 1930-31*, Roma, 15 = *Rivista di filosofia neoscolastica* 24 (1932): 216. However, it seems impossible to find Ussani's note following Thomson's reference. See, S. Harrison Thomson, "A note on Grosseteste's Work of Translation", *Journal of Theological Studies* 34 (1933): 48-52, especially 52.

[15] Thomson, "A note on Grosseteste's Work of Translation"; S. Harrison Thomson, *The Writing*

However, Ussani's attempt of attributing DLI to Grosseteste was based only on an ascription found in one manuscript from the middle of the 13th century: Milan, Ambrosian Library, E. 71 Sup. Since then, the research on DLI has been quite latent. Therefore, it is still necessary to re-examine this attribution and provide further evidence for Grosseteste's authorship by focusing on linguistic aspects (e.g., the translation method used) and other substantial arguments (e.g., philosophical references to DLI in Grosseteste's works).

Although this claim on the attribution to Grosseteste needs further verification, Grosseteste's acquaintance with DLI is corroborated by some of his philosophical tenets. For instance, Cecilia Panti has shown that Grosseteste refers to DLI in his *De luce*, seemingly endorsing the theory criticized in it.[16] In the *De luce*, Grosseteste develops his original theory combining the metaphysics of light and hylomorphism: the whole physical universe is produced through the infinite self-multiplication of the dimensionless and incorporeal point of light. First matter and first form are the ontological elements structuring every physical body. Light, identified with the first form, provides matter with spatial extension and stretches it into a three-dimensional body. A reasonable consequence that one might draw is that everything is ultimately composed of points of light, that is, infinite, indivisible elements which are also responsible for dimensions of bodies:[17]

> It is clear that light, by multiplying itself infinitely, extends the matter into greater and lesser finite dimensions, according to whatever their mutual proportions, that is, both numerical and non-numerical. For if light, by multiplying itself infinitely, extends the matter into a two-cubic dimension, by doubling that same infinite multiplication, it extends the same [matter] into a four-cubic dimension, and by halving that same infinite multiplication [it extends matter] into a one-cubic dimension, and so forth according to other numerical and non-numerical proportions. This is, I believe, the

of Robert Grosseteste, Bishop pf Lincoln (1235-1253) (Cambridge: Cambridge University Press, 1940), 67-68; Ezio Franceschini, "Roberto Grossatesta, vescovo di Lincoln e le sue traduzioni latine", reprinted in Ezio Franceschini, *Scritti di filologia Latina medievale*, 2 vols. (Padova: Antenore, 1976), II, 409-544; Auguste Mansion, "Quelques travaux récents sur les versions latines des Éthiques et d'autres ouvrages d'Aristote", *Revue néo-scolastique de philosophie* 39 (1936): 78-94.

[16] Robert Grosseteste, *De luce*, edited by C. Panti (Pisa: Pisa University Press, 2011), 126-128.

[17] For Grosseteste's original indivisibilism, see Neil Lewis, "Robert Grosseteste and the Continuum", in *Albertus Magnus and the Beginning of the Medieval Reception of Aristotle in the Latin West. From Richardus Rufus to Franciscus de Mayronis*, edited by L. Honnefelder, R. Wood, M. Dreyer and M.-A. Aris (Münster: Aschendorff, 2005), 159-187. Neopythagorean elements lying beneath Grosseteste's account of indivisibilism can be found in his *Commentary on Aristotle's Physics*. This has been studied in Aurélien Robert, "Atomisme pythagoricien et espace géométrique", in *Lieu, éspace, mouvement: physique, métaphysique et cosmologie (XII – XVI siècles). Actes du Colloque International Université de Fribourg (Suisse), 12-14 mars 2015*, edited by T. Suarez Nani, O. Ribordy and A. Petagine (Turnhout: Brepols, 2017), 181-206.

> account of the philosophers who held that everything is composed of atoms and argued that bodies are composed of surfaces, the surfaces of lines, and the lines of points.[18]

From this perspective, the process of the infinite multiplication of light leads to the stretching of matter into finite dimensions: proportions among specific amounts of material quantity correspond to proportions of such infinite multiplications of light. As Panti explains, according to Grosseteste, a dimensionless thing, such as light in this case, can produce a *quantum* of given dimension only when it is multiplied infinite times – and this is analogous to how the dimensionless geometrical point 'generates' lines, surfaces, and solids.[19] Therefore, Grosseteste links his theory of the infinite self-multiplication of light to the one according to which bodies are made of surfaces, surfaces of lines, and lines of points, stating that this is how atomism can be conceived. All this presupposes precisely one aspect of the theory criticized in DLI, which envisages a 'jump' or reduction of a bodily quantity to indivisible geometrical constituents.

DLI did not allow a univocal reading. If Grosseteste used it – however marginally – as a source for the indivisibilist theory and to argue about his metaphysics of light in the *De luce*, other philosophers and theologians, such as Albert the Great (1206 – 1280), endorsed DLI's original purpose, i.e., the critique of atomistic doctrine. Based on Grosseteste's (supposed) translation, Albert developed his own commentary on DLI, that he inserted at the end of book 6 of his *Physics* – the *terminus ante quem* for DLI being therefore the writing of Albert's *Physics*, that is, around 1250. Albert's refusal of atomism in the light of his interpretation of DLI has not yet been examined, but a preliminary reading of his introduction to DLI clarifies the specific kind of indivisibilism he aims to confute:[20]

> You shall know that many ancient thinkers agreed on this, namely, that neither a line nor a body is made of points. However, they maintained that quantified bodies are made of bodily atoms, and the bodily atoms are made of indivisible surfaces, and the indivisible surfaces are made of indivisible lines, and the line is constituted by a flow of points and not made by points, since the line is the first according to kind of the quantified continuum. [...] As to those who say that indivisible lines are the first components of the continuum, they are not content with the first part of the sixth book

[18] Grosseteste, *De luce*, 78-79, ll. 67-74. My translation.

[19] Cecilia Panti, "La moltiplicazione infinita della luce e la sua funzione nel De luce di Roberto Grossatesta", in *Immagini della luce. Dimensioni di una metafora assoluta*, edited by S. Lavecchia (Rome: Mimesis, 2019), 97-122, esp. 109.

[20] I am currently preparing an article dedicated to Albert's use of geometrical arguments against atomism, comparing his view with Roger Bacon's account – such comparison being carried out with regard to their philosophy of mathematics. Analyzing Albert's *Physics*, a brief reference to his reading of DLI has been made by Steven C. Snyder in a collective contribution: David Twetten, Steven Baldner and Steven C. Snyder, "Albert's Physics", in *A Companion to Albert the Great*, edited by I. M. Resnick (Leiden and Boston: Brill, 2013), 173-221, esp. 202.

> of the *Physics*, where it is shown that points, instants, and moments are not first components – although this indeed is enough. Thus, I hereby present this discussion, to perfectly satisfy also them, once they will be confuted in every way as opponents of the truth.[21]

In Albert's account of the theory of indivisible lines, we find the same principle Grosseteste refers to in his *De luce*, namely that bodies come from surfaces, lines, and points – although differently from Grosseteste, Albert specifies that this indivisibilist theory holds that bodies are reduced in the first place to bodily atoms and then to geometrical constituents. Moreover, Albert denotes that the advocates of such theory held that the line is indeed to be conceived as the first of the quantified continua, for it is not really made out of points, but rather is composed from a flow of points. Albert's use of DLI is intended to complement the book 6 of Aristotle's *Physics*: although Aristotle already ruled out that points and instants do not compose the magnitudes of bodies and time, the DLI confutes 'in every way' those maintaining the anti-Aristotelian stance. Confuting it in 'every way' means also to expand on the concise geometrical arguments offered by the pseudo-Aristotelian author. And so does Albert, with a view of showing the paradoxes that arise from assuming that indivisible constituents are at the basis of geometrical constructions.

4. Later Latin reception: Henry of Harclay and Adam of Wodeham

Besides Grosseteste and Albert the Great, we have less than a partial mastery of the way DLI impacted the later medieval debate about atomism, and a poor awareness of the extent the medieval geometrical reasonings were based on DLI. More generally, whether and how DLI was known and problematized by both indivisibilists and divisibilists is still an open issue. In the following lines I will (partially) answer only this last question by referring to two thinkers that show their concern for DLI. Let us move from the 13th century to the early 14th century to focus on Henry of Harclay (1270 – 1317) and Adam of Wodeham (d. 1358). References to DLI made by them is quite emblematic because they, as in the case of Grosseteste and Albert, can be identified as an indivisibilist and a divisibilist, respectively.

Henry of Harclay began his theological education at the University of Paris while Duns Scotus was lecturing on the *Sentences* (between 1302 and 1305), becoming a master at Oxford, where later in 1312 he was confirmed as chancellor of the English university. His *Ordinary Questions* are the result of his teaching as Master of Theology at Oxford, and in this work, more precisely in question 29, he refers to DLI. Question 29 is concerned with whether the world is eternal in respect to the future.[22] Assuming that infinites

[21] Albert the Great, *Scientia libri De lineis indivisibilibus*, 1, in Albert the Great, *Physica*, edited by P. Hossfeld Alberti Magni Opera Omnia IV, 1 (Münster: Aschendorff, 1987), 498.

[22] On this question, see John Murdoch, "Henry of Harclay and the Infinite", in *Studi sul XIV secolo in memoria di Anneliese Maier*, edited by A. Maierù and A. Paravicini Bagliani (Rome: Edizioni

exist in act and that unequal infinities are possible, Henry's response to this question lays the foundation for his claims on indivisibilism: there can be two unequal infinities as much as there can be infinite years and twelve instances of infinite months. In the same way, a two-foot line and one-foot line are both composed of infinite points and yet they are unequal. Arguments against unequal infinities assume that a continuum cannot be made from indivisibles, says Henry. More precisely, according to Henry, continua are composed of an infinite number of indivisibles that are *immediately* conjoined to one another, meaning that the indivisibles are discretely ordered in a continuum.[23] DLI is explicitly evoked by Henry himself as an argument against his own view– an argument that we already recalled above when retracing the contents of DLI:

> To the other argument maintaining that a quantity could not be divided into two equal parts if it were to consist of an odd number of points, it should be said that in the book *On Indivisible Lines* Aristotle uses such reasoning to prove that a line is not [composed of] points. For he argues: suppose a line consists of points; therefore, it [is composed] of either a perfect and even or a superfluous and odd number of points. If the second alternative were granted, then the intended point is made. If the first alternative were admitted, then a point would necessarily be divided, since [any] line can have two halves. Furthermore, later in the same opuscule he shows that a [single] point cannot be removed from a line, since if this were the case, he argues, then on the same grounds a point could be added to a line. Then he deduces the following contradictions: a line would be greater than [another] line by a single point, which is, as he says, impossible. Nevertheless, he claims that a point can be removed from a line incidentally, in the sense that when some whole [line segment] is removed from another [such whole], a point would also be removed. And this can occur in the manner we have argued for above in proving (in our second argument) that point is immediate to point.[24]

Henry recalls two arguments by Pseudo-Aristotle against the lines being composed of points. First, division into two equal parts is proper to every continuous magnitude, including lines. If a line were composed of an odd number of points, the 'superfluous' point, that is, the remainder from the division into two, should be divided, which contradicts the assumption that points are indivisible. Second, a single point cannot be

di Storia e Letteratura, 1981), 219-261.

[23] Henry of Harclay's indivisibilism is different from Grosseteste's, as the latter maintains that continua are composed of an infinite number of indivisibles that are *mediately* conjoined to one another. At least, this is the account of Henry's and Grosseteste's forms of indivisibilism found in Thomas Bradwardine's *Tractatus de continuo*; John Murdoch, "Geometry and the Continuum in the Fourteenth Century: A Philosophical Analysis of Thomas Bradwardine's Tractatus de continuo", PhD diss., University of Winsconsin, 1957, 380: "Alii autem ex infinitis, et sunt bipartiti, quia quidam eorum, ut Henricus modernus, dicit ipsum [continuum] componi ex infinitis indivisibilibus immediate coniunctis; alii autem, ut Lyncuf, ex infinitis ad invicem mediatis". See Lewis, "Robert Grosseteste and the Continuum", 162-164.

[24] Henry of Harclay, *Ordinary Questions*, q. 29, 94, edited by M. G. Henninger, translated by R. Edwards and M. G. Henninger (Oxford: Oxford University Press, 2008), 1073-1075.

subtracted nor added to a line segment; otherwise, there would be lines bigger than others by a single point. From this perspective, a point can be subtracted from a line segment only accidentally, namely, when one removes a smaller line segment from a greater one. Henry does not limit himself to report this critique, but he rather elaborates a threefold answer:

> Reply: I claim that [although] it is true that it is not possible for us to remove a point [from a line] unless we remove [a segment of] the line at the same time, non the less it is possible for God. Then, when it is argued that a line may be divided into equal parts, one must reply that not every line can be divided into two halves, just as [no] odd number can [be so divided]. This notwithstanding, since a point [added to one of two halves of a line] causes no sensible difference or increase to the line, the two [half-]lines are judged equal even though one exceeds the other by one point. For example, a quantity [existing] in a natural thing (for instance, in wood) cannot be divided into two equal parts that we can perceive to be equal, no matter what sense is involved.[25]

Henry's reply considers three issues. First, God is indeed able to remove a single point from a line segment, and it is only human beings that cannot do so. Second, one must accept that some lines cannot be divided into two equal parts just as odd numbers cannot be so divided. Third, a point added or subtracted from a line segment does not cause any sensible difference, for human beings cannot perceive those halves of a natural thing that is equally partitioned to be perfectly equal. Central to this last remark is the human perception: none of the senses can be trusted to determine whether a point has been added to a line or to assess whether a piece of wood is perfectly divided into two equal parts.

Henry's indivisibilism was later criticized by Adam of Wodeham in his *Tractatus de indivisibilibus*.[26] Such treatise is articulated into five questions, whose redaction was greatly influenced by William of Ockham's *Expositio Physicorum*. As Rega Wood, editor of Adam's *Tractatus* suggests, this helps to determine the date of its composition, that is, around 1323.[27] In the first *quaestio*, Adam delves into the issue of the composition of continua in order to assess whether some incorruptible form is composed of indivisible forms. The second article is devoted to presenting twelve arguments in favor of indivisibilism, mainly taken from Henry of Harclay and Walter Chatton (d. 1344). Among these, Adam refers to Chatton's response to the issue of the division into halves of a line composed of either an even or odd number of points. One might easily retrace the link between Adam's mention of Chatton's response and the exposition of DLI's critique by Henry of Harclay. This is revealing that Chatton, and consequently Adam,

[25] Henry of Harclay, *Ordinary Questions*, q. 29, 95, 1075.

[26] For Wodeham's refutation of Henry's arguments, see Edith D. Sylla, "God, Indivisibles, and Logic in the Later Middle Ages: Adam Wodeham's Response to Henry of Harclay", *Medieval Philosophy and Theology* 7 (1998): 69-87.

[27] Rega Wood, introduction to Adam of Wodeham, *Tractatus de indivisibilibus*, edited and translated by R. Wood (Dordrech, Boston and London: Kluwer Academic Publishers, 1988), 15-16.

surely had DLI in mind, although it is not explicitly mentioned in this passage of the *Tractatus*:

> My sixth principal argument is this: If a continuum is composed of indivisibles, and a finite [entity] from a finite number of these, then not every line would be divisible in two equal halves. The opposite is shown by the 10th [proposition] of Euclid's first book. The inference is evident, since the multitude of indivisibles from which [a continuum] is composed are either even or odd. If they are odd, what we proposed to show is evident. If they are even, one can be removed, and we have what we intended to show for the remainder. And though Chatton concedes this conclusion, it is absurd.[28]

Henry of Harclay discussed DLI's objection to the composition of the line out of indivisibles, namely: if a line were composed of an odd number of points and divided into two equal parts, one would necessarily admit the division of the remainder point into two, and therefore accept the divisibility of the point. Chatton must have had in mind such objection elaborated on in DLI, thus he offered another solution: if a continuum, consisting of an odd number of indivisibles were divided into halves, one would identify the remainder as indivisible, having indeed what is considered not to be divisible any further. To Adam, however, this solution does not disprove DLI's argument.

An explicit reference to DLI is found in the first article of the same question, where Adam presents twelve other arguments against indivisibilism, mainly taken from Aristotle. As for DLI, the following argument is precisely ascribed to Aristotle:

> Again, Aristotle argues in numerous ways in *On Indivisible Lines* using the arguments already discussed. And he adds another argument: If a body were composed of surfaces, and surfaces of lines and lines of points, points would be the elements of bodies, and they would belong to the same species as bodies, which is impossible. He also adds many other arguments.[29]

Endorsing Pseudo-Aristotle's account, Adam argues that the assumption of bodies being composed of surfaces, lines, and points would lead to the absurd conclusion that bodies and points are of the same species. Therefore, points cannot be conceived as elements of bodies.

5. Conclusions

This survey of the reception of DLI within the Latinate world has shown that this pseudo-Aristotelian text entered the debate about atomism from the middle of the 13th century onward. A first reference to it – although not explicit – is made by Grosseteste. In his *De luce* Grosseteste seems to endorse the theory criticized by the Pseudo-Aristotle,

[28] Adam of Wodeham, *Tractatus de indivisibilibus*, q. 1, a. 1, 72, 79.
[29] Adam of Wodeham, *Tractatus de indivisibilibus*, q. 1, a. 1, 41-43.

i.e., that bodily quantities can be reduced to geometrical, indivisible entities. The same aspect of this theory is openly criticized by Albert the Great in the introduction to his commentary on DLI: his commentary is precisely intended to refute those who defend that bodies are reduced to indivisible surfaces, indivisible lines, and indivisible points. Therefore, his commentary complements book 6 of Aristotle's *Physics*. Grosseteste's and Albert's use of DLI shows that this pseudo-Aristotelian work could be taken as both a source for where to find information about the indivisibilist tenet, and as an expansion of Aristotle's anti-atomistic critiques scattered throughout his authentic works.

One of the arguments developed by Pseudo-Aristotle to criticize the reduction of bodies to indivisible entities consists of showing the absurd conclusion of such a tenet in this way: with points being the ultimate 'elements' of bodies, elements themselves would all be the same, also according to species. This reasoning is taken up in the 14th century by Adam of Wodeham, who rejects the indivisibilist theory in his *Tractatus de indivisibilibus*. In the same work, Adam borrows another reasoning from DLI, namely, the division of all line segments into halves, showing that indivisibles have to indeed be divisible. Adam's discussion can be seen as a reaction to Walter Chatton's response to this DLI critique, although we have seen that Henry of Harclay, too, considers this issue in order to defend indivisibilism in his *Ordinary Questions*. Also in this case, the use of DLI is twofold according to the tenet defended: on the one hand, Henry takes DLI's argument of the division of lines into equal parts and replies to it; on the other hand, Adam expands on this argument showing the incongruities implied by his indivisibilist adversaries.

This reassessment of the state of the art about the Latin reception of DLI shows that Pseudo-Aristotle's text influenced and oriented the medieval debate about the continuum. Yet many points are still obscure, also due to the lack of a critical edition of the Latin text of DLI. In addition, a thorough investigation of Albert the Great's commentary will allow for a better understanding of the modalities of DLI's circulation and attribution to Aristotle, and for a systematic study of the medieval philosophical arguments that, relying on DLI, spread cross-disciplinarily to other philosophical domains.

Clelia Crialesi
clelia.crialesi@kuleuven.be

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