A SURVEY ON THE FIXED POINT THEOREMS VIA ADMISSIBLE MAPPING

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Reception: 21/08/2022 **Acceptance:** 05/08/2022 **Publication:** 29/12/2022

Suggested citation:

Karapınar, E.(2022). A Survey on the Fixed Point Theorems via Admissible Mapping. *3C TIC. Cuadernos de desarrollo aplicados a las TIC*, *11*(2), 26-50. https://doi.org/10.17993/3ctic.2022.112.26-50

ABSTRACT

In this survey, we discuss the crucial role of the notion of admissible mapping in the metric fixed point theory. Adding admissibility conditions to the statements leads not only to generalizing the existing results but also unifying several corresponding results in different settings. In particular, a contraction via admissible mapping involves and covers contractions defined on partially ordered sets, and contractions forming cyclic structure.

KEYWORDS

Admissible mapping, fixed point

1 INTRODUCTION

The metric fixed point theory is one of the most attractive research topics that lies in the intersection of three disciplines of the mathematics; topology, applied mathematics and nonlinear functional analysis. In the literature, it was assumed that the first metric fixed point theorem was proved by Banach [18] in 1922. On the other hand, it was noted that the idea of the fixed point theorem was used before Banach's paper. Indeed, the fixed point results were used to prove the existence and uniqueness solution of the initial value problems in the some research papers of Liouville, Picard, Poincaré and so on. Despite this fact, the sole purpose of Banach's theorem is the indicate the existence and uniqueness of the fixed point.

The Banach's fixed point theorem stated perfectly and its proof is also awesome: Every contraction in a complete metric space guarantee not only the existence but also uniqueness of a fixed point. In the proof, Banach indicates how the desired fixed point should be found. Regarding that it is a core tool in the solution of the certain differential equation, the existence and uniqueness of the solution of the differential equations are induced to the existence and uniqueness of a fixed point. Indeed, this observation can be the main reason why the metric fixed point theory has been investigated heavily.

In the last three decades, thousands of new results have been announced in the framework of the metric fixed point theory. Most of them claimed that it was the generalization of the existing results. Mainly, these announced results are slight generalizations or extensions of the existing ones. Indeed, predominantly, most of the proofs are the mimic of the proof of the pioneer fixed point theorem of Banach: Construct a sequence (usually, by using the Picard operator); indicate that this sequence is convergent and finally prove that the limit of this recursive sequence is the required fixed point of the considered operator. The fact that existing so many publications on this topic has encouraged researchers to organize and unify this scattered literature. One of the best examples of this trend was given by Samet *et al.* [61] in 2012 by involving the notion of admissible mappings.

Now, we shall explain why the new notion of Samet *et al.* [61], an admissible mapping, is important. First of all, we need to underline that Banach's famous fixed point theorem has been generalized and extended in many different ways in the literature. The most classic approach for generalization and extension is to change the definitions of contraction. The other approach and method are that a given contraction function is in cyclic form. One of the other methods for generalization and extension is to add a partial order on the structure where contraction is defined. Admissible mappings make it possible to put together these three approaches in a single statement. For the clarification of the fixed point theory literature, admissible mapping plays one of the key roles.

In this manuscript, we shall revisit the literature by using the auxiliary function: admissible mapping. We shall provide examples and put several consequences to illustrate how this approach works successfully.

2 Preliminaries

In this section, we collect some necessary tools (notions, notations) that are essential to express the results.

First of all, we shall fixed some basic notations: Hereafter, $\mathbb{N}$ and $\mathbb{N}_0$ denote the set of positive integers and the set of nonnegative integers. Furthermore, the symbols $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{R}_0^+$ represent the set of reals, positive reals and the set of nonnegative reals, respectively. Throughout the manuscript, all considered sets are non-empty.

We shall first recall the admissible mapping defined by Samet *et al.* [61]. Let $\alpha : X \times X \to [0, \infty)$ be a function. Then, the mapping $T : X \to X$ is said to be α-admissible [61] if, for all $x, y \in X$,

$$\alpha(x, y) \geq 1 \Longrightarrow \alpha(Tx, Ty) \geq 1.$$

Next, we recollect the notion of triangular α-admissible that plays crucial rules in usage of triangle inequality axiom of the metric.

 https://doi.org/10.17993/3ctic.2022.112.26-50

Definition 1. *[41] A self-mapping $T : X \to X$ is called triangular α-admissible if*

$$(T_1)\ \ T \text{ is } \alpha - \text{admissible},$$
$$(T_2)\ \ \alpha(x,z) \geq 1, \quad \alpha(z,y) \geq 1 \Longrightarrow \alpha(x,y) \geq 1, \quad x,y,z \in X.$$

Example 1. *Suppose that $M = (0, +\infty)$. Define $T : M \to M$ and $\alpha : M \times M \to [0, \infty)$ by*

(1) $T(x) = \ln(x+1)$, for all $x \in M$ and $\alpha(x,y) = \begin{cases} 1, & \text{if } x \geq y; \\ 0, & \text{if } x < y. \end{cases}$
Then T is α-admissible.

(2) $T(x) = \sqrt[3]{x}$, for all $x \in M$ and $\alpha(x,y) = \begin{cases} e^{x-y}, & \text{if } x \geq y; \\ 0, & \text{if } x < y. \end{cases}$
Then T is α-admissible.

For more examples of such mappings are presented in [30, 40, 41, 46–48, 61] and references therein.

In 2014, Popescu [54] conclude that the notion of triangular α-admissible can be refined slightly, as follows:

Definition 2. *[54] Let $T : X \to X$ be a self-mapping and $\alpha : X \times X \to [0, \infty)$ be a function. Then T is said to be α-orbital admissible if*

$$(T3)\ \ \alpha(x, Tx) \geq 1 \Rightarrow \alpha(Tx, T^2x) \geq 1.$$

Definition 3. *[54] Let $T : X \to X$ be a self-mapping and $\alpha : X \times X \to [0, \infty)$ be a function. Then T is said to be triangular α-orbital admissible if T is α-orbital admissible and*

$$(T4)\ \ \alpha(x,y) \geq 1 \text{ and } \alpha(y, Ty) \geq 1 \Rightarrow \alpha(x, Ty) \geq 1.$$

As it is expected, each α-admissible mapping is an α-orbital admissible mapping and each triangular α-admissible mapping is a triangular α-orbital admissible mapping. Notice also that the converse is false, see e.g. ([54] Example 7).

A metric space (X, d) is said α-regular [54], if for every sequence $\{x_n\}$ in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k.

Lemma 1. *[54] Let $T : X \to X$ be a triangular α-orbital admissible mapping. Assume that there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for each $n \in \mathbb{N}_0$. Then we have $\alpha(x_n, x_m) \geq 1$ for all $m, n \in \mathbb{N}$ with $n < m$.*

In what follows, we recall the following auxiliary family of the functions, namely, comparison and c-comparison functions. A mapping $\psi : [0, \infty) \to [0, \infty)$ is called a *comparison function* if it is increasing and $\psi^n(t) \to 0$, $n \to \infty$, for any $t \in [0, \infty)$. We denote by Φ, the class of the corporation function $\psi : [0, \infty) \to [0, \infty)$. For more details and examples , see e.g. [21, 58]. Among them, we recall the following essential result.

Lemma 2. *(Berinde [21], Rus [58]) If $\psi : [0, \infty) \to [0, \infty)$ is a comparison function, then:*

(1) each iterate ψ^k of ψ, $k \geq 1$, is also a comparison function;

(2) ψ is continuous at 0;

(3) $\psi(t) < t$, for any $t > 0$.

Later, Berinde [21] introduced the concept of (c)-*comparison function* in the following way.

https://doi.org/10.17993/3ctic.2022.112.26-50

Definition 4. *(Berinde [21]) A function $\psi : [0, \infty) \to [0, \infty)$ is said to be a (c)-comparison function if*

(c_1) ψ is increasing,

(c_2) there exists $k_0 \in \mathbb{N}$, $a \in (0, 1)$ and a convergent series of nonnegative terms $\sum_{k=1}^{\infty} v_k$ such that $\psi^{k+1}(t) \le a\psi^k(t) + v_k$, for $k \ge k_0$ and any $t \in [0, \infty)$.

From now on, the letter Ψ is reserved to indicate the family of functions (c)-comparison function. It is evident that Lemma 2 is valid for $\psi \in \Psi$. Further, (c_2) yields

$$\sum_{n=1}^{+\infty} \psi^n(t) < \infty,$$

for all $t > 0$, where ψ^n is the n^{th} iterate of ψ.

Samet *et al.* [61] prove the first fixed point result via admissible mapping by introducing the following concepts.

Definition 5. *Let (X, d) be a metric space and $T : X \to X$ be a given mapping. We say that T is an $\alpha - \psi$ contractive mapping if there exist two functions $\alpha : X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that*

$$\alpha(x, y)d(Tx, Ty) \le \psi(d(x, y)), \text{ for all } x, y \in X.$$

It is obvious that, any contractive mapping forms an $\alpha - \psi$ contractive mapping with $\alpha(x, y) = 1$ for all $x, y \in X$ and $\psi(t) = kt$, $k \in (0, 1)$.

The following is the interesting fixed point theorem of Samet et al. [61]

Theorem 1. *Let (X, d) be a complete metric space and $T : X \to X$ be an $\alpha - \psi$ contractive mapping. Suppose that*

(i) T is $\alpha-$admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \ge 1$;

(iii) T is continuous, or

(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \ge 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then $\alpha(x_n, x) \ge 1$ for all n.

Then there exists $u \in X$ such that $Tu = u$.

Note that in this setting, for the uniqueness additional condition is considered.

Theorem 2. *Adding to the hypotheses of Theorem 1 (resp. Theorem 1) the condition: For all $x, y \in X$, there exists $z \in X$ such that $\alpha(x, z) \ge 1$ and $\alpha(y, z) \ge 1$, we obtain uniqueness of the fixed point.*

Another successful attempt to simplify and clarify the literature of the metric fixed point theory was done by Khojasteh *et al.* [44] in 2015. In this paper, the authors introduced the notion of the simulation functions to combine the several existing results. In what follows, we recall the definition of this auxiliary function.

Definition 6. *(See [44]) A function $\zeta : [0, \infty) \times [0, \infty) \to \mathbb{R}$ is said to be* simulation *if it satisfies the following conditions:*

https://doi.org/10.17993/3ctic.2022.112.26-50

(ζ_1) $\zeta(0,0)=0$;

(ζ_2) $\zeta(t,s)<s-t$ *for all* $t,s>0$;

(ζ_3) *if* $\{t_n\},\{s_n\}$ *are sequences in* $(0,\infty)$ *such that* $\lim\limits_{n\to\infty} t_n=\lim\limits_{n\to\infty} s_n>0$, *then*

$$\limsup_{n\to\infty}\zeta(t_n,s_n)<0. \tag{1}$$

The family of all simulation functions $\zeta:[0,\infty)\times[0,\infty)\to\mathbb{R}$ will be denoted by $\mathcal{Z}$. On account of (ζ_2), we observe that

$$\zeta(t,t)<0 \text{ for all } t>0,\ \zeta\in\mathcal{Z}. \tag{2}$$

Notice also that the condition (ζ_1) is superfluous due to (ζ_2).

Example 2. *(See e.g. [12, 15, 42–44, 56]) Let* $\zeta_i:[0,\infty)\times[0,\infty)\to\mathbb{R}$, $i\in\{1,2,3\}$, *be mappings defined by*

(*i*) $\zeta_1(t,s)=\psi(s)-\phi(t)$ *for all* $t,s\in[0,\infty)$, *where* $\phi,\psi:[0,\infty)\to[0,\infty)$ *are two continuous functions such that* $\psi(t)=\phi(t)=0$ *if, and only if,* $t=0$, *and* $\psi(t)<t\leq\phi(t)$ *for all* $t>0$.

(*ii*) $\zeta_2(t,s)=s-\dfrac{f(t,s)}{g(t,s)}t$ *for all* $t,s\in[0,\infty)$, *where* $f,g:[0,\infty)\to(0,\infty)$ *are two continuous functions with respect to each variable such that* $f(t,s)>g(t,s)$ *for all* $t,s>0$.

(*iii*) $\zeta_3(t,s)=s-\varphi(s)-t$ *for all* $t,s\in[0,\infty)$, *where* $\varphi:[0,\infty)\to[0,\infty)$ *is a continuous function such that* $\varphi(t)=0$ *if, and only if,* $t=0$.

(*iv*) *If* $\varphi:[0,\infty)\to[0,1)$ *is a function such that* $\limsup\limits_{t\to r^+}\varphi(t)<1$ *for all* $r>0$, *and we define*

$$\zeta_T(t,s)=s\,\varphi(s)-t \qquad \text{for all } s,t\in[0,\infty),$$

then ζ_T *is a simulation function.*

(*v*) *If* $\eta:[0,\infty)\to[0,\infty)$ *is an upper semi-continuous mapping such that* $\eta(t)<t$ *for all* $t>0$ *and* $\eta(0)=0$, *and we define*

$$\zeta_{BW}(t,s)=\eta(s)-t \qquad \text{for all } s,t\in[0,\infty),$$

then ζ_{BW} *is a simulation function.*

(*vi*) *If* $\phi:[0,\infty)\to[0,\infty)$ *is a function such that* $\int_0^\varepsilon\phi(u)du$ *exists and* $\int_0^\varepsilon\phi(u)du>\varepsilon$, *for each* $\varepsilon>0$, *and we define*

$$\zeta_K(t,s)=s-\int_0^t\phi(u)du \qquad \text{for all } s,t\in[0,\infty),$$

then ζ_K *is a simulation function.*

Suppose (X,d) is a metric space, T is a self-mapping on X and $\zeta\in\mathcal{Z}$. We say that T is a $\mathcal{Z}$-*contraction* with respect to ζ [44], if

$$\zeta(d(T(x),T(y)),d(x,y))\geq 0, \text{ for all } x,y\in X.$$

It is evident that renowned Banach contraction forms $\mathcal{Z}$-*contraction* with respect to ζ where $\zeta(t,s)=ks-t$ with $k\in[0,1)$ and $s,t\in[0,\infty)$.

Theorem 3. *On the complete metric space, every* $\mathcal{Z}$-contraction *possesses a unique fixed point.*

 https://doi.org/10.17993/3ctic.2022.112.26-50

3 A theorem with many consequences

In this section, we shall consider a theorem that generalizes and hence unifies a number of existing results. Consequently, we list many corollaries.

Definition 7. *Let (X, d) be a metric space and $T : X \to X$ be a given mapping. We say that T is a generalized Suzuki type $(\alpha-\psi)-\mathcal{Z}$-contraction mapping if there exist two functions $\alpha : X \times X \to [0, \infty)$, $\zeta \in \mathcal{Z}$ and $\psi \in \Psi$ such that for all $x, y \in X$, we have*

$$\frac{1}{2}d(x, Tx) \leq d(x, y) \text{ implies } \zeta(\psi(M(x, y)), \alpha(x, y)d(Tx, Ty)) \geq 0, \tag{3}$$

where $M(x, y) = \max\left\{d(x, y), \frac{d(x, Tx) + d(y, Ty)}{2}, \frac{d(x, Ty) + d(y, Tx)}{2}\right\}$.

Theorem 4. *Let (X, d) be a complete metric space. Suppose that $T : X \to X$ is a generalized Suzuki type $(\alpha - \psi) - \mathcal{Z}$-contraction mapping and satisfies the following conditions:*

(i) T is triangular α-orbital admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;

(iii) T is continuous.

Then there exists $u \in X$ such that $Tu = u$.

Proof. On account of the assumption (ii) of the theorem, there is a point $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$.

Starting with this initial point, we shall built-up a recursive sequence $\{x_n\}$ in X by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}_0$. We, first, observe that incase of $x_{n_0} = x_{n_0+1}$ for some n_0, we conclude that $u = x_{n_0}$ is a fixed point of T. Accordingly, we presume that $x_n \neq x_{n+1}$ for all n. Hence, we find that

$$0 < \frac{1}{2}d(x_n, x_{n+1}) = \frac{1}{2}d(x_n, Tx_n) \leq d(x_n, x_{n+1}),$$

for all n.

On the other hand, employing that T is $\alpha-$admissible, we derive that

$$\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq 1 \Rightarrow \alpha(Tx_0, Tx_1) = \alpha(x_1, x_2) \geq 1.$$

Inductively, we have

$$\alpha(x_n, x_{n+1}) \geq 1, \text{ for all } n = 0, 1, \ldots \tag{4}$$

From (3) and (4), it follows that for all $n \geq 1$, we have

$$\frac{1}{2}d(x_n, Tx_n) \leq d(x_n, x_{n+1}),$$

implies that

$$\zeta(\psi(M(x_n, x_{n+1})), \alpha(x_n, x_{n+1}))d(Tx_n, Tx_{n+1}))) \geq 0,$$

which is equivalent to

$$d(x_{n+1}, x_n) = d(Tx_n, Tx_{n-1}) \leq \alpha(x_n, x_{n-1})d(Tx_n, Tx_{n-1}) \leq \psi(M(x_n, x_{n-1})). \tag{5}$$

Now, we shall simplify the right hand side of the inequality above, as follows

$$\begin{aligned} M(x_n, x_{n-1}) &= \max\left\{d(x_n, x_{n-1}), \frac{d(x_n, Tx_n) + d(x_{n-1}, Tx_{n-1})}{2}, \frac{d(x_n, Tx_{n-1}) + d(x_{n-1}, Tx_n)}{2}\right\} \\ &= \max\left\{d(x_n, x_{n-1}), \frac{d(x_n, x_{n+1}) + d(x_{n-1}, x_n)}{2}, \frac{d(x_{n-1}, x_{n+1})}{2}\right\} \\ &\leq \max\left\{d(x_n, x_{n-1}), \frac{d(x_n, x_{n+1}) + d(x_{n-1}, x_n)}{2}\right\} \\ &\leq \max\{d(x_n, x_{n-1}), d(x_n, x_{n+1})\}. \end{aligned}$$

https://doi.org/10.17993/3ctic.2022.112.26-50

Consequently, regarding (5) together with the fact that ψ is a nondecreasing function, we derive

$$d(x_{n+1}, x_n) \leq \psi\left(\max\{d(x_n, x_{n-1}), d(x_n, x_{n+1})\}\right), \tag{6}$$

for all $n \geq 1$. If for some $n \geq 1$, we have $d(x_n, x_{n-1}) \leq d(x_n, x_{n+1})$, from (6), we obtain that

$$d(x_{n+1}, x_n) \leq \psi(d(x_n, x_{n+1})) < d(x_n, x_{n+1}),$$

a contradiction. Thus, for all $n \geq 1$, we have

$$\max\{d(x_n, x_{n-1}), d(x_n, x_{n+1})\} = d(x_n, x_{n-1}). \tag{7}$$

Using (6) and (7), we get that

$$d(x_{n+1}, x_n) \leq \psi(d(x_n, x_{n-1})) < d(x_n, x_{n-1}), \tag{8}$$

for all $n \geq 1$. By induction, we get

$$d(x_{n+1}, x_n) \leq \psi^n(d(x_1, x_0)), \text{ for all } n \geq 1. \tag{9}$$

From (9) and using the triangular inequality, for all $k \geq 1$, we have

$$\begin{aligned} d(x_n, x_{n+k}) &\leq d(x_n, x_{n+1}) + \ldots + d(x_{n+k-1}, x_{n+k}) \\ &\leq \sum_{p=n}^{n+k-1} \psi^n(d(x_1, x_0)) \\ &\leq \sum_{p=n}^{+\infty} \psi^n(d(x_1, x_0)) \to 0 \text{ as } n \to \infty. \end{aligned}$$

This implies that $\{x_n\}$ is a Cauchy sequence in (X, d). Since (X, d) is complete, there exists $u \in X$ such that

$$\lim_{n\to\infty} d(x_n, u) = 0. \tag{10}$$

Since T is continuous, we obtain from (10) that

$$\lim_{n\to\infty} d(x_{n+1}, Tu) = \lim_{n\to\infty} d(Tx_n, Tu) = 0. \tag{11}$$

From (10), (11) and the uniqueness of the limit, we get immediately that u is a fixed point of T, that is, $Tu = u$.

In what follows, the continuity of the contraction in Theorem 4 is refined.

Theorem 5. *Let (X, d) be a complete metric space. Suppose that $T : X \to X$ is a generalized Suzuki type $(\alpha - \psi) - \mathcal{Z}$*-contraction *mapping and satisfies the following conditions:*

(i) T is triangular α-orbital admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;

(iii) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k.

Then there exists $u \in X$ such that $Tu = u$.

 https://doi.org/10.17993/3ctic.2022.112.26-50

Proof. Following the proof of Theorem 4 line by line, we deduce that the recursive sequence $\{x_n\}$ defined by $x_{n+1} = Tx_n$ for all $n \geq 0$, converges for some $u \in X$. From (4) and condition (iii), there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, u) \geq 1$ for all k.

Now, we use (3) to deduce $u \in X$ is the required fixed point. For this purpose, we need to show that $\frac{1}{2}d(x_{n(k)}, x_{n(k)+1}) = \frac{1}{2}d(x_{n(k)}, Tx_{n(k)}) \leq d(x_{n(k)}, u)$ or $\frac{1}{2}d(x_{n(k)+1}, x_{n(k)+2}) = \frac{1}{2}d(x_{n(k)+1}, Tx_{n(k)+1}) \leq d(x_{n(k)+1}, u)$. Suppose, on the contrary, that $\frac{1}{2}d(x_{n(k)}, x_{n(k)+1}) > d(x_{n(k)}, u)$ and $\frac{1}{2}d(x_{n(k)+1}, x_{n(k)+2}) > d(x_{n(k)+1}, u)$. On account of the triangle inequality, we have

$$\begin{aligned} d(x_{n(k)}, x_{n(k)+1}) &\leq d(x_{n(k)}, u) + d(u, x_{n(k)+1}) \\ &< \tfrac{1}{2}d(x_{n(k)}, x_{n(k)+1}) + \tfrac{1}{2}d(x_{n(k)+1}, x_{n(k)+2}) \\ \text{on account of (8)} & \\ &< \tfrac{1}{2}d(x_{n(k)}, x_{n(k)+1}) + \tfrac{1}{2}d(x_{n(k)}, x_{n(k)+1}) = d(x_{n(k)}, x_{n(k)+1}), \end{aligned} \tag{12}$$

a contradiction. Thus, we have

$$\frac{1}{2}d(x_{n(k)}, Tx_{n(k)}) \leq d(x_{n(k)}, u) \text{ or } \frac{1}{2}d(x_{n(k)+1}, Tx_{n(k)+1}) \leq d(x_{n(k)+1}, u).$$

After this observation, by applying (3), for all k, we find

$\frac{1}{2}d(x_{n(k)}, Tx_{n(k)}) \leq d(x_{n(k)}, u)$ implies $\zeta(\psi(M(x_{n(k)}, u)), \alpha(x_{n(k)}, u)d(Tx_{n(k)}, Tu)) \geq 0$ which yields that

$$d(x_{n(k)+1}, Tu) = d(Tx_{n(k)}, Tu) \leq \alpha(x_{n(k)}, u)d(Tx_{n(k)}, Tu) \leq \psi(M(x_{n(k)}, u)). \tag{13}$$

On the other hand, we have

$$M(x_{n(k)}, u) = \max\left\{d(x_{n(k)}, u), \frac{d(x_{n(k)}, x_{n(k)+1}) + d(u, Tu)}{2}, \frac{d(x_{n(k)}, Tu) + d(u, x_{n(k)+1})}{2}\right\}.$$

Letting $k \to \infty$ in the above equality, we obtain that

$$\lim_{k\to\infty} M(x_{n(k)}, u) = \frac{d(u, Tu)}{2}. \tag{14}$$

Suppose that $d(u, Tu) > 0$. From (14), for k large enough, we have $M(x_{n(k)}, u) > 0$, which yields that $\psi(M(x_{n(k)}, u)) < M(x_{n(k)}, u)$. Hence, from (13), we find

$$d(x_{n(k)+1}, Tu) < M(x_{n(k)}, u).$$

Letting $k \to \infty$ in the above inequality, using (14), we obtain that

$$d(u, Tu) \leq \frac{d(u, Tu)}{2},$$

which is a contradiction. Thus we have $d(u, Tu) = 0$, that is, $u = Tu$.

Notice that the proved theorems above are valid when we replace Instead of $M(x, y)$ with $N(x, y)$, where

$$N(x, y) = \max\left\{d(x, y), \frac{d(x, Tx) + d(y, Ty)}{2}\right\}.$$

The following example indicate that the hypotheses in Theorem 4 and Theorem 5 do not guarantee uniqueness of the fixed point.

Example 3. *Consider the set* $X = \{(2, 1), (1, 2)\} \subset \mathbb{R}^2$ *endowed with the standard Euclidean distance*

$$d((x, y), (u, v)) = |x - u| + |y - v|,$$

 https://doi.org/10.17993/3ctic.2022.112.26-50

for all $(x,y),(u,v) \in X$. *It is evident that* (X,d) *is a complete metric space. It is clear that* $T(x,y) = (x,y)$ *that is trivially continuous. On account of the inequality* $0 = d(T(x,y),T(u,v)) \leq d((x,y),(u,v))$ *we have*

$$\zeta(\psi(M((x,y),(u,v))),\alpha((x,y),(u,v))d(T(x,y),T(u,v))) \geq 0,$$

for any $\psi \in \Psi$. *Consequently, we have*

$$\alpha((x,y),(u,v))d(T(x,y),T(u,v)) \leq \psi(M((x,y),(u,v))),$$

for all $(x,y),(u,v) \in X$, *where*

$$\alpha((x,y),(u,v)) = \begin{cases} 1 & \textit{if} \quad (x,y) = (u,v), \\ 0 & \textit{if} \quad (x,y) \neq (u,v). \end{cases}$$

Consequently, T *is a generalized Suzuki type* $(\alpha - \psi) - \mathcal{Z}$-contraction *mapping. On the other hand, for all* $(x,y),(u,v) \in X$, *we have*

$$\alpha((x,y),(u,v)) \geq 1 \to (x,y) = (u,v) \to T(x,y) = T(u,v) \to \alpha(T(x,y),T(u,v)) \geq 1.$$

Thus, the mapping T *is* $\alpha-$*admissible. Furthermore, for all* $(x,y) \in X$, *we have* $\alpha((x,y),T(x,y)) \geq 1$. *Hence, the assumptions of Theorem 4 are fulfilled. Note that the assumptions of Theorem 5 are also satisfied, indeed if* $\{(x_n,y_n)\}$ *is a sequence in* X *that converges to some point* $(x,y) \in X$ *with* $\alpha((x_n,y_n),(x_{n+1},y_{n+1})) \geq 1$ *for all* n, *from the definition of* α, *we have* $(x_n,y_n) = (x,y)$ *for all* n, *that yields* $\alpha((x_n,y_n),(x,y)) = 1$ *for all* n. *Notice that, in this case,* T *has two fixed points in* X.

For the uniqueness of a fixed point of such mappings, we need to the following additional condition:

(H) For all $x,y \in \mathrm{Fix}(T)$, there exists $z \in X$ such that $\alpha(x,z) \geq 1$ and $\alpha(y,z) \geq 1$.

Here, $\mathrm{Fix}(T)$ denotes the set of fixed points of T.

Theorem 6. *Adding condition (H) to the hypotheses of Theorem 4 (resp. Theorem 5), we obtain that* u *is the unique fixed point of* T.

Proof. Suppose, on the contrary, that v is another fixed point of T. From (H), there exists $z \in X$ such that

$$\alpha(u,z) \geq 1 \text{ and } \alpha(v,z) \geq 1. \tag{15}$$

Due to the fact that T is $\alpha-$admissible together with (15), we find

$$\alpha(u,T^n z) \geq 1 \text{ and } \alpha(v,T^n z) \geq 1, \text{ for all } n. \tag{16}$$

Construct a sequence $\{z_n\}$ in X by $z_{n+1} = Tz_n$ for all $n \geq 0$ and $z_0 = z$.

Taking (16) into account, for all n, we have

$$0 = \frac{1}{2}d(u,Tu) \leq d(u,z_n) \text{ implies } \zeta(\psi(M(u,z_n)),\alpha(u,z_n)d(Tu,Tz_n)) \geq 0, \tag{17}$$

which is equivalent to

$$d(u,z_{n+1}) = d(Tu,Tz_n) \leq \alpha(u,z_n)d(Tu,Tz_n) \leq \psi(M(u,z_n)). \tag{18}$$

On the other hand, we find

$$\begin{aligned} M(u,z_n) &= \max\left\{d(u,z_n), \frac{d(z_n,z_{n+1})}{2}, \frac{d(u,z_{n+1}) + d(z_n,u)}{2}\right\} \\ &\leq \max\left\{d(u,z_n), \frac{d(z_n,u) + d(u,z_{n+1})}{2}\right\} \\ &\leq \max\{d(u,z_n), d(u,z_{n+1})\}. \end{aligned}$$

 https://doi.org/10.17993/3ctic.2022.112.26-50

On account of the inequality above, the expression (18) and the monotone property of ψ, we derive that

$$d(u, z_{n+1}) \leq \psi \left(\max\{d(u, z_n), d(u, z_{n+1})\}\right), \tag{19}$$

for all n. Without restriction to the generality, we can suppose that $d(u, z_n) > 0$ for all n. If $\max\{d(u, z_n), d(u, z_{n+1})\} = d(u, z_{n+1})$, we get from (19) that

$$d(u, z_{n+1}) \leq \psi(d(u, z_{n+1})) < d(u, z_{n+1}),$$

which is a contradiction. Thus we have $\max\{d(u, z_n), d(u, z_{n+1})\} = d(u, z_n)$, and

$$d(u, z_{n+1}) \leq \psi(d(u, z_n)),$$

for all n. This implies that

$$d(u, z_n) \leq \psi^n(d(u, z_0)), \text{ for all } n \geq 1.$$

Letting $n \to \infty$ in the above inequality, we obtain

$$\lim_{n\to\infty} d(z_n, u) = 0. \tag{20}$$

Similarly, one can show that

$$\lim_{n\to\infty} d(z_n, v) = 0. \tag{21}$$

From (20) and (21), it follows that $u = v$. Thus we proved that u is the unique fixed point of T.

3.1 Immediate consequences

. The first immediate consequence is obtained by removing the Suzuki condition, as in the following definition.

Definition 8. *Let (X, d) be a metric space and $T : X \to X$ be a given mapping. We say that T is a generalized type $(\alpha - \psi) - \mathcal{Z}$-*contraction *mapping if there exist two functions $\alpha : X \times X \to [0, \infty)$, $\zeta \in \mathcal{Z}$ and $\psi \in \Psi$ such that for all $x, y \in X$, we have*

$$\zeta(\psi(M(x, y)), \alpha(x, y)d(Tx, Ty)) \geq 0, \tag{22}$$

where $M(x, y) = \max\left\{d(x, y), \dfrac{d(x, Tx) + d(y, Ty)}{2}, \dfrac{d(x, Ty) + d(y, Tx)}{2}\right\}$.

Theorem 7. *Let (X, d) be a complete metric space. Suppose that $T : X \to X$ is a generalized Suzuki type $(\alpha - \psi) - \mathcal{Z}$-*contraction *mapping and satisfies the following conditions:*

(i) T is triangular α-orbital admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;

(iii) T is continuous*
or

*(iii)** if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k,*

(iv) the condition (H) holds.

Then there exists $u \in X$ such that $Tu = u$.

We skipped the proof. Indeed, it is verbatim of the combinations of the proofs of Theorem 4, Theorem 5 and Theorem 6, by removing the related lines about the Suzuki condition.

Taking Example 2 into account, both Theorem 6 and Theorem 7 yields several consequences. In this direction, one of the first example, by considering the case (i) Example 2 is the following theorem

 https://doi.org/10.17993/3ctic.2022.112.26-50

Definition 9. *Let (X,d) be a metric space and $T : X \to X$ be a given mapping. We say that T is a generalized $\alpha - \psi$ contractive mapping if there exist two functions $\alpha : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that for all $x, y \in X$, we have*

$$\alpha(x,y)d(Tx,Ty) \leq \psi(M(x,y)), \tag{23}$$

where $M(x,y) = \max\left\{d(x,y), \frac{d(x,Tx)+d(y,Ty)}{2}, \frac{d(x,Ty)+d(y,Tx)}{2}\right\}$.

Theorem 8. *Let (X,d) be a complete metric space. Suppose that $T : X \to X$ is a generalized $\alpha - \psi$ contractive mapping and satisfies the following conditions:*

(i) T is $\alpha-$admissible ;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;

(iii) T is continuous*
or

*(iii)** if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k,*

(iv) the condition (H) holds.

Then there exists $u \in X$ such that $Tu = u$.

It is evident that by taking, the other cases of Example 2, into account, one can further consequences. We prefer to skip these consequences by the sake of the length of the manuscript.

4 Further Consequences

In this section, we shall indicate that several existing results in the literature can be deduced easily from our Theorem 6.

4.1 Standard fixed point theorems

Taking Theorem 6 into account, employing $\alpha(x,y) = 1$ for all $x, y \in X$, we obtain immediately the following fixed point theorem.

Corollary 1. *Let (X,d) be a complete metric space and $T : X \to X$ be a given mapping. Suppose that there exists a function $\psi \in \Psi$ such that*

$$\frac{1}{2}d(x,Tx) \leq d(x,y) \text{ implies } \zeta(\psi(M(x,y)), d(Tx,Ty)) \geq 0,$$

for all $x, y \in X$, where $M(x,y) = \max\left\{d(x,y), \frac{d(x,Tx)+d(y,Ty)}{2}, \frac{d(x,Ty)+d(y,Tx)}{2}\right\}$. *Then T has a unique fixed point.*

In the same way, by letting $\alpha(x,y) = 1$, for all $x, y \in X$, in Theorem 7, we find the following result:

Corollary 2. *Let (X,d) be a complete metric space and $T : X \to X$ be a given mapping. Suppose that there exists a function $\psi \in \Psi$ such that*

$$\zeta(\psi(M(x,y)), d(Tx,Ty)) \geq 0,$$

for all $x, y \in X$, where $M(x,y) = \max\left\{d(x,y), \frac{d(x,Tx)+d(y,Ty)}{2}, \frac{d(x,Ty)+d(y,Tx)}{2}\right\}$. *Then T has a unique fixed point.*

 https://doi.org/10.17993/3ctic.2022.112.26-50

Analogously, by letting $\alpha(x,y)=1$, for all $x,y\in X$, in Theorem 8, we find the following result:

Corollary 3. *Let (X,d) be a complete metric space and $T:X\to X$ be a given mapping. Suppose that there exists a function $\psi\in\Psi$ such that*

$$d(Tx,Ty)\leq\psi(M(x,y)),$$

for all $x,y\in X$. Then T has a unique fixed point.

The following fixed point theorems follow immediately from Corollary 3.

Corollary 4 (see Berinde [22])**.** *Let (X,d) be a complete metric space and $T:X\to X$ be a given mapping. Suppose that there exists a function $\psi\in\Psi$ such that*

$$d(Tx,Ty)\leq\psi(d(x,y)),$$

for all $x,y\in X$. Then T has a unique fixed point.

Corollary 5 (see Ćirić [26])**.** *Let (X,d) be a complete metric space and $T:X\to X$ be a given mapping. Suppose that there exists a constant $\lambda\in(0,1)$ such that*

$$d(Tx,Ty)\leq\lambda\, max\left\{d(x,y),\frac{d(x,Tx)+d(y,Ty)}{2},\frac{d(x,Ty)+d(y,Tx)}{2}\right\},$$

for all $x,y\in X$. Then T has a unique fixed point.

Corollary 6 (see Hardy and Rogers [33])**.** *Let (X,d) be a complete metric space and $T:X\to X$ be a given mapping. Suppose that there exist constants $A,B,C\geq 0$ with $(A+2B+2C)\in(0,1)$ such that*

$$d(Tx,Ty)\leq Ad(x,y)+B[d(x,Tx)+d(y,Ty)]+C[d(x,Ty)+d(y,Tx)],$$

for all $x,y\in X$. Then T has a unique fixed point.

Corollary 7 (Banach Contraction Principle [18])**.** *Let (X,d) be a complete metric space and $T:X\to X$ be a given mapping. Suppose that there exists a constant $\lambda\in(0,1)$ such that*

$$d(Tx,Ty)\leq\lambda\, d(x,y),$$

for all $x,y\in X$. Then T has a unique fixed point.

Corollary 8 (see Kannan [36])**.** *Let (X,d) be a complete metric space and $T:X\to X$ be a given mapping. Suppose that there exists a constant $\lambda\in(0,1/2)$ such that*

$$d(Tx,Ty)\leq\lambda\,[d(x,Tx)+d(y,Ty)],$$

for all $x,y\in X$. Then T has a unique fixed point.

Corollary 9 (see Chatterjea [24])**.** *Let (X,d) be a complete metric space and $T:X\to X$ be a given mapping. Suppose that there exists a constant $\lambda\in(0,1/2)$ such that*

$$d(Tx,Ty)\leq\lambda\,[d(x,Ty)+d(y,Tx)],$$

for all $x,y\in X$. Then T has a unique fixed point.

 https://doi.org/10.17993/3ctic.2022.112.26-50

Corollary 10 (Dass-Gupta Theorem [29]). *Let (X, d) be a metric space and $T : X \to X$ be a given mapping. Suppose that there exist constants $\lambda, \mu \geq 0$ with $\lambda + \mu < 1$ such that*

$$d(Tx, Ty) \leq \mu d(y, Ty)\,\frac{1 + d(x, Tx)}{1 + d(x, y)} + \lambda d(x, y), \quad \textit{for all } x, y \in X. \tag{24}$$

Then T has a unique fixed point.

Sketch of the Proof. Consider the functions $\psi : [0, \infty) \to [0, \infty)$ and $\alpha : X \times X \to \mathbb{R}$ defined by

$$\psi(t) = \lambda t, \;\; t \geq 0 \tag{25}$$

and

$$\alpha(x, y) = \begin{cases} 1 - \mu\dfrac{d(y, Ty)(1 + d(x, Tx))}{(1 + d(x, y))d(Tx, Ty)}, & \text{if } Tx \neq Ty, \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

The rest is simple evaluation. For the detailed proof, we refer to Samet [60].

Corollary 11 (Jaggi theorem [35]). *Let (X, d) be a metric space and $T : X \to X$ be a given mapping. Suppose there exist constants $\lambda, \mu \geq 0$ with $\lambda + \mu < 1$ such that*

$$d(Tx, Ty) \leq \mu\frac{d(x, Tx)d(y, Ty)}{d(x, y)} + \lambda d(x, y), \quad \textit{for all } x, y \in X, \; x \neq y. \tag{27}$$

Then there exist $\psi \in \Psi$ and $\alpha : X \times X \to \mathbb{R}$ such that T is an α-ψ contraction. Then T has a unique fixed point.

Sketch of the Proof.

Consider the functions $\psi : [0, \infty) \to [0, \infty)$ and $\alpha : X \times X \to \mathbb{R}$ defined by

$$\psi(t) = \lambda t, \;\; t \geq 0 \tag{28}$$

and

$$\alpha(x, y) = \begin{cases} 1 - \mu\dfrac{d(x, Tx)d(y, Ty)}{d(x, y)d(Tx, Ty)}, & \text{if } Tx \neq Ty, \\ 0, & \text{otherwise.} \end{cases} \tag{29}$$

The rest is a simple evaluation. For the detailed proof, we refer to Samet [60].

Theorem 9 (Berinde Theorem [23]). *Let (X, d) be a metric space and $T : X \to X$ be a given mapping. Suppose that there exists $\lambda \in (0, 1)$ and $L \geq 0$ such that*

$$d(Tx, Ty) \leq \lambda\, d(x, y) + L\, d(y, Tx), \quad \textit{for all } x, y \in X. \tag{30}$$

Then T has a fixed point.

Sketch of the Proof.

Consider the functions $\psi : [0, \infty) \to [0, \infty)$ and $\alpha : X \times X \to \mathbb{R}$ defined by

$$\psi(t) = \lambda t, \;\; t \geq 0$$

and

$$\alpha(x, y) = \begin{cases} 1 - L\dfrac{d(y, Tx)}{d(Tx, Ty)}, & \text{if } Tx \neq Ty, \\ 0, & \text{otherwise.} \end{cases} \tag{31}$$

The rest is straightforward. For more details for the proof, we refer to Samet [60].

 https://doi.org/10.17993/3ctic.2022.112.26-50

Theorem 10 (Ćirić's non-unique fixed point theorem [27].). *Let (X,d) be a metric space and $T : X \to X$ be a given mapping. there exists $\lambda \in (0,1)$ such that for all $x, y \in X$, we have*

$$\min\{d(Tx,Ty), d(x,Tx), d(y,Ty)\} - \min\{d(x,Ty), d(y,Tx)\} \leq \lambda\, d(x,y). \tag{32}$$

Then T has a fixed point.

*S*ketch of the Proof. Consider the functions $\psi : [0,\infty) \to [0,\infty)$ and $\alpha : X \times X \to \mathbb{R}$ defined by

$$\psi(t) = \lambda\, t, \;\; t \geq 0 \tag{33}$$

and

$$\alpha(x,y) = \begin{cases} \min\left\{1, \frac{d(x,Tx)}{d(Tx,Ty)}, \frac{d(y,Ty)}{d(Tx,Ty)}\right\} - \min\left\{\frac{d(x,Ty)}{d(Tx,Ty)}, \frac{d(y,Tx)}{d(Tx,Ty)}\right\}, & \text{if } Tx \neq Ty, \\ 0, & \text{otherwise.} \end{cases} \tag{34}$$

The rest is simple evaluation. For the detailed proof, we refer to Samet [60].

Theorem 11 (Suzuki Theorem [63]). *Let (X,d) be a metric space and $T : X \to X$ be a given mapping. Suppose that there exists $r \in (0,1)$ such that*

$$(1+r)^{-1} d(x,Tx) \leq d(x,y) \Longrightarrow d(Tx,Ty) \leq r\, d(x,y), \quad \textit{for all } x, y \in X. \tag{35}$$

Then T has a unique fixed point.

*S*ketch of the Proof. Consider the functions $\psi : [0,\infty) \to [0,\infty)$ and $\alpha : X \times X \to \mathbb{R}$ defined by

$$\psi(t) = r\, t, \;\; t \geq 0$$

and

$$\alpha(x,y) = \begin{cases} 1, & \text{if } (1+r)^{-1} d(x,Tx) \leq d(x,y), \\ 0, & \text{otherwise.} \end{cases} \tag{36}$$

From (35), we have

$$\alpha(x,y) d(Tx,Ty) \leq \psi(d(x,y)), \quad \text{for all } x, y \in X.$$

Then T is an α-ψ contraction.

4.2 Fixed point theorems on metric spaces endowed with a partial order

In the last two decades, one of the trend in fixed point theory is the revisit the well-known fixed point theorem on metric spaces endowed with partial orders [65]. Among all, Ran and Reurings in [55] revisited the Banach contraction principle in partially ordered sets with some applications to matrix equations. Another version of the generalization of Banach contraction principle in partially ordered sets was proposed by Nieto and Rodríguez-López in [49]. Later, this trend was supported by several authors, see e.g. [2, 13, 28, 34, 53, 59] and the references cited therein . In this section, we shall show that Theorem 6 implies easily various fixed point results on a metric space endowed with a partial order. At first, we need to recall some concepts.

Definition 10. *Let $(X, \preceq)$ be a partially ordered set and $T : X \to X$ be a given mapping. We say that T is nondecreasing with respect to $\preceq$ if*

$$x, y \in X, \;\; x \preceq y \Longrightarrow Tx \preceq Ty.$$

Definition 11. *Let $(X, \preceq)$ be a partially ordered set. A sequence $\{x_n\} \subset X$ is said to be nondecreasing with respect to $\preceq$ if $x_n \preceq x_{n+1}$ for all n.*

 https://doi.org/10.17993/3ctic.2022.112.26-50

Definition 12. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X. We say that $(X, \preceq, d)$ is regular if for every nondecreasing sequence $\{x_n\} \subset X$ such that $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $x_{n(k)} \preceq x$ for all k.*

We have the following result.

Corollary 12. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a function $\psi \in \Psi$ such that*

$$\frac{1}{2}d(x, Tx) \leq d(x, y) \text{ implies } \zeta(\psi(M(x, y)), d(Tx, Ty)) \geq 0,$$

for all $x, y \in X$ with $x \succeq y$. Suppose also that the following conditions hold:

- *(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*
- *(ii) T is continuous or $(X, \preceq, d)$ is regular.*

Then T has a fixed point. Moreover, if for all $x, y \in X$ there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, we have uniqueness of the fixed point.

Proof. Define the mapping $\alpha : X \times X \to [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} 1 \text{ if } x \preceq y \text{ or } x \succeq y, \\ 0 \text{ otherwise.} \end{cases}$$

Clearly, T is a generalized $\alpha - \psi$ contractive mapping, that is,

$$\alpha(x, y)d(Tx, Ty) \leq \psi(M(x, y)),$$

for all $x, y \in X$. From condition (i), we have $\alpha(x_0, Tx_0) \geq 1$. Moreover, for all $x, y \in X$, from the monotone property of T, we have

$$\alpha(x, y) \geq 1 \Longrightarrow x \succeq y \text{ or } x \preceq y \Longrightarrow Tx \succeq Ty \text{ or } Tx \preceq Ty \Longrightarrow \alpha(Tx, Ty) \geq 1.$$

Thus T is $\alpha-$admissible. Now, if T is continuous, the existence of a fixed point follows from Theorem 4. Suppose now that $(X, \preceq, d)$ is regular. Let $\{x_n\}$ be a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$. From the regularity hypothesis, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $x_{n(k)} \preceq x$ for all k. This implies from the definition of α that $\alpha(x_{n(k)}, x) \geq 1$ for all k. In this case, the existence of a fixed point follows from Theorem 5. To show the uniqueness, let $x, y \in X$. By hypothesis, there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, which implies from the definition of α that $\alpha(x, z) \geq 1$ and $\alpha(y, z) \geq 1$. Thus we deduce the uniqueness of the fixed point by Theorem 6.

Corollary 13. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a function $\psi \in \Psi$ such that*

$$\zeta(\psi(M(x, y)), d(Tx, Ty)) \geq 0,$$

for all $x, y \in X$ with $x \succeq y$. Suppose also that the following conditions hold:

- *(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*
- *(ii) T is continuous or $(X, \preceq, d)$ is regular.*

Then T has a fixed point. Moreover, if for all $x, y \in X$ there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, we have uniqueness of the fixed point.

https://doi.org/10.17993/3ctic.2022.112.26-50

Corollary 14. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a function $\psi \in \Psi$ such that*

$$d(Tx, Ty) \leq \psi(M(x, y)),$$

for all $x, y \in X$ with $x \succeq y$. Suppose also that the following conditions hold:

(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;

(ii) T is continuous or $(X, \preceq, d)$ is regular.

Then T has a fixed point. Moreover, if for all $x, y \in X$ there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, we have uniqueness of the fixed point.

The following results are immediate consequences of Corollary14.

Corollary 15. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a function $\psi \in \Psi$ such that*

$$d(Tx, Ty) \leq \psi(d(x, y)),$$

for all $x, y \in X$ with $x \succeq y$. Suppose also that the following conditions hold:

(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;

(ii) T is continuous or $(X, \preceq, d)$ is regular.

Then T has a fixed point. Moreover, if for all $x, y \in X$ there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, we have uniqueness of the fixed point.

Corollary 16. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a constant $\lambda \in (0, 1)$ such that*

$$d(Tx, Ty) \leq \lambda\, max \left\{ d(x, y), \frac{d(x, Tx) + d(y, Ty)}{2}, \frac{d(x, Ty) + d(y, Tx)}{2} \right\},$$

for all $x, y \in X$ with $x \succeq y$. Suppose also that the following conditions hold:

(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;

(ii) T is continuous or $(X, \preceq, d)$ is regular.

Then T has a fixed point. Moreover, if for all $x, y \in X$ there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, we have uniqueness of the fixed point.

Corollary 17. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exist constants $A, B, C \geq 0$ with $(A + 2B + 2C) \in (0, 1)$ such that*

$$d(Tx, Ty) \leq Ad(x, y) + B[d(x, Tx) + d(y, Ty)] + C[d(x, Ty) + d(y, Tx)],$$

for all $x, y \in X$ with $x \succeq y$. Suppose also that the following conditions hold:

(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;

(ii) T is continuous or $(X, \preceq, d)$ is regular.

 https://doi.org/10.17993/3ctic.2022.112.26-50

Then T has a fixed point. Moreover, if for all $x, y \in X$ there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, we have uniqueness of the fixed point.

Corollary 18 (see Ran and Reurings [55], Nieto and López [49])**.** *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a constant $\lambda \in (0, 1)$ such that*

$$d(Tx, Ty) \leq \lambda\, d(x, y),$$

for all $x, y \in X$ with $x \succeq y$. Suppose also that the following conditions hold:

(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;

(ii) T is continuous or $(X, \preceq, d)$ is regular.

Then T has a fixed point. Moreover, if for all $x, y \in X$ there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, we have uniqueness of the fixed point.

Corollary 19. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a constant $\lambda \in (0, 1/2)$ such that*

$$d(Tx, Ty) \leq \lambda\, [d(x, Tx) + d(y, Ty)],$$

for all $x, y \in X$ with $x \succeq y$. Suppose also that the following conditions hold:

(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;

(ii) T is continuous or $(X, \preceq, d)$ is regular.

Then T has a fixed point. Moreover, if for all $x, y \in X$ there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, we have uniqueness of the fixed point.

Corollary 20. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a constant $\lambda \in (0, 1/2)$ such that*

$$d(Tx, Ty) \leq \lambda\, [d(x, Ty) + d(y, Tx)],$$

for all $x, y \in X$ with $x \succeq y$. Suppose also that the following conditions hold:

(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;

(ii) T is continuous or $(X, \preceq, d)$ is regular.

Then T has a fixed point. Moreover, if for all $x, y \in X$ there exists $z \in X$ such that $x \preceq z$ and $y \preceq z$, we have uniqueness of the fixed point.

4.3 Fixed point theorems for cyclic contractive mappings

The notion of "cyclic contraction mapping"to find a fixed point was proposed by Kirk, Srinivasan and Veeramani [45]. In this paper, they revisited the famous Banach Contraction Mapping Principle was proved by Kirk, Srinivasan and Veeramani [45] via cyclic contraction. Following this pioneer work [45], several fixed point theorems in the framework of cyclic contractive mappings have appeared (see, for instant, [1, 37, 39, 51, 52, 57]). In this section, we shall proved that Theorem 6 implies several fixed point theorems in the context of cyclic contractive mappings.

We have the following result.

 https://doi.org/10.17993/3ctic.2022.112.26-50

Corollary 21. *Let $\{A_i\}_{i=1}^2$ be nonempty closed subsets of a complete metric space (X, d) and $T : Y \to Y$ be a given mapping, where $Y = A_1 \cup A_2$ and $\zeta \in \mathcal{Z}$. Suppose that the following conditions hold:*

(I) $T(A_1) \subseteq A_2$ and $T(A_2) \subseteq A_1$;

(II) there exists a function $\psi \in \Psi$ such that

$$\frac{1}{2}d(x, Tx) \leq d(x, y) \text{ implies } \zeta(\psi(M(x, y)), d(Tx, Ty)) \geq 0;$$

Then T has a unique fixed point that belongs to $A_1 \cap A_2$.

Proof. Since A_1 and A_2 are closed subsets of the complete metric space (X, d), then (Y, d) is complete. Define the mapping $\alpha : Y \times Y \to [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} 1 \text{ if } (x, y) \in (A_1 \times A_2) \cup (A_2 \times A_1), \\ 0 \text{ otherwise.} \end{cases}$$

From (II) and the definition of α, we can write

$$\alpha(x, y)d(Tx, Ty) \leq \psi(M(x, y)),$$

for all $x, y \in Y$. Thus T is a generalized $\alpha - \psi$ contractive mapping.

Let $(x, y) \in Y \times Y$ such that $\alpha(x, y) \geq 1$. If $(x, y) \in A_1 \times A_2$, from (I), $(Tx, Ty) \in A_2 \times A_1$, which implies that $\alpha(Tx, Ty) \geq 1$. If $(x, y) \in A_2 \times A_1$, from (I), $(Tx, Ty) \in A_1 \times A_2$, which implies that $\alpha(Tx, Ty) \geq 1$. Thus in all cases, we have $\alpha(Tx, Ty) \geq 1$. This implies that T is $\alpha-$admissible.

Also, from (I), for any $a \in A_1$, we have $(a, Ta) \in A_1 \times A_2$, which implies that $\alpha(a, Ta) \geq 1$.

Now, let $\{x_n\}$ be a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$. This implies from the definition of α that

$$(x_n, x_{n+1}) \in (A_1 \times A_2) \cup (A_2 \times A_1), \text{ for all } n.$$

Since $(A_1 \times A_2) \cup (A_2 \times A_1)$ is a closed set with respect to the Euclidean metric, we get that

$$(x, x) \in (A_1 \times A_2) \cup (A_2 \times A_1),$$

which implies that $x \in A_1 \cap A_2$. Thus we get immediately from the definition of α that $\alpha(x_n, x) \geq 1$ for all n.

Finally, let $x, y \in \text{Fix}(T)$. From (I), this implies that $x, y \in A_1 \cap A_2$. So, for any $z \in Y$, we have $\alpha(x, z) \geq 1$ and $\alpha(y, z) \geq 1$. Thus condition (H) is satisfied.

Now, all the hypotheses of Theorem 6 are satisfied, we deduce that T has a unique fixed point that belongs to $A_1 \cap A_2$ (from (I)).

Corollary 22. *Let $\{A_i\}_{i=1}^2$ be nonempty closed subsets of a complete metric space (X, d) and $T : Y \to Y$ be a given mapping, where $Y = A_1 \cup A_2$ and $\zeta \in \mathcal{Z}$. Suppose that the following conditions hold:*

(I) $T(A_1) \subseteq A_2$ and $T(A_2) \subseteq A_1$;

(II) there exists a function $\psi \in \Psi$ such that

$$\zeta(\psi(M(x, y)), d(Tx, Ty)) \geq 0;$$

Then T has a unique fixed point that belongs to $A_1 \cap A_2$.

Corollary 23. *Let $\{A_i\}_{i=1}^2$ be nonempty closed subsets of a complete metric space (X, d) and $T : Y \to Y$ be a given mapping, where $Y = A_1 \cup A_2$. Suppose that the following conditions hold:*

 https://doi.org/10.17993/3ctic.2022.112.26-50

(I) $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$*;*

(II) there exists a function $\psi \in \Psi$ *such that*

$$d(Tx, Ty) \leq \psi(M(x, y)), \text{ for all } (x, y) \in A_1 \times A_2.$$

Then T *has a unique fixed point that belongs to* $A_1 \cap A_2$*.*

The following results are immediate consequences of Corollary 23.

Corollary 24 (see Pacurar and Rus [51])**.** *Let* $\{A_i\}_{i=1}^2$ *be nonempty closed subsets of a complete metric space* (X, d) *and* $T : Y \to Y$ *be a given mapping, where* $Y = A_1 \cup A_2$*. Suppose that the following conditions hold:*

(I) $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$*;*

(II) there exists a function $\psi \in \Psi$ *such that*

$$d(Tx, Ty) \leq \psi(d(x, y)), \text{ for all } (x, y) \in A_1 \times A_2.$$

Then T *has a unique fixed point that belongs to* $A_1 \cap A_2$*.*

Corollary 25. *Let* $\{A_i\}_{i=1}^2$ *be nonempty closed subsets of a complete metric space* (X, d) *and* $T : Y \to Y$ *be a given mapping, where* $Y = A_1 \cup A_2$*. Suppose that the following conditions hold:*

(I) $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$*;*

(II) there exists a constant $\lambda \in (0, 1)$ *such that*

$$d(Tx, Ty) \leq \lambda \max\left\{d(x, y), \frac{d(x, Tx) + d(y, Ty)}{2}, \frac{d(x, Ty) + d(y, Tx)}{2}\right\}, \text{ for all } (x, y) \in A_1 \times A_2.$$

Then T *has a unique fixed point that belongs to* $A_1 \cap A_2$*.*

Corollary 26. *Let* $\{A_i\}_{i=1}^2$ *be nonempty closed subsets of a complete metric space* (X, d) *and* $T : Y \to Y$ *be a given mapping, where* $Y = A_1 \cup A_2$*. Suppose that the following conditions hold:*

(I) $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$*;*

(II) there exist constants $A, B, C \geq 0$ *with* $(A + 2B + 2C) \in (0, 1)$ *such that*

$$d(Tx, Ty) \leq Ad(x, y) + B[d(x, Tx) + d(y, Ty)] + C[d(x, Ty) + d(y, Tx)], \text{ for all } (x, y) \in A_1 \times A_2.$$

Then T *has a unique fixed point that belongs to* $A_1 \cap A_2$*.*

Corollary 27 (see Kirk et al. [45])**.** *Let* $\{A_i\}_{i=1}^2$ *be nonempty closed subsets of a complete metric space* (X, d) *and* $T : Y \to Y$ *be a given mapping, where* $Y = A_1 \cup A_2$*. Suppose that the following conditions hold:*

(I) $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$*;*

(II) there exists a constant $\lambda \in (0, 1)$ *such that*

$$d(Tx, Ty) \leq \lambda\, d(x, y), \text{ for all } (x, y) \in A_1 \times A_2.$$

https://doi.org/10.17993/3ctic.2022.112.26-50

Then T has a unique fixed point that belongs to $A_1 \cap A_2$.

Corollary 28. *Let $\{A_i\}_{i=1}^2$ be nonempty closed subsets of a complete metric space (X, d) and $T : Y \to Y$ be a given mapping, where $Y = A_1 \cup A_2$. Suppose that the following conditions hold:*

(I) $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$;

(II) there exists a constant $\lambda \in (0, 1/2)$ *such that*

$$d(Tx, Ty) \leq \lambda \left[d(x, Tx) + d(y, Ty)\right], \text{ for all } (x, y) \in A_1 \times A_2.$$

Then T has a unique fixed point that belongs to $A_1 \cap A_2$.

Corollary 29. *Let $\{A_i\}_{i=1}^2$ be nonempty closed subsets of a complete metric space (X, d) and $T : Y \to Y$ be a given mapping, where $Y = A_1 \cup A_2$. Suppose that the following conditions hold:*

(I) $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$;

(II) there exists a constant $\lambda \in (0, 1/2)$ *such that*

$$d(Tx, Ty) \leq \lambda \left[d(x, Ty) + d(y, Tx)\right], \text{ for all } (x, y) \in A_1 \times A_2.$$

Then T has a unique fixed point that belongs to $A_1 \cap A_2$.

5 Conclusion

The fixed point theorem is one of the most actively studied research fields of recent times. Naturally, there are many publications on this subject and several new results are announced. This causes the literature to become rather disorganized and dysfunctional. The more troublesome situation is that the existing theorems are rediscovered again and again due to this messiness. More accurately, the results have been repeated. It is therefore essential to organize the fixed-point theory literature, weeding out false and/or repetitive results, and, if possible, combining and unifying existing results into a more general framework. In this work, we show that using admissible mapping, many existing fixed point theorems can be written as a consequence of the main theorem we have given.

Note that the consequences of the main result of the paper, Theorem 6 is not complete. It is possible to add several corollaries. On the other hand, we prefer to skip these possible consequences, since it is clear how the possible result can be concluded from our main theorem and how can be proved. Further, we underline that the main theorem can be derived in the distinct abstract spaces, such as, partial metric space, b-metric space, quasi-metric space, and so on.

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