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Cristo Mark D. Ancheta[1]

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[1]Faculty member, Nueva Ecija University of Science and Technology, Philippines

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An Error Analysis of Students’ Misconceptions and Skill Deficits in Pre-Calculus Subjects

Cristo Mark D. Ancheta
Faculty member, Nueva Ecija University of Science and Technology, Philippines
Email: c_mark14@yahoo.com

ABSTRACT

The study determined the students’ misconceptions and skill deficits in Pre-Calculus subjects utilizing error analysis. It determined the students’ misconceptions, skill deficits, and their reasons, and propose reinforcement activities addressing the problem in Pre-Calculus subjects. The quantitative and qualitative approaches were conducted to come up with a high reliability score result. The respondents were the 50 second-year Engineering students of Nueva Ecija University of Science and Technology who have difficulty in pre-requisite subjects of Calculus. Based on the findings, students had misconceptions about the basic concepts and laws in Algebra necessary for performing fundamental algebraic operations. They also lacked skills in performing algebraic operations. Students also had misconceptions about Trigonometry, particularly on trigonometric functions and their inverses which were pre-requisite concepts and skills in higher mathematics subjects. Moreover, students had misconceptions about Analytic Geometry, particularly in the slope of a line, equations of a line, and graphing functions and conics. The reasons for their misconceptions and skill deficits were due to their lack of knowledge retention, teachers’ methodology, teachers’ lack of knowledge, and lots of school activities.

Keywords: Analytic Geometry, Calculation Error, Deficit, Error Analysis, Pre-Calculus

INTRODUCTION

In the mathematics curriculum, subjects are sequenced in order of requisites to prepare students to be more receptive to new concepts, approaches, and applications in mathematics offered in the upcoming years.

Calculus is one of the subjects is taught in engineering, education, and other technical courses requiring pre-requisite mathematics subjects. The success or failure of any Calculus student is dependent on the extent of competency gained in Pre-Calculus subjects like Algebra, Trigonometry, and Analytic Geometry. Students’ prior knowledge of Pre-Calculus is essential in the learning of concepts, definitions, theorems, and applications of Calculus (Beng, YH. and Yunus, A. A., 2013). Students who have a deficiency in relevant prior knowledge will have difficulty in acquiring new knowledge and new understandings. Inadequate content knowledge of students in these subjects may cause their pitfall in Calculus. Students build more advanced knowledge from prior understandings(Smith III, J., Sessa, A. and Roschelle, J., 2009). According to Mwavita (2005), the background characteristics of students that have been examined by researchers is the role of prior mathematics preparation in relation to calculus performance.

Students who passed Pre-Calculus subjects with mediocrity are likely to grapple with Calculus as this subject requires prior knowledge and skill on these subjects. In the study of Beng and Yumus’ (2013) the students who passed calculus were the students with high marks in pre-calculus subjects and the students with low marks in pre-calculus failed in calculus. Relative to this, a mathematics teacher must know how mathematical instruction in pre-Calculus subjects is received, processed, and retained in the students’ minds. By doing so, appropriate teaching and learning strategies can be planned and executed to attain more meaningful student learning outcomes.

Viable approaches can be employed to probe students’ conceptions of mathematical knowledge and skills deficit. According to Kingsdorf and Krawec (2014) the error analysis approach has proven to be an effective tool for analyzing students’ errors in mathematics. Error analysis is a type of diagnostic assessment that can help a teacher determine what types of errors a student is making and why. More specifically, it is the process of identifying and reviewing students’ errors to determine whether an error pattern exists—that is, whether a student is making the same type of error consistently. Error analysis involves the analysis of error patterns to identify about students have with facts, concepts, strategies, and procedures. Identifying the type of error allows teachers to address the needs of the learners more efficiently (Kundu, U. and Sengupta, D, 2014). If a pattern does exist, the teacher can identify students’ misconceptions or skill deficits. A student has skill deficits if he does not know how to perform the desired skills.

Students who are found weak in basic conceptual knowledge in mathematics commit errors in solving problems (Kaur, N. A, 2011) and (Luneta, 2015). According to Elbrink(2008) and Marshall(1983), the general error categories in mathematics include processing language information, interpreting spatial information, selecting appropriate procedures, making concept associations, and using irrelevant rules or information, calculation errors, procedural errors, and symbolic errors.

From the foregoing discussion, it is evident that students commit errors in mathematics due to their misconceptions and skill deficits, and thus important to analyze the nature and sources of these errors. If this is done, measures and interventions can be adopted to overcome them. Hence, a study is needed to analyze students' misconceptions and skill deficits about subjects.

This study analyzed the students' misconceptions and skill deficits in Pre-Calculus subjects using an error analysis approach. Specifically, it answered the following research objectives, identify students' misconceptions and skill deficits in pre-Calculus subjects, determine the reasons causing students' misconceptions and skill deficits in Pre-Calculus subjects, and propose reinforcement activities to help students with mathematical misconceptions and skill deficits.

The students' misconceptions and skill deficits in pre-calculus subjects are two hindrances to the success of students in the calculus subject (Jones, 2016). To address such problems, identification, and analysis of students' specific needs in every pre-calculus subject must ensure. Also, the factors influencing the students' misconceptions and skill deficits need to be analyzed.

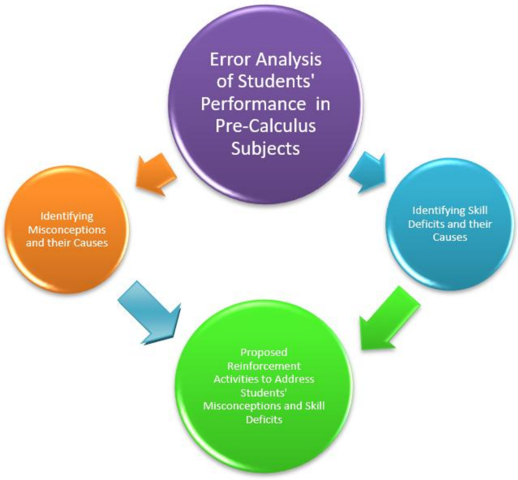

Figure 1. Research Paradigm

Figure 1 shows the paradigm of the study. The conceptual framework of the study used the diagram of the Error Analysis Approach. The diagram starts with the error analysis of the students' performance in the pre-calculus subjects which include the subjects of Algebra, Trigonometry, and Analytic Geometry. The students' errors were described and analyzed based on the result of the Pre-Calculus Test.

From the analysis, the misconceptions and skill deficits of the students were identified. The identified misconceptions and skill deficits served as the basis for proposing reinforcement activities in addressing their misconceptions and skill deficits.

Research on error analysis is not new. Researchers around the world have been conducting studies on this topic for decades. Error analysis is an effective method for identifying patterns of mathematical errors for any student, with or without disabilities, who are struggling in mathematics[4]. Error analysis can identify the patterns of errors or mistakes that students make in their work, understand why students make the errors, and provide targeted instruction to correct the errors (Cohen and Spenciner, 2016).

This study adopted quantitative and qualitative approaches. Quantitative research specifically the descriptive method was used in the description of misconceptions and skill deficits of the students. Quantitative research is the numerical representation and manipulation of observations for the purpose of describing and explaining the phenomena that those observations reflect (Beng, Y. H and Yumus A.S, 2013).

Moreover, the study also made use of a qualitative research design, a case study type. When: (a) the purpose of the study is to answer "how" and "why" questions; (b) you cannot manipulate the behavior of those involved in the study; (c) you want to cover contextual conditions because you believe they are relevant to the phenomenon under study; or (d) the boundaries between the phenomenon and context are unclear. This method is required when analyzing the underlying causes of students' misconceptions and skill deficits.The above premises justified the use of quantitative and qualitative approaches since they determined the students' misconceptions and skill deficits in Pre-Calculus subjects and the underlying reasons behind these were analyzed.

The participants of the study were 50 second-year Engineering students; 30 Civil, 10 Electrical, and 10 Mechanical. They were purposely chosen from a population of 450 students who have difficulties in Algebra, Trigonometry, and Analytic Geometry.

Three sets of instruments were used in gathering the data needed. One is the Pre-Calculus Test (PCT), which consisted of three sub-tests on Algebra, Trigonometry, and Analytic Geometry. The contents of the test were drawn from the learning competencies in the syllabi of instruction in these subjects. Mapping of competencies and skills developed among students in Pre-Calculus and Calculus subjects served as the first step in the development of the test. This mapping was the guide in doing the questions in the Pre-Calculus Test. The items were constructed to measure misconceptions and skill deficits.

The second is the interview guide for students. It was used to determine the underlying reasons for their misconceptions and skill deficits. This was administered after the results of the Pre-Calculus test were initially analyzed. The answers and solutions provided by the students were also clarified during the interview. The third is the interview guide for teachers. It was utilized to clarify the identified misconceptions and skill deficits identified from the results of the Pre-Calculus test. The results of the interview provided consistency with the answers drawn from the other instruments.

METHODS

The figure below is the block diagram used as the error analysis model of the study.

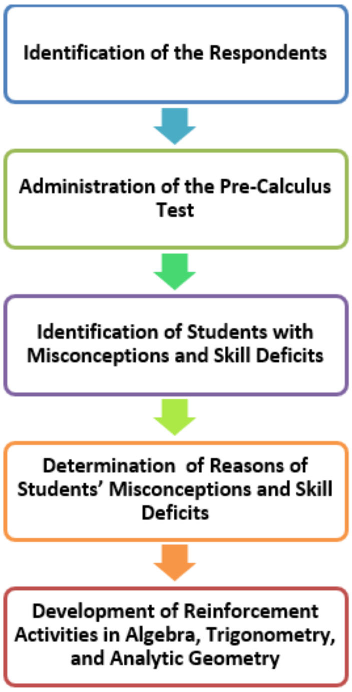

Figure 2. The Error Analysis Model

The first step is the identification of participants. Coordination was made among teachers of Calculus to identify students who have difficulty in Pre-Calculus subjects. From a total of 450 engineering students, a total of 50 students comprising 10 electrical, 10 mechanical, and 30 civil engineering students were purposely chosen to compose the samples of the study. The purpose of the study was explained to the students. They were requested to voluntarily participate in the study. The Pre-Calculus test was administered to the participants in coordination with their subject teachers. After the retrieval of the tests, answers were checked to determine the items where the students had misconceptions and skill deficits (Brown, J., and Skow, K., 2014). The answers of the students in each item were analyzed by identifying the errors committed in Pre-Calculus subjects. Interviews with the participants were conducted after the test was checked to clarify their solutions and obtain relevant information from their answers. Moreover, interviews with the mathematics teachers were also conducted to gather additional information regarding the errors committed pertaining to their misconceptions and skill deficits in Pre-Calculus subjects. Interviews with students and teachers were conducted and analyzed to determine the reasons causing the misconceptions and skill deficits. Their answers were categorized according to the themes that emerged from their answers. The classification of answers by themes was done by analyzing their responses individually. Reinforcement activities in each of the Pre-requisite subjects were made based on the identified students' misconceptions and skill deficits (Dawkins, P., 2016).

RESULTS AND DISCUSSIONS

Students' Misconceptions and Skill Deficits in Pre-Calculus Subjects

The students' misconceptions and skill deficits in Pre-Calculus subjects were presented and discussed according to the subject: Algebra, Trigonometry, and Analytic Geometry.

Students' Misconceptions in Algebra

The students' misconceptions in Algebra were categorized into algebraic expressions and functions, special products and factoring rational expressions, exponents and radicals, linear equations, quadratic and word problems, and exponential and logarithmic functions.

Table 1 Number and Percentage of Students with Misconceptions in Algebra

MAIN TOPIC	EE	ME	CE	TOTAL	%	Qualitative Description of Students with Misconceptions
	n =10	n= 10	n=30	n=50		
Algebraic Expressions and Functions	7.7	7.0	16.3	31	62	Many
Special Products and Factoring	7.3	8.0	19.3	35	69	Many
Rational Expressions	6.8	5.0	19.3	31	62	Many
Exponents and Radicals	7.0	6.0	20.3	33	67	Many
Linear Equations, Quadratic and Word Problems	6.5	6.5	18.5	32	63	Many
Logarithms	6.0	6.5	18.0	31	61	Many
Average	**6.9**	**6.5**	**18.6**	**32**	**64**	**Many**

Table 1 shows the number and percentage of students with misconceptions on selected topics in Algebra. On Algebraic Expressions and Functions, students had misconceptions in identifying similar terms, identifying algebraic expressions which were not polynomials, and defining relation and function. Many of the students (n = 31, 62%) in general have misconceptions about algebraic expressions and functions. Many of the students (n = 35, 69%) have misconceptions about special products and factoring. Many of the students from electrical engineering, mechanical engineering, and civil engineering have misconceptions about rational expression (n = 31, 62%), exponents and radicals (n = 33, 67%), linear equations, quadratic, and word problems (n = 32, 63%), and logarithms (n = 31, 61%). The results would indicate that many of the respondents have misconceptions about each of the topics in Algebra (n = 32, 64%). Some of them did not realize the difference between polynomial expression and algebraic expression, they did not distinguish that function is a type of relation (Kaur, 2013). The students easily forgot the patterns in factoring as they did not identify polynomial expressions which are not factorable and many other reasons why there is a big percentage of students who have misconceptions in algebra (Prudchenko, K., 2012).

Students' Skill Deficits in Algebra

The students' skill deficits in Algebra are categorized into algebraic expressions and operations, special products and factoring, rational expressions, exponents, and radicals, solving of linear, quadratic, rational, exponential, and logarithmic equations, and solving of word problems (Seed, 2014).

Table 2 Number and Percentage of Students with Incorrect Answers in the Skills of Algebra

MAIN TOPIC	EE	ME	CE	TOTAL	%	Qualitative Description of Students with Skill Deficits
	n =10	n= 10	n=30	n=50		
Algebraic Expressions and Functions	9.0	5.5	16.5	31	62	Many
Special Products and Factoring	8.8	9.6	28.4	47	94	Almost All
Rational Expressions	4.0	7.7	15.7	27	55	Some
Exponents and Radicals	7.2	7.5	19.7	34	69	Many
Solving of Exponential and Logarithmic Equations	9.0	9.0	21.5	40	79	Many
Solving worded Problems	9.7	7.3	25.7	43	85	Almost All
Average	**7.9**	**7.8**	**21.2**	**37**	**74**	**Many**

Table 2 shows the number of students who got incorrect answers to skill items on selected topics in Algebra. Some of the respondents have a deficit in rational expression (n = 27, 55%). Many (n = 31, 62%) have a deficit in Algebraic Expressions and Operations, exponents, and radicals (n = 34, 69%), and solving of exponential and logarithmic equations (n = 40, 79%). The reason for their mistakes is, sign problems, they did not simplify the results, putting a group symbol, they did not recall the properties of the exponential function, and they did not know what to do in solving the problem. Almost all respondents have a deficit in special products and factoring (n = 47, 94%), and solving worded problems (n = 43, 85%). Most of them do not know what to do in factoring and solving word problems. The student had an incomplete solution and only wrote the first equation but failed to write the second (Jones, 2016) and (Vella, 2014). Students formed an incorrect equation and translated the words in math symbols incorrectly. Lastly, most of the students did not know how to translate the words in math symbols correctly due to the wrong analysis of the problem[19].

Students' Misconceptions in Trigonometry

The students' misconceptions in Trigonometry are categorized into angle and angle measure, trigonometric functions, trigonometric identities, inverse trigonometric functions, graphs of trigonometric and inverse trigonometric functions, and solutions of triangles.

Table 3 Number and Percentage of Students with Misconceptions in Trigonometry

MAIN TOPIC	EE	ME	CE	TOTAL	%	Qualitative Description of Students with Misconceptions
	n =10	n= 10	n=30	n=50		
Angle Measure	8.0	6.0	24.0	38	76	Many
Trigonometric Functions	6.0	7.4	18.7	32	64	Many
Trigonometric Identities	3.0	7.0	20.5	31	61	Many
Inverse Trigonometric Functions	8.0	7.0	23.6	39	77	Many
Graph of Trigonometric and Inverse Trigonometric Functions	7.2	7.3	19.2	34	67	Many
Solutions of Triangles	8.0	7.7	21.0	37	73	Many
Average	**6.7**	**7.1**	**21.2**	**35**	**70**	**Many**

Table 3 shows the number and percentage of students with misconceptions on selected topics in Trigonometry. Many students have misconceptions about angle measure (n = 38, 76%), trigonometric functions (n = 32, 64%), trigonometric identities (n = 31, 61%), inverse trigonometric functions (n = 39, 77%), graph of trigonometric and inverse trigonometric functions (n = 34, 67%), and solutions of triangle (n = 37, 73%). The reasons are, that they assumed that the representation of radian is π which is wrong since decimal can also be used to represent a radian measure. Students were not comfortable using the radian measure since they did not understand its meaning. In measuring angle measures, they frequently used degree measure instead of radian measure. Students were always uncertain when it comes to trigonometric functions. They have difficulty recalling the concept of even function. Students always forgot the values of trigonometric functions since they did not understand how to obtain the values (Seed, 2014). They only memorized it for the sake of compliance. Students said that they did not consider the reference angle and they only assumed that the horizontal side was the adjacent side while the vertical side of the triangle was the opposite side. They easily forget the trigonometric functions (Dawkins, 2016). The students admitted that they were confused about the signs of trigonometric functions in quadrants. For them, the co-function of sine is cosecant. The students always use this in their solutions to trigonometric functions (Seed, 2014). They also added that the co-function of cosine is secant. These students did not realize that what they did was reciprocal functions and not co-functions. The co-function of sine is cosine since if their corresponding angles are complementary then their values are equal. They said also that the reciprocal of cosecant was the secant function. students said that these functions are always partners. Students did not realize what they thought was the co-function. The reciprocal of cosecant is the sine function since cosecant is equal to 1 over the sine function. The students always forget not only the range but also the domain of trigonometric functions. The students said that the inverse of the cosine function is the sine function. They were not aware of the relations between the trigonometric functions and their inverse. The students forgot the trigonometric function values, so they tend to only guess their answers. The students do not understand the concept of symmetry. They always forget the conditions when using the different laws of trigonometric functions. They added that aside from the conditions, students also forgot the formulas of the laws of trigonometry.

Students' Skill Deficits in Trigonometry

The students' skill deficits in Trigonometry are categorized into trigonometric functions, inverse trigonometric functions, proving identities, graphs of trigonometric and inverse trigonometric functions, and application of trigonometric functions (Dawkins, 2016).

Table 4 Number and Percentage of Students with Incorrect Answers in Skills of Trigonometry

MAIN TOPIC	EE	ME	CE	TOTAL	%	Qualitative Description of Students with Skill Deficits
	n =10	n= 10	n=30	n=50		
Trigonometric Functions	5.7	9.3	22.3	37	75	Many
Inverse Trigonometric Functions	6.7	9.7	23.7	40	80	Many
Trigonometric Identities	4.7	7.7	16.7	29	58	Some
Graph of Trigonometric and Inverse Trigonometric Functions	8.6	10.0	28.4	47	94	Almost All
Application of Trigonometric Functions	9.0	9.7	28.0	47	93	Almost All
Average	**6.9**	**9.3**	**23.8**	**40**	**80**	**Many**

Table 4 shows the number of students who got incorrect answers to items on selected topics in Trigonometry. Some of them have skills deficits in trigonometric identities (n = 29, 58%).

Many of the have skills deficits in trigonometric functions (n = 37, 75%), and inverse trigonometric functions (n = 40, 80%).

Almost all of them have skills deficits in graphing trigonometric and inverse trigonometric functions (n = 47, 94%), and in the application of trigonometric functions (n = 47, 93%). The reasons are, that students forgot the trigonometric values as they assumed the value of cos π as zero instead of -1. The students cannot recall the trigonometric function values for sine and cosine function in degree measure while taking the examination.

They forgot the steps in finding the value of the trigonometric function. The students obtained an error in evaluating trigonometric values for sine and cosine using the operation of addition (Ozkan and Ozkan, 2012). The students need to apply first the inverse sine function since the condition was to find an angle x for the sine function that makes it equal to one half. The students also need to recall the values of trigonometric functions to obtain the correct answer.

The students also forgot the steps in solving inverse trigonometric functions. In proving trigonometric equations, students need to recall the different fundamental identities which in this case, the use of Pythagorean and ratio identities. The students also need to have a critical observation of the problem so that the problem will be solved. In proving trigonometric equations, students need to recall the different fundamental identities which in this case, the use of double angle identity for the sine function[20]. The students also need to have a critical observation of the problem so that the problem will be solved. In proving trigonometric equations, students need to recall the different fundamental identities which in this case, the use of Pythagorean identity (Idris, 2011).

The students also need to have a critical observation of the problem so that the problem will be solved. The students were not able to graph the function since they forgot the steps in graphing trigonometric functions. They were able to solve the hypotenuse of the triangle but did not obtain the final answer since they were taught that this was the final answer. The students need to obtain the given information and determine what is being asked in the problem[16]. The use of an oblique triangle involving cosine law is needed to obtain the correct answer. The student can draw the figure correctly, but he used the right triangle instead of the oblique triangle (Idris, 2011).

Students' Misconceptions in Analytic Geometry

The students' misconceptions in Analytic Geometry are categorized into a rectangular coordinate system, the slope of a line, equations of a line, conic sections, and plane and solid figures.

Table 6 shows the number and percentage of students with misconceptions on selected topics in Analytic Geometry.

Table 5 Number and Percentage of Students with Misconceptions in Analytic Geometry

MAIN TOPIC	EE	ME	CE	TOTAL	%	Qualitative Description of Students with Misconceptions
	n =10	n= 10	n=30	n=50		
Rectangular Coordinate System	6.8	5.8	16.8	29	59	Some
Slope of a Line	6.2	6.3	19.5	32	64	Many
Equations of a Line	5.3	6.7	18.9	31	62	Many
Conic Sections	6.5	6.5	21.1	34	68	Many
Plane and Solid Figures	5.0	5.5	17.5	28	56	Some
Average	**5.9**	**6.2**	**18.7**	**31**	**62**	**Many**

Some of the respondents have misconceptions about the rectangular coordinate system (n = 29, 59%), and the plane and solid figures (n = 28, 56%). Many of the respondents have misconceptions about the slope of a line (n = 32, 64%), the equations of a line (n = 31, 62%), and the conic sections (n = 34, 68%). The reason is, that the distance of a point P from the y-axis is the y-coordinate which is wrong since it is the x-coordinate. They did not analyze the problem carefully. Since the y-axis is on the problem, they already assumed that it is the y-coordinate in complement of the y-axis. The students have misconceptions about defining the x-coordinate (Jones, 2016). They said that students frequently interchanged the definition of x and y-coordinates. The students were not aware of the abscissa and ordinate. They could hardly recognize which is the x or the y-coordinate. Often, the two are interchanged. They could not define the origin. If the definition is given to them, they would surely memorize it without understanding[16]. The students did not understand the concept of negative slope in a graph of a line. They were merely making a wild guess in identifying the slope of a line. The students were not aware of the sign of the slope of the line whether it is parallel or perpendicular to the axes. They were not aware of the slope of a line whether it is parallel or perpendicular to the axes. The students frequently made a mistake in finding the slope of a line since they used incorrect formula for slope (Ozkan and Ozkan, 2012). It simply shows that students did not have a strong conceptual understanding of the slope of a line. The students were confused about the different equations of a line. They did not recognize the specific formula to be used in finding the equation of a line. The students were confused about zero slopes since they did not understand the concept of slope in an equation (Idris, 2011). The students have misconceptions in identifying the perimeter of rectangles since they were always confused about formulas. They used to forget the formula by not having twice its length and width. Instead, they only take its product as in the formula of area (Beng, Y. H and Yumus A.S, 2013). They were confused about the formula of perimeter and area.

Students' Skill Deficits in Analytic Geometry

Students' skill deficits in Analytic Geometry are categorized into the slope of a line, graphing of functions and relations, equation of a line, and curve sketching.

Table 6 Number and Percentage of Students' Skill Deficits in Analytic Geometry

MAIN TOPIC	EE	ME	CE	TOTAL	%	Qualitative Description of Students with Skills Deficits
	n =10	n= 10	n=30	n=50		
Finding Slope of a Line	7.0	7.3	21.0	35	71	Many
Graphing of Functions and Relations	7.6	9.0	27.6	44	88	Almost All
Solving Equations of a Line	8.0	9.0	26.0	43	86	Almost All
Drawing of Curves	4.0	3.0	18.0	25	50	Some
Average	**6.7**	**7.1**	**23.2**	**37**	**74**	**Many**

Table 6 shows the number of students who got incorrect answers to the items on selected topics in Analytic Geometry. Some of the respondents have skills deficits in drawing curves (n = 25, 50%). Many of the respondents have skills deficits in finding the slope of a line (n = 35, 71%). Almost all the respondents have skills deficits in graphing functions and relations (n = 44, 88%), and solving equations of a line (n = 43, 86%). The reasons are, that the student assumed that the x and y-axis was the slope and used the derivative in finding the slope but failed to obtain the answer since the derivative of a constant was incorrect. The students forgot the steps in finding the slope of a line. The students don't know how to manipulate the equation in different forms to obtain the slope (Kaur, 2013). The equation is required to change into slope-intercept form. They solved the x and y-intercepts and made these the x and y coordinates of a point. They don't know how to graph quadratic equations (Prudchenko, 2012). The students forgot the steps in graphing rational functions. The students solved for x and y intercepts and plotted this to the Cartesian plane and mistakenly connected the points as a line instead of the parabola. They forgot the steps in finding the equation of a line and were able to find the slope but did not find the equation (Vella, 2014).

Students' Reasons that Caused Misconceptions and Skill Deficits in Algebra
The reasons for students' misconceptions and skill deficits in Algebra were categorized based on the description of students' reasons.

Table 7 Students' Reasons for Misconceptions and Skill Deficits in Algebra

MAIN TOPIC	Description of Students' Reason
Algebraic Expressions and Functions	Students' Poor Knowledge Retention
	Students' Attitude Toward the Lesson
Special Products and Factoring	Students' Readiness
	Students' Absences
	Teachers' methodology
	Students' Poor Knowledge Retention
Rational Expressions	Students' Poor Knowledge Retention
	Time of Schedule of Classes
	Teachers' methodology
Exponents and Radicals	Students' Poor Knowledge Retention
	Students' Tardiness
Linear Equations, Quadratic and Word Problems	Students' Poor Knowledge Retention
	Teachers' methodology
	Curriculum/Time of Classes
	Students' Language Deficit
	Students' Poor Analysis
Logarithms	Students' Poor Knowledge Retention
	Teachers' methodology

Table 7 shows the students' reasons for misconceptions and skill deficits in Algebra. Generally, the students' reasons for causing misconceptions and skill deficits were poor knowledge retention and attitudes toward the lesson (Tall, 2002).
In Algebra, the students' reason was due to poor knowledge retention. It was evident in the statement "I forgot the topic" which means that they had already forgotten the concepts and skills in identifying polynomial expressions. One teacher suggested that the students should always review their notes and stay focused. The other statements were "hindi naintindihan", "nakakalito" and "a bit confusing". The student's poor knowledge retention was observed in translating words to math symbols. They have also problems understanding the English language used in problem-solving.

Students' Reasons Causing Misconceptions and Skill Deficits in Trigonometry
The reasons for students' misconceptions and skill deficits in Trigonometry were categorized based on the description of students' reasons.

Table 8 Students' Reasons for Misconceptions and Skill Deficits in Trigonometry

MAIN TOPIC	Description of Students' Reason
Angle Measures	Students' Poor Knowledge Retention
Trigonometric Functions	Students attitude toward listening
	School Activities/Programs
	Students' Poor Knowledge Retention
Trigonometric Identities	Students' Poor Knowledge Retention
	Time of Schedule of Classes
	Teachers' methodology
Inverse Trigonometric Functions	Students' Poor Knowledge Retention
	Students attitude toward listening
	Teachers' Methodology
Graph of Trigonometric and Inverse Trigonometric Functions	Students' Poor Knowledge Retention
	Teachers' methodology
	Students attitude toward listening
Solutions of Triangles	Students' Poor Knowledge Retention
	Students attitude toward listening
	Students' Poor Analysis

On Trigonometry, the reason for misconceptions and skill deficits was due to students' poor knowledge retention.

In trigonometry, the statements "I forgot the topic", "nakalimutan", "nakalimutan na", "hindi ko po kabisado yung conversion formula" and "medyo nakalimutan na rin" mean that the students' retention of the concepts and skills was poor. This was evident in the statements of the students "I forgot the topic", "nakalimutan", "hindi ko sure kung napagbaliktad ko na yung co-function at reciprocal", "hindi ko po kabisado, lagi kong nakakalimutan", "nakalimutan na sa dami ng formula", "it always rumbles in my mind", "I forgot", and "nakalimutan ko na dahil karaming rules na dapat tandaan at nakakalito". Another reason for students' misconceptions and skill deficits was due to teachers' methodology. From the statements of the student, "nalilito po ako hindi po kasi tinuro ng prof" and "because I don't know to my teacher, she always said the topics are easy, but she can't teach these to us", it can be deduced that the methods of teaching were not fitted to them.

Students' Reasons Causing Misconceptions and Skill Deficits in Analytic Geometry

The reasons for students' misconceptions and skill deficits in Analytic Geometry were categorized based on the description of students' reasons.

Table 9 Students' Reasons for Misconceptions and Skill Deficits in Analytic Geometry

MAIN TOPIC	Description of Students' Reason
Rectangular Coordinate System	Students' Poor Knowledge Retention
Slope of a Line	Students' Poor Knowledge Retention
	School Activities/Programs
Equations of a Line	Students' Poor Knowledge Retention
	Teachers' methodology
Conic Sections	Students' Poor Knowledge Retention
	Students attitude toward listening
	Students' Lack Concentration
	Students' Absences
	Students' Poor Graphic Skills
	Students Lack of Vocabulary
	Teachers' Methodology
Plane and Solid Figures	Students' Poor Knowledge Retention

In Analytic Geometry, the reason for misconceptions and skill deficits was due to students' poor knowledge retention.

The students' poor knowledge retention was observed in defining the second coordinate, identifying the first coordinate, defining the origin, and identifying the term for the first coordinate. It was evident in their statements such as "I forgot" and "nalilito".

The students' poor knowledge retention was observed in identifying the slope formula, identifying the slope of perpendicular lines, identifying the slope of the horizontal line, identifying the slope of vertical lines, and identifying the negative slope. The statements of the students such as "nakalimutan po", "I forgot the lesson", "nalilimutan yung mga iba", "naghalo-halo yung mga formulas, hindi ko na alam kung ano gagamitin" means that the students did not recall the concepts and skill needed in the question. The many school activities can be seen also in identifying negative slopes. The statement "we didn't meet our teacher because of the school activities" means that they did not tackle the lesson due to the school events.

Proposed Reinforcement Activities

The proposed reinforcement activities in Pre-Calculus were divided into subject categories, namely: Algebra Trigonometry and Analytic Geometry. Each subject was composed of different activities to reinforce the concepts and skills found to have misconceptions and skill deficits. The approach of the activities was made funny and enjoyable for the benefit of the students with difficulties in understanding concepts and skills in Pre-Calculus subjects[22].

Each activity has this format:(1) introduction, which will give students the idea about the activity they will deal with, (2) learning outcome/s, that will need to meet at the end of the activity, and (3) list of materials needed in the activity, (4) a step-by-step procedure, that serves as a guide for the teachers, (5) assessment, that will assess the retention of the concepts and skills gained by the students in the activity, and (6) references for additional search about the lesson.

In Algebra, there were 12 activities which include the following: Pick Me Up! Factoring Battle, Common Multiple War, Pick Me Up 2!, Pick a Statement, Ex-Log War, Polynomial Division Flip, Polynomial Tag, Speed Dating, Row Game, Rational and Quadratic Equation Hunt, and Word Problem Battle.

In Trigonometry, the six (6) activities include the following: Measure Me! Follow me! The Biorhythm Cycle, Trigonometry Square 1 and 2, and Graphing Inverse Trigonometric Functions.

Analytic Geometry also has six (6) activities namely: Solve a graph Puzzle, Slope of a Line, Story of My Equation, Camera Activity, Human Conics, and Measure Me2!

CONCLUSIONS AND RECOMMENDATIONS

Based on the findings, the following conclusions are drawn:

Students have misconceptions about the basic concepts and laws in Algebra that are necessary for performing fundamental algebraic operations. They also lack the required skills in performing algebraic operations.

Students have misconceptions about the basic concepts of Trigonometry. These basic concepts are necessary in the study of Analytic Geometry.

The students' misconceptions and skill deficits in Analytic Geometry are attributed to the students' misconceptions and skill deficits in Algebra and Trigonometry.

The underlying reasons for students' misconceptions are due to students' lack of conceptual understanding and retention in their schema. The students easily forget what they previously learned because of rote learning. They tend to memorize the concepts needed for skillful mathematical operation instead of grasping the meaning of these concepts. Some skill deficits are caused by carelessness and incorrect application of laws and principles.

Mathematics teachers are encouraged to provide immediate feedback to students and their peers on students' misconceptions and skill deficits so that these can be addressed earlier. In doing so, the escalation of these problems in higher mathematics subjects can be minimized.

Additional reinforcement activities in Algebra, Trigonometry, and Analytic Geometry are proposed to be given in addition to the learning tasks and examples provided during classes. Mathematics teachers are encouraged to reflect on and or revisit their teaching practices as these may provide them sound bases in utilizing other teaching methods that may redound to improve students' mathematics performance. A remediation program can be designed to help students with misconceptions and skill deficits in Pre-calculus subjects. The underlying reasons causing students' misconceptions and deficit skills in Pre-Calculus subjects be shared with the other mathematics teachers and students so that they may know what to focus on and emphasize during teaching and learning. The students may also be given training/seminar on time management so they can balance their studies and other extra-curricular activities participated in. The proposed reinforcement activities are recommended for implementation in mathematics classes to improve identified deficiencies in content. Similar studies are recommended to be conducted on the students' misconceptions and skill deficits in other subject areas of mathematics.

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