CONCAVITY METHOD: A CONCISE SURVEY

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ABSTRACT

This short review article discusses the concavity method, one of the most effective ways to deal with parabolic equations with unbounded solutions in finite time. If the solution ceases to exist for some time, we say it blows up. The solution or some of its derivatives become singular depending on the equation. We focus on situations where the solution becomes unbounded in finite time, and our objective is to review some of the key blowup theory papers utilising the concavity method.

KEYWORDS

Parabolic equations, Concavity method, weak solutions, blowup, variable exponent spaces, $p(x)$-Laplacian operator.

1 INTRODUCTION

The idea of unbounded solutions, known as blowup theory, holds a specific place in the study of nonlinear equations. Blow-up is a phenomenon where solutions of differential equations cease to exist because of the infinite growth of the variables describing the evolution processes. Before the successful calculation of mathematical methods to deal with the unboundedness of solutions, the physical significance of blowup was understood because it occurs in processes such as heat conduction, combustion, volcanic eruption, gas dynamics, etc. In addition, some of the current notable works in this field include the blowing up of cancer cells, nuclear blowup, electrical blow up, laser fusion, blow up in pandemic simulations, etc.

The solutions to a problem can be unbounded at a finite or infinite time. In this work, we only deal with papers studying finite time blowup of solutions using the concavity method. Finite time blowup is a sufficient condition for the nonexistence of global solutions, since the solutions grow without bound in finite time intervals. There are nonlinear PDEs, with local solutions for time $t < T^\star$, which blowup at a finite time $T^\star$. Thus, we can give a formal definition to finite time blowup as, if $T^\star < \infty$ and

$$\limsup_{t \to T^\star} \|z(t)\| = \infty, \tag{1}$$

then we say the solution z of a given problem blows up at a finite time $T^\star$.

From 1960s onward, the following equations,

$$z_t = \Delta z + |z|^{p-1} z, \tag{2}$$

and

$$z_t = \Delta z + \lambda e^z, \lambda > 0 \tag{3}$$

have become fundamental models for blow-up study [4,20,38,52]. Fujita, Hayakawa [16,17,22], Kaplan [24] and Friedman [18] studied these problems and obtained several critical results on blowup of solutions. These fundamental results from the 1960s initiated deep research of blowup solutions for various nonlinear evolution PDEs in the next decade [2, 3, 25, 28–30, 48, 49]. Some of the fascinating problems in this area are finding whether the solutions blow up at a finite time, obtaining lower and upper bounds for blowup time and getting the blowup rate. Challenges in this theoretical study have attracted many scientists, and useful techniques have been developed to deal with certain nonlinear parabolic problems. They include eigenfunction method, explicit inequality method, logarithmic convexity method, Fourier coefficient method, comparison method, concavity method and differential inequality techniques. We solely concentrate on the concavity approach out of all these methods.

Concavity Method

The concavity method was introduced by H. A. Levine [28] and proved very successful with a wide range of applications. The method uses the concavity of an auxiliary functional, say $M^{-\eta}$, with $\eta > 0$. Here $M(t)$ is a positive energy functional of the solution to a PDE. Since $M^{-\eta}$ is concave, we get

$$\frac{d^2 M(t)}{dt^2} \leq 0. \tag{4}$$

Hence by integrating the above inequality, one can arrive at

$$M^\eta(t) \geq \frac{M^{\eta+1(t)}}{M(0) - t\eta M'(0)}, \tag{5}$$

this implies if $M'(0) > 0$, then $M^\eta(t)$ is bounded below by a function which becomes unbounded at a finite time. To apply the concavity method, we need to show that $M(t)$ obeys the inequality (4). Hence we have the following inequality,

$$\frac{d^2 M^{-\eta}(t)}{dt^2} = -\eta M^{-\eta-2}(t)[M(t)M''(t) - (1+\eta)(M'(t))^2]. \tag{6}$$

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Now, for $M(t) > 0$ we get

$$M(t)M''(t) - (1+\eta)(M'(t))^2 > 0. \tag{7}$$

Hence this inequality (7) is a sufficient condition for the existence of blowup. Moreover, the inequality (5) helps to derive an upper bound for blowup time.

In [28], Levine studied an abstract parabolic equation

$$\begin{cases} P\dfrac{dz}{dt} = -A(t)z + f(z(t)), t \in [0,\infty) \\ z(0) = z_0, \end{cases} \tag{8}$$

where P and A are positive linear operators defined on a dense subdomain D of a real or complex Hilbert space. In which he obtained blowup results under the conditions

$$2(\alpha+1)F(x) \leq (x, f(x)), F(z_0(x)) > \frac{1}{2}(z_0(x), Az_0(x)), \tag{9}$$

for every $x \in D$, where $F(x) = \int_0^1 (f(\rho x), x)d\rho$. This study has been acknowledged as an innovative and elegant way, known as "the concavity method,"for providing criteria for the blowup.

Later, Philippin and Proytcheva [39] transformed the procedure from its abstract form into a concrete form and used it to solve the same equation with $P = 2$. Since then, the concavity method has been used for several variations of the equations (8) or other equations to get the blowup solutions. Levine, together with Payne [29, 30], established nonexistence theorems for the heat equation with nonlinear boundary conditions and for the porous medium equation backwards in time using the concavity argument. In [37] Payne, Philippin and Piro dealt with the blowup of the solutions to a semilinear second-order parabolic equation with nonlinear boundary conditions. They demonstrated that blowup would occur at some finite time under specific conditions on the nonlinearities and data. Junning [23] investigated the following initial boundary value problem

$$\begin{cases} z_t = \operatorname{div}(|\nabla z|^{(p-2)}\nabla z) + f(\nabla z, z, x, t), & (x,t) \in \Omega \times (0,t) \\ z(x,0) = z_0(x), & x \in \Omega \\ z(x,t) = 0, & x \in \partial\Omega \end{cases} \tag{10}$$

and obtained results on the existence and nonexistence of solutions under specific conditions. Erdem [15] estabilished sufficient conditions for the global nonexistence of solutions of a second order quasilinear parabolic equations.

In 2017, the finiteness of the time for blow up of a semilinear parabolic equation with Dirichlet boundary conditions discussed by Chung and Choi [9]. They continued the study and obtained a new condition for the concavity method of blowup solutions to p-Laplacian parabolic equations [10]. The authors then developed a condition for blowup solutions to discrete p-Laplacian parabolic equations under the mixed boundary conditions on networks with Hwang [11]. For a reaction-diffusion equation with a generalised Lewis function and nonlinear exponential growth, Dai and Zhang established results on nonexistence of solutions using concavity method [12]. In [47], the authors took into account the results of finite time blowup for a parabolic equation coupled with a superlinear source term and a local linear boundary dissipation. The adequate conditions for the solutions to blow up in a finite time were deduced using the concavity argument. The existence of finite time blowup solutions with arbitrarily high initial energy and the upper and lower bound of the blowup time were specifically obtained. Galaktionov [19] provide the sufficient conditions for the unboundedness of the solutions of boundary value problems for a class of quasilinear equations and systems of parabolic type, describing the propagation of heat in media with nonlinear heat conduction and volume liberation of energy.

Blow up of solutions for a semilinear heat equation with a viscoelastic term was studied in [21] for nonlinear flux on the boundary. The authors obtained blowup results for initial negative energy by

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employing the concavity method. Li and Han [32] then improved these results for positive initial energy with the support of the potential well method. Sun et. al. [46] investigated an initial boundary value problem for a pseudo-parabolic equation under the influence of a linear memory term and a nonlinear source term and obtained results on finite time blowup of solutions under suitable assumptions on the initial data and the relaxation function.

The existence of solutions for a pseudo-parabolic equation with memory was deduced by Di and Shang [13] using Galerkin method and potential well theory. Then using the concavity method, derived finite time blowup results for both negative and non-negative initial energy. Sun et. al. [45] studied the problem and came up with existence and finite time blow-up results using Galerkin method, concavity argument and potential well theory by making a slight change in the source term. They derived an upper bound for the blowup time and obtained the existence of solutions which blow up in finite time with arbitrary initial energy conditions. Di and Shang [14] worked on a class of nonlinear pseudo-parabolic equations with a memory term under Dirichlet boundary condition. They proved a finite time blowup result for specific initial energy and relaxation function. In 2019, Messaoudi and Talahmeh [33] studied a semilinear viscoelastic pseudo-parabolic problem with variable exponent and demonstrated any weak solution with initial data at arbitrary energy level blows up in finite time. Furthermore, they obtained an upper bound for the blowup time using the concavity method. Chen and Xu [5] studied a finitely degenerate semilinear pseudo-parabolic problem and showed the global existence and blowup in finite time of solutions with sub-critical and critical initial energy. The asymptotic behaviour of the global solutions and a lower bound for blowup time of the local solution are also obtained. A pseudo-parabolic equation with variable exponents under initial and Dirichlet boundary value conditions is the subject of study in [53]. In [53], Zhou et. al. established the global existence and blowup results of weak solutions with arbitrarily high initial energy.

Existence and blow-up studies of the following $p(x)$-Laplacian parabolic equation with memory was studied by Lakshmipriya and Gnanavel [34],

$$\begin{cases} z_t - \Delta z - \mu \nabla(|\nabla z|^{p(x)-2}\nabla z) + \int_0^t h(t-\tau)\Delta z(x,\tau)d\tau = \beta |z|^{b(x)-2}z, & x \in \Omega, t \geq 0 \\ z(x,t) = 0, & x \in \partial\Omega, t \geq 0 \\ z(x,0) = z_0(x), & x \in \Omega \end{cases} \tag{11}$$

where $\Omega \subset \mathbb{R}^N, (N \geq 1, N \neq 2)$ is a bounded domain with smooth boundary $\partial\Omega$. $\beta > 0, \mu \geq 0$ are constants. The authors established the existence and finite time blow up of weak solutions of the problem. Further, obtained upper and lower bounds for the blowup time of solutions, by employing the concavity method and differential inequality technique, respectively. In [35], the authors analysed and interpreted unbounded solutions of a viscoelastic $p(x)-$ Laplacian parabolic equation with logarithmic nonlinearity. Here the problem was considered for initial data corresponding to the sub-critical initial energy. In this attempt, Lakshmipriya and Gnanavel obtained the local existence of solutions on an interval $[0, T)$. Moreover, it extracted an upper bound for the blowup time by applying the concavity method.

The paper [27] deals with the existence and blowup of weak solutions of the following pseudo-parabolic equation with logarithmnonlinearityity

$$\begin{cases} w_t - \Delta w_t - div(|\nabla w|^{p(x)-2}\nabla w) = |w|^{s(x)-2}w + |w|^{h-2}w \log|w|, & (x,t) \in \Omega \times (0,\infty) \\ w(x,t) = 0, & (x,t) \in \partial\Omega \times [0,\infty) \\ w(x,0) = w_0(x), & x \in \overline{\Omega} \end{cases} \tag{12}$$

where $\Omega \subset \mathbb{R}^n (n \geq 1)$ is a bounded domain with smooth boundary $\partial\Omega$. The model consider is used to describe the non-stationary process in semiconductors in the presence of sources; the first two terms represent the free electron density rate and logarithmic and polynominonlinearityity stands for the source of free electron current [26]. Lakshmipriya and Gnanavel [36] analysed the blowup of solutions to the following problem

$$\begin{cases} z_t(x,t) = \Delta_{p(x)} z(x,t) + g(z(x,t)), & (x,t) \in \Omega \times (0,\infty) \\ z(x,t) = 0, & (x,t) \in \partial\Omega \times [0,\infty) \\ z(x,0) = z_0(x) \geq 0, & x \in \overline{\Omega} \end{cases} \tag{13}$$

where $\Omega \subset \mathbb{R}^N (N \geq 1)$ is a bounded domain with smooth boundary $\partial\Omega$. The model is involved in image processing, elastic mechanics and electro-rheological fluids [1, 40, 42]. The authors considered a condition on the nonlinear function $g(z)$ given by,

$$\varsigma \int_0^z g(s)ds \leq zg(z) + \eta z^{b(x)} + \mu, z > 0.$$

Obtained results on blowup and established an upper bound for the blowup time with the help of the concavity method.

Now, we consider the most recent works involving the Concavity method. Ruzhansky et.al. [41] proved a global existence and blowup of the positive solutions to the initial-boundary value problem of the nonlinear porous medium equation and the nonlinear pseudo-parabolic equation on the stratified Lie groups based on the concavity argument and the Poincare inequality. A nonlinear porous medium equation under a new nonlinearity condition is considered in a bounded domain by Sabitbek and Torebek [43]. They presented the blowup of the positive solution to the considered problem for the negative initial energy. For the subcritical and critical initial energy cases, obtained a global existence, asymptotic behaviour and blowup phenomena in a finite time of the positive solution to the nonlinear porous medium equation. In [31], Li and Fang are concerned with the blowup phenomena for a semilinear pseudo-parabolic equation with general nonlinearity under the null Dirichlet boundary condition. When the nonlinearity satisfies a new structural condition, they obtain some new blowup criteria with different initial energy levels. They derived the growth estimations and life span of blowup solutions.

The concavity method is also used to understand the blowup behaviour of system of nonlinear parabolic equations [8]. Apart from parabolic and pseudo-parabolic equations, the method plays a significant role in studying unbounded solutions of hyperbolic equations and systems. Some of the latest works are as follows [6, 7, 44, 50, 51]. Hence the Concavity method is a simple and powerful tool to use in the blow up studies of solutions to differential equation problems.

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