Implementation of a new matheuristic to solve the vehicle routing problem with simultaneous deliveries and pick-ups - VRPSPD

Implementación De Una Nueva Matheuristica Para Resolver El Problema De Ruteo De Vehículos Con Entregas Y Recogidas Simultáneos - VRPSPD

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Abstract— **The objective of this article is to present a new methodology for the solution of the homogeneous vehicle routing problem with simultaneous pickups and deliveries (VRPSPD) incorporating a matheuristic integrated by the Chu-Beasley genetic algorithm and mixed integer linear programming, based on the Branch-and-Bound procedure, applied to the best configuration obtained from the genetic algorithm with the support of constructive heuristic methods in the determination of the sub problems, which make part of the generation of the initial population, necessary in the stage of local improvement. The problem is that, from a group of geographically dispersed customers, with known pick- up and delivery demands, minimum cost routes must be established that guarantee the satisfaction of customer demand, considering the restrictions of the system and the number of vehicles required to provide the service. The methodology developed is implemented in C++, and a solver CPLEX software is used to find the solution. It is demonstrated in this article that the efficiency of the application of this new hybrid model is evidenced using test instances disclosed in the specialized literature, achieving good results in very short computation times. This article is one of the scientific products derived from my doctoral thesis in Engineering and from the presentation of similar topics in international congresses, therefore, for obvious reasons there is a certain similarity in the bibliographical references consulted.**

Index Terms—**Genetic algorithm of Chu–Beasley, heuristics, optimization, pick-up and delivery, vehicles routing.**

Resumen—**El objetivo de este artículo es presentar una nueva metodología para la solución del problema de ruteo homogéneo de vehículos con recogidas y entregas simultáneas (VRPSPD) incorporando una matheurística integrada por el algoritmo genético de Chu-Beasley y programación lineal entera mixta, basada en el método Branch- Procedimiento and-Bound, aplicado a la mejor configuración obtenida del algoritmo genético con apoyo de métodos heurísticos constructivos en la determinación de los subproblemas, que hacen parte de la generación de la población inicial, necesaria en la etapa de mejoramiento local. El problema es que, a partir de un grupo de clientes dispersos geográficamente, con demandas conocidas de retiro y entrega, se deben establecer rutas de costo mínimo que garanticen la satisfacción de la demanda de los clientes, considerando las restricciones del sistema y la cantidad de vehículos requeridos para atender. el servicio. La metodología desarrollada está implementada en C++, y se utiliza un software Solver CPLEX para encontrar la solución. En este artículo se demuestra que la eficiencia de la aplicación de este nuevo modelo híbrido se evidencia utilizando instancias de prueba divulgadas en la literatura especializada, logrando buenos resultados en tiempos de cómputo muy cortos. Este artículo es uno de los productos científicos derivado de mi tesis doctoral en Ingeniería y de la presentación de temas similares en congresos internacionales, por lo tanto, por obvias razones hay cierta similitud en las referencias bibliográficas consultadas.**

Palabras claves— **Algoritmo genético de Chu – Beasley, entregas y recogidas, heurísticas, optimización, ruteo de vehículos.**

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I. Introduction

THE content of the research on the vehicle routing problem - VRP comprises an important series of events that include very brief and simple cases to situations that, due to their complication and complexity, deserve a greater level of depth in the scientific context.

It is well known to all that the practical solution of this class of routing problems focuses on determining a viable set of routes served by a set of vehicles to satisfy the delivery and pick-up service requested by customers, at a reasonable cost of transport, which is caused in the development of the distribution processes of supply of products, people and merchandise.

Structurally, this routing problem aims to find a set of solutions that can be good or optimal, achieved with the use of heuristic, metaheuristic or matheuristics techniques that are normally affected by restrictions associated with the number of vehicles used, their capacity, the location of pick-up and delivery sites, preparation times, route duration times, use of a single depot or multiple depots, topography of the land, etc.

All routes begin and end at the depot. The load of the vehicles is made up of the products or merchandise that must be delivered and what must be collected simultaneously according to the services requested by the clients. It is convenient to specify that in this case combinatorial optimization is applied, where there are many variables accompanied by a good number of parameters and in addition, the different versions of the vehicle routing problem are of NP-Hard class because polynomial time is not used in the solution. One of the pioneers in this kind of routing problem was H. Min [1], who devised a three-stage constructive heuristic to solve the case of a distribution system for the public library in Franklin County, Ohio. His solution was reflected in saving time and distance using a mathematical model.

Below is a list of the most relevant variants of this routing problem from 1989 to 2018:

Application of time windows in the solution of vehicle routing problems -VRP with division of deliveries and pickups of goods [2]; VRP pickup and delivery with time windows [3]; VRP of pickups and deliveries with waiting times and time windows [4]; VRP for delivery and pickup and customer satisfaction using a single vehicle [5]; VRP selective pickup and delivery using a single vehicle [6]; VRP with simultaneous pickup and delivery with single vehicles and multiple vehicles [7]; Multi-vehicle VRP and multi-vehicle split VRP with simultaneous pickup and delivery [8]; VRP with cargo splitting into deliveries and pick-ups and delivery using multiple vehicles[9]; VRP with simultaneous pickup and delivery techniques [10]; VRP solution with flexible deliveries and pick-ups, using time windows [11]; VRP with multi-product inventory for deliveries and pick-ups with transfer and IRP green approach [12]; VRP solution with deliveries, pickups and transfer PDPT [13]; VRP with pickup and delivery, using TWPDPRP time and contamination windows [14]; VRP with programming in the pickup and fractional delivery of GPDP oil [15]; VRP solution with mixed pickups and deliveries SPD [16]; VRP solution with deliveries and collections with full loads FTPDP [17]; VRP solution with simultaneous deliveries and pickups with limited time [18]; VRP solution with multi-product stack deliveries and pick-ups using PDPTWMS time windows [19]; VRP solution with simultaneous deliveries and pick-ups considering two-dimensional load constraints VRP2LSPD [20]; Mobile vendor VRP solution with deliveries and pick-ups of different m-PDTSP products [21]; VRP solution with deliveries and pick-ups, using time windows and reservation request PDPTWPR [22].

The effort of authors such as [23] stands out, who, through a classification scheme of the deliveries and pick-ups of the VRP, make a proposal defined in the following groups:

- Group 1: made up of the set of “many-to-many” problems. Here any vertex can be taken as source or destination. Structurally it is like the VRP with simultaneous deliveries and pick-ups – VRPDPD.
- Group 2: is made up of the “one-to-many-to-one” group of problems. Its analog is the route generation VPR when mixed vehicles are used – MVRP. In this case, the customer requires one of two services: delivery or pick-up.
- Group 3: is made up of “one by one” VPRs where each product is treated as a requirement that comes from an origin and has a defined destination. Examples of this group are courier operations and the provision of door-to-door transport.

In addition, it is convenient to take into account that the use of exact techniques in the solution of vehicle routing problems with simultaneous deliveries and pick-ups - VRPSPD has presented difficulties related to the use of a considerable number of variables and restrictions [24][25].

Cases like these are easily solved with the use of the matheuristics proposed in this article, which allows finding good solutions, usually close to the optimal solution in relatively short computation times.

It has been shown that most of the applications of the VRPSPD are presented in the different processes of direct and reverse logistics, because the organizations in their mission must carry out operations and activities related to the direct and reverse flow applied for the distribution of finished products, semi-finished products and raw materials. The activities carried out in vehicle assembly plants, soft drink, beer, or soft drink bottling plants are cited as examples. In the latter, they occur when visits to customers are scheduled, full bottles are delivered to them and empty bottles are collected, simultaneously fulfilling deliveries and pick-ups, satisfactorily meeting customer requirements by applying the optimal route with minimum costs in the supply chain.

Another example of the VRPSPD is illustrated in: the transport of passengers either in the transfer or in the collection in different places; in home services, where goods and documents are delivered and money is collected; in the transmission and reception of information and data; in transmission and reception of electrical energy in urban, commercial and industrial sectors; in the different production systems when supplies and raw materials are delivered and finished or semi-finished products are collected. The VRPSPD is also applied in cases in which new products are delivered and obsolete products or products whose expiration date has already passed are collected, with the purpose of giving them an adequate final disposal.

In practical terms, a large part of the processes in organizations where VRP is applied are developed empirically and, in general, transportation costs are high without ignoring the environmental impact and the questionable level of service to the end customer. That is why, the efforts oriented to the improvement and the search for a scientific solution of the VRP constitute another contribution of this article.

The structure of this article is made up of the following parts: 1. Introduction. 2. Description of the problem. 3. Formulation of the mathematical model. 4. Description of the application of the Chu-Beasley genetic algorithm for the solution of VRPSPD. 5. Experimental results of the applications of the proposed matheuristics formed by the Chu-Beasley genetic algorithm and exact techniques for a depot, fifty customers and four vehicles. 6. Sensitivity analysis of the results obtained with the Chu-Beasley genetic algorithm on varying the population size. 7. Conclusions and the list of bibliographical references consulted.

II. Description Of The Problem

It is very important to bear in mind that, in this case, the different scenarios where the deliveries and pickups of products or people that must be transported between an origin and a destination are presented are part of the VRP and the system restrictions must be complied with, including the capacity of the vehicles used. Because of the bibliographic review carried out, it was evidenced that there are three groups of VRPs as described below:

- VRP with backhauls VRPBH: in this version, deliveries are served first and then pick-ups [4].
- VRP with generation of routes using mixed vehicles MVRP: here the clients request one of the two services [8].
- VRP with simultaneous deliveries and pickups – VRPSPD: in this version customers request both services, deliver and collect products or people, guaranteeing a 100% level of service and using a vehicle on each route [25] [26] [27] [28].

For the construction of this article, the latter group was chosen, characterized by the presence of a warehouse, several vehicles and customers who request delivery and pickup services simultaneously. Therefore, the problem is solved when the set of routes that satisfy the constraints of the system is determined, which are listed below:

- Only one visit is made to each client.
- The total load transported in each vehicle must not exceed its capacity. That is, infeasibility is not allowed. In Fig. 1 you can see the situation of the VRPSPD using three vehicles and the corresponding routes:

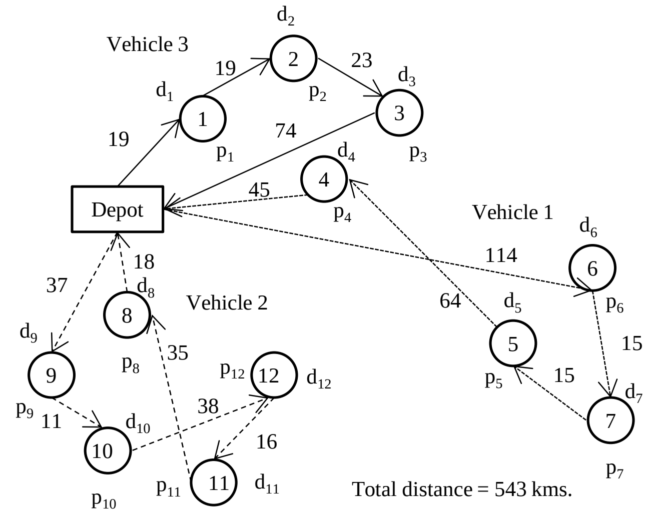


Fig. 1. Graphic structure of the VRPSD for three vehicles

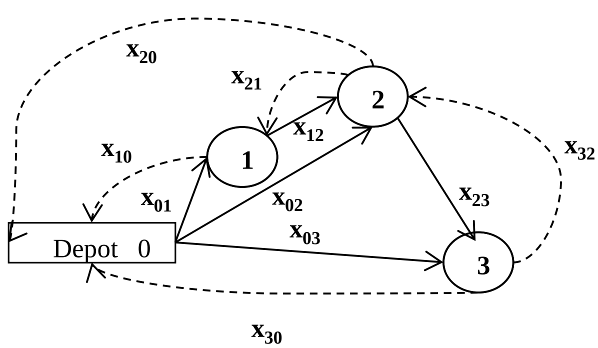
III. Formulation of the mathematical model

Another significant aspect in the feasibility of the VRPSPD is the determination of the sequence of the chosen route, which must be viable if, when verifying the services required by the client, these do not exceed the capacity of the vehicles.

For a better understanding of this version of the VRPSPD, you can see Fig. 2, where the respective feasible or infeasible routes are presented:

- Route 1: 0 → 1 → 2 → 3 → 0 feasible route, distance traveled 135-length unit.
- Route 2: 0 → 1 → 3 → 2 → 0 infeasible route, distance traveled 136-length unit.
- Route 3: 0 → 2 → 1 → 3 → 0 infeasible route.
- Route 4: 0 → 2 → 3 → 1 → 0 infeasible route.
- Route 5: 0 → 3 → 1 → 2 → 0 infeasible route.
- Route 6: 0 → 3 → 2 → 1 → 0 infeasible route.

LOAD INITIAL SITUATION		
Customer	d_i	p_i
1	540	480
2	380	420
3	280	300
	1200	1200



C_{ij}	0	1	2
0	-	19	47
1	19	-	19
2	47	19	-
3	74	47	23

Fig. 2. Example of VRPSPD

The shortest route in this case is 0 → 1 → 2 → 3 → 0, which corresponds to the optimal solution.

The mathematical model applied in this article for the solution of the VRPSPD was proposed by Dell'Amico et al [28] and used by [29].

Its description is:

A = is the set of arcs formed by the pairs (i, j) and (j, i) in each of the edges {i, j} ε E_k.

$G = (V_k, E_k)$ = represents the complete graph that includes the vertices V= {0, 1, 2… n}, where vertex 0 identifies the deposit and the other vertices belong to the respective clients.

Each edge {i, j} ε is assigned a cost and each of the customers i ε V'=V- {0} = {1, 2, 3…... n}.

d_i = corresponds to the quantity of merchandise or product that must be delivered to customer i.

p_i = is the amount of merchandise or product that must be picked up from customer i.

c_{ij} = is the matrix of costs or distances of the trips made by the vehicles used, i, j ε V.

C = {1, 2… m} = represents all the m homogeneous vehicles of capacity Q.

E_k = is the subset of V_k*V_k that includes all possible arcs in the mathematical model used.

The decision variables are:

$$x_{ij} \begin{cases} 1, \text{ when the vehicle } k \text{ transits the arc } (i, j) \text{ ε V of the chosen route} \\ 0, \text{ in the other cases} \end{cases}$$

D_{ij} = corresponds to the quantity of pending products or merchandise that must be delivered, that must be transported in the arc (i, j).

P_{ij} = is the quantity of pending products or merchandise that must be collected, and that are transported in the arc (i, j).

Q = is the load capacity of homogeneous vehicles.

For this mathematical model, both the objective function and its respective constraint are:

The objective function (1): allows the minimization of all travel costs or distances traveled on the chosen route.

Constraint (2) considers that each client must be visited only once on the route determined through the mathematical model.

Constraint (3) allows each vehicle to leave each node or client once on the established route.

Compliance with constraint (4) is a guarantee to verify that each vehicle is used only once on the route.

Constraints (5), (6) and (7) represent the conditions that ensure the preservation of the flow of products delivered and collected in the routes determined by the mathematical model.

The non-negativity conditions and the characteristics of the decision variables are represented in the constraints (8), (9) and (10).

If what you are looking for is to apply a stronger inequality due to the non-negativity of the constraint (8), you can use the constraint (11), as supported by Gouveia in his published research [30], whose characteristic is directly associated with employment of limits.

For the purpose of applying stronger inequalities for the quantities of outstanding products or goods to be collected, Pij, the constraint (9) can be replaced by the constraint (12) and the constraint (7) can be replaced by the inequality (13).

In the constraint (14) it is inferred that in each edge not adjacent to the warehouse merchandise is only transported once at most.

$$Z_{min} = \min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \tag{1}$$

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' \tag{2}$$

$$\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V' \tag{3}$$

$$\sum_{j \in V'} x_{0j} \leq m \tag{4}$$

$$\sum_{j \in V} D_{ji} - \sum_{j \in V} D_{ij} = d_i \quad \forall i \in V' \tag{5}$$

$$\sum_{j \in V} P_{ij} - \sum_{j \in V} P_{ji} = p_i \quad \forall i \in V' \tag{6}$$

$$D_{ij} + P_{ij} \leq Q x_{ij} \quad \forall (i, j) \in A \tag{7}$$

$$D_{ij} \geq 0 \quad \forall (i, j) \in A \tag{8}$$

$$P_{ij} \geq 0 \quad \forall (i, j) \in A \tag{9}$$

$$x_{ij} \in \{0,1\} \quad \forall (i, j) \in A \tag{10}$$

$$(Q - d_i) x_{ij} \geq D_{ij} \geq d_j x_{ij} \quad \forall (i, j) \in A \tag{11}$$

$$(Q - p_j) x_{ij} \geq P_{ij} \geq p_i x_{ij} \quad \forall (i, j) \in A \tag{12}$$

$$(Q - max\{0, p_j - d_j, d_i - p_i\}) x_{ij} \geq D_{ij} + P_{ij} \quad \forall (i, j) \in A \tag{13}$$

$$1 \geq x_{ij} + x_{ji} \quad \forall i, j, \; i < j, \in V' \tag{14}$$

IV. Description Of The Application Of The Chu-Beasley Genetic Algorithm For The Solution Of VRPSPD.

It is known by all that the application of the exact methods to solve NP-Hard type problems such as the VRPSPD is that they require a lot of computation time due to the considerable number of decision variables, system restrictions, population size and number of customers who must be served in delivery and pickup services. Therefore, the use of metaheuristics, in this case, the Chu-Beasley genetic algorithm, allows obtaining good answers, without being optimal in relatively short computation times compared to the exact methods.

It can be seen in [31] that the Chu-Beasley genetic algorithm has certain characteristics that make it more effective, such as: it uses the objective function to determine the value of the best quality solution and recognizes the infeasibility in the substitution process of a solution generated in the execution of the algorithm; replaces one individual at a time in each generation cycle; to avoid the premature tendency to optimal local solutions, each individual that joins the population must be different from all those that make up the current population; contains an aspiration criterion, even if the new individual does not meet the controlled diversity condition; It includes an improvement stage, after recombination, and based on conclusive intra-route and inter-route strategies, a feasible solution is evaluated before deciding if it can be part of the current population.

The application of the Chu-Beasley genetic algorithm contains the following stages:

A. *Construction of the Initial Population*

This stage is composed of: a first component that corresponds to the configurations of the routes obtained from certain constructive heuristics, and a second component is the configurations of the routes obtained in a random controlled manner. For each configuration, the objective function (fitness) and the infeasibility associated with the load of each vehicle on the routes are assessed. Fig. 3 shows a VRPSPD configuration or solution for 20 nodes or clients and its coding.

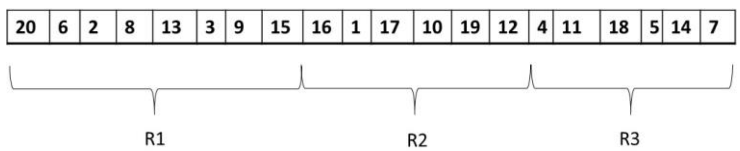


Fig.3 Genetic representation of a configuration of 20 nodes or customers.

The length of the path of the configuration or solution is determined by the number of clients or nodes that must be served by the vehicles. It is a priority to consider that the different routes chosen are limited or restricted by the capacities of the vehicles and by the quantities of the products that must be delivered and collected from each client or node.

The input information that is real is made up of: the cost or distance matrix (c_{ij}); the number of products or people (d_i), which must be delivered or moved from an origin to a destination; the number of products or people (p_i), which must be collected or transferred from one destination to another destination or to the origin; the defined number of vehicles (k) with homogeneous capacity (Q).

In this article the parameters of the Chu-Beasley genetic algorithm that are applied are the recombination rate 0.80 and the mutation rate 0.05.

In the process of searching for solutions in each configuration obtained from the Chu-Beasley genetic algorithm, the value of the objective function (fitness) is obtained, the feasibility or infeasibility of the route is shown, and the load transported by each vehicle that runs through the different sub-routes or vehicles of the configuration. Based on the values of the objective function achieved and the feasibility of the configurations provided by the Chu-Beasley genetic algorithm, it is necessary to apply the genetic operators explained in section B of this article.

B. *Genetic Operators*

Three genetic operators are applied in this article: selection, recombination and mutation. Following is its description:

1) *Selection*

In this stage operator, it is recommended to use the tournament selection method. For this, two tournaments are held where the intervention of all the individuals (customers) of the current population k is necessary. Their selection is random, then the values of their objective functions and their infeasibility are compared, if they occur, deciding on the one that has the best value of the objective function if the k individuals are feasible, or selecting the one with the least infeasibility. if the k individuals are not feasible.

Since the presence of feasible and infeasible individuals is possible in this operator, the feasible individual with the best objective function is preferred. Of these two tournaments, two parents remain, which pass to the recombination stage.

2) *Recombination*

This operator provides the exchange of information present in the two parents, and two descendants are originated that possess genetic material from parent 1 and parent 2. Since the algorithm is elitist, only one of the children passes into the current population.

To carry out the recombination in the specialized literature there are techniques such as the “single point crossover", the "two-point crossover", the "uniform crossover", the "partially mapped crossover (PMX), the "supported crossover in order (OBX), the "cycle cross (CX)", the "multi-parent cross (MPX)". The "longest common subsequence crossover (LCSX)", to name a few. This article applies various forms of recombination that describe the PMX method, the procedure of which is:

Starting with the configuration of the parents P_1 and P_2, a common random fragment or segment is chosen and copied to the child of P_1. Starting from the first crossing point, the elements *i* of this segment that have not been copied are in P_2. For each of these elements *i*, it is sought which elements *j* have been copied into position from P_1.

Then i is put at position *j* taken from P_2, and it turns out that this element is not there, because it has been copied before.

If the place occupied by *j* in P_2 has already been filled in the child *k*, *i* is installed in the place occupied by *k* in P_2. The other remaining elements are obtained from P_2, thus ending the procedure (See Fig. 4).

Various rates were applied to recombination, with the value 0.8 being the best results achieved.

															f.o.
Pi	0	1	8	9	10	11	12	5	7	6	3	2	4	0	372
Pj	0	10	9	6	5	7	8	12	11	4	1	2	3	0	633
Hk	0	1	8	9	10	11	12	6	5	4	7	2	3	0	609

Fig. 4 Application of the recombination PMX in a configuration of twelve customers.

P_i, P_j are the parents and H_k is the child who is the result of the recombination.

3) Mutation

With the application of this genetic operator some changes can be made, or parts or fragments of the solution obtained in the recombination can be altered. That is, the mutation can improve the current solution, incorporating small modifications in the process. The mutation rate used in this article is 0.05. Some strategies are known that facilitate the performance of the mutation, such as:

- Determination of local inter-route sequence using the strategies shift (1, 0), shift (2, 0), shift (3, 0), which allow the transfer or movement of 1, 2, 3 customers from one route to another.
- Determination of inter-route local sequence using the swap (1, 1), swap (2, 1), swap (2, 2) strategies, which make it possible to exchange 1, 2 or 3 customers between one route and another.
- Local intra-route assignment. In this situation, customer neighborhood criteria are applied to make transfers on the same route. Useful strategies in this assignment are rotation and 2-Opt. The latter has been considered as a local search algorithm that was designed by Croes in 1958 to solve the problem of a travel agency. It starts from the base of a route that crosses itself and can be reorganized, avoiding the presence of a crossing.

For the purpose of this article, some exchange strategies between customers have been used and it is illustrated with an example of swap (1, 1), where the exchange of client 3 of route 3 for client 4 of route 1 is evidenced. See Fig. 5.

R1					R2					R3				f.o.
0	6	7	5	4	9	10	11	12	8	1	2	3	0	549
0	6	7	5	3	9	10	11	12	8	1	2	4	0	499

Fig. 5 Swap used (1,1) in the local determination between routes.

It can be verified that when applying the swap strategy (1, 1) in the mutation, another configuration is obtained with an incumbent that is guaranteed to go to the improvement stage.

C. *Process of improvement of an individual*

After selection, recombination and mutation are applied, the resulting configurations are subjected to a local improvement phase, which allows the separation of the individual routes and the determination or construction of subproblems of a single route each with a single deposit, with less customers than the complete configuration. The solution of this problem is achieved through the exact branch-and-cut method, and the partial routes are established by means of the optimal solutions. The configuration obtained becomes part of the replacement stage.

D. Replacement stage

All the configurations obtained in the previous stages are compared with the individuals (customers) of the current population. A substitution of one of the individuals (customers) of the population is carried out, considering the feasibility over the infeasibility and the best objective function when feasible solutions are compared.

V. EXPERIMENTAL RESULTS OF THE APPLICATION OF THE PROPOSED MATHEURISTICS FORMED BY THE CHU- BEASLEY GENETIC ALGORITHM AND EXACT TECHNIQUES FOR A DEPOT, FIFTY CUSTOMERS AND FOUR VEHICLES.

During the bibliographic review of the literature referring to the VRPSPD, three classes of test problems have been established:

- Dethloff's proposal: it provides 40 reference instances with 50 customers and the number of vehicles used were 4, 9 and 10 [5].
- Salhi and Nagy's proposal: They suggest 14 instances, the number of customers is between 50 and 199 and the
- number of vehicles used in the process was 3, 4, 5, 6, 7 and 10 [32].
- Proposal by Montané and Galvão: they consider 12 instances. The customers are between 100-200 and the number of vehicles were 3, 5, 9, 10, 12, 16, 23 and 28 [33].

In this article, some of the Dethloff instances were used for the experimental tests, whose solutions, including the optimal limits, have been widely published in scientific literature [34] [35].

As it has been shown, the methodology used by the authors of the article is the mathematical one integrated by the Chu-Beasley genetic algorithm and the exact techniques such as mixed integer linear programming.

The results of the data processing for this experiment can be seen in Table I, with the use of the Dethloff CON 3-8 instance.

Observing table I, it is evident that, out of 24 experiments, two configurations were obtained with values of the objective function of 538.77 units of length (experiment 8) and 537.35 units of length (experiment 21), which if compared with the value 523.05 length units reported by [20] as the optimal value of the instance, are very good results in relatively short processing times (13.77 min and 14.45 min respectively) compared to the 32.05 minutes that the processing reported by [20].

Based on the configuration achieved through the Chu-Beasley genetic algorithm, whose objective function value is 537.35 distance units, the four routes traveled by the four vehicles are determined.

TABLE I

RESULTS OF DATA PROCESSING USING THE CHU-BEASLEY GENETIC ALGORITHM WITH THE DETHLOFF CON3-8 INSTANCE

Number of experiments	Population size	Last generation with the best objective function	A	B	C(s)
1	200	9,546,567	539.27	539.21	826.57
2	200	169,195,901	631.46	634.55	6,022.12
3	200	14,797,375	604.4	606.85	621.7
4	200	18,281,903	601.58	604.65	865.82
5	200	25,751,134	566.39	569.14	1,296.98
6	200	3,875,937	618	620.87	272.65
7	200	21,615,847	606.91	608.97	750.12
8	200.	13,634,094	538.77	543.37	612.27
9	200	6,847,951	543.44	545.58	413.03
10	200	2,136,192	690.18	692.51	196.77
11	200	1,557,887	601.53	604.32	101.56
12	200	1,795,436	610.4	614.68	107.83
13	200	139,028	676.66	701.17	51.39
14	200	7,860,845	612.68	613.76	216.71
15	200	6,991,853	546.81	550.74	411.32
16	200	594,773	605.86	609.9	134.54
17	200	16,078,476	644.00	646.69	523.77
18	200	41,765,528	593.75	597.37	1131.6
19	200	44,159,791	551.32	553.52	1,378.15
20	200	12,396,610	584.13	586.63	803.84
21	200	17,381,285	537.35	541.39	866.86
22	200	19,271,995	602.19	604.84	851.24
23	200	105,346,306	567.75	579.50	4,786.88
24	200	50,972,210	571.4	573.78	1,836.31

A: Value best achieved by the objective function in the respective generation.

B: Worst value achieved by the objective function in the respective generation.

C: Processing time for the respective generation (in seconds).

TABLE II:

SOLUTIONS ACHIEVED BY APPLYING MIXED INTEGRAL LINEAR PROGRAMMING

Vehicle /Routes	Continuity of established routes	Matheuristic solution	Chu - Beasley solution	Best possible solution
Vehicle /route 1	dep 25 27 42 36 37 24 4 9 2 43 50 31 29 39 dep	194.7	196.63	190.26
Vehicle /route 2	dep 14 21 22 12 6 41 18 15 26 19 48 49 3 dep	128.7	126.73	116.52
Vehicle /route 3	dep 35 45 16 5 10 30 1 13 17 32 44 28 33 7 46 20 47 8 11 dep	183.73	184.62	183.73
Vehicle /route 4	dep 40 23 38 34 dep	31.2	31.2	31.20
	Total distance traveled obtained by matheuristic	**538.33**	**539.08**	
	Total distance traveled achieved by the Chu - Beasley genetic algorithm	**539.08**		
	Optimal solution of the VRPSPD according to Subramanian (2012)	**523.05**		

Under these conditions, each of these four routes is considered as a small linear programming problem, and its solution is very easily obtained by applying the respective mathematical model. That is, the proposed matheuristic is applied, which was supported by a good ordering generated by the application of the Chu-Beasley genetic algorithm, with two constructive algorithms that were designed by the authors:

- An algorithm that generates the route matrices and the quantity of products that must be delivered and collected during the visit to each customer or node of each of the routes.
- A second algorithm that controls the load on each route, avoiding infeasible configurations.

With the data from the previous algorithms, the mixed integer linear mathematical programming model is applied. The solutions obtained for the four routes can be seen in Table II

VI. SENSIBILITY ANALYSIS OF THE RESULTS OBTAINED WITH THE CHU-BEASLEY GENETIC ALGORITHM ON VARYING THE POPULATION SIZE

It is good to remember that a sensitivity analysis can focus on different scenarios such as:

- Sensitivity analysis to assess the performance of the Chu-Beasley genetic algorithm when recombination and mutation rates are modified.
- Sensitivity analysis to assess the performance of the Chu-Beasley genetic algorithm when the size of the population is varied.
- Sensitivity analysis to assess the performance of the Chu-Beasley genetic algorithm when recombination and mutation rates are modified and the variation in population size is also included.

In this article, this analysis is made for the second option: varying the size of the population and its results can be seen in Table IV for three sizes of the population with configurations of 100, 200 and 300 clients.

Analyzing the results of the sensitivity analysis shown in Table III and the performance of the objective function for different population sizes in Fig. 6, the following aspects stand out:

For a population size of 100 clients, the objective function stabilizes with a value of 550.81 units of length between 2,500,000 and 7,500,000 generations and from 8,000,000 generations another incumbent appears with a better value of the objective function, 537.63 units of length.

For a population size of 200 clients, the objective function is fixed with a value of 537.35 units of length from 6,000,000 generations, which in any case is very close to the optimal solution reported by [23].

For a population size of 300 clients, the objective function is ensured with a value of 650.17 length units from 5,000,000 generations.

TABLE III
SENSITIVITY ANALYSIS FOR DIFFERENT POPULATION SIZES IN MULTIPLE GENERATIONS IN THE IMPLEMENTATION OF THE CHU – BEASLEY ALGORITHM USING THE DETHLOFF INSTANCE WITH 3-8

Population size			Generation of evolutive process	Objective function - OF(100)	Objective function - OF(200)	Objective function - OF(300)
100	200	300	500,000	668.06	670.73	677.91
100	200	300	1,000,000	594.55	591.68	665.57
100	200	300	1,500,000	589.76	589.35	658.70
100	200	300	2,000,000	555.06	589.35	652.20
100	200	300	2,500,000	550.81	589.35	651.09
100	200	300	3,000,000	550.81	589.48	651.09
100	200	300	3,500,000	550.81	579.25	651.09
100	200	300	4,000,000	550.81	572.81	650.88
100	200	300	4,500,000	550.81	571.37	650.88
100	200	300	5,000,000	550.81	555.82	650.17
100	200	300	5,500,000	550.81	547.78	650.17
100	200	300	6,000,000	550.81	537.35	650.17
100	200	300	6,500,000	550.81	537.35	650.17
100	200	300	7,000,000	550.81	537.35	650.17
100	200	300	7,500,000	550.81	537.35	650.17
100	200	300	8,000,000	537.63	537.35	650.17
100	200	300	8,500,000	537.63	537.35	650.17
100	200	300	9,000,000	537.63	537.35	650.17

The coding of the Chu-Beasley genetic algorithm was carried out in C++ considering the following aspects:

- The configurations achieved in the process with the best objective functions.

- Each of the routes determined in the solution was treated as a subproblem that was solved by the exact method, using CPLEX Studio Optimization version 12.4.
- The process of applied matheuristic was carried out on three computers with the following technical specifications as can be seen in Table IV.

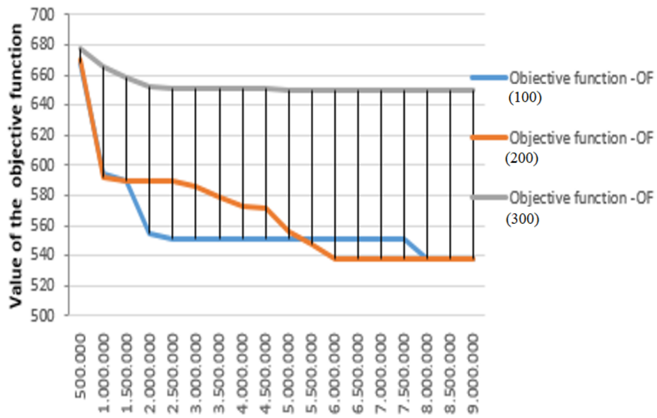


Fig. 6 Performance of the objective function for different population sizes

TABLE IV TECHNICAL SPECIFICATIONS OF THE COMPUTER EQUIPMENT USED

Lenovo B40 Laptop	Dell Latitude E6500 Laptop	Lenovo Personal Computer
Intel processor core (TM) 1.70 GHz- 2.40 GHz x4. RAM memory: 4:00 GB 64 bits OS	Intel processor core (TM) 2 Duo- 2.8 GHz – 2.80 GHz RAM memory: 4:00 GB 64 bits OS	Intel processor core (TM) 3.00 GHz – 3.00GHz x4 RAM memory: 8:00 GB 64 bits OS

VII. CONCLUSIONS

The proposed matheurístic, resulting from the integration of the Chu-Beasley genetic algorithm and the exact techniques supported by the branch-and-cut method, becomes a very good option to solve the VRPSPD for large customers or nodes, because for the same nature of NP-Hard, the mixed integer linear programming - MILP, does not solve it within a reasonable time horizon.

The incorporation of the constructive heuristic algorithms designed by the authors of the article, which facilitate the generation of the matrix of the routes from the configuration with the best incumbent, and the determination of the number of customers per route, from the control of load per vehicle, together with the matheuristic described in this article, show that very good results are obtained in relatively short processing times, ratified by the sensitivity analysis carried out for different sizes of the customer population.

The scope of the proposed matheurístic to solve this class of routing problems is promising and is conditioned to the knowledge and skills of the researchers, to achieve an excellent execution, making the corresponding adjustments and ensure a good performance in the simulations without ignoring that the matheuristics do not guarantee the global optimal solution of routing problems.

On the other hand, the challenge arises of integrating the environmental effects into this situation and carrying out the respective tests, exploring the instances available in other works published in indexed journals or creating the instances themselves.

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