On the description of the movement and the frame of reference

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Abstract

This work aims to explain to the general public, in an accessible way and avoiding advanced mathematical language, some concepts about the description of movement and its relationship with the frame of reference used. To arouse the reader's interest, we initially analyze the situation that we have all perceived at some time when traveling by bus or car, distracted from the outside looking at the cell phone or reading a book, that suddenly our transport travels in reverse. Then, looking directly out the window, we realize that this is not the case. Essential concepts such as the Galilean transformations, the inertial and non-inertial reference frames, non-inertial forces, and Lorentz transformations are presented and discussed. In the end, a brief mention is made of the theory of relativity.

Keywords: Relative motion, reference frames, non-inertial forces.

Resumen

Este trabajo pretende explicar al público en general, en una forma accesible y evitando el lenguaje matemático avanzado algunos conceptos sobre la descripción del movimiento y su relación con el marco de referencia utilizado. Para despertar el interés al lector, se analiza inicialmente la situación que todos hemos percibido alguna vez al viajar en autobús o automóvil, distraídos del exterior observando el celular o leyendo un libro, que de pronto nuestro transporte viaja en reversa. Luego, al mirar directamente por la ventanilla nos damos cuenta qué no es así. Se presentan y discuten algunos conceptos básicos como las transformaciones Galileanas, los marcos de referencia inerciales y no inerciales, las fuerzas no inerciales, las transformaciones de Lorentz, finalizando con una breve mención a la teoría de la relatividad.

Palabras clave: Movimiento relativo, marcos de referencia, fuerzas no inerciales. **ISSN 1870-9095**

I. RELATIVE MOTION

When we travel by bus or car, distracted from the outside looking at our cell phone or reading a book, many of us have experienced the alarming sensation that suddenly our transport is going in reverse. Then, looking directly out the window, we realize that this is not the case. This is a consequence of the relative movement between us and the car next to us moving faster than ours and in the same direction.

To explain the above in more detail, let's consider two bodies, *A* and *B*, moving in the same direction. Common sense tells us that the position of body *B* from the point of view of *A* (fixed reference frame at *A*) is

$$x_{BA} = x_B - x_A \,, \qquad (1)$$

where x_A and x_B represent the distance or position of bodies *A* and *B* from an observer or fixed reference on the floor O located on the line that passes through *A* and *B*. This situation is described in figure 1 for two cars labeled *A* and *B*. The position of *A* from the referential of *B* will be

$$x_{AB} = x_A - x_B \,, \qquad (2)$$

From these two relationships, it follows

$$x_{AB} = -x_{BA} \,. \qquad (3)$$

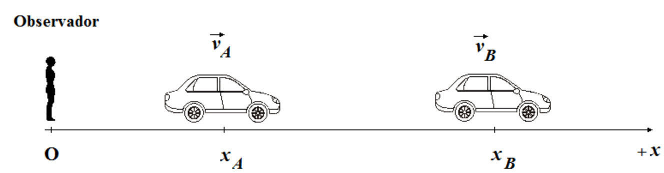

Figure 1. Two cars move straight with velocities v_A and v_B from an observer O fixed on the ground.

Taking the rate of change with time in (1), we obtain that the velocity of *B* observed from *A* is

$$v_{BA} = v_B - v_A. \qquad (4)$$

And the velocity of *A* observed from *B* is

$$v_{AB} = v_A - v_B \,. \qquad (5)$$

From these last two relations, we obtain

$$v_{AB} = -v_{BA}. \quad (6)$$

At a given instant, A and B observe each other traveling at the same speed but in opposite directions.

For example, if A carries a constant velocity relative to observer O of v_A = 60 km/h and B carries a constant velocity relative to observer O of v_B = 40 km/h, by (4) the velocity of B observed from A is v_{BA} = 40 km/h - 60 km/h = - 20 km/h, and by (5) the speed of A observed from B is v_{AB} = 60 km/h - 40 km/h = + 20 km/h.

A passenger at A, aware of the movement of B will observe that, from their point of view, B is moving in reverse, approaching him at a speed of -20 km/h. A passenger at B, aware of A's movement, will observe that, from their point of view, A is moving in the same direction approaching him with a speed of +20 km/h.

On the other hand, a passenger in B distracted, reading a book or on their cell phone, perceive car A out of the window and suppose that this car is stopped. Therefore, he feels with alarm that their car is moving in reverse. This feeling is more common when we are stopped at a traffic light with other cars. Being distracted, suddenly we feel that our car is going backward, but this is not the case; our car is still stopped, and the car next to us that we assumed is stopped like ours is already moving forward.

II. GALILEAN TRANSFORMATIONS

The expressions used in our analysis are derived from the so-called Galilean transformations of the coordinates that link the coordinates and time (x, y, z, t) of an object measured from an observer fixed in O with the corresponding (x', y', z', t') of the same object measured from an observer O' that moves with constant speed u concerning the observer O along the line represented by the +x-axis. These are [1]

$$\begin{aligned} x' &= x - ut, \\ y' &= y, \\ z' &= z, \\ t' &= t. \end{aligned} \quad (7)$$

In our example, if we place the referential O' in car B and car A as the object under observation, then $x_{AB} = x'$, $x_A = x$, and $x_B = ut$. So, the first equation in (7) is equivalent to equation (2). Likewise, now choosing the referential O' in car A and car B as the object under observation, it follows that the first equation of (7) is equivalent to equation (3).

Taking the rate of change with the time in (7), we obtain the transformation of speeds,

$$\begin{aligned} v_x' &= v_x - u, \\ v_y' &= v_y, \\ v_z' &= v_z. \end{aligned} \quad (8)$$

Again, if we choose the referential O' in car B and car A as the object under observation, then $v_{AB} = v_x'$, $v_A = v_x$, and $v_B = u$. Therefore, the first equation in (8) is equivalent to equation (5). In the same way, now choosing the referential O' in car A and car B as the object under observation can be shown that the first equation of (8) is equivalent to equation (4).

It is worth asking the following question: Are the Galilean transformations fulfilled generally, or do they have limitations? The answer is: Galilean transformations are valid in most common situations but not in general. They have the limitations of being valid only in inertial reference systems and at ordinary speeds, that is, speeds much lower than the speed of light of 300 thousand km/s or $3\text{x}10^8$ m/s.

III. INERTIAL REFERENCE SYSTEM

An inertial reference system (IRS) is the system that complies with the principle of inertia, proposed by the Italian philosopher, mathematician, and physicist Galileo Galilei (1564-1642) [2]: when an object lets itself go and is not disturbed, it remains in uniform motion with constant speed in a straight line if it initially moves, or it will continue at rest if it was at the beginning. Thus, an IRS is one where the acceleration of an object, on which no net force acts, is zero.

For most practical purposes, a fixed reference frame on the earth is a sufficient approximation to an IRS since the accelerations due to its rotation about its axis and to its revolution around the sun are less than 0.01 m/s^2, that is, a thousand times less than the acceleration due to gravity (~ 9.81 m/s^2) of the force of attraction of the earth on the bodies on its surface. In other situations, such as geophysical or astronomical studies, it is necessary to consider an IRS located in the most distant galaxies (considered "fixed") [3].

IV. NON-INERTIAL REFERENCE SYSTEM

A non-inertial reference system (NIRS) is an accelerated reference system relative to an IRS. A reference frame fixed to a rotating body is a NIRS. A fixed reference frame on the earth is a NIRS since the earth is rotating; however, as already discussed, we can consider a fixed reference frame on it with good approximation as an IRS for most practical purposes.

From a NIRS, it is necessary to introduce fictitious forces (non-inertial forces), which depend on the type of non-inertiality of the system, to justify the movement of the bodies, in addition to the real forces. The centrifugal force and the Coriolis force are two examples of non-inertial forces [4, 5].

V. CENTRIFUGAL AND CORIOLIS FORCES

Centrifugal force is that which appears to act on a body when its motion is described according to a rotating frame of reference. Its apparent sense is radial outward, and its magnitude is equal to the centripetal force, which is an inertial force. We perceive the centrifugal force when we travel in a car that takes a curve. We feel that "something" pushes us in the opposite direction as the car turns. The car is a NIRS in a circular motion concerning a fixed system on the ground during the turn. For an external observer, there is no such centrifugal force, it is the inertia of the passenger that tends to maintain its movement in a straight line, but as a result of the turn, the passenger needs to exert a force leaning on somewhere inside the car in a radial and opposite direction of the turn to hold your position. The support point used by the passenger responds to it with a force of equal magnitude, but in the opposite direction; that is, in the radial direction and towards the axis of rotation. This is the centripetal force that gives rise to the curvilinear motion of the passenger. The centripetal force responsible for the car's turn is the friction between the tires and the ground.

The Coriolis force, also called the Coriolis effect, is a fictitious or apparent force used to explain the anomalous motion that describes an object moving within a rotating non-inertial frame of reference. This effect is seen in the whirl of a hurricane. If a center of low pressure develops in the atmosphere, the wind flow toward the center. The Coriolis force deflects air molecules to the right of their trajectories in northern latitudes. In the southern hemisphere, the air molecules are deflected to the left of their trajectories.

VI. LORENTZ TRANSFORMATIONS

Returning to IRSs, at high speeds close to one-tenth the speed of light or greater, the Galilean transformations fail and must be replaced by the Lorentz transformations (Hendrik Antoon Lorentz, 1853-1928. Nobel Prize-winning Dutch physicist Physics of the year 1902):

$$\begin{gathered} x' = \frac{x-ut}{\sqrt{1-u^2/c^2}}, \\ y' = y, \\ z' = z, \\ t' = \frac{t-(u/c^2)x}{\sqrt{1-u^2/c^2}}. \end{gathered} \tag{9}$$

At low speeds, much slower than the speed of light (denoted by the letter c), these equations reduce to the Galilean transformation equations (7). Figure 2 shows a graph of factor $1/(1-u^2/c^2)^{1/2}$, called *the Lorentz factor* (denoted by the Greek letter *gamma*), as a function of the relative speed of light u/c (commonly denoted by the Greek letter *beta*). It is observed that at speeds greater than 0.1c, this factor increases its value from unity, firing its growth after 0.9c [1].

Unlike the Galilean transformations, the Lorentz transformations do not fit our common sense used to ordinary speeds. For speeds where the Lorentz factor departs from unity (u > 0.1c), the Lorenz transformation equations lead to the following consequences:

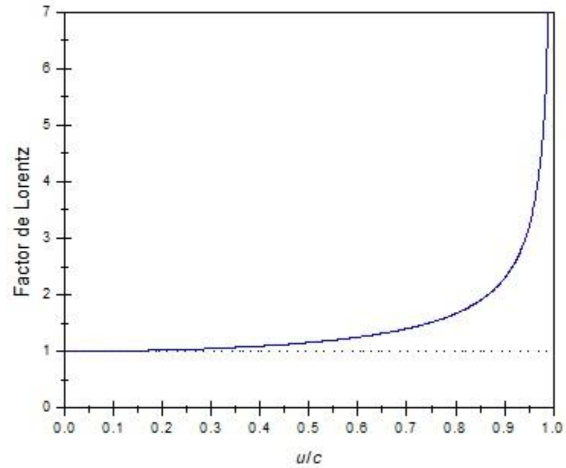

Figure 2. Lorentz factor vs *u*/*c*.

Length contraction. The length of a body measured is greater when it is at rest concerning the observer. When the body moves at speed *u* relative to the observer, its length contracts in the direction of its motion by the factor $(1-u^2/c^2)^{1/2}$, while its dimensions perpendicular to the direction of movement remain unchanged.

Time dilation. The maximum speed with which the time measured in a clock passes occurs when it is at rest relative to the observer. When the clock moves at speed *u* relative to the observer, the observer will notice that the elapsed time measured on the clock has decreased by the factor $(1-u^2/c^2)^{1/2}$.

VII. RELATIVITY THEORY

In 1905, the German physicist Albert Einstein (1879-1955) released his special theory of relativity that describes only events seen by observers in an IRS. This theory is based on two postulates, the *principle of relativity* and the *principle of th*e constancy of the speed of light. According to the principle of relativity, any experiment carried out in an IRS will develop identically in any other inertial system. The principle of the constancy of the speed of light postulates that light propagates in empty space with a constant speed *c* at any IRS [1].

Gravity does not intervene in the theory of special relativity. To include it, Einstein later developed the theory of general relativity which generalizes the theory of special relativity and considers accelerated reference frames [6].

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