RECIBIDO EL 25 DE MAYO DE 2020 - ACEPTADO EL 26 DE SEPTIEMBRE DE 2020

MODEL APPLIED TO EXPONENTAL GROWTH OF COVID 19

EL MODELO DE MALTHUS APLICADO AL CRECIMIENTO EXPONENCIAL DE COVID 19

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ABSTRACT

The Malthus growth model is the most widely used law to model dynamic processes. In this work, we use the Malthusian theory to estimate the growth rate of new daily cases of COVID-19 infection and two periods of time in which this type of growth occurred, the first of 41 days and the second of 101 days. In the first one, the growth rate was 10 times greater than in the second one. From the results, it is concluded that the United States, Spain, France, Italy, Germany and the United Kingdom were the countries that had the greatest impact on exponential growth during the first period, while the Americas, Russia and India were the ones that contributed the most in the second one.[4]

RESUMEN

El modelo de crecimiento de Malthus es la ley más utilizada para modelar procesos dinámicos. En este trabajo utilizamos la teoría maltusiana para estimar la tasa de crecimiento de los nuevos casos diarios de infección por COVID-19 y dos períodos de tiempo en los que se produjo

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4 *Traducción realizada con la versión gratuita del traductor www.DeepL.com/Translator*

este tipo de crecimiento, el primero de 41 días y el segundo de 101 días. En el primero, la tasa de crecimiento fue 10 veces mayor que en el segundo. De los resultados se concluye que Estados Unidos, España, Francia, Italia, Alemania y el Reino Unido fueron los países que tuvieron mayor impacto en el crecimiento exponencial durante el primer período, mientras que América, Rusia e India fueron los que más contribuyeron en el segundo.

1. INTRODUCTION

Among the functions of the population dynamics is the population growth of species, which is of vital importance in various areas of knowledge (life sciences, humanities, social sciences, engineering, among others). This process depends on various factors (climate, temperature, physicochemical, density, regulatory mechanisms, compensatory mechanisms, among others) that determine growth dynamics [1].

There is a vast literature of more than two centuries of research focused on modelling the variation of populations caused by biological and physical processes. Mathematical modelling has contributed significantly to the solution of real problems, and from them theories have been generated that have subsequently been applied to several areas of knowledge [2]. Among the classics, we can mention the Lotka-Volterra model in ecology [2, 3, 4, 5], Sir Ross McDonald model in epidemiology [6], Monod model in microbiology [7], and Malthus model in demography [8].

A common characteristic in all dynamic processes is the period of exponential growth, a stage in which the highest rate of reproduction of the species occurs. For this reason, estimating the exponential growth rate is of vital importance. The model formulated by Thomas Robert Malthus is the most adequate to carry out this task, in 1798 he calculated the exponential growth rate of the world population and predicted that in 1981 world overpopulation would be reached [9].

At present, Malthus exponential law or Malthus growth model is best known principle of population dynamics, Malthusian theory is applied to multiple sciences, the investigations generally focus on establishing the growth rate based on the factors involved in the physical phenomenon, estimating the rate and predicting the increase or decrease exponential growth of the population under study. In [10], conducted a study on food engineering in which they define the growth rate in terms of physical factors temperature, pressure, electric fields, and electromagnetic fields to develop innovative processing technologies. In [11, 12] carried out an investigation on virus inactivation in which they defined the growth rate in terms of reactive oxygen species and active iron to determine for catalytic activity of hydrogen peroxide. A key part of these investigations lies in estimating the constant with the best possible confidence interval, for this purpose numerical and statistical methods are used that allow us to fit empirical data to the mathematical model. In order to achieve this goal, we must choose the most suitable regression.

One of the most relevant phases in the global health emergency produced by the pandemic caused by the coronavirus disease COVID 19, is the exponential growth phase of new daily cases of infected people. In this work, we apply the Malthus model and regressions to estimate the rates and periods of exponential growth of new daily cases of infected people that have occurred to date. From the results, the proportion of susceptible people who were infected daily in these periods was established.

2. METHOD

2.1 Malthus Model

In 1798 when Thomas Robert. Malthus wrote his famous work 'An Essay on the Principle of Population' generated all kinds of feelings both admirers and fans, as well as adversaries. Through well-founded and discussed arguments, he verified that the propagation of the human species is faster than food supply.

Furthermore, under the assumption that the spread of human species follows a geometric progression while the means of subsistence in the most favorable circumstances for the industry follows an arithmetic progression, he concluded that in 1981 world overpopulation would be reached [9]. From the Malthus hypothesis of the geometric progression it is deduced that the population follows an exponential growth. Let P(t) the population at time t and suppose that the instant variation of the population with respect to time is proportional to the current population, under above assumption we obtain the following differential equation

$$\frac{dP}{dt} = kP, \qquad (1)$$

Where k is the constant of proportionality, models the Malthus hypothesis. Under assumption that at the initial time t_0 the initial population is P_0, we obtain the following initial condition

$$P(t_0) = P_0, \qquad (2)$$

is obtained. The solution of the initial value problem (IVP) defined by (1) and (2) is given by

$$P(t) = P_0 e^{k(t-t_0)}. \qquad (3)$$

If the parameters P_0, t_0 y k are known, then from $P(t)$ defined in (3) the population at time t can be determined.

2.2 Regressions

Usually, the proportionality constant or exponential growth rate k is unknown. If we have a data set, a natural way to estimate k is through of statistical methods that estimate the relationships between a dependent variable and one or more independent variables called regressions. For example, the equation (3) define an exponential regression at which is possible estimate P_O, t_0 and k. Let $Y(t)= P(t)/P_0$, then from (3) we obtain the following linear regression in (4) the parameters k and t_0 can be estimated.

Depending on the type of data we can use other regressions such as logistic. There are different methods for regression analysis such as ordinary least squares which is used for estimating the unknown parameter in a linear regression

$$Y(t) = k(t - t_0), \qquad (4)$$

3. RESULTS

On the incidence worldwide of COVID-19, at the end of December 2019, cases of a new pneumonia of unknown etiology were identified in the city of Wuhan China. In the 44 cases reported until January 3, the causal agent was not identified. However, on January 7, 2020, it was established that the severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) was the cause of the disease COVID-19, which was declared a pandemic on March 11, 2020 by World Health Organization (WHO) [12,13]. Currently, COVID-19 has spread worldwide and its incidence continues growing at an exponential rate. The growth rate is associated with social, cultural, and economic factors, among others. Until an effective treatment or vaccine is developed, the strategies that are implemented to reduce the growth rate will determine the outcome of the pandemic.

Estimating of the rate of daily new cases in data used to calibrate the Malthus model defined in (1) and (2) were obtained from the website of the Johns Hopkins University. To date, there have been two periods of exponential growth in the incidence of COVID 19 worldwide, the first one between February 23-April 4, 2020. In this case, Figure 1 shows data were adjusted to the following exponential regression $Y_1(t)=11,376e^{0.1299t}$ with a determination coefficient $R^2=0,9734$. In consequence, the growth rate is $k=0.1299$ which implies that for every 1000 people approximately 130 people were infected worldwide.

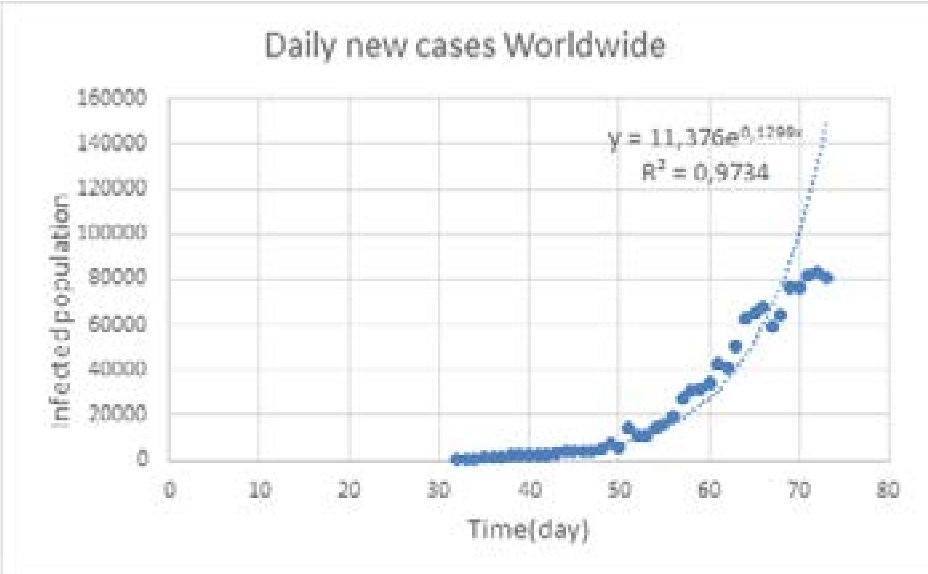

Figure 1. Day 32 (23/02/2020) to day 73 (4/04/2020) of the COVID 19 pandemic.

Figure 2 shows the second one, between April 13-July 23, 2020, for above period data were fitted by the regression $Y_2(t)=224026e^{0.0129t}$ with $R^2=0.9138$. In consequence, the growth rate is $k=0.0129$ which implies that for every 1000 people approximately 13 people were infected worldwide.

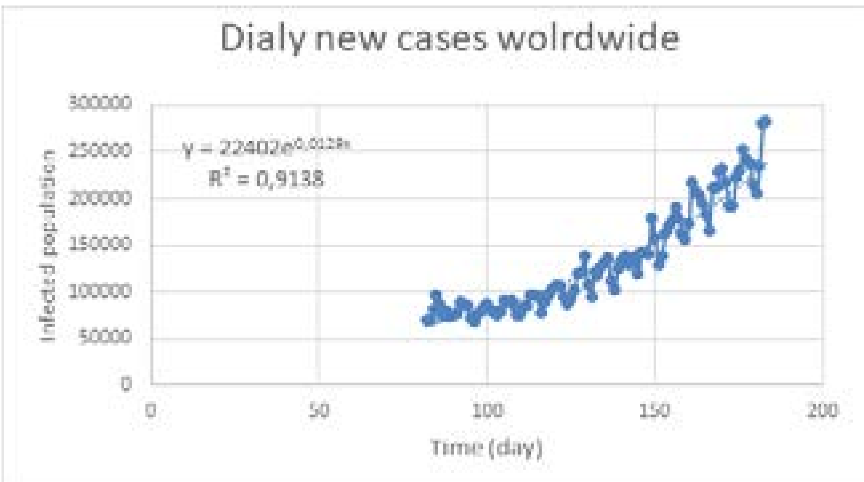

Figure 2. Day 82 (13/04/2020) to 183 (23/07/2020) of the COVID 19 pandemic.

2. DISCUSSION

To date, there have been two periods of exponential growth in the number of daily new cases of infected people during the COVID 19 pandemic, the first one of 41 days between February 23, 2020 and April 4, 2020, and the second one of 101 days was presented between April 13, 2020 and July 23, 2020. In these time periods, data showed a good goodness of fit to the Malthus model, 97% in the first period and 91% in the second one. This implies that in both periods of time more than 90% of the data adjusted to the Malthusian dynamics. However, in the first exponential phase the growth rate was 10 times greater than in the second phase, which means that the force of infection was much greater in the first time period than in the second one. Above leads us to the question: What were the factors that influenced the reduction of the exponential growth constant? its answer is not obvious, a more in-depth study is needed. However, we could assume the reduction in the force of infection is associated with factors such as preventive, containment, mitigation and suppression strategies implemented by the different countries of the world.

It is important to note that in the first period of exponential growth, COVID 19 had already spread throughout much of the global world. However, China and North Korea had already controlled its propagation, while the United States, Canada, Australia, Italy, Spain, Germany, France, the United Kingdom, Turkey, Switzerland, Belgium, Austria, Holland, Portugal, Israel, Sweden, Norway, Ireland, Japan, Singapore and Iran were the countries that presented a dynamics of exponential growth of daily cases of infected [13,14]. In the second period, most of the countries mentioned above had overcome the exponential growth phase and had started the pandemic control phase.

Countries with more than 50,000 infected that presented exponential growth in some range of the second period of time are India, Brazil, Russia, Peru, Colombia, Mexico, South Africa, Argentina, Chile, Bangladesh, Saudi Arabia, Pakistan, Iraq, Philippines, Indonesia, Ukraine, Bolivia, Qatar, Kazakhstan, Romania, Dominican Republic, Egypt, Panama, Kuwait, Oman, Morocco, Guatemala, Poland, Honduras, Ethiopia, Bahrain, Venezuela, Nigeria, Costa Rica, Nepal. Countries such as the United States, Israel, Spain, the United Kingdom, Sweden, Japan and Iran decreased the rate of infections per day, in the second period. However, they contributed a significant number of daily cases to the global growth rate. Finally, the exponential growth phase of United Arab Emirates and Belarus did not coincide with either of the two time periods.

3. CONCLUSIONS

In this investigation, Malthusian dynamics estimate the exponential growth rate of new daily cases of COVID 19 infected worldwide. In addition, two time periods in which there was exponential growth of infected were determined. In the first period of time, the value of the growth rate was obtained from the cases reported in the United States, France, Italy, Spain, Germany, the United Kingdom, and Israel. While in the second one, the value is mainly due to the Americas, Russia and India.

Although the growth rate in the first period was 10 times greater than the rate in the second period, the accumulated number of infected was much lower in the first period, apparently the length of the periods influenced this result, the first period was 41 days, while the second one was 101 days.

Foregoing leads us to wonder ¿Why the greatest period of exponential growth occurred in the Americas? What were the factors that influenced this result? Attempting to answer these questions could contribute to the planning of strategies for the control of epidemics.

ACKNOWLEDGMENTS

This work is supported by Universidad de Nariño (Project No 114-19/10/2017 VIPRI-UDENAR)

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