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Elementa Curvarum Linearum more Apollonius that Descartes

Elementa Curvarum Linearum más Apolonio que Descartes

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ABSTRACT:

This document shows an analysis of the second book Elementa Curvarum Linearum written by Jan De Witt, published for the first time in the second edition of Geometry [1]. This writing is considered the first analytical geometry textbook. The influence of the work carried out by Apollonius in his conics book is studied, the use and interpretation of diagrams is debated. The development of the analytical method and the generation of curves by means of movement are also studied. Some propositions were renewed versions in terms of eighteenth-century mathematics, they used symbology, algebraic techniques and curves were classified by means of their symbolic representations, in these propositions a work closer to Apollonius is seen, the conic is not generated, it is assumed its existence, its nature is geometric. The study concludes that, although the textbook was published in the second edition of Geometry, the genesis of the curves remains geometric. The conics appear as objects of study in action immersed in the symbolic and algebraic practice characteristic of the time.

RESUMEN

Este documento muestra un análisis del segundo libro Elementa Curvarum Linearum escrito por Jan de Witt, publicado por primera vez en la segunda edición de la Geometría [1]. Este escrito es considerado el primer libro de texto de geometría analítica. Se estudia la influencia del trabajo realizado por Apolonio en su libro cónicas, se debate el uso e interpretación de los diagramas. También se estudia el desarrollo del método analítico y la generación de curvas por medio de movimiento. Algunas proposiciones fueron versiones renovadas en términos de las matemáticas del siglo XVll, usaron simbología, técnicas algebraicas y se clasificaron curvas por medio de sus representaciones simbólicas, en estas proposiciones se ve un trabajo más cercano a Apolonio, la cónica no se genera, se supone su existencia, su naturaleza es geométrica. El estudio concluye que, aunque el libro de texto se publicó en la segunda edición de la Geometría la génesis de las curvas sigue siendo geométrica. Las cónicas aparecen como objetos de estudio en acto inmersas en la práctica simbólica y algebraica característica de la época.

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1. Introduction

After the French publication in 1637 of the book of the Discourse on the Method that included as one of its appendices a treaty called Geometry, the academic community of the Netherlands dedicated efforts to study and communicate the mathematical ideas raised by Descartes (1596 - 1650). It was at the University of Leyden where these studies were carried out that allowed the publication in Latin of Geometry 1 as an independent text. The translation of the document, the edition and election of the annexes that compose the second edition were directed by Frans Van Schooten (1615 - 1660).

This document is about one of the texts that complement and interpret the text of Descartes, Elementa Curvarum Linearum, written by the Dutch mathematician Jan de Witt (1625 - 1672). The complete document has two volumes; the first part is considered an introduction to the classification of quadratic curves that is worked on in the second part. For some historians 3 the organization and content of the text became the first book of analytical geometry that took Cartesian ideas into account.

The book that dealt with the conics and most influential until the seventeenth century was the one made by Apollonius of Perga (c.262 BC - c.190 BC), in this text we can find a classification of conics through their properties geometric [3], making use of the definition in act, product of a cut in a cone. Descartes in several sections of Geometry resorts to the work of Apollonius, however its definition of conic was related to the construction by points and the footprint left by a controlled movement; as a geometric place. It was not an act that defined the curve, it is not an action on the cone, the conic is produced from fulfilling a geometric property that ends up represented in a symbolic form.

2. Elementa Curvarum Linearum. Book II

The structure of the textbook was arranged in chapters for each of the conics. As in the book of Apollonius and unlike the Geometry of Descartes, the conical object is studied from its properties. In Geometry, conics are used as tools not as objects, their nature was the solution of geometric problems, there is no conic as an object of study; if it was in the treaty of Apollonius and De Witt.

2.1. The symbolic to classify

One of the aspects for which Descartes's mathematical work is known is the relationship it builds between geometry and algebra. However, the strongest epistemological status was that given to the geometric and not to the algebraic. It is shared with Dennis [5] and Bos [6] that, as in ancient times, the state of truth of knowledge remained anchored to geometric aspects and not to algebraic representations. In the interpretations that are made of the curves: solution of a problem, set of points that meet a property and representative of the movement of a point; The symbolic depends on the construction. In book three of Geometry [2] the quantity of solutions of an equation is related to the degree of the equation, however the geometric representational exercise of the solutions has already been given in the previous books.

In the case of book II by Jan de Witt, the presentation of each conic was made through symbolic representation. The style that was maintained was, for example, Proposition 1 of Chapter I, indicates: if the equation is $y = \left(\frac{b}{a}x\right) + c$ then the required place is a straight line. [3]

The diagram associated with this proposal was based on the definition of Descartes product, generalized property to define the locus by choosing a point to chance.

Let, in fact, A be the immutable initial point of x and let it be understood that x extends indefinitely along the line AB. Then we select at random a point on this AB, let us say B, and we draw BC at the angle ABC, equal to the given or chosen angle such that the ratio of the intercepted AB to the draw BC is the same as thar of a know b; this means that AB is to BC as a is to b. Next, let line AC be drawn through A and C, infinitely produced, then this line will be the required locus.[3]

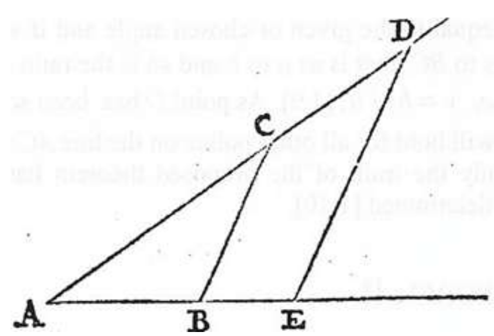

Figure 1. Diagram associated with proposition 1 of chapter I. [3]

In fact, if we select a point at random on *AC* let us say *D* and if we draw *DE* at the angle *DEA* equal to the given or chosen angle and if we call this *DE* y them *E*A will be to *ED as AB* is to *BC* that is as *a* to *b* and so is the ratio of *x* to *y*.Thus *ay* = *bx that is*, after divining by *a*, $y = \frac{bx}{a}$ **[3].**

In this proposition the relation of the symbolic representation with the geometric construction and the synthetic form used for the argumentation of the proposition is clearer. In Descartes the process of building the product equation depended on the unit. In this case a is an amount of any magnitude.

The interpretation of each conic from the symbolic representation allowed a definition more in act than constructive. However, the arguments that supported the demonstration were a more current view in terms of the symbology and the parameters that had been given up to now of the algebraic percentages. The existence of the conic was still linked to the criteria and properties tested by Apollonius. For several propositions what is found in this analytical geometry textbook were translations of some propositions from the book of *Conics* of antiquity [4]

De Witt in chapter II, theorem VII, proposition 7, proves the following statement: "If the equation is $y^2=ax$ or becomes $ay=x^2$ then the required place is a parable" [3]. The analysis of the diagram that accompanies the construction uses the same argument as proposition 20 of book I of Apollonius [4].

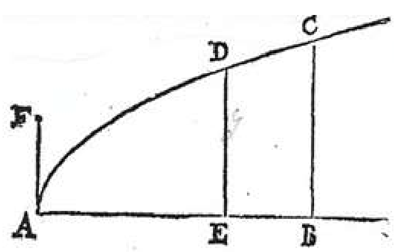

Figure 2. Proposition diagram VII. Chapter II. [3]

Apollonius shows the proportionality between the square of the length of the straight lines that go from the curve to the diameter, maintaining the same angle and the distance of the vertex to each of the straight lines.

> If in a parabola two straight lines are dropped ordinatewise to the diameter, the squares on them will be to each other as the straight lines cut off by them on the diameter beginning from the vertex are to each other [4]

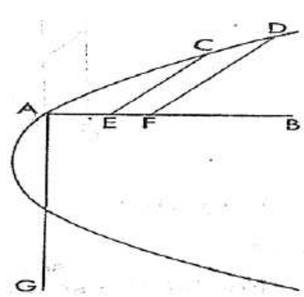

Figure 3. Proposition diagram 20. [4]

This fact is fundamental in the prove presented by De Witt. Specifically, he constructed two random points on the. *ADC c*urve and two straight lines with the same angle, from them, he maintained the argument of Apollonius, saying that by the characteristics of the parabola the square in *ED* is equal to a rectangle in *FAE* of Figure 2.

In De Witt's text it is reflected that both the classification of conics and their instance as an object depend on the symbolic representation, the construction takes another place in the description of the conic. However, it is the propositions of Apollonius translated or reinterpreted in the language of the algebra of the time that allows us to justify its existence

.A large part of the propositions presented in De Witt's book are based on taking a point *x* that "goes out indefinitely along the line *AB" "*, referring to the possible quantities represented by the variable *x* in a universe limited by the segment *AB*. A translation of the phrase "any point in *AB* is taken" used in ancient times by Apollonius, to the elements that were arranged in the seventeenth century through the Cartesian plane and the beginnings of the variables.

2.2. Problem solving

At the end of each chapter, a situation was resolved where the theorems presented were applied. The problems generalized the properties of the conic treated. These problems were accompanied by a diagram and had as a solution a geometric place described by means of symbolic representation.

> If, then, we wish to solve any problem, we first suppose the solution already effected, and give names to all the lines that seem needful for its construction, to those that are unknown as well as to those that are known. Then, making no distinction between know and unknown lines, we must unravel the difficulty in any way that shows most naturally the relations between these lines, until we find it possible to express a single quantity in two ways. This will constitute an equation, since the terms of one of these two expressions are together equal to the terms of the other.[2]

In a diagram he established a position for the known and unknown and from this geometric place, limited and experimental, proceeds to the solution of the problem:

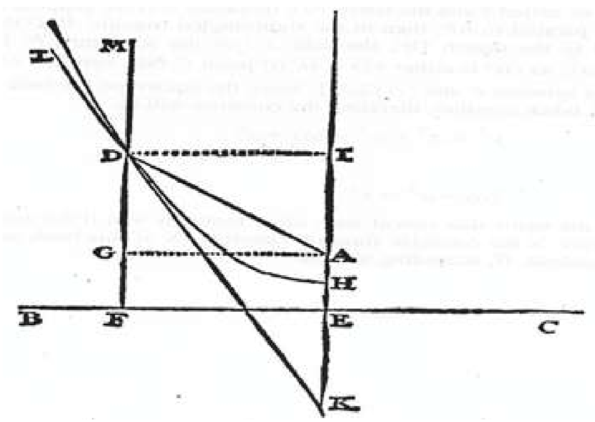

Figure 4. Problem I. [3]

Let *A* be the given point and *C* the straight line position is given and let it be reauired to find anoter point in the plane passing through both say *D*, so that the segment *DA* and *DF* are mutually equal; the latter being supposed to be perpendicualr to the given *BC*
In the same way, he assumed the right angle *EFD* and the parallelism between *AG* and *EF* to establish the relationships that allowed us to deduce that the geometric place is a parabola. The generalizing nature of the problems and the use of the fundamentals of analytical technique are inheritance of Cartesian work.

3. Conclusions

After the revision of the textbook *Elementa Curvarum Linearum Liber Secundus* that appears in the Latin edition of Descartes *Geometry*, it can be concluded that his reflections were more linked to a modernization, for the time, of the properties that Apollonius presented in his Conics book [4]. The anatomy of the text in propositions and a primordial use of symbolic representation as a definition allowed a translation of the most important properties of geometric places. For the uses that Grootendorst, Aarts, Bakker, and Erné, [3] report, he had the textbook on the appropriation, communication and dissemination of Cartesianism, in the case of mathematics, we can conclude that aspects such as: the use of mechanical instruments for the drawing of curves, the construction of geometric places through the intersection of curves, the role of geometry in the instrumentalization of algebra and the epistemic status of geometry, lost importance within the mathematical culture of the time and did not give the step to the education system. It remains the model of justification of the conics more linked to Apollonius than to Descartes. From the Geometry treatment, the resolution of problems through the Cartesian analytical method is seen as the aspect that was inherited in the educational processes.

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