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Comparative Solutions of Exact and Approximate Methods for Traveling Salesman Problem

Soluciones comparativas de métodos exáctos y aproximados para el problema de los vendedores ambulantes

Artículo de investigación

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Abstract:

There are two major optimization methods: Exact and Approximate methods. A well known exact method, Branch and Bound algorithm (B&B) and approximate methods, Elimination-based Fruit Fly Optimization Algorithm (EFOA) and Artificial Atom Algorithm (A^3) are used for solving the Traveling Salesman Problem (TSP). For 56 destinations, the results of total distance, processing time, and the deviation between exact and approximate method will be compared where the distance between two destinations is a Euclidean distance and this study shows that the distance of B&B is 270 , EFOA is 270 and A^3 is 288.38 which deviates 6.81%. For time processing aspect, B&B needs 12.5 days to process, EFOA needs 36.59 seconds, A^3 needs 35.34 seconds. But for 29 destinations, exact method is more powerful than approximate method.

Keywords:

Branch and Bound, Elimination-based Fruit Fly Optimization, Arti icial Atom Algorithm, Traveling Salesman Problem.

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Resumen:

Hay dos métodos principales de optimización: Exact y aproximate. Un método exacto bien conocido, algoritmo de rama y atado (B&B) y métodos aproximados, el algoritmo de optimización de la mosca de la fruta basada en la eliminación (EFOA) y el algoritmo de átomo artificial (A3) se utilizan para resolver el problema del vendedor ambulante (TSP). Para 56 destinos, se compararán los resultados de la distancia total, el tiempo de procesamiento y la desviación entre el método exacto y el aproximado donde se encuentre la distancia entre dos destinos es una distancia euclídea y este estudio muestra que la distancia de B&B es 270, la EFOA es 270 y A3 es 288,38, lo que se desvía un 6,81%. Para el aspecto de procesamiento de tiempo, B&B necesita 12,5 días para procesar, EFOA necesita 36,59 segundos, A3 necesita 35,34 segundos. Pero para 29 destinos, el método exacto es más poderoso que el método aproximado.

Palabras clave:

Rama y atado, eliminación basada en la optimización de la mosca de la fruta, algoritmo de átomo artificial, problema de vendedor ambulante.

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1. INTRODUCTION

In a fierce market environment, every profit organization is trying to be a competitive one. The goal is the same: Maximize profit and minimize cost. In big cities, we are dealing with a large concentration of transport needs in time and space that occur with specific periodicity, especially in rush hours [1]. Transport cost such as fuel consumption is one of the factors which affects financial condition and economic aspects [2]. Travel distance and travel time are two factors that have a great impact on transport cost. Jorgensen and Preston [3] concluded that there is a relationship between travel distance and fare. Rietveld, et.al [4] in their paper also stated that distance relates to fuel cost and cost of maintenance, repair, and depreciation. Those are the reasons why professionals and researchers are searching the methods for minimizing the travel distance, which is known as a traveling salesman problem (TSP). TSP [5] is the most popular combinatorial optimization problem, proven to be NP-Hard and the goal is to find a tour that minimizes the total distance. The tour visits every location only once and called a Hamiltonian cycle [6], [7], [8]. Many real-world problems can be modeled as variants of TSP as real-world problems are often more complicated than TSP [9], [10]. There are two major optimization methods: exact and approximate. Branch and bound and dynamic programming algorithms are categorized as exact methods. Metaheuristics such as single solution-based and population-based are categorized as approximate methods [11]. Each of these methods has some characteristics which exact method explores each and every possible solution to find the exact optimal solution, needs a mathematical convergence proof, computationally less efficient and on the other hand, the approximate method seeks to find a near-optimal solution, usually has natural, physical and biological principles, balancing between exploration (diversification) and exploitation (intensification), and computationally faster than exhaustive search [12]. The branch and Bound (B&B) algorithm as a first and well-known exact algorithm is a desirable algorithm to get an exact solution [13]. In the meanwhile, there are numerous comparative studies on metaheuristics algorithms. Yildirim and Karci

[14] shows that Artificial Atom Algorithm (A^3) as a nature-inspired algorithm is better than Genetic Algorithm (GA), Particle Swarm Algorithm (PSO), Artificial Bee Colony (ABC) in their study of traveling salesman problem (TSP) for 81 provinces in Turkey. Huang, L., et al [15] have proved that the Elimination-based Fruitfly Optimization Algorithm (EFOA) has a better convergence rate and precision than other algorithms: RABNET, HACO, CGAS, ACOTM, HA, DWIO. Of course, the question arises: “How large is the gap for the results between exact and approximate method?” or in other words: “how large is the deviation between optimal and near-optimal solution?”. In a complex world, one tends to solve the problem faster, more accurately, and with shorter distances. A shorter distance means less fuel consumption, less carbon dioxide emissions, and a greener environment [16], [17]. In this paper, the authors do a comparative study based on 56 locations between an exact – branch and bound (B&B) algorithm and approximate method – (A^3) and EFOA algorithm to get the value about distance, processing time by MATLAB software, and deviation between two optimization methods.

2. METHODOLOGY AND THEORY

Branch and Bound Algorithm

The term branch and bound was coined in conjunction with the TSP algorithm by Little, et.al in 1963. B&B methods solve a discrete optimization problem by breaking up its feasible set into successively smaller subsets, calculating bounds on the objective function, and using them to discard certain subsets from further consideration. The procedure ends when there is no better solution than the existing solution. [18]. There are several steps in this algorithm [19]:

Step 1: Collect the distance data between locations, $d_{ij,}$ and arrange in the table, where i indicates the row and j indicate the column

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Step 2: Reducing each element (d_{ij});

__$d'_{ij} = d_{ij} - r_i - c_j$ where $r_i = \min_j d_{ij}$ and $c_j = \min_i (d_{ij} - r_i)$

$$b = \sum_{i=1}^{n} r_i + \sum_{j=1}^{n} c_j$$

Step 4: Branching: The penalty for not utilizing trajectory

$$\pi_{ij} = \min_j d'_j + \min_i d'_j \ .$$

The branching is the maximum penalty:

$$\pi = \max_{ij} \pi_j$$

Step 5: Etiquette Calculating: Remove p line and q column in which related to position where element $_{ij}$ which has a maximum penalty. This step is repeatedly using step 2 and step 3. The sum of reduced elements is marked with σ.

Step 6: Drawing the branching tree: assign b etiquette to the knot from which branching started.

$b+\pi b+\pi$ to the non (p, q)
$b+\sigma b+\sigma$ to the (p, q)

Step 7: Locating the knot with the smallest etiquette. And algorithm is completed when the table of distance contains routes only.

Artificial Atom Algorithm (A^3)

A^3 is a metaheuristic algorithm and inspired by chemical compounding processes. This algorithm has beed developed by modelling chemical ionic bond and covalent bond processes. The difference from other meteheuristic algorithm is the effect of parameter values on the result separately. There are three concepts: first is electron which represents each parameter value that has an effect on the solution, second is atoms which consist of electrons and indicate candidate solutions and third is atom set which consists of atoms and is determined according to the size of the problem. A^3 uses two basic operators: covalent bond and ionic bond [20].

Pseudo code for covalent bond operator is as follow:

```
i ←1,2,..., βn          // i ≤ βn
If E[A_j[i]] is better than E[A_r[i]]
        Copy value of A_j[i] to A_r[i]
Else
        Copy value of A_r[i] to A_j[i]
```

Pseudo code for ionic bond operator is as follow:

```
Ionic bond (AtomSet, m, n, β)
j ←1, ...m // m: number of atoms
i ← βn + 1, ... n // β: Covalent rate
                 // n: Number of electrons
A_j[i] ← L_i + η * (U_i – L_i)
// A_j [i] ϵ AtomSet
// η : a randomm number generated between (0-1)
// U_i : upper bound for i^th attribute
// L_i : lower bound for i^th attribute
```

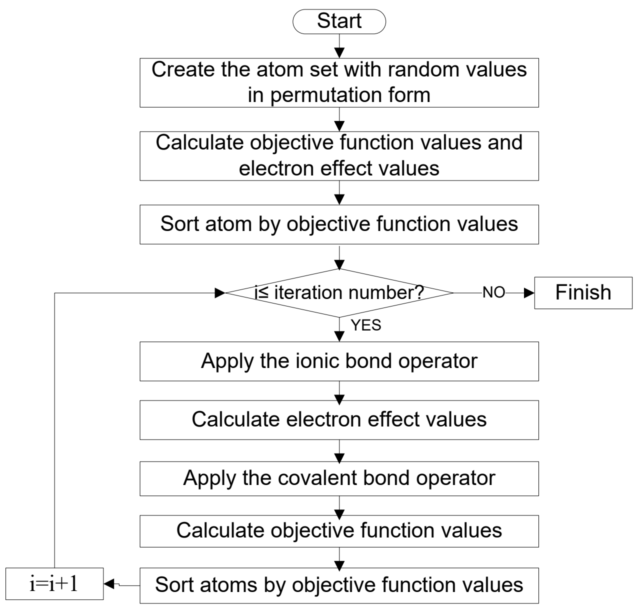

Figure 1. The Algorithm Steps of A^3 Source: [14]

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Elimination-based Fruitfly Optimization Algorithm (EFOA)

In 2011, Fruit fly algorithm as a approximate algorithm is introduced by Wen Tsao Pan and originated from foraging behavior of fruit flies which have a keen vision and smell to find food quickly by following the odor concentration in the air. This algorithm has a simple structure and easily understood [21]. Because its simple structure then it can easily fall into the local optimum and produces low optimization precision. Then Huang, L., et al proposed improved fruit fly algorithm that eliminates some individuals – weak fruit flies and some new individuals are generated in fruit fly foraging process, and this proposed algorithm is called Elimination based Fruit Fly Optimization Algorithm–EFOA. In EFOA algorithm, the vision search algorithm is enhanced as follow:

$$X_i = c\ x\ X_i + (1-c)\ x\ X_{best}$$
$$Y_i = c\ x\ Y_i + (1-c)\ x\ Y_{best}$$

Where (X_{best}, Y_{best}) indicates the current optimal individual and c is a random number from zero to one. After getting this, then calculate the odor concentration value smell:

$$S_i = \frac{1}{\sqrt{X_i^2 + Y_i^2}}$$
$$Smell = F(S_i)$$
$$Smell_{best} = \max(Smell)$$

The proposed algorithm is following:

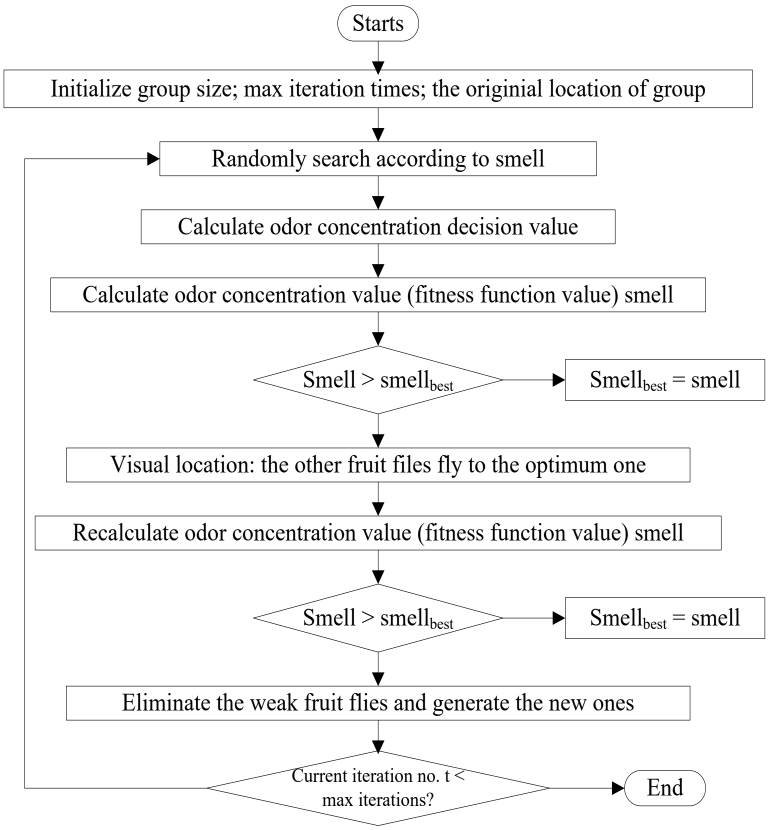

Figure 2. The Algorithm Steps of EFOA Source: [15]

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3. RESEARCH FRAMEWORK – COMPARATIVE STUDY EXACT AND APPROXIMATE METHODS

In this research, we use 56 locations include coordinates and distance between two locations for both exact and approximate methods and software MATLAB R2015a and LINGO 18.0, Intel Core i5 7200 U CPU 2.5 GHz, 32 bit ACPI x64 based PC will be used to calculate the total distance and total processing time. MATLAB as a powerful software package is used for Approximate method – A^3 and EFOA algorithm and LINGO as a comprehensive tool for building and solving mathematical optimization more easier and more efficient which is used for Exact method – B&B algorithm in this study. This software is similar to CPLEX. These results will be compared and the deviation between two methods is calculated. From this, we will know how large the gap is and which approximate method is closer to exact one. LINGO software is available on the LINDO system website [22].

4. RESULTS

B&B algorithm as an exact method produces 270 in distance in 1,088,494.17 seconds. On the other hand, approximate methods, A3 produces 288.38 in distance in average time 35.34 seconds, and EFOA produces 270 in distance in average time 36.59 seconds.

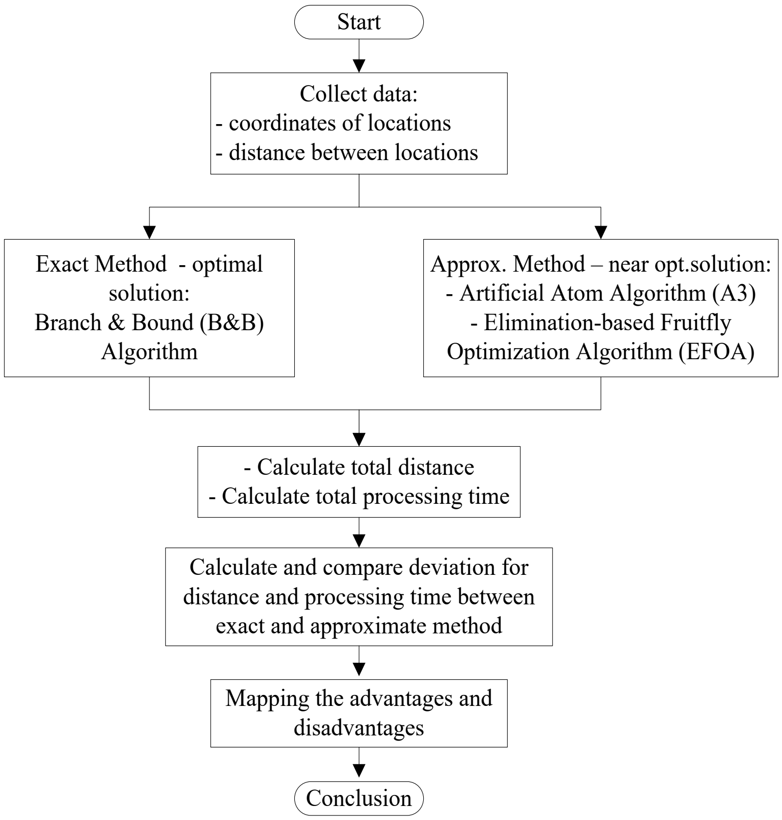

Figure 3. Research Framework

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Here are the results:

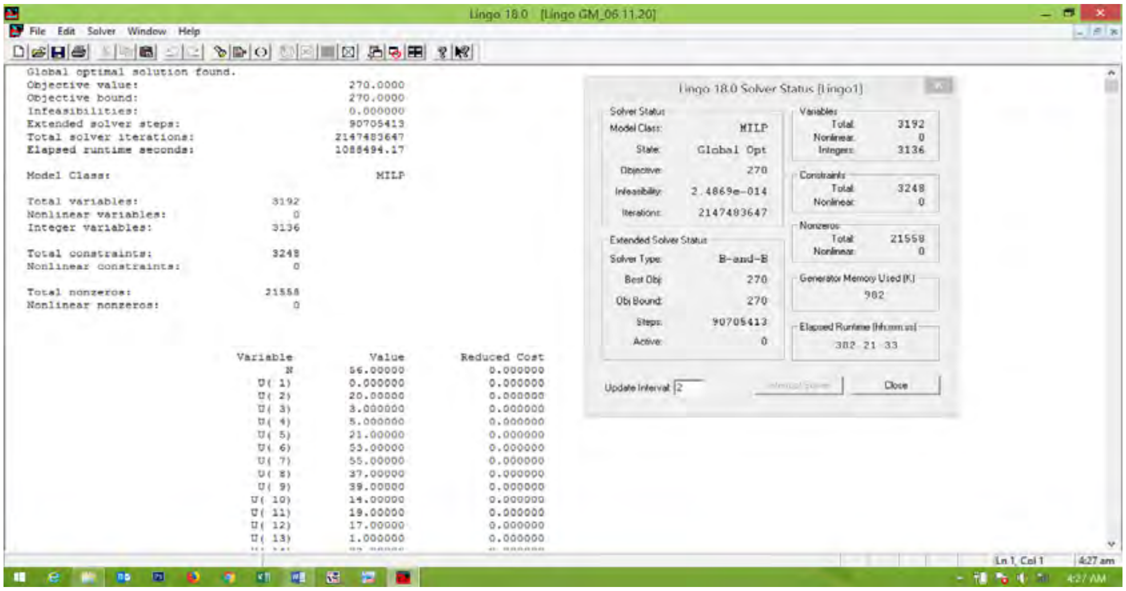

Figure 4. B&B Algorithm for 56 destinations by Lingo18.0

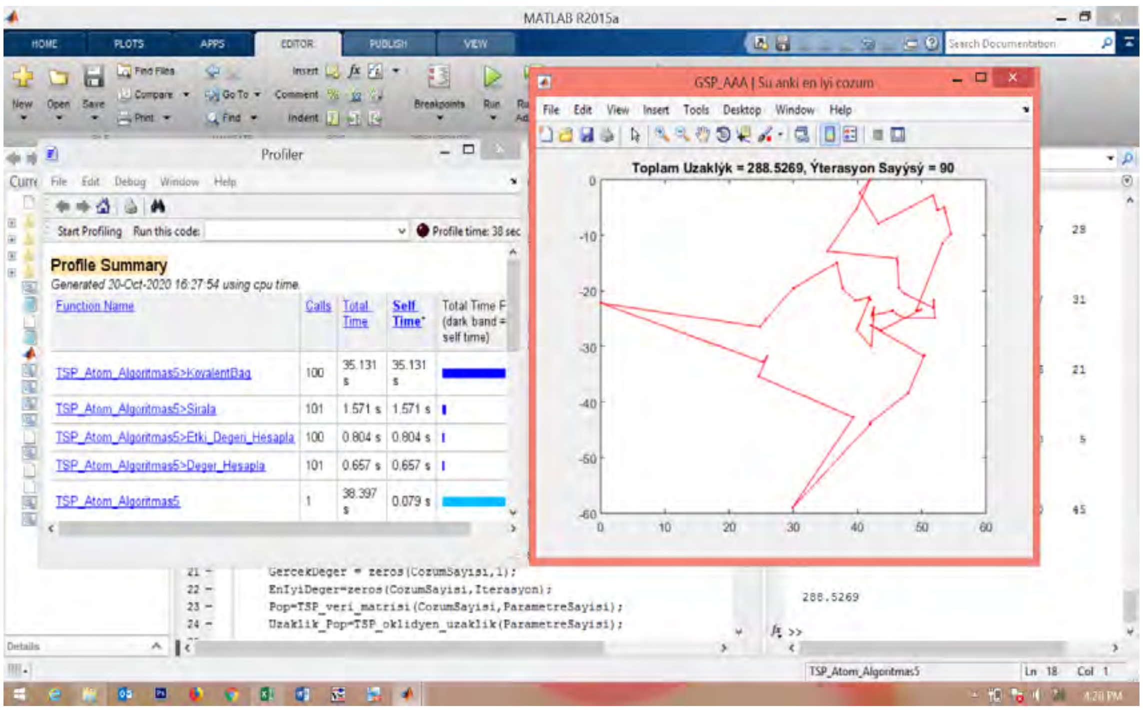

Figure 5. A3 algorithm for 56 destinations by Matlab

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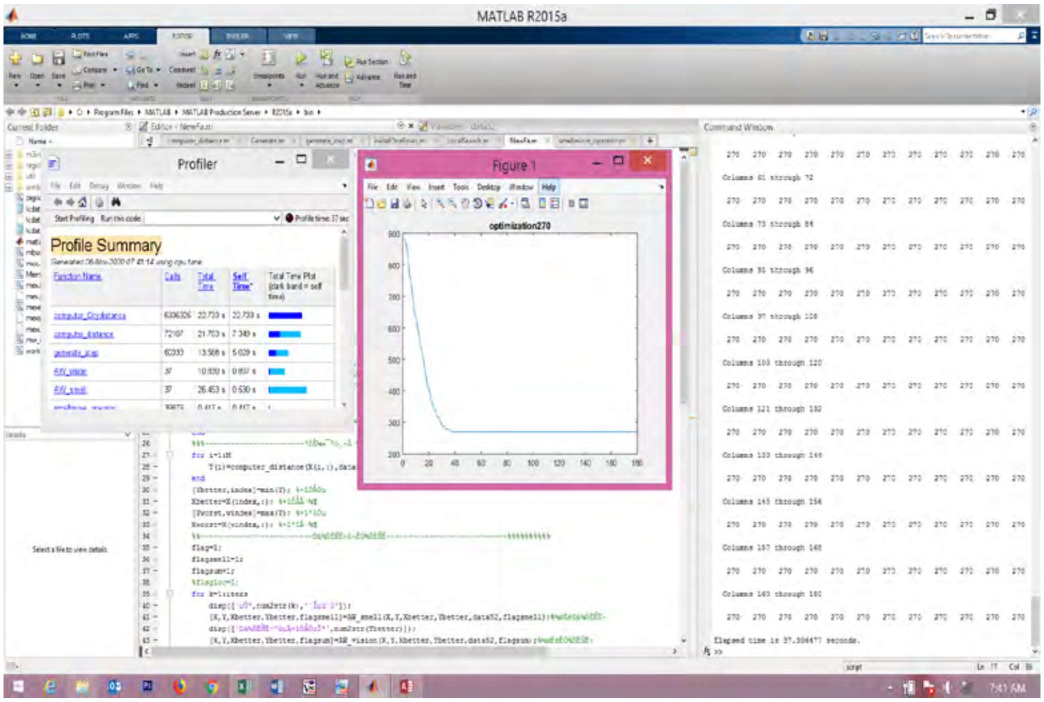

Figure 6. EFOA algorithm for 56 destinations by Matlab

The following table 1 shows the results:

Table 1. The Result of Algorithm for 56 destinations

No	Algorithm	Method	BKS	Average (distance)	Error (%)	Average time (seconds)
1	B&B	Exact	270	270	0	1,088,494.17
2	A3	Approximate	270	288.38	6.81	35.34
3	EFOA	Approximate	270	270	0	36.59

Note: BKS is Best Known Solution

These algorithms is also tested on tsplib_bays29 (29 cities in Bavaria, street distances (Groetschel,Juenger,Reinelt), the result is:

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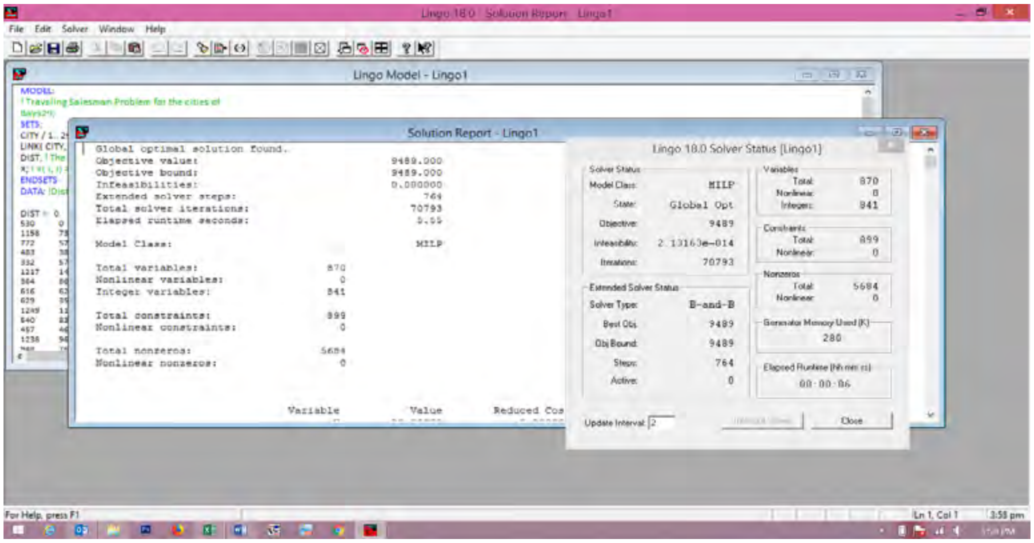

Figure 7. B&B algorithm for tsplib_bays29

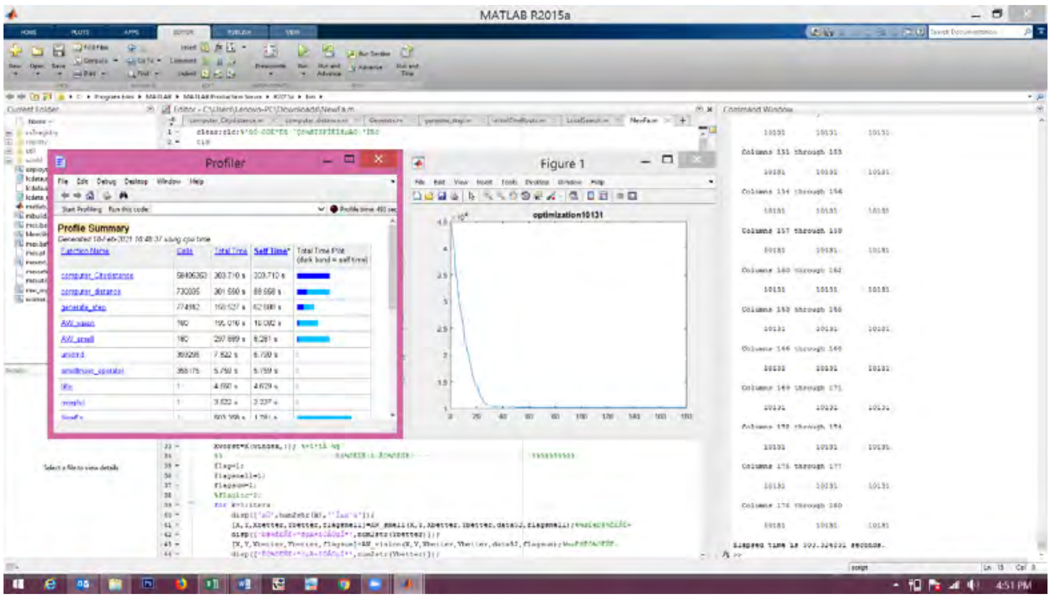

Figure 8. EFOA algorithm for tsplib_bays29

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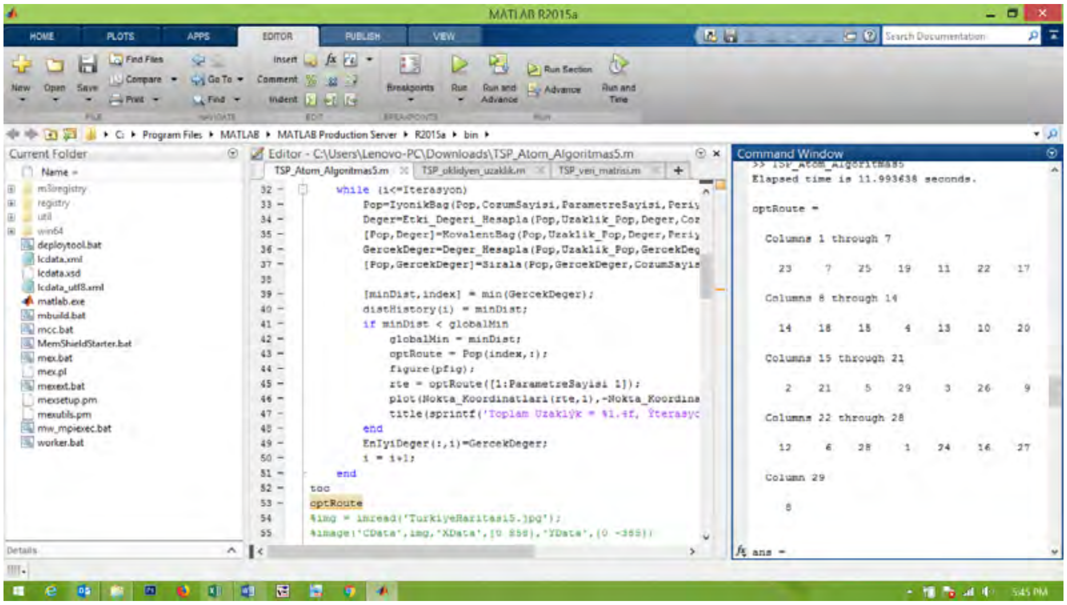

Figure 9. A3 algorithm for tsplib_bays29

Table 2. The Result of Algorithm for 29 cities (tsplib_bays29.tsp)

No	Algorithm	Method	BKS	Average (distance)	Error (%)	Average time (seconds)
1	B&B	Exact	9,489	9,489	0	5.65
2	A3	Approximate	9,489	10,689	12.65	11.99
3	EFOA	Approximate	9,489	10,135	6.81	415.5

From this case study fo 56 destinations and tsplib_bays29, the results show EFOA algorithm produces better total distance than A3 algorithm with deviation 0% and 6.81% from exact algorithm, but for A3 algorithm deviates 6.81% and 12.65% from exact algorithm. Time comparison is also produced and compared by these three methods, B&B spent million seconds which equals to twelve days, but for appoximate methods produced only around thirty seconds for 56 destinations, but in less destinations, exact algorithms – B&B algorithm seems faster than approximate method.

5. DISCUSSION

One need to consider using exact method – B&B algorithm in solving traveling salesman problem, it is because the B&B method only suitable for solving less than 60 locations (Mataija, M., et al: 2016, p.261). To overcome this obstacle of B&B method, one may consider to use approximate method, such as EFOA and A3, but for the number of locations is less than 30 points, exact method is more powerful than approximate method.

6. CONCLUSIONS

The results of searching the shortest distance for 56 locations, exact method – B&B algorithm is 270, and approximate method – EFOA algorithm has the same result with exact method, 270 in distance, another approximate method – A3 algorithm is 288.38 in distance, only deviates around 6.81% from exact method. From processing time aspect, both approximate method produce around 35 seconds and exact method in 12.5 days. But for tsplib_bays29, where the number of locations is less than 30, exact method is powerful than approximate method

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CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this article.

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