REDUCING THE PROBLEM OF WAVEGUIDE EXCITATION BY CURRENTS IN CROSS-SECTION TO A SYSTEM OF INTEGRAL VOLTERRA EQUATIONS

Angelina Markina
Kazan Federal University, Kazan, Russia.

Nikolai Pleshchinskii
Kazan Federal University, Kazan, Russia.

Dmitrii Tumakov
Kazan Federal University, Kazan, Russia.

E-mail: m8angelina@gmail.com

Recepción: 05/08/2019 **Aceptación:** 09/09/2019 **Publicación:** 23/10/2019

Citación sugerida:
Markina, A., Pleshchinskii, N. y Tumakov, D. (2019). Reducing the problem of waveguide excitation by currents in cross-section to a system of integral volterra equations. *3C TIC. Cuadernos de desarrollo aplicados a las TIC. Edición Especial, Octubre 2019*, 106-125. doi: https://doi.org/10.17993/3ctic.2019.83-2.106-125

Suggested citation:
Markina, A., Pleshchinskii, N. & Tumakov, D. (2019).Reducing the problem of waveguide excitation by currents in cross-section to a system of integral volterra equations. *3C TIC. Cuadernos de desarrollo aplicados a las TIC. Special Issue, October 2019*, 106-125. doi: https://doi.org/10.17993/3ctic.2019.83-2.106-125

ABSTRACT

The problem of excitation of a cylindrical metal waveguide by a source located in the cross section is considered. We assume that the source is surface currents on a flat, infinitely thin metal plate with a smooth boundary. The plate is connected to the generator of non-harmonic oscillations. The boundary of the cross section of a waveguide filled with a homogeneous dielectric is a closed piecewise-smooth contour. The initial physical problem is formulated as a mixed boundary problem for the system of the Maxwell equations. Components of the desired solution for the problem is presented in the form of a series in two sets of two-dimensional eigenfunctions of the Laplace operator. The first set of the eigenfunctions corresponds to the operator with Dirichlet boundary conditions, the second set to the operator with Neumann boundary conditions. We show that the expansion coefficients of the longitudinal components (components directed along the waveguide axis) of the electric and magnetic intensity vectors must be solutions to the jump problem for a system of telegraph equations. The problem of finding the unknown coefficients of the expansion of the longitudinal component of the vector of electric intensity is reduced to solving a system of the Volterra integral equations of the first kind with respect to the derivatives of the desired coefficients. The unknown coefficients of the expansion of the longitudinal component of the vector of magnetic intensity are found by solving a system of the Volterra integral equations of the second kind.

KEYWORDS

Metal waveguide, Wave excitation, Telegraph equation, Cross-sectional source, Volterra equation.

Edición Especial Special Issue Octubre 2019
DOI: https://doi.org/10.17993/3ctic.2019.83-2.106-125

1. INTRODUCTION

Metal waveguides are widely used in electronics and engineering. The study of such waveguide structures includes both the description of the set of eigenwaves and the search for the conditions of their excitation (Barybin, 2007). In particular, the excitation of oscillatory processes with specified characteristics in such structures is one of the tasks facing engineers.

In the case of a harmonic non-stationary electromagnetic field, the fundamentals of the theory of waveguides with metal walls were created in the middle of the last century (see, for example, works) (Samarskii & Tikhonov, 1948; Samarskii & Tikhonov, 1947). The problem of field excitation by currents given inside the waveguide was investigated in enough detail. The modern theory of excitation of waveguides of various types is presented in the review article (Solncev, 2009; Ghaderi & Mahdavi Panah, 2018). For metal waveguides, there are cases when solutions to the problems of propagation and diffraction of eigenwaves can be obtained analytically (Collin, 1960; Mittra, 1971).

Various methods are used to excite waveguides. For example, in optical waveguides, geometric inhomogeneities on a dielectric are often used to excite oscillations by an incident external wave (Sun & Wu, 2010; Shapochkin *et al.*, 2017; Kheirabadi & Mirzaei, 2019; Kashisaz & Mobarak, 2018). For metal waveguides, adjacent transducer waves are used or, more often, probes inside the waveguide (Yirmiyahu, Niv, Biener, Kleiner, & Hasman, 2007; Kong, 2002; Pan & Li, 2013; Eslami & Ahmadi, 2019; Jabbari *et al.*, 2019; Nakhaee & Nasrabadi, 2019). In this case, the probes can have both a simple dipole shape and a loop shape. Also, the natural waves are excited through the slits of the waveguide or through another conjugate waveguide (Sadiku, 2014). In this case, the waveguide itself can be both homogeneous and inhomogeneous filling (Bogolyubov *et al.*, 2003; Islamov *et al.*, 2017; Sailaukyzy *et al.*, 2018).

In the present work, we consider the problem of the excitation of a cylindrical metal waveguide by currents on an infinitely thin metal plate located in cross section and connected to a generator. We assume that the waveguide cross section is bounded by a piecewise smooth curve. The non-harmonic electromagnetic field excited in the waveguide is sought as a solution to the jump problem for the Maxwell equations. We show that the longitudinal components of the field must be solutions of the system of telegraph equations. The jump problem for such the system of equations is reduced to the system of the Volterra integral equations.

2. PROBLEM STATEMENT

Let an infinite cylindrical waveguide with metal walls (Figure 1a) is filled with a homogeneous isotropic dielectric, and the z axis is the longitudinal axis of the waveguide. Let its cross section Ω $(z = 0)$ be bounded by a piecewise-smooth contour C and consists of two parts: M and N (Figure 1b), moreover, $\Omega := \bar{M} \cup N$. Part M is an infinitely thin ideally conducting plate connected to a generator of high-frequency non-harmonic oscillations. The currents arising on the plate excite an electromagnetic field in the waveguide. It is necessary to find this field.

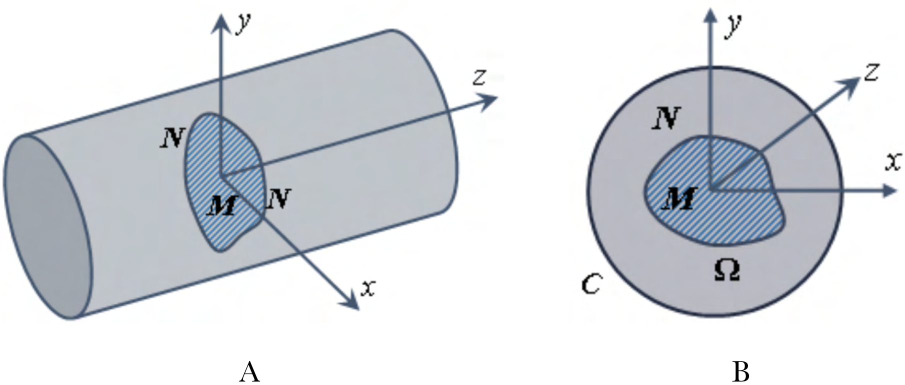

Figure 1. The construction of the cylindrical waveguide with a current source in a cross section in a plane *z* = 0

As is known (Nikolskij & Nikolskaya, 1989), the following necessary conjugation conditions (boundary conditions) are fulfilled at the interface: the tangential component of the electric intensity $\vec{E}$ is continuous, the jump of the tangential component of magnetic intensity $\vec{H}$ is equal to the density of the surface current,

DOI: https://doi.org/10.17993/3ctic.2019.83-2.106-125

the jump of the normal component of the electric induction $\vec{D}$ is equal to the density of the surface charge, the normal component of magnetic induction $\vec{B}$ is continuous.

We assume that the field is non-harmonically dependent on time. We search for solutions of the Maxwell equations at $z > 0$ and at $z < 0$:

$$\nabla \times \vec{E} = -\mu\mu_0 \frac{\partial \vec{H}}{\partial t}, \quad \nabla \times \vec{H} = \varepsilon\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad (1)$$

on the set jumps to Ω of the tangent components of the vectors $\vec{E}$ and $\vec{H}$:

$$\begin{aligned} H_x^+(x,y,t) - H_x^-(x,y,t) &= A_x(x,y,t), \quad (x,y) \in \Omega, t > 0, \\ H_y^+(x,y,t) - H_y^-(x,y,t) &= A_y(x,y,t), \quad (x,y) \in \Omega, t > 0, \end{aligned} \qquad (2)$$

$$\begin{aligned} E_x^+(x,y,t) - E_x^-(x,y,t) &= B_x(x,y,t), \quad (x,y) \in \Omega, t > 0, \\ E_y^+(x,y,t) - E_y^-(x,y,t) &= B_y(x,y,t), \quad (x,y) \in \Omega, t > 0. \end{aligned} \qquad (3)$$

Conditions (2) and (3) are slightly more general than required. In the problem of excitation of a waveguide by a source at a cross section, the jump in the tangential component of the magnetic vector is set to M (this is the electric current density) and is zero at N; the jump of the tangent component of the electric vector is everywhere zero.

The desired solutions of the Maxwell equations (1) must also satisfy the initial conditions at $t = 0$:

$$\begin{aligned} H^\pm(x,y,z) = 0 \quad &\text{or} \quad \frac{\partial H^\pm(x,y,z)}{\partial t} = 0, \\ E^\pm(x,y,z) = 0 \quad &\text{or} \quad \frac{\partial E^\pm(x,y,z)}{\partial t} = 0, \end{aligned} \qquad (4)$$

and be sufficiently smooth at $z > 0$ and at $z < 0$. We assume that their limit values are correctly determined at $z \to 0 \pm 0$ in the classical or generalized sense (Pleshchinskii, 2019).

3. JUMP PROBLEM FOR TELEGRAPH EQUATIONS

Let us proceed from the jump problem for the Maxwell equations (1)-(4) in the cylindrical domain $\Omega \times R$ to the jump problem for an infinite system of telegraph equations. The components of the solutions for the Maxwell equations in a cylindrical region that satisfy the boundary conditions on the wall of the waveguide (the tangent component of the electric vector is zero) can be represented for the components $H(x, y, z, t)$ in the following form (Pleshchinskii *et al.*, 2017):

$$H_z^\pm = \sum_m H_{z,m}^\pm(z,t)\lambda_m \varphi_m(x,y),\ m = 0,1,..., \tag{5}$$

$$H_x^\pm = \varepsilon_0 \varepsilon \sum\nolimits_m \frac{\partial E_{z,m}^\pm}{\partial t}(z,t)\frac{\partial \psi_m}{\partial y}(x,y) + \sum\nolimits_m \frac{\partial H_{z,m}^\pm}{\partial z}(z,t)\frac{\partial \varphi_m}{\partial x}(x,y), \tag{6}$$

$$H_y^\pm = -\varepsilon_0 \varepsilon \sum\nolimits_m \frac{\partial E_{z,m}^\pm}{\partial t}(z,t)\frac{\partial \psi_m}{\partial x}(x,y) + \sum\nolimits_m \frac{\partial H_{z,m}^\pm}{\partial z}(z,t)\frac{\partial \varphi_m}{\partial y}(x,y) \tag{7}$$

and for components $E(x, y, z, t)$ as follows:

$$E_z^\pm = \sum_m E_{z,m}^\pm(z,t)\chi_m \psi_m(x,y), \tag{8}$$

$$E_x^\pm = \sum\nolimits_m \frac{\partial E_{z,m}^\pm}{\partial z}(z,t)\frac{\partial \psi_m}{\partial x}(x,y) - \mu_0 \mu \sum\nolimits_m \frac{\partial H_{z,m}^\pm}{\partial t}(z,t)\frac{\partial \varphi_m}{\partial y}(x,y), \tag{9}$$

$$E_y^\pm = \sum\nolimits_m \frac{\partial E_{z,m}^\pm}{\partial z}(z,t)\frac{\partial \psi_m}{\partial y}(x,y) + \mu_0 \mu \sum\nolimits_m \frac{\partial H_{z,m}^\pm}{\partial t}(z,t)\frac{\partial \varphi_m}{\partial x}(x,y), \tag{10}$$

where ε is the dielectric constant, and μ is the magnetic permeability of the substance. The functions $\psi_m(x,y)$ and $\varphi_m(x,y)$ are orthonormal sets of eigenfunctions for the Laplace operator in the domain Ω, with Neumann and Dirichlet boundary conditions, respectively. Moreover, λ_m and χ_m are eigenvalues of the Laplace operator. We assume that a piecewise-smooth contour is such that the eigenfunctions exist in simple cases; and in simple cases, as a circle or a rectangle, they are constructed analytically, in other cases, they are constructed

DOI: https://doi.org/10.17993/3ctic.2019.83-2.106-125

numerically. The expansion coefficients $E_{z,m}(z,t)$ and $H_{z,m}(z,t)$ are new unknown functions and must be solutions to jump problems for telegraph equations $H^{\pm}_{z,m}(z,t)$:

$$\frac{\partial^2 H^{\pm}_{z,m}}{\partial t^2}(z,t) = \frac{1}{\kappa^2}\frac{\partial^2 H^{\pm}_{z,m}}{\partial z^2}(z,t) - \frac{\lambda_m}{\kappa^2} H^{\pm}_{z,m}(z,t), \qquad m = 0,1,\ldots \tag{11}$$

$$\begin{gathered} H^{+}_{z,m}(0,t) - H^{-}_{z,m}(0,t) = H_0(x,y,0,t), \quad (x,y) \in \Omega, \\ \frac{\partial H^{+}_{z,m}}{\partial z}(0,t) - \frac{\partial H^{-}_{z,m}}{\partial z}(0,t) = H_1(x,y,0,t), \quad (x,y) \in \Omega, \end{gathered} \tag{12}$$

$$H^{\pm}_{z,m}(z,0) = 0, \qquad \frac{\partial H^{\pm}_{z,m}}{\partial t}(z,0) = 0, \tag{13}$$

and for telegraph equations $E^{\pm}_{z,m}(z,t)$:

$$\frac{\partial^2 E^{\pm}_{z,m}}{\partial t^2}(z,t) = \frac{1}{\kappa^2}\frac{\partial^2 E^{\pm}_{z,m}}{\partial z^2}(z,t) - \frac{\chi_m}{\kappa^2} E^{\pm}_{z,m}(z,t), \qquad m = 0,1,\ldots \tag{14}$$

$$\begin{gathered} E^{+}_{z,m}(0,t) - E^{-}_{z,m}(0,t) = E_0(x,y,0,t), \quad (x,y) \in \Omega, \\ \frac{\partial E^{+}_{z,m}}{\partial z}(0,t) - \frac{\partial E^{-}_{z,m}}{\partial z}(0,t) = E_1(x,y,0,t), \quad (x,y) \in \Omega, \end{gathered} \tag{15}$$

$$E^{\pm}_{z,m}(z,0) = 0, \qquad \frac{\partial E^{\pm}_{z,m}}{\partial t}(z,0) = 0, \tag{16}$$

where κ is the wave number ($\kappa^2 = \mu_0\varepsilon_0\mu\varepsilon$). The functions $H_0(x,y,0,t)$, $H_1(x,y,0,t)$, $E_0(x,y,0,t)$, $E_1(x,y,0,t)$ are some known functions in domains Ω. We show below what these functions will be equal to. The functions are defined on M as functions of the current density, and on N they are equal to zero.

Now we show that the desired expansion coefficients for the longitudinal field components in the problems (11)-(13) and (14)-(16) can be found after solutions for some integral equations are obtained.

For this purpose, we consider the jump problem for telegraph equation in the general case with respect to the auxiliary function $u(z, t)$ in the half-plane $t > 0$:

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial z^2} - b^2 u, \quad z \neq 0 \tag{17}$$

with boundary conditions:

$$u(0+0,t)-u(0-0,t)=A(t),$$
$$\frac{\partial u}{\partial z}(0+0,t)-\frac{\partial u}{\partial z}(0-0,t)=B(t),$$

and initial conditions:

$$u(z,0)=f(z),\ \ \frac{\partial u}{\partial t}(z,0)=F(z).$$

We note that the functions $f(z)$ and $F(z)$ can have discontinuities at the point z=0:

$$f(z)=\{f^{-}(z),z<0\,;\ \ f^{+}(z),z>0\},$$
$$F(z)=\{F^{-}(z),z<0\,;\ \ F^{+}(z),z>0\}.$$

We seek a solution of the jump problem in the space of continuously differentiable functions. Let $F^{\pm}(z), f^{\pm}(z), g^{\pm}(t), G^{\pm}(t)$ be some known functions defined in the spaces $C^1(R)$ and $C^1(R^+)$.

Earlier in (16), we considered the over-determined boundary problem for telegraph equation (17) in the right quarter of the plane ($z > 0$, $t > 0$) with initial and boundary conditions (see Figure 2):

$$u(z,0)=f^{+}(z),\qquad \frac{\partial u}{\partial t}(z,0)=F^{+}(z),$$
$$u(0+0,t)=g^{+}(t),\qquad \frac{\partial u}{\partial z}(0+0,t)=G^{+}(t).$$

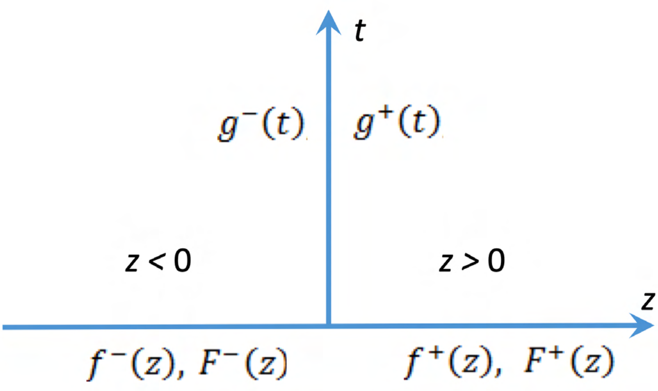

Figure 2. Traces of functions of the over-determined boundary problem for telegraph equation in the upper half-plane t>0.

DOI: https://doi.org/10.17993/3ctic.2019.83-2.106-125

We obtained in (16) the condition for the solvability of this problem in the following form:

$$\frac{1}{2}f^{+}(2at)+\frac{1}{2}\int_{0}^{2at}\Phi^{+}(at,t,\xi)d\xi=\frac{1}{2}g^{+}(2t)+\frac{1}{2}\int_{0}^{2t}\Psi^{+}(t,at,\xi)d\xi$$

or

$$f^{+}(2at)+\frac{1}{2}\int_{0}^{2at}\left[\frac{1}{a}F^{+}(\xi)J_{0}\left(\frac{b}{a}\sqrt{(2at-\xi)\xi}\right)-btf^{+}(\xi)\frac{J'_{0}\left(\frac{b}{a}\sqrt{(2at-\xi)\xi}\right)}{\sqrt{(2at-\xi)\xi}}\right]d\xi$$
$$=g^{+}(2t)+\frac{1}{2}\int_{0}^{2t}\left[aG^{+}(\xi)J_{0}\left(b\sqrt{(\xi-2t)\xi}\right)+btg^{+}(\xi)\frac{J'_{0}\left(b\sqrt{(\xi-2t)\xi}\right)}{\sqrt{(\xi-2t)\xi}}\right]d\xi.$$

Note that this equality establishes the relationship between the boundary functions in a mixed boundary problem for telegraph equation in a quarter of a plane. If the condition of solvability of the over-determined problem is executed, then it can be extended to the whole quarter-plane and even to the half-plane.

Now we consider the over-determined problem in the left quarter of the plane ($z < 0$, $t > 0$) with initial and boundary conditions

$$u(z,0)=f^{-}(z),\qquad \frac{\partial u}{\partial t}(z,0)=F^{-}(z),$$
$$u(0-0,t)=g^{-}(t),\qquad \frac{\partial u}{\partial z}(0-0,t)=G^{-}(t).$$

We replace z with $\tilde{z}=-z$ and seek the function $\tilde{u}(\tilde{z},t)=u(-z,t)$. The equation itself does not change, and the initial and boundary conditions take the form

$$\tilde{u}(\tilde{z},0)=f^{-}(-\tilde{z}),\qquad \frac{\partial\tilde{u}}{\partial t}(\tilde{z},0)=F^{-}(-\tilde{z})$$

and

$$\tilde{u}(0+0,t)=g^{-}(t),\qquad \frac{\partial\tilde{u}}{\partial\tilde{z}}(0+0,t)=-G^{-}(t).$$

Then, in a similar way as in (16), we obtain the solvability condition for this problem in the following form:

$$f^{-}(-2at)+\frac{1}{2}\int_{0}^{2at}\left[\frac{1}{a}F^{-}(-\xi)J_0\left(\frac{b}{a}\sqrt{(2at-\xi)\xi}\right)-btf^{-}(-\xi)\frac{J'_0\left(\frac{b}{a}\sqrt{(2at-\xi)\xi}\right)}{\sqrt{(2at-\xi)\xi}}\right]d\xi$$

$$=g^{-}(2t)+\frac{1}{2}\int_{0}^{2t}\left[-aG^{-}(\xi)J_0\left(b\sqrt{(\xi-2t)\xi}\right)+btg^{-}(\xi)\frac{J'_0\left(b\sqrt{(\xi-2t)\xi}\right)}{\sqrt{(\xi-2t)\xi}}\right]d\xi.$$

We use the solvability conditions and reduce solving the jump problem with zero initial conditions to solving the system of four equations:

$$g^{+}(\eta)+\frac{1}{2}\int_{0}^{\eta}\left[aG^{+}(\xi)J_0\left(b\sqrt{(\xi-\eta)\xi}\right)+btg^{+}(\xi)\frac{J'_0\left(b\sqrt{(\xi-\eta)\xi}\right)}{\sqrt{(\xi-\eta)\xi}}\right]d\xi=0,$$

$$g^{-}(\eta)+\frac{1}{2}\int_{0}^{\eta}\left[-aG^{-}(\xi)J_0\left(b\sqrt{(\xi-\eta)\xi}\right)+btg^{-}(\xi)\frac{J'_0\left(b\sqrt{(\xi-\eta)\xi}\right)}{\sqrt{(\xi-\eta)\xi}}\right]d\xi=0,$$

$$g^{+}(\eta)-g^{-}(\eta)=A(\eta),\quad G^{+}(\eta)-G^{-}(\eta)=B(\eta).$$

We denote $\eta=2t$. If the solvability conditions are added and subtracted, then we get two integral equations:

$$g^{+}(\eta)+g^{-}(\eta)+\eta\frac{b}{2}\int_{0}^{\eta}[g^{+}(\eta)+g^{-}(\eta)]\frac{J'_0\left(b\sqrt{(\xi-\eta)\xi}\right)}{\sqrt{(\xi-\eta)\xi}}d\xi$$

$$=-a\int_{0}^{\eta}B(\xi)J_0\left(b\sqrt{(\xi-\eta)\xi}\right)d\xi,$$

$$a\int_{0}^{\eta}[G^{+}(\xi)+G^{-}(\xi)]J_0\left(b\sqrt{(\xi-\eta)\xi}\right)d\xi$$

$$=-A(\eta)-\eta\frac{b}{2}\int_{0}^{\eta}A(\xi)\frac{J'_0\left(b\sqrt{(\xi-\eta)\xi}\right)}{\sqrt{(\xi-\eta)\xi}}d\xi.$$

If $A(\eta)=0$ and $B(\eta)$ is different from 0, then the function $g^{+}(\eta)=g^{-}(\eta)=g(\eta)$ is found as a solution for the integral equation:

$$2g(\eta)+\eta b\int_{0}^{\eta}g(\xi)\frac{J'_0\left(b\sqrt{(\xi-\eta)\xi}\right)}{\sqrt{(\xi-\eta)\xi}}d\xi=-a\int_{0}^{\eta}B(\xi)J_0\left(b\sqrt{(\xi-\eta)\xi}\right)d\xi. \qquad (18)$$

DOI: https://doi.org/10.17993/3ctic.2019.83-2.106-125

If $(\eta)=0$ $A(\eta)\neq 0$, then the function $G^{+}(\eta)=G^{-}(\eta)=G(\eta)$, and is found as a solution for the integral equation:

$$2a\int_{0}^{\eta} G(\eta)J_0\left(b\sqrt{(\xi-\eta)\xi}\right)d\xi=-A(\eta)-\eta\frac{b}{2}\int_{0}^{\eta} A(\xi)\frac{J_0'\left(b\sqrt{(\xi-\eta)\xi}\right)}{\sqrt{(\xi-\eta)\xi}}d\xi. \qquad (19)$$

Thus, we reduced solving the jump problem for telegraph equation to solving the Volterra integral equations (18) and (19). Solving the first equation, we can find the longitudinal components in the jump problem for $H_{z,m}(z,t)$, which corresponds to the jump problem for the equation (17) with $A(\eta)=0$ in the boundary conditions. The second equation can be used to determine $E_{z,m}(z,t)$ with $B(\eta)=0$.

4. BOUNDARY CONDITIONS OF THE JUMP PROBLEM FOR TELEGRAPH EQUATIONS

We express the right parts in the formulas (12) and (15) through known functions. From the conditions of the jump problem for the tangential components of the electric and magnetic vectors (2), (3), we obtain the conjugation conditions for the normal components of these vectors. We express the terms of the left-hand sides under conjugation conditions through $E_{z,m}(z,t)$ and $H_{z,m}(z,t)$. For this, we use the representations of the transverse field components in formulas (6), (7), (9), (10) $H_x(x, y, z, t)$, $H_y(x, y, z, t)$, $E_x(x, y, z, t)$, $E_y(x, y, z, t)$, we substitute them into the corresponding conditions (2), (3) and for any $(x,y)\in\Omega$ we get:

$$\begin{gathered}
\varepsilon_0\varepsilon\sum_m\left[\frac{\partial E_{z,m}^{+}}{\partial t}(0,t)-\frac{\partial E_{z,m}^{-}}{\partial t}(0,t)\right]\frac{\partial\psi_m}{\partial y}(x,y)\\
+\sum_m\left[\frac{\partial H_{z,m}^{+}}{\partial z}(0,t)-\frac{\partial H_{z,m}^{-}}{\partial z}(0,t)\right]\frac{\partial\varphi_m}{\partial x}(x,y)=A_x(x,y,t),\\
-\varepsilon_0\varepsilon\sum_m\left[\frac{\partial E_{z,m}^{+}}{\partial t}(0,t)-\frac{\partial E_{z,m}^{-}}{\partial t}(0,t)\right]\frac{\partial\psi_m}{\partial x}(x,y)\\
+\sum_m\left[\frac{\partial H_{z,m}^{+}}{\partial z}(0,t)-\frac{\partial H_{z,m}^{-}}{\partial z}(0,t)\right]\frac{\partial\varphi_m}{\partial y}(x,y)=A_y(x,y,t),
\end{gathered} \qquad (20)$$

$$\begin{gathered}\sum_m\left[\frac{\partial E^+_{z,m}}{\partial z}(0,t)-\frac{\partial E^-_{z,m}}{\partial z}(0,t)\right]\frac{\partial \psi_m}{\partial y}(x,y)-\mu_0\mu\sum_m\left[\frac{\partial H^+_{z,m}}{\partial t}(0,t)\right.\\ \left.-\frac{\partial H^-_{z,m}}{\partial t}(0,t)\right]\frac{\partial \varphi_m}{\partial y}(x,y)=B_x(x,y,t),\\ \sum_m\left[\frac{\partial E^+_{z,m}}{\partial z}(0,t)-\frac{\partial E^-_{z,m}}{\partial z}(0,t)\right]\frac{\partial \psi_m}{\partial y}(x,y)\\ +\mu_0\mu\sum_m\left[\frac{\partial H^+_{z,m}}{\partial t}(0,t)-\frac{\partial H^-_{z,m}}{\partial t}(0,t)\right]\frac{\partial \varphi_m}{\partial x}(x,y)=B_y(x,y,t).\end{gathered} \tag{21}$$

In the system (20), we differentiate the first equation with respect to x, the second with respect to y and add them. Then we get:

$$\begin{gathered}\sum_m\left[\frac{\partial H^+_{z,m}}{\partial z}(0,t)-\frac{\partial H^-_{z,m}}{\partial z}(0,t)\right]\left[\frac{\partial^2 \varphi_m}{\partial x^2}(x,y)+\frac{\partial^2 \varphi_m}{\partial y^2}(x,y)\right]\\ =\frac{\partial A_x}{\partial x}(x,y,t)+\frac{\partial A_y}{\partial y}(x,y,t).\end{gathered}$$

Now let us differentiate the first equation with respect to y, the second one with respect to x, and we subtract the second equation from the first equation. Then we obtain:

$$\begin{gathered}\varepsilon_0\varepsilon\sum_m\left[\frac{\partial E^+_{z,m}}{\partial t}(0,t)-\frac{\partial E^-_{z,m}}{\partial t}(0,t)\right]\left[\frac{\partial^2 \psi_m}{\partial x^2}(x,y)+\frac{\partial^2 \psi_m}{\partial y^2}(x,y)\right]\\ =\frac{\partial A_x}{\partial y}(x,y,t)-\frac{\partial A_y}{\partial x}(x,y,t).\end{gathered}$$

We perform similar transformations in the system (21) and obtain the following equations:

$$\begin{gathered}\sum_m\left[\frac{\partial E^+_{z,m}}{\partial z}(0,t)-\frac{\partial E^-_{z,m}}{\partial z}(0,t)\right]\left[\frac{\partial^2 \psi_m}{\partial x^2}(x,y)+\frac{\partial^2 \psi_m}{\partial y^2}(x,y)\right]\\ =\frac{\partial B_x}{\partial x}(x,y,t)+\frac{\partial B_y}{\partial y}(x,y,t),\\ -\mu_0\mu\sum_m\left[\frac{\partial H^+_{z,m}}{\partial t}(0,t)-\frac{\partial H^-_{z,m}}{\partial t}(0,t)\right]\left[\frac{\partial^2 \varphi_m}{\partial x^2}(x,y)+\frac{\partial^2 \varphi_m}{\partial y^2}(x,y)\right]\\ =\frac{\partial B_x}{\partial y}(x,y,t)-\frac{\partial B_y}{\partial x}(x,y,t).\end{gathered}$$

DOI: https://doi.org/10.17993/3ctic.2019.83-2.106-125

In square brackets, the expressions for the sum of the derivatives of the functions $\phi_m(x, y)$ and $\psi_m(x, y)$ represent the Laplace operator applied to these functions, respectively. We use this property of the eigenfunctions. Next, we scalar multiply both sides of the equations on $\phi_k(x, y)$ and $\psi_k(x, y)$, and, using the orthogonality of these functions, we obtain:

$$\begin{aligned} &\left[\frac{\partial H_{z,k}^{+}}{\partial z}(0,t) - \frac{\partial H_{z,k}^{-}}{\partial z}(0,t)\right]\lambda_k \\ &= -\iint_{\Omega}\left[\frac{\partial A_x}{\partial x}(x,y,t) + \frac{\partial A_y}{\partial y}(x,y,t)\right]\varphi_k(x,y)dxdy, \\ &\lambda_k\mu_0\mu\left[\frac{\partial H_{z,k}^{+}}{\partial t}(0,t) - \frac{\partial H_{z,k}^{-}}{\partial t}(0,t)\right] \\ &= \iint_{\Omega}\left[\frac{\partial B_x}{\partial y}(x,y,t) - \frac{\partial B_y}{\partial x}(x,y,t)\right]\varphi_k(x,y)dxdy, \end{aligned} \tag{22}$$

$$\begin{aligned} &\left[\frac{\partial E_{z,k}^{+}}{\partial z}(0,t) - \frac{\partial E_{z,k}^{-}}{\partial z}(0,t)\right]\chi_k \\ &= -\iint_{\Omega}\left[\frac{\partial B_x}{\partial x}(x,y,t) + \frac{\partial B_y}{\partial y}(x,y,t)\right]\psi_k(x,y)dxdy, \\ &\varepsilon_0\varepsilon\left[\frac{\partial E_{z,k}^{+}}{\partial t}(0,t) - \frac{\partial E_{z,k}^{-}}{\partial t}(0,t)\right]\chi_k \\ &= -\iint_{\Omega}\left[\frac{\partial A_x}{\partial y}(x,y,t) - \frac{\partial A_y}{\partial x}(x,y,t)\right]\psi_k(x,y)dxdy. \end{aligned} \tag{23}$$

We express the time derivatives in the systems of equations (22) and (23) as follows:

$$\begin{aligned} &\int_{\tau}\left[\frac{\partial H_{z,k}^{+}}{\partial t}(0,t) - \frac{\partial H_{z,k}^{-}}{\partial t}(0,t)\right]d\tau \\ &= \frac{1}{\lambda_k\mu_0\mu}\int_{\tau}\iint_{\Omega}\left[\frac{\partial B_x}{\partial y}(x,y,t) - \frac{\partial B_y}{\partial x}(x,y,t)\right]\varphi_k(x,y)dxdy\,d\tau, \\ &\int_{\tau}\left[\frac{\partial E_{z,k}^{+}}{\partial t}(0,t) - \frac{\partial E_{z,k}^{-}}{\partial t}(0,t)\right]d\tau \\ &= -\frac{1}{\varepsilon_0\varepsilon\chi_k}\int_{\tau}\iint_{\Omega}\left[\frac{\partial A_x}{\partial y}(x,y,t) - \frac{\partial A_y}{\partial x}(x,y,t)\right]\psi_k(x,y)dxdy\,d\tau. \end{aligned}$$

We consider that the initial conditions are zero, and we get:

$$H_{z,k}^{+}(0,t) - H_{z,k}^{-}(0,t)$$
$$= \frac{1}{\lambda_k \mu_0 \mu} \int_{\tau} \iint_{\Omega} \left[\frac{\partial B_x}{\partial y}(x,y,t) - \frac{\partial B_y}{\partial x}(x,y,t)\right] \varphi_k(x,y) dx dy d\tau,$$
$$E_{z,k}^{+}(0,t) - E_{z,k}^{-}(0,t) = -\frac{1}{\varepsilon_0 \varepsilon \chi_k} \int_{\tau} \iint_{\Omega} \left[\frac{\partial A_x}{\partial y}(x,y,t) - \frac{\partial A_y}{\partial x}(x,y,t)\right] \psi_k(x,y) dx dy d\tau.$$

The last two expressions and the first equations in the systems of equations (22), (23) are the conjugation conditions on the waveguide cross section in the jump problem for $H_{z,m}(z, t)$ and $E_{z,m}(z, t)$, respectively.

We use the obtained results and write down the jump problem for telegraph equation with respect to $H_{z,m}(z, t)$. We assume that $A_x(x, y, t) = j_x(x, y, t)$ and $A_y(x, y, t) = j_y(x, y, t)$, $B_x(x, y, t) = 0$ and $B_y(x, y, t) = 0$. Then we get:

$$\frac{\partial^2 H_{z,k}^{\pm}}{\partial t^2}(z,t) = \frac{1}{\kappa^2} \frac{\partial^2 H_{z,k}^{\pm}}{\partial z^2}(z,t) - \frac{\lambda_k}{\kappa^2} H_{z,k}^{\pm}(z,t),$$
$$H_{z,k}^{+}(0,t) - H_{z,k}^{-}(0,t) = 0,$$
$$\frac{\partial H_{z,k}^{+}}{\partial z}(0,t) - \frac{\partial H_{z,k}^{-}}{\partial z}(0,t)$$
$$= -\frac{1}{\lambda_k} \iint_{\Omega} \left[\frac{\partial j_x}{\partial x}(x,y,t) + \frac{\partial j_y}{\partial y}(x,y,t)\right] \varphi_k(x,y) dx dy, \qquad (x,y) \in M.$$

Since the first condition is homogeneous, we denote $H_{z,k}^{+}(0,t) = H_{z,k}^{-}(0,t) = H_{z,k}(t)$, and then the new desired function is the solution for the Volterra integral equation of the second kind (for the function $g(h)$ by the formula (18)

$$2H_{z,k}(t) + bt \int_0^t H_{z,k}(\xi) \frac{J'_0(b\sqrt{(\xi - t)\xi})}{\sqrt{(\xi - t)\xi}} d\xi$$
$$= \frac{a}{\lambda_k} \int_0^t J_0(b\sqrt{(\xi - t)\xi}) \iint_M \left[\frac{\partial j_x}{\partial x}(x,y,\xi) + \frac{\partial j_y}{\partial y}(x,y,\xi)\right] \varphi_k(x,y) dx dy d\xi, \qquad (24)$$

where $a = 1/\kappa, b = \sqrt{\lambda_k}/\kappa$.

After calculating the trace of the function $H_{z,k}(t)$ on the waveguide cross section, solving the jump problem is reduced to the recovery of two functions $H_{z,k}^{-}(z,t)$ and $H_{z,k}^{+}(z,t)$ by the following formulas from [16]:

DOI: https://doi.org/10.17993/3ctic.2019.83-2.106-125

$$H_{z,k}^{-}(z,t) = H_{z,k}(t+z/a) + b\frac{z}{a}\int_{0}^{t+z/a} H_{z,k}(\xi)\frac{J'_0\left(b\sqrt{(\xi-t)^2 - z^2/a^2}\right)}{\sqrt{(\xi-t)^2 - z^2/a^2}}d\xi,$$

$$H_{z,k}^{+}(z,t) = H_{z,k}(t-z/a) - b\frac{z}{a}\int_{0}^{t-z/a} H_{z,k}(\xi)\frac{J'_0\left(b\sqrt{(\xi-t)^2 - z^2/a^2}\right)}{\sqrt{(\xi-t)^2 - z^2/a^2}}d\xi.$$

The jump problem for $E_{z,m}(z, t)$ takes the following form:

$$\frac{\partial^2 E_{z,k}^{\pm}}{\partial t^2}(z,t) = \frac{1}{\kappa^2}\frac{\partial^2 E_{z,k}^{\pm}}{\partial z^2}(z,t) - \frac{\chi_k}{\kappa^2}E_{z,k}^{\pm}(z,t),$$

$$E_{z,k}^{+}(0,t) - E_{z,k}^{-}(0,t)$$

$$= -\frac{1}{\varepsilon_0\varepsilon\chi_k}\int_{\tau}\iint_{\Omega}\left[\frac{\partial A_x}{\partial y}(x,y,t) - \frac{\partial A_y}{\partial x}(x,y,t)\right]\psi_k(x,y)dxdy\,d\tau,$$

$$\frac{\partial E_{z,k}^{+}}{\partial z}(0,t) - \frac{\partial E_{z,k}^{-}}{\partial z}(0,t) = 0.$$

In this problem, we have a homogeneous second condition, then $\frac{\partial E_{z,k}^{+}}{\partial z}(0,t) = \frac{\partial E_{z,k}^{-}}{\partial z}(0,t) = \frac{\partial E_{z,k}}{\partial z}(0,t)$ and the limit value of the function $\frac{\partial E_{z,k}}{\partial z}(0,t)$ on the cross section of the waveguide we find as a solution for the Volterra integral equation of the first kind by formula (19):

$$2a\int_{0}^{t}\frac{\partial E_{z,k}}{\partial z}(0,\xi)J_0\left(b\sqrt{(\xi-t)\xi}\right)d\xi$$

$$= \frac{1}{\varepsilon_0\varepsilon\chi_k}\int_{0}^{t}\iint_{M}\left[\frac{\partial j_x}{\partial y}(x,y,\tau) - \frac{\partial j_y}{\partial x}(x,y,\tau)\right]\psi_k(x,y)dxdy\,d\tau$$

$$+\frac{b}{2}\frac{t}{\varepsilon_0\varepsilon\chi_k}\int_{0}^{t}\frac{J'_0\left(b\sqrt{(\xi-t)\xi}\right)}{b\sqrt{(\xi-t)\xi}} \qquad (25)$$

$$\times\int_{0}^{t}\iint_{M}\left[\frac{\partial j_x}{\partial y}(x,y,\tau) - \frac{\partial j_y}{\partial x}(x,y,\tau)\right]\psi_k(x,y)dxdy\,d\tau\,d\xi.$$

The values of the functions $E_{z,k}^{+}(z,t)$ and $E_{z,k}^{-}(z,t)$ in the entire waveguide are determined by the following two formulas from [16]:

$$E^{+}_{z,k}(z,t) = -a \int_{0}^{t-z/a} \frac{\partial E_{z,k}}{\partial z}(0,\xi) J_0\left(b\sqrt{(\xi-t)^2 - z^2/a^2}\right) d\xi,$$
$$E^{-}_{z,k}(z,t) = a \int_{0}^{t-z/a} \frac{\partial E_{z,k}}{\partial z}(0,\xi) J_0\left(b\sqrt{(\xi-t)^2 - z^2/a^2}\right) d\xi.$$

Thus, all components of the electromagnetic field can be expressed through solutions for the Volterra equations (24) and (25).

5. CONCLUSIONS

The problem of the excitation of a cylindrical waveguide by the surface currents on infinitely thin metal plate located in the cross section is considered. The components of the excited electromagnetic field in the waveguide are searched in the form of series in the eigenfunctions of the Laplace operator. The jump problem for searching the unknown coefficients of these series is reduced to solving a system of the Volterra integral equations.

6. ACKNOWLEDGEMENTS

The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

7. REFERENCES

Barybin, A. A. (2007). *Electrodynamics of the waveguide structures. Theory of excitation and wave connection*, Moscow: Fizmatlit. [In Russian]

Bogolyubov, A. N., Delitsyn, A. L., & Sveshnikov, A. G. (2003). Excitation of Waveguides with a Nonhomogeneous Filling. *Computational Mathematics and Modeling*, *14*(1), 64-73. doi: https://doi.org/10.1023/A:1022029918613

DOI: https://doi.org/10.17993/3ctic.2019.83-2.106-125

Collin, R. E. (1960). Field theory of guided waves. New York: John Wiley & Sons, Inc. Retrieved from https://es.scribd.com/doc/183903942/Field-Theory-of-Guided-Waves-Collin-pdf

Eslami, R., & Ahmadi, S. (2019). Investigating the Role of Educational Media on Secondary School Students' Learning Process Improvement in Jahrom City. *Journal of Humanities Insights,* 3(01), 13-6. doi: https://doi.org/10.22034/jhi.2019.80890

Ghaderi, M. A. & Mahdavi Panah, M. (2018). Studying The Importance And Application Of The "Tradition" Viewpoint In The Islamic Law Based On The Viewpoint Of Imam Khomeini (RA) In The Book Of "Tahrir Ol-Wasilla". *UCT Journal of Social Sciences and Humanities Research, 6*(1), 6-8.

Islamov, A. E., Rassolov, I. M., Petunova, S. A., Albov, A. P., Zaikina, I. V., & Shulga, T. I. (2017). Students' Tolerant Behavior Formation Mechanisms. *International Electronic Journal of Mathematics Education, 12*(1), 43-50. Retrieved from https://www.iejme.com/article/students-tolerant-behavior-formation-mechanisms

Jabbari, E., Charbaghi, Z., & Dana A. (2019). Investigating the Effects of Educational and Motivational Education at Different Levels on the Performance and Application of dart throwing. *Journal of Humanities Insights, 3*(02), 37-44. doi: https://doi.org/10.22034/jhi.2019.80896

Kashisaz, S., & Mobaraki, E. (2018). The Effects of Private Education Institutes in Providing Modern Financial Knowledge in Developing Countries. *Journal of Humanities Insights, 02*(04), 172-178. doi: https://doi.org/10.22034/jhi.2018.80887

Kheirabadi, M. A., & Mirzaei, Z. (2019). Descriptive valuation pattern in education and training system: a mixed study. *Journal of Humanities Insights, 3*(01), 7-12. doi: https://doi.org/10.22034/jhi.2019.80889

Kong, J. A. (2002). Electromagnetic Wave Theory. EMW Publishing.

Mittra, R. (1971). Analytical techniques in the theory of guided waves. Macmillan.

Nakhaee, J., & Nasrabadi, M. A. (2019). Strategies for Research-Centered Education of Architectural Designing by Examining the Research-Centered Activities of the Top Universities. *Journal of Humanities Insights, 3*(02), 50-56. doi: https://doi.org/10.22034/jhi.2019.80898

Nikolskij, V. V., & Nikolskaya, T. I. (1989). *Electrodynamics and propagation of radiowaves*, Moscow, Nauka. [In Russian]

Pan, W., & Li, K. (2013). *Propagation of SLF/ELF electromagnetic waves*. Springer Science & Business Media.

Pleshchinskii, N. B. (2019). On Generalized Solutions of Problems of Electromagnetic Wave Diffraction by Screens in the Closed Cylindrical Waveguides. *Lobachevskii Journal of Mathematics*, *40*(2), 201-209. doi: https://doi.org/10.1134/S1995080219020124

Pleshchinskii, N. B., Markina, A. G., & Tumakov, D. N. (2017). On Waveguide Excitation By Source Placed On The Lateral Cross-Section. *Revista Publicando*, *4*(13(2)), 351-364. Retrieved from https://revistapublicando.org/revista/index.php/crv/article/view/891

Sadiku, M. N. (2014). *Elements of electromagnetics*. Oxford university press.

Sailaukyzy, A., Shakuova, R., Sak, K., & Lebedeva, T. (2018). Contemporary view to the history of Kazakhstan's democratic journalism and publicism. *Opción*, *34*(85-2), 774-799.

Samarskii, A. A., & Tikhonov, A. N. (1947). On the excitation of the radio-waveguides. *Zhurnal Tekhnicheskoi Fiziki, 17*(11), 1283. [In Russian]

Samarskii, A. A., & Tikhonov, A. N. (1948). The representation of the field in waveguide in the form of the sum of TE and TM modes. *Zhurnal Tekhnicheskoi Fiziki*, *18*, 971–985. [In Russian]

DOI: https://doi.org/10.17993/3ctic.2019.83-2.106-125

Shapochkin, P. Y., Kapitonov, Y. V., Poltavtsev, S. V., Lozhkin, M. S., Lozhkina, O. A., Manukhova, A. D., ... & Solov'ev, I. A. (2017). A theory of excitation of a planar semiconductor optical waveguide using a diffraction grating: Single-scattering approximation. *Optics and Spectroscopy*, *122*(2), 303-314. doi: https://doi.org/10.1134/S0030400X17020254

Solncev, V. A. (2009). Theory of excitation of the wavegides. *Applied Nonlinear Dynamics*, *17*(3), 55-89.

Sun, T., & Wu, D. (2010). Guided-mode resonance excitation on multimode planar periodic waveguide. *Journal of Applied Physics*, *108*(6). doi: https://doi.org/10.1063/1.3483958

Yirmiyahu, Y., Niv, A., Biener, G., Kleiner, V., & Hasman, E. (2007). Excitation of a single hollow waveguide mode using inhomogeneous anisotropic subwavelength structures. *Optics express*, *15*(20), 13404-13414. doi: https://doi.org/10.1364/OE.15.013404