



Random distribution of topological defects in mesoscopic superconducting samples

Distribución aleatoria de defectos topológicos en muestras superconductoras mesoscópicas

Oscar Silva-Mosquera¹, Omar Yamid Vargas-Ramírez², José José Barba-Ortega^{3*}

¹Licenciado en Física, osilvam@correo.udistrital.edu.co, Orcid: 0000-0001-7168-5009, Universidad Distrital Francisco José de Caldas, Bogotá, Colombia.

²Licenciado en Física, osyvargasr@correo.udistrital.edu.co, Orcid: 0000-0002-7697-3250, Universidad Distrital Francisco José de Caldas, Bogotá, Colombia.

³Doctor en Física, jjbarbao@unal.edu.co, Orcid: 0000-0003-3415-1811, Universidad Nacional de Colombia, Bogotá, Colombia.

How to cite: O. Silva-Mosquera, O.Y. Vargas-Ramírez, J.J. Barba-Ortega, "Random distribution of topological defects in mesoscopic superconducting samples". *Respuestas*, vol. 25, no. 1, pp. 178-183, 2020.

Received on July 6, 2019; Approved on November 17, 2019

ABSTRACT

Keywords:

Superconductor,
Ginzburg-Landau,
Mesoscopic,
Vortices.

In the present work we analyze the effect of topological defects at different temperatures in a mesoscopic superconducting sample in the presence of an applied magnetic field H . The time-dependent Ginzburg-Landau equations are solved with the method of link variables. We study the magnetization curves $M(H)$, number of vortices $N(H)$ and Gibbs $G(H)$ free energy of the sample as a applied magnetic field function. We found that the random distribution of the anchor centers for the temperatures used does not cause strong anchor centers for the vortices, so the configuration of fluxoids in the material is symmetrical due to the well-known Beam-Livingston energy barrier.

RESUMEN

Palabras clave:

Superconductor,
Ginzburg-Landau,
Mesoscópico,
Vórtices.

En el presente trabajo analizamos el efecto de defectos topológicos a diferente temperatura en una muestra superconductoras mesoscópicas en presencia de un campo magnético aplicado H . Se solucionan las ecuaciones Ginzburg-Landau dependientes del tiempo con el método de variables de enlace. Estudiamos las curvas de magnetización $M(H)$, número de vórtices $N(H)$ y energía libre de Gibbs $G(H)$ de la muestra como función del campo magnético aplicado. Encontramos que la distribución aleatoria de los centros de anclaje para las temperaturas utilizadas no origina centros de anclaje fuertes para los vórtices, por lo cual la configuración de los fluxoides en el material es simétrica debido a la bien conocida barrera de energía de Beam-Livingston.

Introduction

The loss of the superconducting state in materials can be caused by external magnetic field strengths above the limit posed by the critical magnetic field H_c , or by the temperature increase exceeding the limit given by the critical temperature T_c or by a current density increase above the limit J_c . It is interesting to note that in the mixed state superconductivity coexists with magnetism minimizing Gibbs free energy, this state occurs in a range given by an external critical magnetic field value H_{c1} (lower critical field) and another H_{c2} (upper critical field). In this range of magnetic field occurs the phenomenon in which quanta of magnetic flux (Lines of force of the external magnetic field) cross the material generating vortices inside it. The increase

of vortices by cause as the external magnetic field increases will cause the decrease of the diamagnetism in the material and consequently of its superconducting state [1] - [3]. A sample is considered mesoscopic if its size is comparable to the coherence length ξ or to the penetration length λ , thermodynamic properties such as vortex configuration or arrangement, in many cases, are specific to the geometrical structure analyzed. Several works have been done on vortex configuration with structural or topological defects in superconducting thin films, finding that superconductor/dielectric boundary conditions lead to the well known Beam-livingston surface barrier, which is responsible for the hysteresis in the magnetization curve. In turn, a superconducting/superconducting interface or temperature gradients increase the transition field H_{c2} [4] - [7]. The effect of

*Corresponding author.

E-mail Address: jjbarbao@unal.edu.co (José José Barba-Ortega)



Peer review is the responsibility of the Universidad Francisco de Paula Santander.
This is an article under the license CC BY-NC-ND 4.0

the extrapolation length of deGennes b on the critical temperature T_c for various sample geometries was studied by Fink et al. They found that the parameter of deGennes can be used to describe a reduction of T_c in small superconductors [8]. Other authors found that T_c can be modified by applying electric field in a reversible way [9] - [11]. To study the effect of topological defects on the superconducting state, we solve the Ginzburg-Landau time-dependent equations (TDGL) in a mesoscopic square where the temperature T is modified locally within the sample as $T(x,y)=\delta\text{Random}$, where δ is a constant that identifies the maximum intensity of the generated numbers and Random is a function that takes random values between 0 and 1, (see references [2] and [7]). The paper is organized as follows: in section 2, we write the dimensionless TDGL equations in an invariant zero gauge form using the auxiliary field in Cartesian coordinates. In Section 3 we present the results of numerical simulations for certain values from δ .

Results and Discussion

In this work, a superconducting square immersed in a magnetic field perpendicular to its surface $H=H_z$, is modeled using the time-dependent Ginzburg Landau theory for the order parameter $\psi(x,y)$ and the potential vector $A(x,y)$ related to the magnetic induction $B=\nabla\times A$. The TDGL equations take the form [12]-[17]:

$$\dot{\psi} = -(i\nabla + A)^2\psi + (1 - T(x,y))\psi(|\psi|^2 - 1) \quad (1)$$

$$\dot{A} = (1 - T(x,y))\text{Re}[\bar{\psi}(-i\nabla - A)\psi] - \kappa^2\nabla\times\nabla\times A \quad (2)$$

We also take a superconductor/vacuum interface: $n\cdot(-i\nabla-A)\psi=0$, where n is the unit vector directed out of the interface. Equations (1) and (2) are written in their non-dimensional form as follows: ψ in units of $(\alpha\beta)(1/2)$, where α and β are two phenomenological parameters of the material, distances in units of the coherence length ξ , times in units of $\pi\hbar/(96\mu K_B)$ T_c , vector potential A in units of $Hc_2 \xi$, where Hc_2 is the second critical field. Magnetization $-4\pi M=B-H=\nabla\times A-H$, where B is the magnetic induction. We simulate a mesoscopic square of lateral size $L_x=L_y=12\xi$, $\kappa=5.0$, typical value for a sample of Pb-In [17].

We studied the influence of topological defects at different temperatures randomly located in the sample of the shape [18]:

$$T(x,y)=\delta * \text{Random} \quad (3)$$

Where Random is a function that generates random numbers between 0 and 1:

$$\text{Case a: } 0\leq T_a\leq\delta; y \delta=0.10 \quad (4)$$

$$\text{Case b: } 0\leq T_b\leq\delta; y \delta=0.25 \quad (5)$$

$$\text{Case c: } 0\leq T_c\leq\delta; y \delta=0.50 \quad (6)$$

Where equation (3) shows the simulated temperature settings in the sample for each case with a maximum amplitude shown in the equations (4,5,6). Figure 1 shows how the magnetization of the material varies as a function of the external magnetic field applied for each temperature range. The blue curve corresponds to case a, the yellow to case b and the red to case c. From here we can observe that H_1 depends on δ , while H_2 remains invariant, in this case H_1 is the penetration field of the first vortex and H_2 the superconducting transition field in the normal state.

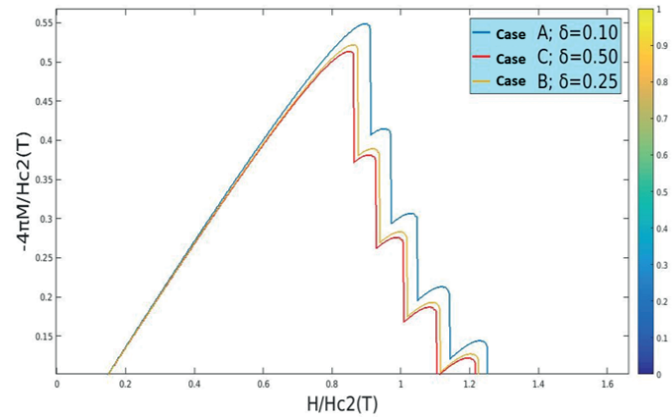


Figure 1. Magnetization as a function of the magnetic field for the three cases studied

In addition to observing a symmetry in the magnetization curves for the three temperature ranges studied (Figure 1) we see a higher $M_{\text{Mayor}}=0.5491$ and $M_{\text{Menor}}\approx 0$ magnetization value for case a, with critical fields $H_1=0.894$, with respect to the other two temperature ranges. The value for the second critical field is given in $H_2=2.418$ where the superconducting state of the material is completely lost. In turn, $H_1=0.848$ and $H_2=2.314$ for case b, and magnetization $M_{\text{Mayor}}=0.5137$ and $M_{\text{Menor}}\approx 0$; and finally for case c we obtain $H_1=0.860$ and $H_2=2.338$ magnetization $M_{\text{Mayor}}=0.5221$ y $M_{\text{Menor}}\approx 0$. In Figures 2, 3 and 4, we plot the core of the vortices.

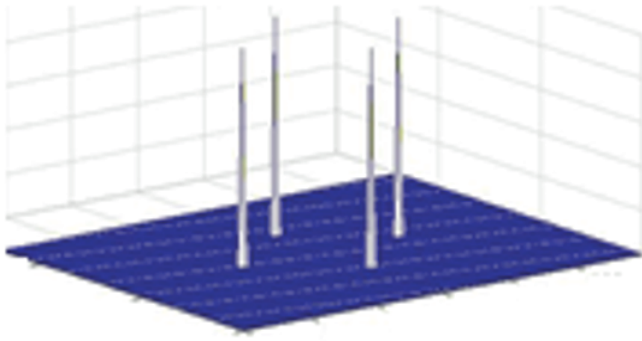


Figure 2. Schematic position of the vortex core for $N = 4$ at $H = 2.394$

It can be seen that the first $N=4$ vortices are entered at an applied field value $H=0.912$ (Figure 2). For the largest number of vortices, $N=44$, a field value of $H=2,394$ is given. As shown, within the range of the critical fields already shown By increasing the magnetic field in which we have $N=44$ vortices, we can see that these begin to move to generate a larger vortex in the center of the sample, which as the value of the applied field increases, it increases in size until finally we have a giant vortex for $H>2,418$ value corresponding to H_2 .

As already mentioned, the increase of the applied external magnetic field causes the material to lose its superconducting state, but before this happens completely there is a smooth transition from the superconducting state to the normal state with an intermediate state where vortices exist.

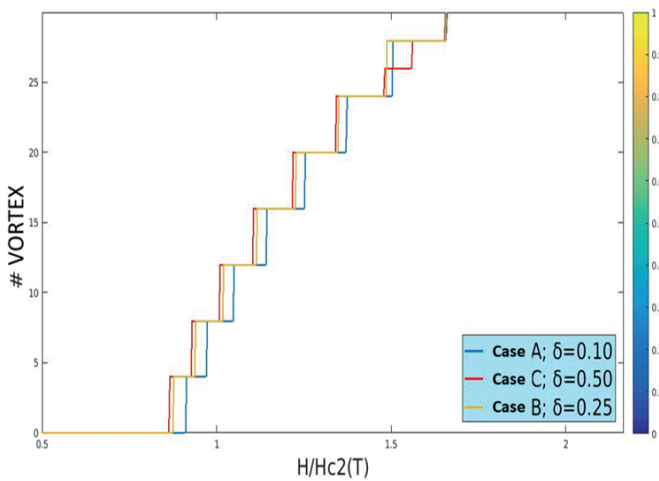


Figure 3. Number of vortices (N) as a function of the applied magnetic field (H) for the cases indicated.

Figure 2 shows the core of the vortices for a stationary co-figuration with $N=4$. Figure 3 shows the vorticity as a function of the magnetic field applied for each case studied. We can see that the number of vortices increases proportionally to the applied field in regular transitions from N to $N+4$.

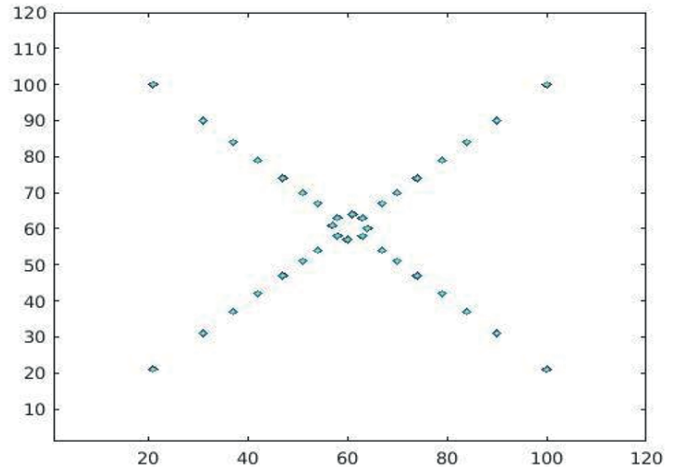


Figure 4. Distribution of vortex centres towards high magnetic fields

In Figure 4, the symmetry with which the vortices are distributed can be seen, showing a fairly regular and equal shape for the three cases of temperature studied. This indicates that the anchoring force of the topological defects is low compared to the magnetic pressure. The supercurrent density is of great relevance in our study, since as we said before the random distribution of the vortices depends on this one, besides that in the Ginzburg-Landau theory we have as a range J_{c1} to J_{c2} , in which the value of this one should be neither below nor above the limit values that these establish, since if this way it happens the superconducting state in the sample will disappear. Below are the supercurrent densities for some cases of vortex generation (Figure 5). This figure shows how the supercurrent flows swirl around the vortices entering the sample. These flows generate by electromagnetic induction magnetic fields that make the vortices repel each other generating the already mentioned symmetry.

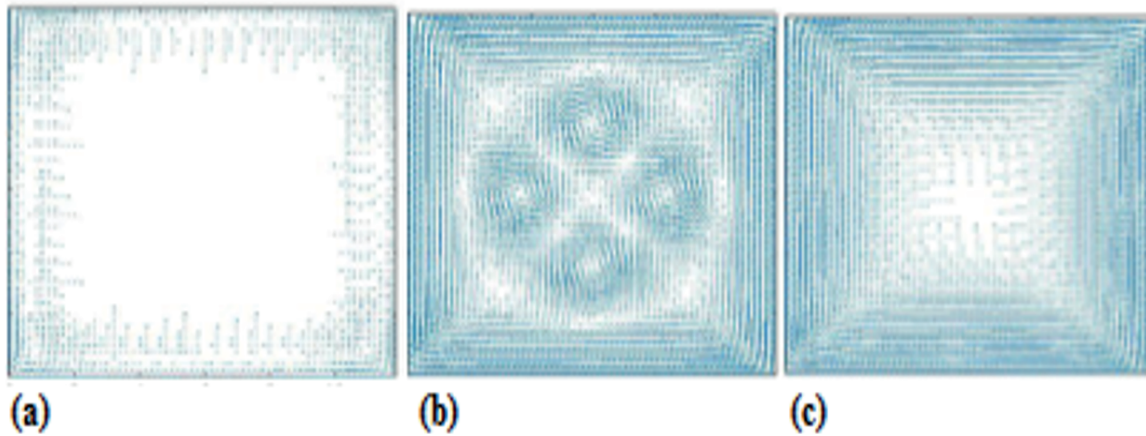


Figure 5. Supercurrent Density for a) $N = 0$, b) $N=4$, and c) $N= 30$.

Perhaps the most important concept in the Ginzburg-Landau theory is that of the internal energy change of the system in the phase transitions of the normal-superconducting states and vice versa. In Figure 6, we plot the Gibbs energy change of the system ($\Delta G = G_n - G_s$) as a function of the applied external magnetic field strength. We see positive and negative values in this graph when the applied magnetic field strength increases. The positive parts ($\Delta G > 0, G_s < G_n$) of the energy show the system in a normal non-conventional state, while ($\Delta G < 0, G_s > G_n$) are typical energy values of the superconducting state. The vortex states in the sample are included in the negative energy values, as well as the Meissner state. According to the Ginzburg-Landau theory, the Gibbs G free energy in superconducting systems is a serial expansion of the order parameter ψ , a variable describing the supercondensity of load carriers, this can be visualized in figure 7, for some data taken from case a.

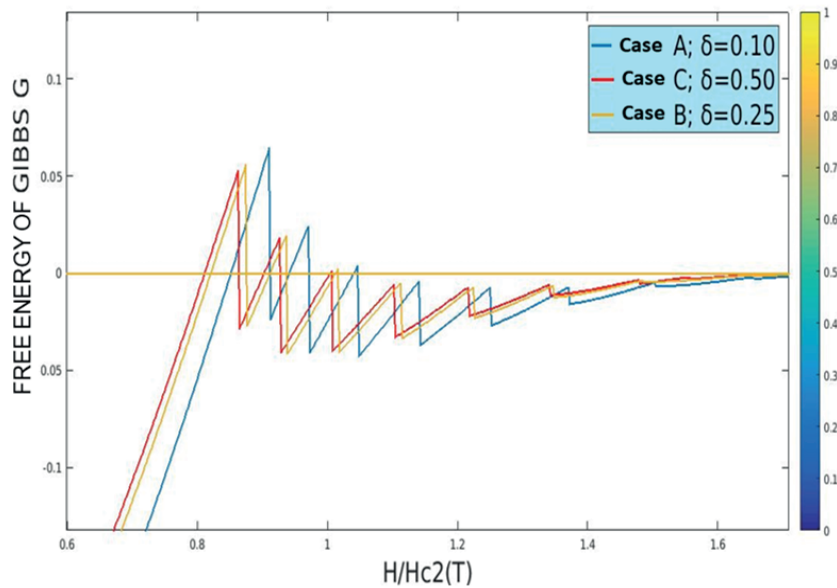


Figure 6. Gibbs free energy as a function of the magnetic field for the cases studied

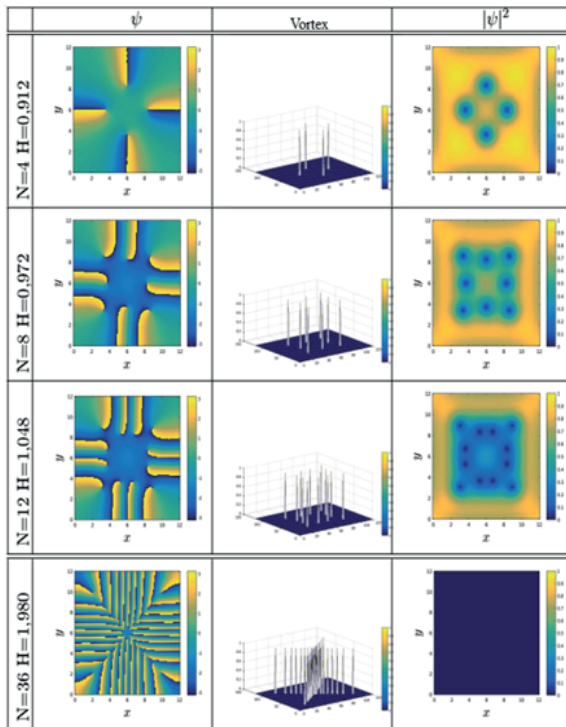


Figure 7. (Left) Phase of the order parameter, (center) vortex core and (right) charge carrier density for the indicated magnetic field and vorticity values.

Conclusion

Solving the Ginzburg-Landau time-dependent equations, we find that the magnetization of the system decreases as the temperature range increases, i.e. the larger temperature range has a lower magnetization value. For case a there was a maximum number of vortices, $N=44$, in the form of an X for an applied magnetic field strength greater than $H>2.3$. From this value the material begins with the loss of the superconducting state decreasing its value in vortices, but generating a larger vortex in its central part that grows until it finally occupies the whole sample and its superconducting state disappears completely. It was possible to corroborate the superconducting phenomenon and its different characteristics, such as the magnetization and the free energy of Gibbs, among others, according to the works and texts studied previously

References

- [1] M. Thinkham, “Introduction to Superconductivity”, 2nd ed. (McGraw-Hill, New York, 1996.
- [2] E. D. V. Niño, A. Díaz-Lantada and J. Barba-Ortega, “Vortex matter in a superconducting square under 2D thermal gradient”, *Journal of Low Temperature Physics*, vol. 195, pp. 202-210, 2019.
- [3] L. Komendova, M. V. Milošević, A. A. Shanenko and F. M. Peeters, “Different length scales for order parameters in two-gap superconductors: Extended Ginzburg-Landau theory”, *Physical Review B*, vol. 84, no. 064522, pp.1-5, 2011
- [4] R. Fazio, V. F. Gantmakher and Y. Imry, “New Directions in Mesoscopic Physics”, edited by (SpringerVerlag, Berlin), 2003.
- [5] J. Barba-Ortega and M. R. Joya, “Nucleación de vórtices y antivórtices en películas superconductoras con nanoestructuras magnéticas”, *Revista Respuestas*, vol. 16, no. 1, pp. 45-49, 2011.
- [6] C. A. Aguirre, E. Sardella and J. Barba-Ortega, “Length dependence of the number of phase slip lines in a superconducting strip”, *Solid State Communications*, vol. 306, no. 113799, pp. 1-7, 2020.
- [7] J. Barba-Ortega, H. M. Carrillo and C.A. Sjögreen-Blanco “Unidimensional thermal gradients T^n in a nanoscopic superconductor”, *Physica C: Superconductivity*, vol. 525, pp. 61-64, 2016.
- [8] T. N. Jorge, C. A. Aguirre, A. S. de Arruda and J. Barba-Ortega, “Two-band superconducting square with a central defect: Role of the deGennes extrapolation length”, *European Journal of Physics B*, vol. 93, no. 69, pp.1-7, 2020.
- [9] P. G. deGennes and J. Matricon, “Collective modes of vortex lines in superconductors of the second kind”, *Review Modern Physics*, vol. 36, no. 1, pp. 45-49, 1964.

- [10] W. D. Lee, J. L. Chen, T. J. Yang and B. Chioua, “Influence of an external electric field on high-Tc superconductivity”, *Physica C: Superconductivity*, vol. 261, no.1, pp. 167-172, 1996.
- [11] N. V. Orlova, A. A. Shanenko, M. V. Milošević, F. M. Peeters, A. V. Vagov and V. M. Axt, “Ginzburg-Landau theory for multiband superconductors: microscopic derivation” *Physical Review B*, vol. 87, no. 134510, pp. 1-8, 2013.
- [12] C. A. Aguirre, Q. Martins and J. Barba-Ortega, “Analytical development of Ginzburg-Landau equations for superconducting thin film in presence of currents”, *Revista UIS Ingeniería*, vol. 18, no. 2, pp. 213-220, 2019.
- [13] Q. Du, “Numerical approximations of the Ginzburg-Landau models for superconductivity”, *Journal of Mathematical Physics*, vol. 46, no. 095109, pp.1-7, 2005.
- [14] O. Vargas-Ramírez, O. Silva-Mosquera and J. Barba-Ortega, “Respuesta magnética de un superconductor inmerso en un campo de calor”, *Aibi Revista de Investigación, Administración e Ingeniería*, Vol. 8, no. 1, pp. 86-90, 2020.
- [15] G. C. Buscaglia, C. Bolech and A. Lopez, “Connectivity and Superconductivity”, (Eds.), *Springer*, 2000.
- [16] M. V. Milošević and R. Geurts, “The Ginzburg–Landau theory in application”, *Physica C, Physica C: Superconductivity*, vol. 470, no. 19, pp. 791-795, 2010.
- [17] A. I. Gubin, K. S. Il'ĭn, S. A. Vitusevich, M. Siegel and N. Klein, “Dependence of magnetic penetration depth on the thickness of superconducting *Nb thin films*” *Physical Review B*, vol. 72, no. 064503, pp. 1-6, 2005.
- [18] E. C. S Duarte, A. Presotto, D. Okimoto, V. S Souto, E. Sardella and R. Zadorosny, “Use of thermal gradients for control of vortex matter in mesoscopic superconductors”, *Journal of Physics: Condensed Matter*, vol. 31, no. 405901, pp. 1-8, 2019.