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MULTI-ITEM A SUPPLY CHAIN PRODUCTION INVENTORY MODEL OF TIME DEPENDENT PRODUCTION RATE AND DEMAND RATE UNDER SPACE CONSTRAINT IN FUZZY ENVIRONMENT

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ABSTRACT

In this paper, we have developed an integrated production inventory model for two echelon supply chain consisting of one vendor and one retailer. Production rate and demand rate of retailer and customer are time dependent. Idle time cost of the vendor has been considered. Multi-item inventory has been considered. In integrated inventory model average cost has been calculated under limitation on stroge space. Two echelon supply chain fuzzy inventory model has been solved by various techniques like as Fuzzy programming technique with hyperbolic membership functions (FPTHMF), Fuzzy non-linear programming technique (FNLP) and Fuzzy additive goal programming technique (FAGP), weighted Fuzzy non-linear programming technique (WFNLP) and weighted Fuzzy additive goal programming technique (WFAGP). A numerical example is illustrated to test the model. Finally to make the model more realistic, sensitivity analysis has been shown.

Keywords: Inventory, Supply Chain, Production, Multi-item, Fuzzy Technique

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1. INTRODUCTION

A Supply Chain inventory model deal with decision that minimum the total average cost or maximum the total average profit. In that way to construct a real life mathematical inventory model on base on various assumptions and notations and approximations. Supply Chain management has taken a very important and critical role for any company in the increase globalization and competition in the business.

The success of any supply chain system in any business depends on its level of integration. Idle time cost is the very important function or role in the supply chain inventory model. In the real field inventory costs like as raw material holding cost, finished goods holding cost, production cost etc. are not always fixed. Therefore consideration of fuzzy number for all cost parameters is more realistic and practical the model.

In the real life business transaction the demand rate of any product is always varying. Several inventory models have been established by considering time-dependent demand. Silver and Meal (1969) first established the inventory for the case of a varying demand. Dave and Patel (1981) developed inventory models for deteriorating items with time-proportional demand. Sana and Chaudhuri (2004) presented inventory model with time dependent demand for of deteriorating items.

Tripathi, Kaur And Pareek (2016) considered a model on inventory model with exponential time-dependent demand rate, variable deterioration, shortages and production cost. Chung and Ting (1994) formulated a model on replenishment schedule for deteriorating items with time-proportional demand. Lee and Ma (2000) studied on optimal inventory policy for deteriorating items with two-warehouse and time-dependent demands. Tripathi and Kaur (2018) discussed on a linear time-dependent deteriorating inventory model with linearly time-dependent demand rate and inflation.

Multi items and limitations of space are the very important part in the business world. Cárdenas-Barrón, Sana (2015), established multi-item EOQ inventory model in a two-layer supply chain while demand varies with promotional effort. Islam and Roy (2010) studied on multi-objective geometric-programming problem and its application. Islam and Mandal(2017) presented a fuzzy inventory model with unit production cost, time depended holding Cost, with-out shortages under a space constraint: a parametric geometric programming approach. Islam(2008) developed multi-objective marketing planning inventory model. Islam and Roy (2006) established a fuzzy EPQ model with flexibility and reliability consideration and demand

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depended unit Production cost under a space constraint: A fuzzy geometric programming approach.

Two echelon supply chain inventory model and it in fuzzy environment is very interesting. To solve supply chain inventory types of problems fuzzy set theory are used. The fuzzy set theory was introduced by Zadeh (1965). Afterward Zimmermann (1985) applied the fuzzy set theory concept with some useful membership functions to solve the linear programming problems with some objective functions. Wang and Shu (2005) established fuzzy decision modeling for supply chain management. Islam and Mandal (2019) have written a book on fuzzy geometric programming techniques and applications. Cárdenas-Barrón and Sana (2014) developed a production inventory model for a two echelon supply chain when demand is dependent on sales teams' initiatives.

Yang (2006) considered a two-echelon inventory model with fuzzy annual demand in a supply chain. Sana (2010) presented a paper on a collaborating inventory model in a supply chain. Thangam and Uthayakumar (2009) formulated on two-echelon trade credit financing for perishable items in a supply chain when demand depends on both selling price and credit period. Lee and Wu (2006) established a study on inventory replenishment policies in a two-echelon supply chain system.

In this article, consider an integrated production inventory model for a two echelon supply chain consisting of one vendor and another retailer. Production rate of the vendor and demand rate of retailer and customer are assumed dependent on time. Idle time cost of the vendor has been considered. Multi-item inventory has been considered under space constraint.

Two echelon supply chain fuzzy inventory model has been formulated due to uncertainty of the cost parameters and solve by various techniques like as Fuzzy programming technique with hyperbolic membership functions (FPTHMF), Fuzzy non-linear programming technique (FNLP) and Fuzzy additive goal programming technique (FAGP), weighted Fuzzy non-linear programming technique (WFNLP) and weighted Fuzzy additive goal programming technique (WFAGP). Numerical example and sensitivity analysis has been shown to illustrate the proposed two echelon supply chain inventory model.

2. MODEL FORMULATION

2.1. Notations

$P_{vi}(t)$: Production rate at time t for the vendor of the i^th item.

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$D_{ri}(t)$: Demand rate at time t for retailer of the ith item.

$D_{ci}(t)$: Demand rate per unit time for customer of the ith item.

$I_{vi}(t)$: Vendor inventory level of the ith item at time t, during the production time.

$I'_{vi}(t)$: Vendor inventory level of the ith item at time t, during the non-production time.

$I_{ri}(t)$: Retailer inventory level of the ith item at time t, during the period of the vendor.

$I'_{ri}(t)$: Retailer inventory level of the ith item at time t, during the idle time of the vendor.

t_1: Production period of the vendor ($t_1 > 0$). (Decision variable)

t_2: Period of the vendor ($t_1 < t_2$). (Decision variable)

T: The length of cycle time of the supply chain inventory model ($t_1 < t_2 < T$). (Decision variable)

h_{vi}: Holding cost per unit per unit time for the vendor of the ith item.

h_{ri}: Holding cost per unit per unit time for the retailer of the ith item.

p_{vi}: Inventory production cost per unit item of the ith item.

Q_i: The production quantity for the duration of a cycle of length T for ith item.

A_{vi}: Set-up cost per order of ith item for the vendor.

A_{ri}: Set-up cost per order of ith item for the retailer.

i_v: Cost per unit idle time of the vendor.

W_i: Storage space per unit for the ith item.

W: Total storage space for all items.

TC_{vi}: Total cost for the vendor of the ith item.

TC_{ri}: Total cost for the retailer of the ith item.

$TAC_i(t_1, t_2, T)$: Joint total average cost of the ith item.

$\widetilde{h_{vi}}$: Fuzzy holding cost per unit per unit time for the vendor of the ith item.

$\widetilde{h_{ri}}$: Fuzzy holding cost per unit per unit time for the retailer of the ith item.

$\widetilde{p_{vi}}$: Fuzzy inventory production cost per unit item of the ith item.

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$\widetilde{A_{vi}}$: Fuzzy set-up cost per order of i^th item for the vendor.

$\widetilde{A_{ri}}$: Fuzzy set-up cost per order of i^th item for the retailer.

$\widetilde{\iota_v}$: Fuzzy cost per unit idle time of the vendor.

$TAC_i\widetilde{(t_1, t_2, T)}$: Joint total fuzzy average cost of the i^th item.

2.2. Assumptions

1. The inventory system is developed for multi item.

2. The replenishment occurs instantaneously at infinite rate.

3. The lead time is negligible.

4. Shortages are not allowed.

5. The cost of idle times in the supply chain inventory model has been considered.

6. The Production rate $P_{vi}(t)$ at time t for the vendor of the i^th item is considered as

$P_{vi}(t) = a_i e^{b_i t}$. Where a_i and $b_i$are the positive constant real numbers.

7. The demand rate $D_{ri}(t)$ at time t of retailer of the i^th item is considered as $D_{ri}(t) = \alpha_i t^{\beta_i - 1}$. Where α_i and β_i are constant real numbers with $\alpha_i > 0, \beta_i > 1$.

8. The demand rate $D_{ci}(t)$ at time t of the customer of the i^th item is considered as $D_{ci}(t) = \gamma_i + \delta_i t$. Where γ_i and $\delta_i$are the positive constant.

9. Deteriorations are not allowed.

2.3. Model formation in crisp of ith item

2.3.1. The vendor individual inventory model:

In the proposed model, in this section, we have developed the mathematical inventory model for the manufacturer. Here the production starts from the time $t = 0$ with the production rate $P_{vi}(t)$. Production occurs during the time interval $[0, t_1]$. After production during the time interval $[t_1, t_2]$ the inventory depletes due to only demand of the retailer.

Then governing differential equations of the inventory system at the time t are given follows:

$$\frac{dI_{vi}(t)}{dt} = a_i e^{b_i t} - \alpha_i t^{\beta_i - 1}, \text{for } 0 \le t \le t_1 \qquad (1)$$

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$\frac{dI'_{vi}(t)}{dt} = -\alpha_i t^{\beta_i - 1}$, for $t_1 \leq t \leq t_2$ (2)

With boundary condition, $I_{vi}(0) = 0$, $I_{vi}(t_2) = 0$. (3)

And $I_{vi}(t_1) = I'_{vi}(t_1)$ (4)

Solving the above differential equation (1) and (2) we get

$I_{vi}(t) = \frac{a_i}{b_i}(e^{b_i t} - 1) - \frac{\alpha_i}{\beta_i} t^{\beta_i}$, for $0 \leq t \leq t_1$ (5)

$I'_{vi}(t) = \frac{\alpha_i}{\beta_i}\left(t_2^{\beta_i} - t^{\beta_i}\right)$, for $t_1 \leq t \leq t_2$ (6)

Using continuity condition (4) we have

$t_2^{\beta_i} = \frac{a_i \beta_i}{b_i \alpha_i}(e^{b_i t_1} - 1)$ (7)

And $Q_i = \frac{a_i}{b_i}(e^{b_i t_1} - 1)$

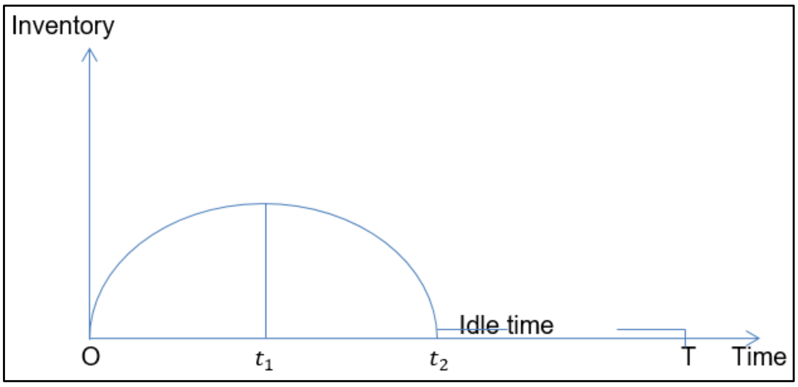

Figure 1: Inventory level of the vendor

Now calculating the various cost as following

i) Set-up-cost per cycle $= A_{vi}$

ii) The inventory holding cost per cycle $= h_{vi}\int_0^{t_1} I_{vi}(t)dt + h_{vi}\int_{t_1}^{t_2} I'_{vi}(t)dt$

$$= h_{vi}\left[\frac{a_i}{b_i}\left\{\frac{1}{b_i}(e^{b_i t_1} - 1) - t_1\right\} + \frac{\alpha_i}{\beta_i + 1} t_2^{\beta_i + 1} - \frac{\alpha_i}{\beta_i} t_2^{\beta_i} t_1\right]$$

iii) The inventory production cost $= p_{vi}\int_0^{t_1} a_i e^{b_i t} dt$

$$= p_{vi}\frac{a_i}{b_i}(e^{b_i t_1} - 1)$$

iv) The vendor idle time cost $= i_v(T - t_2)$

Therefore the vendor total cost is

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$$TC_{vi}(t_1,t_2,T) = \text{< set-up-cost >} + \text{< holding cost >} + \text{< production cost >} + \text{< idle time cost >}$$

$$TC_{vi}(t_1,t_2,T) = A_{vi} + h_{vi}\left[\frac{\alpha_i}{b_i}\left\{\frac{1}{b_i}(e^{b_i t_1}-1)-t_1\right\}+\frac{\alpha_i}{\beta_i+1}t_2^{\beta_i+1}-\frac{\alpha_i}{\beta_i}t_2^{\beta_i}t_1\right]+ p_{vi}\frac{\alpha_i}{b_i}(e^{b_i t_1}-1)+i_v(T-t_2) \quad (8)$$

2.3.2. *The retailer individual inventory model:*

The governing differential equations of the inventory system at the time t are given follows:

$$\frac{dI_{ri}(t)}{dt} = \alpha_i t^{\beta_i-1}-(\gamma_i+\delta_i t), \text{ for } 0 \le t \le t_2 \quad (9)$$

$$\frac{dI'_{ri}(t)}{dt} = -(\gamma_i+\delta_i t), \text{ for } t_2 \le t \le T \quad (10)$$

With boundary condition, $I_{ri}(0) = 0,\ I_{ri}(T) = 0.$ (11)

And $I_{ri}(t_2) = I'_{ri}(t_2)$ (12)

Solving the above differential equation (9) and (10), using boundary conditions, we get

$$I_{ri}(t) = \frac{\alpha_i}{\beta_i}t^{\beta_i}-\left(\gamma_i t+\frac{\delta_i}{2}t^2\right), \text{ for } 0 \le t \le t_2 \quad (13)$$

$$I'_{ri}(t) = \gamma_i(T-t)+\frac{\delta_i}{2}(T^2-t^2), \text{ for } t_2 \le t \le T \quad (14)$$

Using continuity condition (12) we have

$$\frac{\alpha_i}{\beta_i}t_2^{\beta_i} = \gamma_i T+\frac{\delta_i}{2}T^2 \quad (15)$$

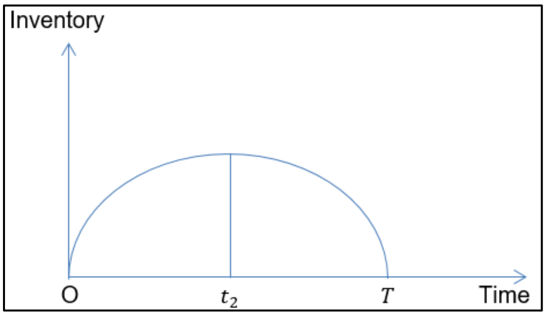

Figure 2: Inventory level of the retailer

Now calculating the various cost as following

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i) Set-up-cost per cycle $= A_{ri}$

ii) The inventory holding cost per cycle $= h_{ri}\int_0^{t_2} I_{ri}(t)dt + h_{ri}\int_{t_2}^{T} I'_{ri}(t)dt$

$$= h_{ri}\left[\frac{\alpha_i}{\beta_i(\beta_i+1)}t_2^{\beta_i+1} + \frac{\gamma_i T}{2}(T-2t_2) + \frac{\delta_i T^2}{6}(2T-3t_2)\right]$$

Therefore the vendor total cost is

$TC_{ri}(t_1,t_2,T) = <\text{ set} - \text{up} - \text{cost} > + < \text{ holding cost } >$

$$TC_{ri}(t_1,t_2,T) = A_{ri} + h_{ri}\left[\frac{\alpha_i}{\beta_i(\beta_i+1)}t_2^{\beta_i+1} + \frac{\gamma_i T}{2}(T-2t_2) + \frac{\delta_i T^2}{6}(2T-3t_2)\right] \quad (16)$$

2.3.3. The integrated inventory model

Therefore the total average cost for i[th] item in integrated model is

$$TAC_i(t_1,t_2,T) = \frac{1}{T}\left(TC_{vi}(t_1,t_2,T) + TC_{ri}(t_1,t_2,T)\right)$$

$$TAC_i(t_1,t_2,T) = \frac{1}{T}\left[A_{vi} + h_{vi}\left\{\frac{a_i}{b_i}\left(\frac{1}{b_i}(e^{b_i t_1}-1) - t_1\right) + \frac{\alpha_i}{\beta_i+1}t_2^{\beta_i+1} - \frac{\alpha_i}{\beta_i}t_2^{\beta_i}t_1\right\} + p_{vi}\frac{a_i}{b_i}(e^{b_i t_1}-1) + i_v(T-t_2) + A_{ri} + h_{ri}\left\{\frac{\alpha_i}{\beta_i(\beta_i+1)}t_2^{\beta_i+1} + \frac{\gamma_i T}{2}(T-2t_2) + \frac{\delta_i T^2}{6}(2T-3t_2)\right\}\right] \quad (17)$$

For $i = 1,2,3,\dots,n$

Therefore, the multi objective inventory model is Minimize

$$\begin{aligned} TAC_i(t_1,t_2,T) = \frac{1}{T}\Bigg[&A_{vi} + h_{vi}\left\{\frac{a_i}{b_i}\left(\frac{1}{b_i}(e^{b_i t_1}-1) - t_1\right) + \frac{\alpha_i}{\beta_i+1}t_2^{\beta_i+1} - \frac{\alpha_i}{\beta_i}t_2^{\beta_i}t_1\right\} \\ &+ p_{vi}\frac{a_i}{b_i}\left(e^{b_i t_1}-1\right) + i_v(T-t_2) + A_{ri} \\ &+ h_{ri}\left\{\frac{\alpha_i}{\beta_i(\beta_i+1)}t_2^{\beta_i+1} + \frac{\gamma_i T}{2}(T-2t_2) + \frac{\delta_i T^2}{6}(2T-3t_2)\right\}\Bigg] \end{aligned}$$

Subject to

$$\frac{a_i}{b_i}(e^{b_i t_1}-1) = \gamma_i T + \frac{\delta_i}{2}T^2$$

$$\sum_1^n W_i Q_i \le W \text{ i.e } \sum_1^n W_i\frac{a_i}{b_i}(e^{b_i t_1}-1) \le W \quad (18)$$

For $i = 1,2,3,\dots,n$

3. FUZZY MODEL:

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Normally the parameters for ordering cost, holding cost, production cost and idle time cost are not particularly known to us. Due to uncertainty, we assume all the parameters $(A_{vi}, A_{ri}, \alpha_i, \beta_i, a_i, b_i, h_{vi}, h_{ri}, p_{vi}, i_v, \gamma_i, \delta_i)$ as generalized trapezoidal fuzzy number (GTrFN) $(\widetilde{A_{vi}}, \widetilde{A_{ri}}, \widetilde{\alpha_i}, \widetilde{\beta_i}, \widetilde{a_i}, \widetilde{b_i}, \widetilde{h_{vi}}, \widetilde{h_{ri}}, \widetilde{p_{vi}}, \widetilde{i_v}, \widetilde{\gamma_i}, \widetilde{\delta_i})$. Let us take,

$\widetilde{A_{vi}} = \left(A_{vi}{}^1, A_{vi}{}^2, A_{vi}{}^3, A_{vi}{}^4; \omega_{A_{vi}}\right), 0 < \omega_{A_{vi}} \le 1;\ \widetilde{\alpha_i} = \left(\alpha_i{}^1, \alpha_i{}^2, \alpha_i{}^3, \alpha_i{}^4; \omega_{\alpha_i}\right), 0 < \omega_{\alpha_i} \le 1$

$\widetilde{\beta_i} = \left(\beta_i{}^1, \beta_i{}^2, \beta_i{}^3, \beta_i{}^4; \omega_{\beta_i}\right), 0 < \omega_{\beta_i} \le 1;\ \widetilde{a_i} = \left(a_i{}^1, a_i{}^2, a_i{}^3, a_i{}^4; \omega_{a_i}\right), 0 < \omega_{a_i} \le 1;$

$\widetilde{b_i} = \left(b_i{}^1, b_i{}^2, b_i{}^3, b_i{}^4; \omega_{b_i}\right), 0 < \omega_{b_i} \le 1;\ \widetilde{h_{vi}} = \left(h_{vi}{}^1, h_{vi}{}^2, h_{vi}{}^3, h_{vi}{}^4; \omega_{h_{vi}}\right), 0 < \omega_{h_{vi}} \le 1$

$\widetilde{h_{ri}} = \left(h_{ri}{}^1, h_{ri}{}^2, h_{ri}{}^3, h_{ri}{}^4; \omega_{h_{ri}}\right), 0 < \omega_{h_{ri}} \le 1; \widetilde{\gamma_i} = \left(\gamma_i{}^1, \gamma_i{}^2, \gamma_i{}^3, \gamma_i{}^4; \omega_{\gamma_i}\right), 0 < \omega_{\gamma_i} \le 1;$

$\widetilde{p_{vi}} = \left(p_{vi}{}^1, p_{vi}{}^2, p_{vi}{}^3, p_{vi}{}^4; \omega_{p_{vi}}\right), 0 < \omega_{p_{vi}} \le 1; \widetilde{\delta_i} = \left(\delta_i{}^1, \delta_i{}^2, \delta_i{}^3, \delta_i{}^4; \omega_{\delta_i}\right), 0 < \omega_{\delta_i} \le 1$

$\widetilde{A_{ri}} = \left(A_{ri}{}^1, A_{ri}{}^2, A_{ri}{}^3, A_{ri}{}^4; \omega_{A_{ri}}\right), 0 < \omega_{A_{ri}} \le 1;\ \widetilde{i_v} = \left(i_v{}^1, i_v{}^2, i_v{}^3, i_v{}^4; \omega_{i_v}\right), 0 < \omega_{i_v} \le 1$

$(\,for\ i = 1,2, \dots\dots\dots, n).$

Then the above crisp inventory model (18) reduces to the fuzzy model as Minimize

$$TAC_i\widetilde{(t_1, t_2, T)} = \frac{1}{T}\Bigg[\widetilde{A_{vi}} + \widetilde{h_{vi}}\left\{\frac{\widetilde{\alpha_i}}{\widetilde{b_i}}\left(\frac{1}{\widetilde{b_i}}\left(e^{\widetilde{b_i}t_1} - 1\right) - t_1\right) + \frac{\widetilde{\alpha_i}}{\widetilde{\beta_i} + 1} t_2^{\widetilde{\beta_i}+1} - \frac{\widetilde{\alpha_i}}{\widetilde{\beta_i}} t_2^{\widetilde{\beta_i}} t_1\right\}$$
$$+ \widetilde{p_{vi}}\frac{\widetilde{\alpha_i}}{\widetilde{b_i}}\left(e^{\widetilde{b_i}t_1} - 1\right) + \widetilde{i_v}(T - t_2) + \widetilde{A_{ri}}$$
$$+ \widetilde{h_{ri}}\left\{\frac{\widetilde{\alpha_i}}{\widetilde{\beta_i}(\widetilde{\beta_i} + 1)} t_2^{\widetilde{\beta_i}+1} + \frac{\widetilde{\gamma_i}T}{2}(T - 2t_2) + \frac{\widetilde{\delta_i}T^2}{6}(2T - 3t_2)\right\}\Bigg]$$

Subject to

$$\frac{\widetilde{\alpha_i}}{\widetilde{b_i}}\left(e^{\widetilde{b_i}t_1} - 1\right) = \widetilde{\gamma_i}T + \frac{\widetilde{\delta_i}}{2}T^2$$

$$\sum_1^n W_i \frac{\widetilde{\alpha_i}}{\widetilde{b_i}}\left(e^{\widetilde{b_i}t_1} - 1\right) \le W \tag{19}$$

For $i = 1,2,3, \dots, n$

In defuzzification of fuzzy number technique, if we consider a GTrFN $\tilde{A} = (a, b, c, d; \omega)$, then the total λ- integer value of $\tilde{A} = (a, b, c, d; \omega)$ is $I_\lambda^w\left(\tilde{A}\right) = \lambda\omega\frac{c+d}{2} + (1 - \lambda)\omega\frac{a+b}{2}$

Taking $\lambda = 0.5$, therefore we get approximated value of a GTrFN $\tilde{A} = (a, b, c, d; \omega)$ is

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$\omega\left(\frac{a+b+c+d}{4}\right)$. Therefore using approximated value of GTrFN, we have the approximated values $(\widehat{A_{v\iota}}, \widehat{A_{r\iota}}, \widehat{\alpha_\iota}, \widehat{\beta_\iota}, \widehat{a_\iota}, \widehat{b_\iota}, \widehat{h_{v\iota}}, \widehat{h_{r\iota}}, \widehat{p_{v\iota}}, \widehat{\iota_v}, \widehat{\gamma_\iota}, \widehat{\delta_\iota})$ of the GTrFN parameters $(\widetilde{A_{v\iota}}, \widetilde{A_{r\iota}}, \widetilde{\alpha_\iota}, \widetilde{\beta_\iota}, \widetilde{a_\iota}, \widetilde{b_\iota}, \widetilde{h_{v\iota}}, \widetilde{h_{r\iota}}, \widetilde{p_{v\iota}}, \widetilde{\iota_v}, \widetilde{\gamma_\iota}, \widetilde{\delta_\iota})$. So the above model (19) reduces to multi objective supply chain inventory model (MOSCIM) as Minimize

$$TAC_\iota\widehat{(t_1, t_2, T)} = \frac{1}{T}\Bigg[\widehat{A_{v\iota}} + \widehat{h_{v\iota}}\left\{\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(\frac{1}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) - t_1\right) + \frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}+1}t_2^{\widehat{\beta_\iota}+1} - \frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}}t_2^{\widehat{\beta_\iota}}t_1\right\}$$
$$+ \widehat{p_{v\iota}}\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) + \widehat{\iota_v}(T - t_2) + \widehat{A_{r\iota}}$$
$$+ \widehat{h_{r\iota}}\left\{\frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}(\widehat{\beta_\iota}+1)}t_2^{\widehat{\beta_\iota}+1} + \frac{\widehat{\gamma_\iota}T}{2}(T - 2t_2) + \frac{\widehat{\delta_\iota}T^2}{6}(2T - 3t_2)\right\}\Bigg]$$

Subject to

$$\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) = \widehat{\gamma_\iota}T + \frac{\widehat{\delta_\iota}}{2}T^2$$

$$\sum_1^n W_i\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) \le W \qquad (20)$$

For $i = 1,2,3, \ldots, n$ we get n objectives.

4. FUZZY PROGRAMMING TECHNIQUE (MULTI-OBJECTIVE ON MAX-MIN AND ADDITIVE OPERATORS)

Solve the MOSCIM (20) as a single objective NLP using only one objective at a time and we ignoring the all others. Repeat the process n times for n different objective functions. So we get the ideal solutions. From the above results, we find out the corresponding values of every objective function at each solution obtained. With these values the pay-off matrix can be prepared as follows:

$$\begin{array}{cc} & \begin{array}{cccc} TAC_1(t_1,t_2,T) & TAC_2(t_1,t_2,T) & \ldots\ldots\ldots & TAC_n(t_1,t_2,T) \end{array} \\ \begin{array}{c} (t_1{}^1,t_2^1,T^1) \\ (t_1{}^2,t_2^2,T^2) \\ \\ \\ (t_1{}^n,t_2^n,T^n) \end{array} & \begin{pmatrix} TAC_1(t_1{}^1,t_2^1,T^1) & TAC_2(t_1{}^1,t_2^1,T^1) & \ldots\ldots & TAC_n(t_1{}^1,t_2^1,T^1) \\ TAC_1(t_1{}^2,t_2^2,T^2) & TAC_2(t_1{}^2,t_2^2,T^2) & \ldots\ldots & TAC_n(t_1{}^2,t_2^2,T^2) \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ TAC_1(t_1{}^n,t_2^n,T^n) & TAC_2(t_1{}^n,t_2^n,T^n) & \ldots\ldots & TAC_n(t_1{}^n,t_2^n,T^n) \end{pmatrix} \end{array} \qquad (21)$$

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Let $U^k = \max\{TAC_k(t_1{}^i, t_2^i, T^i), i = 1,2, \dots, n\}\ for\ k = 1,2, \dots, n$ and (22)

$L^k = \min\{TAC_k(t_1{}^i, t_2^i, T^i), i = 1,2, \dots, n\} for\ k = 1,2, \dots, n.$ (23)

Hence U^k, L^k are identified, $L^k \leq TAC_k(t_1, t_2, T) \leq U^k,\ for\ k = 1,2, \dots, n.$ (24)

For solving MOSCIM (20), in this technique firstly we have to make pay-off matrix which has been shown in the above (21). Then we have to find U^i and L^i , shown in equation no. (22), (23) and (24). In this technique the fuzzy membership function $\mu_{TAC_i}(TAC_i(t_1, t_2, T))$ for the i^{th} objective function $TAC_i(t_1, t_2, T)$ for $i = 1,2, \dots, n$ are defined as follows:

$$\mu_{TAC_i}(TAC_i(t_1, t_2, T)) = \begin{cases} 1 & for\ TAC_i(t_1, t_2, T) < L^i \\ \dfrac{U^i - TAC_i(t_1, t_2, T)}{U^i - L^i} & for\ L^i \leq TAC_i(t_1, t_2, T) \leq U^i \\ 0 & for\ TAC_i(t_1, t_2, T) > U^i \end{cases}$$

$for\ i = 1,2, \dots, n.$ (25)

4.1. Fuzzy non-linear programming technique (FNLP) based on max-min operator

Using the above membership function (25), fuzzy non-linear programming problems are formulated as follows:

$Max\ \alpha'$

Subject to

$TAC_i(t_1, t_2, T) + \alpha'(U^i - L^i) \leq U^i \quad , \quad for\ i = 1,2, \dots, n.$ (26)

$0 \leq \alpha' \leq 1,$

Therefore

$$\frac{1}{T}\left[\widehat{A_{vi}} + \widehat{h_{vi}}\left\{\frac{\widehat{a_i}}{\widehat{b_i}}\left(\frac{1}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1} - 1\right) - t_1\right) + \frac{\widehat{\alpha_i}}{\widehat{\beta_i} + 1}t_2^{\widehat{\beta_i}+1} - \frac{\widehat{\alpha_i}}{\widehat{\beta_i}}t_2^{\widehat{\beta_i}}t_1\right\} + \widehat{p_{vi}}\frac{\widehat{a_i}}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1} - 1\right) + \widehat{\iota_v}(T - t_2)\right.$$
$$\left. + \widehat{A_{ri}} + \widehat{h_{ri}}\left\{\frac{\widehat{\alpha_i}}{\widehat{\beta_i}(\widehat{\beta_i} + 1)}t_2^{\widehat{\beta_i}+1} + \frac{\widehat{\gamma_i}T}{2}(T - 2t_2) + \frac{\widehat{\delta_i}T^2}{6}(2T - 3t_2)\right\}\right] + \alpha'(U^i - L^i)$$
$$\leq U^i$$

$0 \leq \alpha' \leq 1,$

$$\frac{\widehat{a_i}}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1} - 1\right) = \widehat{\gamma_i}T + \frac{\widehat{\delta_i}}{2}T^2$$

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$$\sum_1^n W_i \frac{\widehat{a_i}}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1}-1\right) \leq W \; for \; i=1,2,\ldots,n. \quad (27)$$

The non-linear programming problems (27)can be solved by suitable mathematical programming algorithm and we get the solution of MOSCIM (20).

4.2. Fuzzy additive goal programming technique (FAGP) based on additive operator

In this process, using membership function (25), fuzzy non-linear programming problem is formulated as follows:

$$\text{Max} \sum_{i=1}^n \frac{U^i - TAC_i(t_1,t_2,T)}{U^i - L^i} \quad (28)$$

Subject to $L^i \leq \frac{1}{T}\left[\widehat{A_{vi}} + \widehat{h_{vi}}\left\{\frac{\widehat{a}_i}{\widehat{b}_i}\left(\frac{1}{\widehat{b}_i}\left(e^{\widehat{b}_i t_1}-1\right)-t_1\right) + \frac{\widehat{\alpha}_i}{\widehat{\beta}_i+1} t_2^{\widehat{\beta}_i+1} - \frac{\widehat{\alpha}_i}{\widehat{\beta}_i} t_2^{\widehat{\beta}_i} t_1\right\} + \widehat{p_{vi}} \frac{\widehat{a}_i}{\widehat{b}_i}\left(e^{\widehat{b}_i t_1}-1\right) + \right.$

$\left. \widehat{i_v}(T-t_2) + \widehat{A_{ri}} + \widehat{h_{ri}}\left\{\frac{\widehat{\alpha}_i}{\widehat{\beta}_i(\widehat{\beta}_i+1)} t_2^{\widehat{\beta}_i+1} + \frac{\widehat{\gamma}_i T}{2}(T-2t_2) + \frac{\widehat{\delta}_i T^2}{6}(2T-3t_2)\right\}\right] \leq U^i$

$$\frac{\widehat{a_i}}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1}-1\right) = \widehat{\gamma_i}T + \frac{\widehat{\delta_i}}{2}T^2$$

$$\sum_1^n W_i \frac{\widehat{a_i}}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1}-1\right) \leq W \;\; for \; i=1,2,\ldots,n \quad (29)$$

The non-linear programming problem (29) can be solved by suitable mathematical programming algorithm and we get the solution of MOSCIM (20).

5. FUZZY PROGRAMMING TECHNIQUE (BASED ON WEIGHTED MINIMUM AND ADDITIVE OPERATORS) TO SOLVE MOSCIM (20)

5.1. Fuzzy non-linear programming technique (FNLP) based on weighted max-min operator (WFNLP)

For this process we take positive weights ω_i for each objective $TAC_i(t_1,t_2,T) for \; i = 1,2,\ldots,n$ respectively.

Where $\sum_{i=1}^n \omega_i = 1$.

Using the above membership functions (25), weighted FNLP are stated as follows:

$Max \; \alpha''$

Subject to,

$\omega_i \mu_{TAC_i}\left(TAC_i(t_1,t_2,T)\right) \geq \alpha'' \; , for \; i = 1,2,\ldots,n$

$0 \leq \alpha'' \leq 1,$

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Therefore

$Max\ \alpha''$

$$\omega_i \frac{1}{T}\left[\widehat{A_{vi}} + \widehat{h_{vi}}\left\{\frac{\widehat{\alpha_i}}{\widehat{b_i}}\left(\frac{1}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1}-1\right)-t_1\right)+\frac{\widehat{\alpha_i}}{\widehat{\beta_i}+1}t_2^{\widehat{\beta_i}+1}-\frac{\widehat{\alpha_i}}{\widehat{\beta_i}}t_2^{\widehat{\beta_i}}t_1\right\}+\widehat{p_{vi}}\frac{\widehat{a_i}}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1}-1\right)\right.$$
$$+\widehat{\iota_v}(T-t_2)+\widehat{A_{ri}}$$
$$\left.+\widehat{h_{ri}}\left\{\frac{\widehat{\alpha_i}}{\widehat{\beta_i}(\widehat{\beta_i}+1)}t_2^{\widehat{\beta_i}+1}+\frac{\widehat{\gamma_i}T}{2}(T-2t_2)+\frac{\widehat{\delta_i}T^2}{6}(2T-3t_2)\right\}\right] \geq \alpha''$$

$0 \leq \alpha'' \leq 1,$

And $\sum_{i=1}^{n}\omega_i = 1$

$$\frac{\widehat{a_i}}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1}-1\right) = \widehat{\gamma_i}T + \frac{\widehat{\delta_i}}{2}T^2$$

$\sum_1^n W_i \frac{\widehat{a_i}}{\widehat{b_i}}\left(e^{\widehat{b_i}t_1}-1\right) \leq W\ \ for\ i = 1,2, \dots, n$ (30)

The non-linear programming problem (30) can be solved by favorable mathematical programming algorithm and we get the solution of MOSCIM (20).

5.2. 5.2 Fuzzy additive goal programming technique (FAGP) based on weighted additive operator (WFAGP)

Again using the above membership function (25), weighted FAGP are formulated as follows:

$$Max \sum_{i=1}^{n} \omega_i \mu_{TAC_i}\left(TAC_i(t_1, t_2, T)\right) (31)$$

Subject to,

$0 \leq \mu_{TAC_i}\left(TAC_i(t_1,t_2,T)\right) \leq 1\ , for\ i = 1,2, \dots, n.$

$$\sum_{i=1}^{n}\omega_i = 1$$

Therefore

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$$Max \sum_{i=1}^{n} \frac{\omega_i}{T}\left[\widehat{A_{v\iota}} + \widehat{h_{v\iota}}\left\{\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(\frac{1}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) - t_1\right) + \frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}+1}t_2^{\widehat{\beta_\iota}+1} - \frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}}t_2^{\widehat{\beta_\iota}}t_1\right\}\right.$$

$$+ \widehat{p_{v\iota}}\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) + \widehat{\iota_v}(T - t_2) + \widehat{A_{r\iota}}$$

$$\left. + \widehat{h_{r\iota}}\left\{\frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}(\widehat{\beta_\iota}+1)}t_2^{\widehat{\beta_\iota}+1} + \frac{\widehat{\gamma_\iota}T}{2}(T - 2t_2) + \frac{\widehat{\delta_\iota}T^2}{6}(2T - 3t_2)\right\}\right]$$

Subject to,

$$L^i \leq \frac{1}{T}\left[\widehat{A_{v\iota}} + \widehat{h_{v\iota}}\left\{\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(\frac{1}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) - t_1\right) + \frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}+1}t_2^{\widehat{\beta_\iota}+1} - \frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}}t_2^{\widehat{\beta_\iota}}t_1\right\} + \widehat{p_{v\iota}}\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) +\right.$$

$$\left.\widehat{\iota_v}(T - t_2) + \widehat{A_{r\iota}} + \widehat{h_{r\iota}}\left\{\frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}(\widehat{\beta_\iota}+1)}t_2^{\widehat{\beta_\iota}+1} + \frac{\widehat{\gamma_\iota}T}{2}(T - 2t_2) + \frac{\widehat{\delta_\iota}T^2}{6}(2T - 3t_2)\right\}\right] \leq U^i$$

(32)

and

$$\sum_{i=1}^{n} \omega_i = 1$$

$$\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) = \widehat{\gamma_\iota}T + \frac{\widehat{\delta_\iota}}{2}T^2$$

$$\sum_{1}^{n} W_i \frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1} - 1\right) \leq W \;\; for\; i = 1,2, \ldots., n$$

The non-linear programming problem (32) can be solved by favorable mathematical programming algorithm and we get the solution of MOSCIM (20).

6. FUZZY PROGRAMMING TECHNIQUE WITH HYPERBOLIC MEMBERSHIP FUNCTIONS (FPTHMF) FOR SOLVING MOSCIM (20)

In this technique the fuzzy non-linear hyperbolic membership functions $\mu^H_{TAC_i}(TAC_i(t_1,t_2,T))$ for the ith objective functions $TAC_i(t_1,t_2,T)$ respectively for $i = 1,2, \ldots., n$ are defined as follows:

$$\mu^H_{TAC_i}(TAC_i(t_1,t_2,T)) = \frac{1}{2}tanh\left(\left(\frac{U^i+L^i}{2} - TAC_i(t_1,t_2,T)\right)\rho_k\right) + \frac{1}{2} \quad (33)$$

Where ρ_i is a parameter, $\rho_i = \frac{3}{(U^i - L^i)/2} = \frac{6}{U^i - L^i}$

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Using the above membership function, fuzzy non-linear programming problem is formulated as follows:

$$Max\ \lambda$$

$$\text{Subject to}\quad \frac{1}{2}tanh\left(\left(\frac{U^i+L^i}{2}-TAC_i(t_1,t_2,T)\right)\rho_i\right)+\frac{1}{2}\geq\lambda\,,\,\lambda\geq 0 \qquad (34)$$

And $\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1}-1\right)=\widehat{\gamma_\iota}T+\frac{\widehat{\delta_\iota}}{2}T^2$

$$\sum_1^n W_i\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1}-1\right)\leq W\ \ for\ i=1,2,\dots.,n$$

Now simplifying the above non-linear programming problem (34) and we get

$$Max\ y$$

$$\text{Subject to}\ \ y+\rho_i TAC_i(t_1,t_2,T)\leq\frac{U^i+L^i}{2}\rho_i,\ \ y\geq 0 \qquad (35)$$

$$y+\frac{\rho_i}{T}\left[\widehat{A_{v\iota}}+\widehat{h_{v\iota}}\left\{\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(\frac{1}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1}-1\right)-t_1\right)+\frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}+1}t_2^{\widehat{\beta_\iota}+1}-\frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}}t_2^{\widehat{\beta_\iota}}t_1\right\}+\widehat{p_{v\iota}}\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1}-1\right)+\right.$$

$$\left.\widehat{\iota_v}(T-t_2)+\widehat{A_{r\iota}}+\widehat{h_{r\iota}}\left\{\frac{\widehat{\alpha_\iota}}{\widehat{\beta_\iota}(\widehat{\beta_\iota}+1)}t_2^{\widehat{\beta_\iota}+1}+\frac{\widehat{\gamma_\iota}T}{2}(T-2t_2)+\frac{\widehat{\delta_\iota}T^2}{6}(2T-3t_2)\right\}\right]\leq\frac{U^i+L^i}{2}\rho_i,\ \ y\geq 0,$$

$$\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1}-1\right)=\widehat{\gamma_\iota}T+\frac{\widehat{\delta_\iota}}{2}T^2$$

$$\sum_1^n W_i\frac{\widehat{a_\iota}}{\widehat{b_\iota}}\left(e^{\widehat{b_\iota}t_1}-1\right)\leq W\ \ for\ i=1,2,\dots.,n$$

The programming problem (35) can be solved by suitable mathematical programming algorithm and we get the solution of the MOSCIM (20).

7. NUMERICAL EXAMPLE

We have been considered an inventory model of two items with following parameter values in proper units and total storage area is $W=1000m^2$ and $W_1=3m^2$, $W_2=2m^2$.

Table 1: Input imprecise data for shape parameters

Parameters	Item	
	I	II
$\widetilde{A_{v\iota}}$	$(400,500,550,600;0.9)$	$(500,550,600,650;0.7)$
$\widetilde{A_{r\iota}}$	$(600,650,700,750;0.8)$	$(700,750,800,850;0.9)$

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$\tilde{\alpha}_\iota$	(60,65,70,75; 0.8)	(70,80,90,100,0.7)
$\tilde{\beta}_\iota$	(10.5,15.5,20.5,25.5; 0.6)	(11.5,14.5,17.5,20.5; 0.9)
$\tilde{a}_\iota$	(300,310,320,330; 0.8)	(350,370,390,410; 0.8)
$\tilde{b}_\iota$	(0.6,0.7,0.8,0.9; 0.9)	(0.8,0.9,1.0,1.1; 0.8)
$\widetilde{h_{v\iota}}$	(2,3,4,5; 0.7)	(4,5,6,7; 0.9)
$\widetilde{h_{r\iota}}$	(5,6,7,8; 0.8)	(7,8,9,10; 0.9)
$\widetilde{p_{v\iota}}$	(2,3,4,5; 0.8)	(3,4,5,6; 0.7)
$\tilde{\iota}_v$	(0.6,0.7,0.8,0.9; 0.8)	(0.6,0.7,0.8,0.9; 0.8)
$\tilde{\gamma}_\iota$	(50,60,70,80; 0.8)	(40,50,60,70; 0.9)
$\tilde{\delta}_\iota$	(0.5,0.6,0.7,0.8; 0.9)	(0.7,0.8,0.9,1.0; 0.8)

Approximate value of the above parameters is

Item	Parameters											
	$\widehat{A_{v\iota}}$	$\widehat{A_{r\iota}}$	$\hat{\alpha}_\iota$	$\hat{\beta}_\iota$	$\hat{a}_\iota$	$\hat{b}_\iota$	$\widehat{h_{v\iota}}$	$\widehat{h_{r\iota}}$	$\widehat{p_{v\iota}}$	$\hat{\iota}_v$	$\hat{\gamma}_\iota$	$\hat{\delta}_\iota$
I	461.25	540	54	10.80	252	0.675	2.45	5.20	2.80	0.6	52	0.585
II	402.50	697.50	59.50	10.35	304	0.76	4.95	7.65	3.15	0.6	49.5	0.68

Table 2: Optimal solutions of MOSCIM (20) using different methods

Methods	t_1^*	t_2^*	T^*	TAC_1^*	TAC_2^*
FNLP	0.35	1.14	2.41	585.87	564.52
FAGP	0.35	1.13	2.40	586.45	564.54
FPTHMF	0.35	1.14	2.41	585.87	564.52

Table 3: Optimal solutions of MOSCIM (20) using different weights by WFNLP method

weights	t_1^*	t_2^*	T^*	TAC_1^*	TAC_2^*
$w_1 = 0.4, w_2 = 0.6$	0.38	1.17	2.65	578.42	569.68
$w_1 = 0.5, w_2 = 0.5$	0.35	1.14	2.41	585.87	564.52
$w_1 = 0.6, w_2 = 0.4$	0.35	1.14	2.41	585.87	564.52

Table 4: Optimal solutions of MOSCIM (20) using different weights by WFAGP method

weights	t_1^*	t_2^*	T^*	TAC_1^*	TAC_2^*
$w_1 = 0.4, w_2 = 0.6$	0.35	1.13	2.40	586.45	564.54
$w_1 = 0.5, w_2 = 0.5$	0.35	1.13	2.40	586.45	564.54
$w_1 = 0.6, w_2 = 0.4$	0.35	1.13	2.40	586.45	564.54

From the above table 2, 3 and 4 shows that total average cost of both items is more or less same.

8. SENSITIVITY ANALYSIS

In sensitivity analysis MOSCIM (20) has been solved by using only FPTHMF method.

Table 5: Optimal solutions of MOSCIM (20) for different values of parameters

Parameter	% of the parameter	t_1^*	t_2^*	T^*	TAC_1^*	TAC_2^*
a_1, a_2	-50%	0.63	1.17	2.43	586.33	580.43
	-25%	0.45	1.15	2.45	585.01	570.96
	25%	0.29	1.13	2.42	584.83	560.02
	50%	0.25	1.13	2.48	581.46	557.03
b_1, b_2	-50%	0.37	1.14	2.40	588.09	567.18
	-25%	0.36	1.14	2.41	586.92	565.79
	25%	0.34	1.14	2.42	584.74	563.36
	50%	0.33	1.13	2.42	584.49	562.29

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α_1, α_2	-50%	0.35	1.21	2.42	566.91	537.84
	-25%	0.35	1.17	2.42	577.81	553.67
	25%	0.35	1.11	2.41	591.91	572.73
	50%	0.35	1.09	2.41	597.68	579.31
β_1, β_2	-50%	0.36	1.26	2.54	566.85	557.99
	-25%	0.35	1.18	2.46	577.22	561.56
	25%	0.35	1.11	2.39	591.89	566.62
	50%	0.34	1.09	2.37	591.94	568.20
γ_1, γ_2	-50%	0.25	1.09	3.30	458.40	483.74
	-25%	0.31	1.12	2.76	532.41	534.57
	25%	0.39	1.15	2.17	631.64	580.53
	50%	0.42	1.17	1.99	664.10	586.38
δ_1, δ_2	-50%	0.35	1.14	2.43	583.67	562.50
	-25%	0.35	1.18	2.42	585.06	563.52
	25%	0.35	1.18	2.41	586.88	565.52
	50%	0.35	1.18	2.40	588.27	566.50

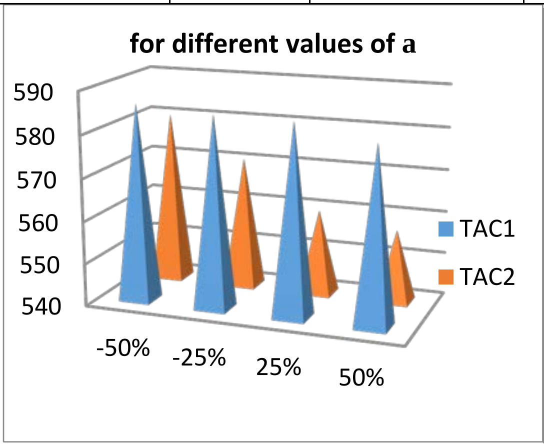

Figure 3: minimum cost of both item for different values of a_1, a_2

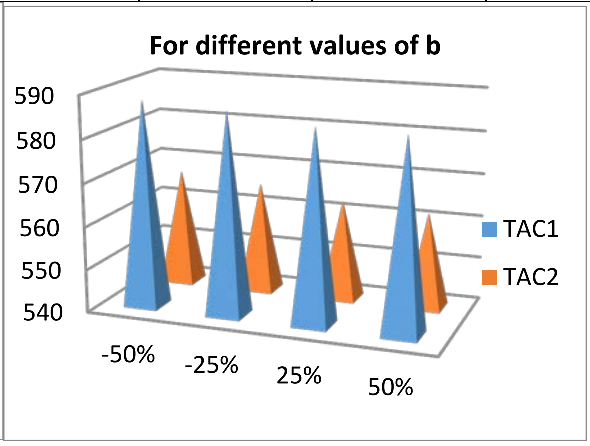

Figure 4: minimum cost of both item for different values of b_1, b_2

From the above figure 5 shows that minimum cost of the both items are decreased when values of a is increased. And figure 6 suggested that the same of a for b.

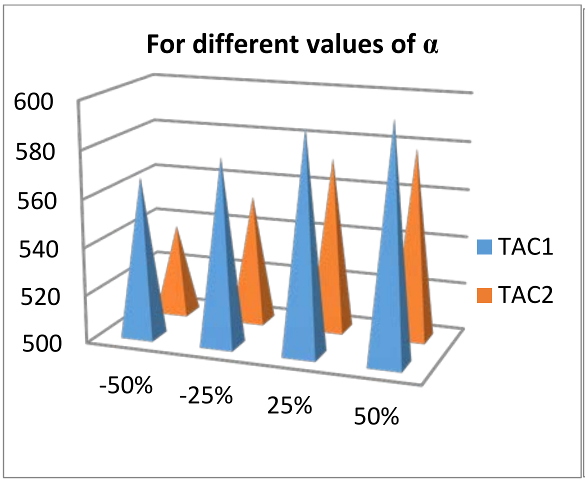

Figure 5: minimum cost of both items for different values of α_1, α_2

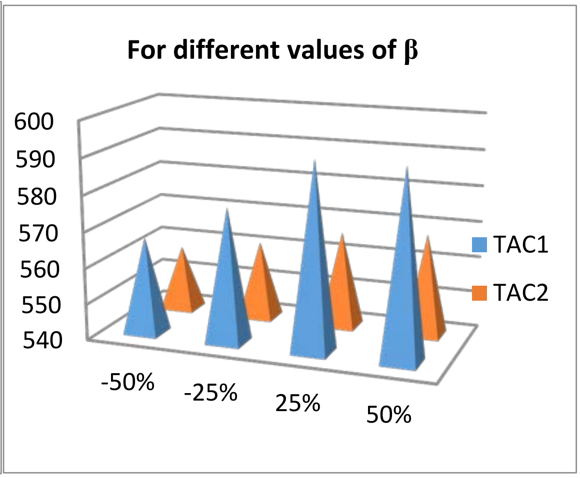

Figure 6: minimum cost of both items for different values of β_1, β_2

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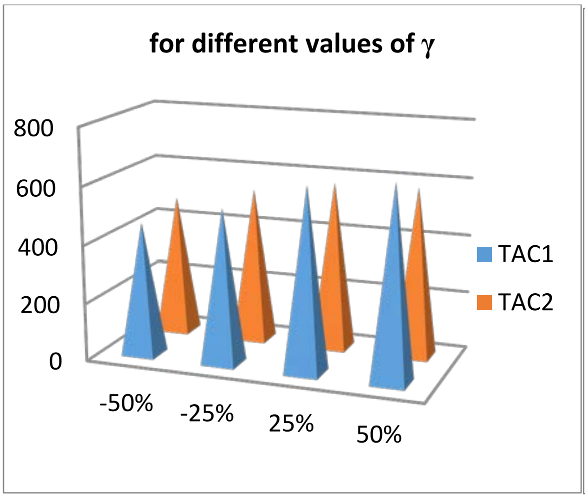

Figure 7: minimum cost of both items for different values of γ_1, γ_2

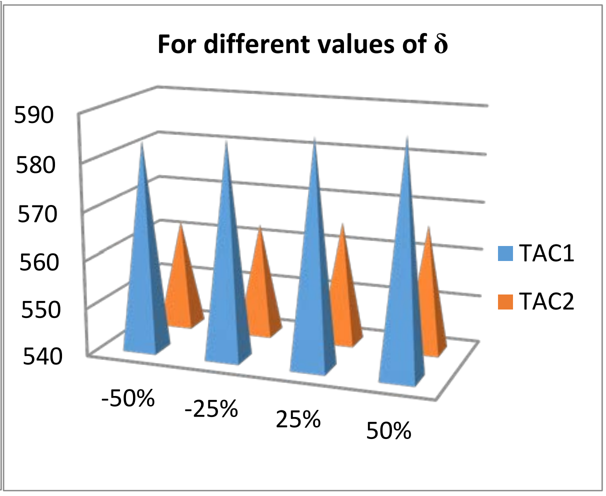

Figure 8: minimum cost of both items for different values of δ_1, δ_2

From the above figure 5, 6, 7, 8 suggests that minimum cost of the both items are increased when values of $\alpha, \beta, \gamma, \delta$ is increased.

9. CONCLUSION:

In this article, we developed an integrated production inventory model for a two echelon supply chain consisting of one vendor and another one retailer. Production rate and demand rate of retailer and customer are considered time dependent. Idle time cost of the vendor has been considered. Multi-item inventory has been considered under limitation on storage space. Due to uncertainty, the cost parameters are taken trapezoidal fuzzy number and the crisp model converted into fuzzy model.

Two echelon supply chain fuzzy inventory model has been solved by various techniques like as Fuzzy programming technique with hyperbolic membership functions (FPTHMF), Fuzzy non-linear programming technique (FNLP) and Fuzzy additive goal programming technique (FAGP), weighted Fuzzy non-linear programming technique (WFNLP) and weighted Fuzzy additive goal programming technique (WFAGP) and found aproximately same results. A numerical example has been provided to test the model.

In the future study, it is hoped to further incorporate the proposed model into more realistic assumption, such as probabilistic demand, introduce shortages, generalize the model under two-level credit period strategy etc. Also other type of membership functions like as triangular fuzzy number, Parabolic flat Fuzzy Number (*PfFN*), Parabolic Fuzzy Number (*pFN*) etc. can be used to form the fuzzy model.

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REFERENCES:

CARDENAS-BARRON, L. E; SANA, S. S. (2014) A Production Inventory Model for a two echelon Supply Chain when demand is dependent on sales teams' initiatives. **Int. J. Production Economics,** http://dx.doi.org/10.1016/j.ijpe.2014.03.007

CARDENAS-BARRON, L. E; SANA, S. S. (2015) Multi-item EOQ inventory model in a two-layer supply chain while demand varies with promotional effort, **Appl. Math. Modell.**http://dx.doi.org/10.1016/j.apm.2015.02.004

CHUNG, K. J.; TING, P. S. (1994) On replenishment schedule for deteriorating items with time-proportional demand, **Production Planning & Control**, n. 5, p. 392–396

DAVE, U.; PATEL, L. K. (1981) policy inventory for deteriorating items with time proportional demand. **J. Oper. Res**. n. 32, p. 137–142

ISLAM, S.; ROY,T. K. (2006) A fuzzy EPQ model with flexibility and reliability consideration and demand depended unit Production cost under a space constraint: A fuzzy geometric programming approach, **Applied Mathematics and Computation**, v. 176, n. 2, p. 531-544

ISLAM S. (2008) Multi-objective marketing planning inventory model, A geometric programming approach, **Appl. Math. Comput**, n. 205, p. 238-246.

ISLAM, S.; ROY, T. K. (2010) Multi-Objective Geometric-Programming Problem and its Application, **Yugoslav Journal of Operations Research**, n. 20, p. 213-227.

ISLAM, S.; MANDAL, W. A. (2017) A Fuzzy Inventory Model with Unit Production Cost, Time Depended Holding Cost, with-out Shortages under A Space Constraint: A Parametric Geometric Programming Approach, **The Journal of Fuzzy Mathematics**, v. 25, n. 3, p. 517-532.

ISLAM, S.; MANDAL, W. A. (2019) Fuzzy geometric programming techniques and applications**, Forum for Interdisciplinary Mathematics: Springer**.

LEE, C. C; MA, C. Y. (2000) Optimal inventory policy for deteriorating items with two-warehouse and time-dependent demands, **Production Planning & Control**, n. 11, p. 689–696.

LEE, H. T.; WU, J. C. (2006) A study on inventory replenishment policies in a two-echelon supply chain system, **Computers & Industrial Engineering**, n. 5, p. 257–263

SANA, S.; CHAUDHURI, K. S. (2004) On a volume flexible production policy for a deteriorating item with time dependent demand and shortage. **Adv. Model. Opt.**, n. 6, p. 57–74

SANA, S. S. (2010) A Collaborating inventory model in a supply chain. **Economic Modeling**, n. 29, p. 2016-2023.

SILVER, E. A.; MEAL, H. C. (1969) A simple modification of the EOQ for the case of a varying demand rate, **Production and Inventory Management**, v. 10, n. 4, p. 52–65.

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THANGAM, A.; UTHAYAKUMAR, R. (2009) Two-echelon trade credit financing for perishable items in a supply chain when demand depends on both selling price and credit period, **Computers & Industrial Engineering**, n. 57, p. 773–786.

TRIPATHI, R. P.; KAUR, M.; PAREEK, S. (2016) Inventory Model with Exponential Time-Dependent Demand Rate, Variable Deterioration, Shortages and Production Cost, **Int. J. Appl. Comput. Math**, DOI 10.1007/s40819-016-0185-4

TRIPATHI, R. P.; KAUR, M. (2018)A linear time-dependent deteriorating inventory model with linearly time-dependent demand rate and inflation, **Int. J. Computing Science and Mathematics**, v. 9, n. 4, p.352–364.

WANG, J.; SHU, Y. F. (2005) Fuzzy decision modelling for supply chain management. **Fuzzy Sets Syst**. n.150, p.107–127.

YANG, M. F. (2006) A Two-echelon inventory model with fuzzy annual demand in a supply chain, **Journal of Information and Optimization Science**, https://doi.org/10.1080/02522667.2006.10699709.

ZADEH, L. A. (1965) Fuzzy sets, **Information and Control**, n. 8, p. 338-353.

ZIMMERMANN, H. J. (1985) Application of fuzzy set theory to mathematical programming, **Information Science**, n. 36, p. 29-58.