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A research vision

Mechanical model of extrinsic muscles of the eyeball

Modelo mecánico de los músculos extrínsecos del globo ocular

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Abstract: The mathematical modeling of the physiological behavior of extraocular muscles in ocular rotation, contributes to the learning of the oculomotor system and the development of technologies for the control of devices, through the precise prediction of the ocular displacement path. We propose in this article a mechanical model of eye movement, which incorporates the physiological properties of extraocular muscles in the force-elongation relationship and the action of agonist and antagonist muscles. An easy to understand model is presented that allows the analysis of forces applied in muscle contraction, the variation of extraocular muscle length, which generates rotation of the eye and the vector interpretation of the direction in the space towards which the eye directs the view, in movements of adduction, abduction, elevation and depression. The proposed model allows a simplification of the mathematical description, compared with other models, in relation to the action of the

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mechanical elements (springs and dampers) that represent the anatomical and physiological components of the contractile mechanics of the extraocular muscles. The implementation of the proposed model could serve in the future in the development of technologies that emulate horizontal or vertical eye movements for the control of devices.

Keywords: Extraocular muscles, eye rotation, mathematical model, oculomotor system.

Resumen: El modelamiento matemático del comportamiento fisiológico de los músculos extraoculares en la rotación ocular, contribuye en el aprendizaje del sistema oculomotor y en el desarrollo de tecnologías para el control de dispositivos, a través de la predicción precisa de la trayectoria de desplazamiento ocular. Nosotros proponemos en el presente artículo un modelo mecánico del movimiento ocular, que incorpora las propiedades fisiológicas de los músculos extraoculares en la relación de fuerza-elongación y la acción de los músculos agonistas y antagonistas. Se presenta un modelo de fácil comprensión que permite el análisis de fuerzas aplicadas en la contracción muscular, la variación de longitud del musculo extraocular, que genera rotación del ojo y la interpretación vectorial de la dirección en el espacio hacia donde el ojo dirige la vista en movimientos de aducción, abducción, elevación y depresión. El modelo propuesto permite una simplificación de la descripción matemática, frente a otros modelos, de la acción de los elementos mecánicos (resortes y amortiguadores) que representan los componentes anatómicos y fisiológicos de los músculos extraoculares. La implementación del modelo propuesto podría servir a futuro en el desarrollo de tecnologías que emulen movimientos oculares horizontales u verticales para el control de dispositivos.

Palabras clave: Músculos extraoculares, rotación ocular, modelo matemático, sistema oculomotor.

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1. Introduction

Mathematical models, related to the prediction of eye movements, are used in the understanding of the oculomotor system, and applied in the development of bioinspired technologies. In the medical field, Hoerantner and Kaltofen in 2007, used mathematical models to create simulators that allowed the analysis of the action of the oblique and rectus muscles of the eye, to plan the treatment in the correction of strabismus and the high axial myopia [1].

In the field of the development of bioinspired technologies, they are frequent the mechanical and electronic conditioning of security cameras, which simulate eye movements in the vision scanning function.

The mathematical model of eye movement considers the biomechanical action of extraocular muscles: rectus superior, inferior, medial and lateral. Bahill in 1980, introduces Hill's muscular model (skeletal muscle represented as a mechanical system), into the mathematical model of eye movements, and defines the values of the passive elements that make up the mechanical system [2]. The oculomotor plant mathematical model, proposed by Holland and Komogorstev in 2013, describes the characteristics of the eye and the extraocular muscles responsible for vertical and horizontal eye movement [3]. Both models are similar, and make use of the state matrix to incorporate the necessary variables that result in vertical and horizontal eye movements. The mathematical analysis of these models can be complex and limit their application in the real field.

In the present study, we propose a mathematical model of eye movement using an alternative model of the classic Hill model, considering the anatomical properties of extraocular muscles,

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simplifying the equations that relate the variables involved in the physiological process of muscle activation, and ending in eye movements in different directions of space.

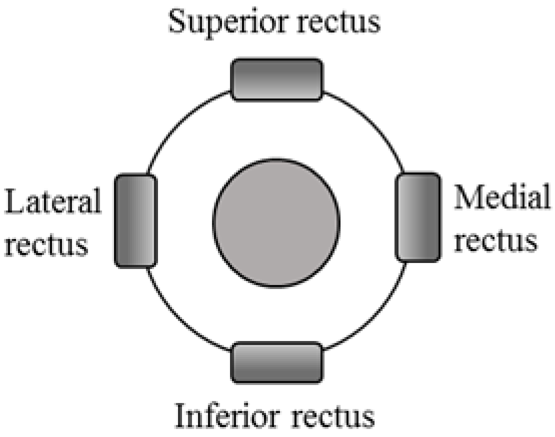
2. Methodology

2.1. Structural representation of the extrinsic muscles of the eyeball

The eyeball was considered a spherical solid structure, to which the extraocular muscles are anatomically inserted: superior, inferior, medial and lateral rectus, which allow the eye to rotate limitedly in the orbital cavity horizontally and vertically.

The representation of the eye in primary position can be seen in Figure 1, which corresponds to a scheme when the eye looks in front of an object.

Figure 1. Extraocular rectus muscles of the human eye.



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The dimensions of the oculomotor system components are also a source of information for the mathematical model proposed in this work. The following anatomical dimensions of the eye were taken into account, according to [4]: eyeball (Radius: 0,012 m), cornea (Radius: vertically 0,005 m and horizontally 0,006 m), pupil (Radius: 0,0015 m – 0,00225 m), and length of

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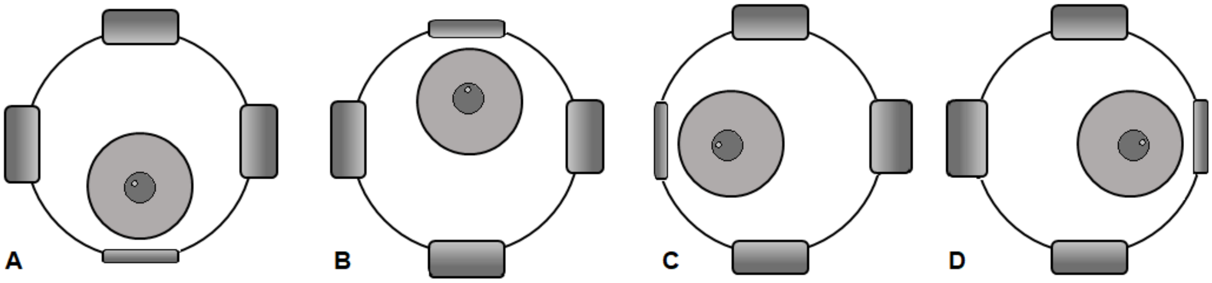
extraocular rectus, according to [5]: medial rectus muscle (Length: 0,041 m), lateral rectus muscle (Length: 0,041 m), superior rectus muscle (Length: 0,042 m) and inferior rectus muscle (Length: 0,042 m).

Figure 2 shows the eye movements considered in this study, according to the muscular behavior of the four extraocular rectus, the eye movement was considered starting from the primary position (Figure 1), changing to a secondary position of the gaze, in a rotation (depression, movement of the pupil downward; elevation, movement of the pupil upward; abduction, movement of the pupil in the lateral direction outward; adduction, movement of the pupil inward or nasal direction).

Figure 2. Eye movement **A.** Depression, **B.** Elevation, **C.** Abduction, **D.** Adduction.



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2.2. Mathematical modeling of rectus muscles of the eyeball

To represent the mechanical action of extraocular muscles, the physiological model of the muscle reported by [6], a variation of the mechanical system of the classic Hill model, was used to determine the relationship of Strength vs. Elongation. Figure 3 shows the muscle model used, it incorporates the same elements, interpretively similar to the Hill muscle model, but in a different configuration.

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The movement in one direction (vertical or horizontal) of the eye depends basically on two muscles. The contraction of a muscle that acts as an agonist or collaborator in the execution of the movement, and the relaxation of a muscle that acts as an antagonist of the movement.

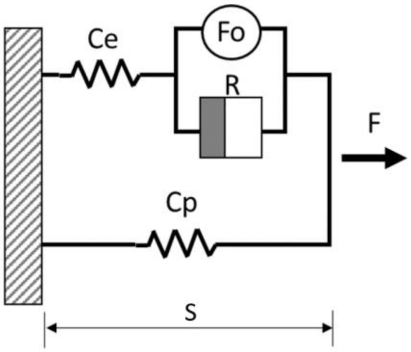
The muscle model used in this work, represents the behavior of a voluntary contraction muscle and relates the real force that occurs in the muscle (F), taking into account its mechanical properties, the value of the maximum contractile force corresponds to 100 gt equivalent to (0,98 N). Fo is the force developed by the active contractile element of the muscle, which physiologically corresponds to the myofibrils and sarcomera of the extraocular muscle. Fo can be considered as a percentage of the maximum force, according to [7], in this study Fo is equal to 0,26% of F.

The series elastic element (Ce) represents the elasticity of the cross actin-myosin junction of the striated fibers and the elastic behavior of the tendons inserted in the anterior part of the sclera, with a value of 2,5 gt / ° [2] equivalent to (125 N / m), while the parallel elastic element (Cp) describes elastic properties of the sarcolemma of the extraocular muscles, with a value of 0,5 gt / ° (25 N / m), and R the viscous damping present in the connective tissue of the eye, equal to 0,06 gt-s / ° [2] equivalent to (3,1 Ns / m).

In this study, the eye rotations performed under the response of the four extraocular lines were considered: in horizontal movements (adduction and abduction), by the mechanical action of the medial and lateral rectus; in main vertical movements (elevation and depression), by the mechanical action of the superior and inferior rectus.

In Figure 3, the representative linear mechanical model of the physiological behavior of extraocular rectus muscles is observed to perform an ocular rotation.

Figure 3. Linear model of muscular mechanics.



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The resulting differential equation of the mechanical system of Figure 3 is described in (1) and represents the behavior of the muscle action when contracted voluntarily. When a force F is applied at the entrance of the mechanical system, a variation of the length of the muscle S is generated, which provides the output of the system.

$$\frac{dF}{dt} + \frac{1}{RCe}F = \left(\frac{1}{Ce} + \frac{1}{Cp}\right)\frac{dS}{dt} + \frac{1}{RCeCp}S + \frac{Fo}{RCe} \tag{1}$$

The Laplace Transform was applied to the differential equation (1) and we find the variable $S(s)$, which corresponds specifically to the shortening length of the agonist muscle in contraction, which acts as an effector by pulling the eyeball in the required direction. While in a proportional way the antagonist muscle stretches the same length S, calculated by (2), presenting a relaxing action.

$$S(s) = \frac{\frac{F(s)}{RCe} + sF(s) - \frac{Fo(s)}{RCe}}{\frac{1}{RCeCp} + \left(\frac{1}{Ce} + \frac{1}{Cp}\right)s} \tag{2}$$

The dynamic balance of the extrinsic muscles of the eyeball allows the eye to maintain a primary position of the gaze, with forward vision. An illustrated example of the action of the superior and inferior rectus muscles is presented in Figure 4A.

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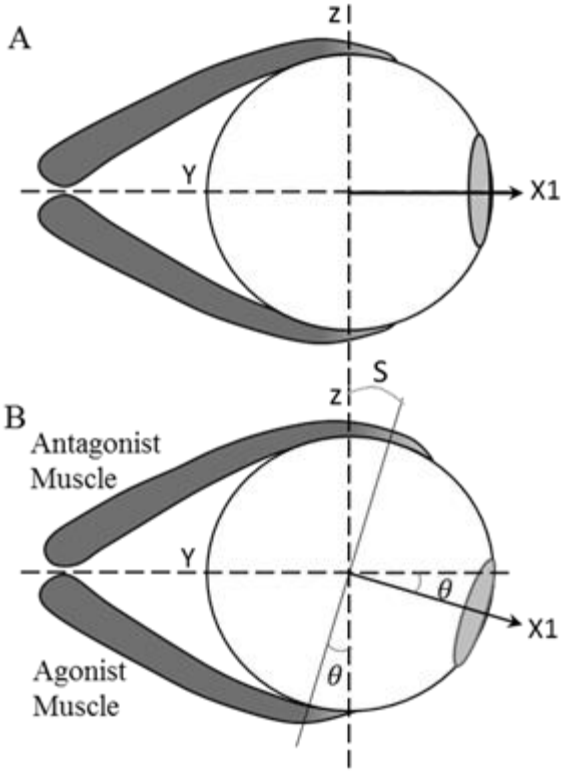
The movement of depression is shown in Figure 4B, which shows the contraction of the agonist muscle (for this example the inferior rectus muscle), the muscle contracts a distance S, while the antagonist relaxes (superior rectus). The action of the mass of the eyeball is considered void, because the mechanical system does not drag the mass, only rotates certain angles, on average, a low value. In addition, the mass is held by the four rectus muscles, which implies a weight distribution.

If we consider the eyeball, seen from the sagittal plane, as a circumference, we can say that, the rotation of the eye is caused by the drag of the muscle when contracting a distance S, without distorting the eyeball.

Figure 4. A. Outline of the eye in dynamic equilibrium (Primary position), **B.** Outline of the eye representative of eye movement.



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Therefore, there is a relationship between S (seen as the arc of the circumference) and the angle that indicates the rotation of the eye with respect to the main axis of equilibrium, see Figure 4B.

The arc length S, is calculated by the inverse Laplace transform of equation (2), R is the radius of the eyeball equal to 0,012 m (ideally considering the eyeball as a perfect circumference), the angle θ can be calculated by the equation (3).

$$\theta = \frac{S * 180^\circ}{\pi * R} \tag{3}$$

The above is described equally for the action of the lateral rectus muscles (medial and lateral) in horizontal rotation movements.

The angle θ obtained by the action of the superior and inferior rectus muscles is called the elevation angle; The angle α obtained by the action of the lateral and medial rectus muscles is called the horizontal scan angle. The conversion of spherical coordinates to Cartesian can be done by the equations (4-6):

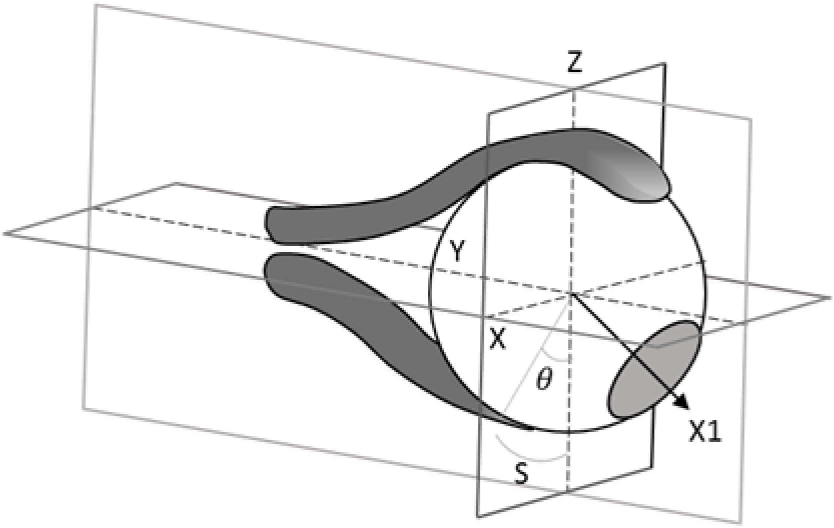
$$X = r * \cos(\theta) * \cos(\alpha) \tag{4}$$

$$Y = r * \cos(\theta) * \sin(\alpha) \tag{5}$$

$$Z = r * \cos(\theta) \tag{6}$$

Figure 5 shows the three-dimensional plane (X, Y, Z) of the eyeball during the execution of a movement and the direction of orientation of the view represented by the vector X1. In horizontal and vertical path movements, one of the three-dimensional axes that is common for eye movements is shared.

Figure 5. Three-dimensional plane in eye movement.



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The superior and inferior rectus muscles allow the pupil to move up and down, coordinates in the Y and Z axes, while the lateral muscles allow the pupil to move to the right and left, coordinates in the X and Y axes.

3. Results

In the model developed, the mechanical behavior of the superior-inferior and lateral-medial rectus muscles was considered. The application of the mathematical model taking into account the mechanical properties of the extraocular muscles allowed to generate a variation of the force at the entrance of the system, which allowed to obtain at the output of the system, the variation of the length of the extraocular muscle that acted as effector of the movement or agonist while elongation of its antagonist was obtained.

For vertical rotation movements, in the case of elevation movement, the maximum angles in the vertical upward direction are; 42º (with a range of 33º to 56º) [8]; while for the depression movement, the maximum angles of the ocular rotation to direct the view downwards are of 50º (with a range of 33º to 62º), for vertical rotation movements the values obtained in angle, applied force, length of contraction, stretching and maximum force ranges can be seen in Table 1.

Table 1. Results obtained after the application of the mechanical model, in vertical movements.

Variable Movement Vertical rotation	Angle	Strength	Final length in contraction	Final stretch length	Maximum force ranges applied
Movement of elevation	42°	0,47 N	0,0344 m Superior rectus	0,0475 m Inferior rectus	0,37 N – 0,63 N.
Movement of depression	50°	0,56 N	0,031 m Inferior rectus	0,0509 m Superior rectus	0,37 N – 0,70 N

Source: own.

In horizontal rotation movements, the maximum angle in the lateral direction for adduction and abduction is approximately 55 °, (with a range of 45 ° to 65 °) [8]. To effect an adduction movement, in which the eye rotates towards the nose, the medial rectus contracted while the lateral rectus stretched. The values obtained in angle, applied force, contraction length, stretching and maximum force ranges can be seen in Table 2.

Table 2. Results obtained after the application of the mechanical model, in horizontal movements.

Variable Movement Horizontal rotation	Angle	Strength	Final length in contraction	Final stretch length	Maximum force ranges applied
Movement adduction	55°	0,61 N	0,0311 m Medial rectus	0,0508 m Lateral rectus	0,51 N – 0,73 N.
Movement abduction	55°	0,62 N	0,031 m Lateral rectus	0,0508 m Medial rectus	0,51 N – 0,73 N

Source: own.

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The analysis of coordinated movements, if both eyes move in the same direction, allowed to identify that, while in the left eye there is an adduction movement by the action of the agonist muscle (medial rectus), in the same situation and simultaneously in the right eye, the medial rectus acts as an antagonist and generates an abduction movement.

The interpretation of the model and the results obtained in ranges of maximum force to perform a movement in a normal way, allowed to theoretically compare the mechanical effects in cases of muscular weakness since coordination deficiencies are generated during the execution of an ocular rotation, as in the case of strabismus.

4. Discussion

During an ocular rotation, starting from the primary position, the movement path can be identified by the movement of the pupil due to the action of four extrinsic muscles (superior, inferior, medial and lateral rectus) allowing movements of: adduction, abduction, elevation and depression. These movements generate angle changes with respect to the main axis, which by compensation correspond to the angle formed between the shortening distance of the agonist muscle with respect to the main axis X1.

The proposed model allowed to identify the relationship between the application of force in a mechanical system to represent the physiological behavior in the response of muscular activation and the analysis of the action of the lateral and medial rectus muscles in horizontal rotation, and the action of superior and inferior rectus, muscles in vertical rotations. The maximum force ranges applicable to the eyeball to perform a movement, may be useful in the medical field for the detection or future treatment of strabismus, in which poor muscle control is generated and the eyes do not align in the same direction.

The model proposed in this work is simple, easy to understand and implement because it incorporates variables from other traditional models, such as the muscle dynamics proposed by Hill (relationship non-linear of force-elongation), and from the Bahill model, variables such as: viscous damping and elastic elements in series and parallel. Thus, the proposed mechanical model could be applied in the control of devices through the prediction of the behavior of the oculomotor system and the development of technology that emulates the ocular movement or that performs ocular tracking to operate systems.

5. Conclusions

The activation of the agonist and antagonist extraocular muscles, generates a dynamic equilibrium position change of the eye, by the mechanical action of the extraocular muscles, giving rise to ocular movements of elevation, depression, adduction or abduction.

The physiological behavior of the extraocular muscle was interpreted through the mechanical model that incorporates the following properties: the viscosity of the connective tissue of the eye, represented in the model as a damping; and the elastic elements present in the muscular system of the eye, were represented as springs series and parallel in the model.

Extraocular muscle activity generates a movement in the eyeball, modeled in this work as a sphere. The effect of only a pair of rectus muscles (superior-inferior or lateral-medial) was considered to calculate the angle formed between the main axis X1 and the equilibrium position using the relationship circumference arc and radius of the eyeball. The rotation movements were analyzed in a coordinate system of the main axes (X, Y, Z) and the angles obtained generated by the activity of rectus muscles, consistently represent the real behavior of the eye movement. The strength of the extraocular muscle originated in the contractile element or sarcomera, is modified by the mechanical process that is generated during an ocular rotation, which influence properties such as tissue viscosity, generating friction and the elastic action of the tendons.

The proposed model allows a simplified understanding of the oculomotor system, compared to other more complex models, and is useful in learning and interpreting the activity of extraocular muscles, through spherical coordinates and analysis in planes (X, Y, Z) .The possible applications of the proposed mechanical model focus on the future development of eye tracking technology to operate systems by predicting the behavior of the oculomotor system and using the maximum force ranges to simulate an eye movement, as well as the effects of eyepieces pathologies or in the optimization of technologies that depend on eye movements or that mimic them.

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