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Latinx Bilingual Students' Perseverance on a Mathematical Task: A Rehumanizing Perspective

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Abstract

We draw on a rehumanizing perspective that covets a student-centered viewpoint around the discipline of mathematics. For Latinx bilinguals, we posit that a translanguaging practice is a vital option by which collective perseverance during problem solving can be sustained and leveraged for meaningful learning. This study explores and examines the collaborative efforts of a group of Latinx twelfth-grade students persevering to make meaning of an exponential relationship. We employed a discursive thematic analysis of this groups' ongoing engagement with a challenging mathematical task, paying specific attention to the ways in which these bilingual students encountered and overcame mathematical obstacles and setbacks. Our findings suggest that Latinx bilingual students can spontaneously and dialogically leverage communicative resources to help persevere with in-the-moment obstacles and build mathematical understandings. We argue for the development of more explicit translanguaging support systems in mathematics classrooms to privilege the viewpoint and experiences of the student, and the ways in which they develop mathematical understandings.

Keywords: mathematics, bilingual, perseverance, translanguaging, rehumanizing.

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La Perseverancia de l@s Estudiantes Bilingües Latin@s en una Tarea Matemática: Una Perspectiva Rehumanizadora

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Resumen

En este trabajo nos basamos en una perspectiva rehumanizadora basada en un punto de vista centrado en @l estudiante en torno a las matemáticas. Creemos que una práctica translingual es un opción vital para l@s estudiantes Latinx bilingües, que junto a la perseverancia colectiva durante la resolución de problemas pueden aprovecharse para y promover un aprendizaje significativo. Este estudio explora y examina los esfuerzos de colaboración de un grupo de estudiantes Latinx del duodécimo grado que perseveran en darle sentido a un problema sobre exponenciación. Por medio de un análisis discursivo, prestando especial atención a los recursos comunicativos que estudiantes bilingües usan, hemos encontrado que ell@s utilizan estos recursos de forma espontánea y dialógica para perseverar ante los obstáculos y el proceso de construir significados matemáticos. Abogamos por el desarrollo de sistemas de apoyo de lenguaje translingües más explícitos en las aulas de matemáticas, que privilegien el punto de vista y las experiencias del estudiante, así como también las formas en las que desarrollan su comprensión matemática.

Palabras clave: matemáticas, bilingüe, perseverancia, Translingual, rehumanización.

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This paper presents a rehumanizing perspective in which students use their agency to develop and affirm their Latinx bilingual mathematical identities in the context of high school mathematics. Although research has opposed deficit perspectives of using native language during mathematics learning, there is still much to learn about the intricate process by which Latinx bilingual students use their language and culture to engage in mathematical problem solving (Razfar, 2013).

Therefore, in the context of working collaboratively on a mathematical task for which a solution path was not known, our aim is to highlight how Latinx bilingual students spontaneously and dialogically leveraged their communicative practices and mathematical resources to persevere past obstacles along their path toward meaning making. Specifically, we hope to reveal the rehumanizing effects of Latinx bilinguals' translanguaging and how they leverage their linguistic resources to persevere while problem solving to help develop their own mathematical meanings. We use the term "bilingual" or "bilinguals" to reflect that the students in this study are not learning English as a second language, but are multilingual along a continuum. Additionally, we use the term "leverage" as students' agency to capitalize on their communicative and linguistic repertoires (see Martinez, Morales, & Aldana, 2017, for a review of how this term is used by scholars).

Positionality of Authors

This study is grounded in the authors' perspectives as former high school mathematics teachers. Taking the lead from Aguirre et al. (2017), we carefully considered our positionalities in doing equity work by "making our ideologies explicit, exploring our lived experiences and identities in relationship to our research and practices, and reflecting on assumptions and decisions we [have made]" (p. 126). This collaboration theoretically bridges the authors' lenses to demonstrate rehumanizing mathematical practices with Latinx bilinguals.

Hector Morales, Jr.

When I began teaching high school mathematics in my hometown urban city in the United States, I was assigned to classes that were created for students who had failed a prior math course. All of the students in my classes were Latinx bilingual students who had recently transitioned out of the bilingual program. I am Puerto Rican and I speak Spanish, but not being able to speak Spanish academically has always been a source of shame and caused me to question my identity as a Latino and my effectiveness as a teacher. It was later I realized the strength of engaging bilingually with my students. I experimented with many accommodating teaching strategies, such as encouraging communication in their first language and creative pairing with bilingual speakers.

I eventually became involved in a funded project at my school to implement the Interactive Mathematics Program (IMP) (Fendel, Resek, Alper, & Fraser, 2015), a secondary mathematics curriculum. IMP is an integrated, problem-based text that emphasizes mathematical discourse and engaging in collaborative problem-solving of demanding tasks that seem to grow in complexity as they unfold. Many IMP tasks value different perspectives that draw on the combined resources of a group, illustrate important mathematical ideas, allow for multiple representations, and have several possible solution pathways. These experiences drove me to pursue questions regarding how interactions between the tasks and communicating mathematically affect Latinx bilinguals' perseverance in order to counter the broader deficit narratives written about Latinx students in educational research.

Joseph DiNapoli

The majority of my research concerns student perseverance. As a White male with a suburban, east coast upbringing in the United States, I have had practically no educational experience with Latinx students before meeting Hector. As a teacher of mathematics in a rural high school for six years, I noticed that my students approached and navigated mathematical obstacles in some situations, yet completely avoided them in others. As a form of action research, I conducted several quasi-experiments to better learn how I could motivate my students to persevere with challenging mathematics. In my classes, I incorporated mathematical activities that provided opportunities for perseverance, while also carefully scaffolding the mathematics to encourage individual students to stay engaged. During my doctoral studies, I began to develop hypotheses about how and why students might persevere with challenging mathematical tasks. Studying productive struggle in the moment, rather than focusing on summative achievements (such as a grit perspective), provided an exciting angle from which to reveal how different students navigate the frustrating times during the learning process that facilitate meaning-making. In my research, as students grew more comfortable persevering, they began to adopt productive struggle as part of their identity and developed a belief that in-the-moment struggle is both normal and helpful for learning mathematics.

Learning from Hector's perspective about how Latinx students persevere, I realized that Latinx bilingual students might navigate situations requiring perseverance differently and there are certain biases from working with monolingual English-speaking students that I did not consider before. Specifically, it became apparent that Latinx bilinguals needed opportunities for discursive collaboration during problem solving while leveraging their communicative repertoires.

Latinx Students and Mathematics

Latinx students are one of the fastest growing school age populations in the U.S. (NCES, 2016). Yet, Moll (2001) posits that classroom practices have continued to create a distance between Latinx students' language, cultural knowledge, and what they know academically. These systems persist in marginalizing, and thus not privileging linguistic, social, and cultural capital to help create dehumanizing school practices (Langer-Osuna, Moschkovich, Norén, Powell & Vazquez, 2016). Moreover, guiding texts like Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014) asserts that all students must have access and opportunity to study mathematics. Further, the Common Core State Standards (CCSSI, 2010) explicitly note the importance of cultivating perseverance for all students to encourage mathematical expertise. This notion of student perseverance exists in the moment, at specific times during problem solving when productive struggle is necessary to overcome obstacles along the path toward developing conceptual knowledge. As such, math classrooms have been encouraged to move from isolated seatwork to more social and verbal activities that require students and teachers to engage in more substantive mathematical discussions and collective practice (Bass & Ball, 2015).

However, there is a concern that mathematics reforms may be in danger of ignoring the needs of Latinx students unless their needs are re-examined in light of the new demands of the mathematics classroom with its increased emphasis on communication and collaboration (Moschkovich, 2000). While this is a generalized perspective of classrooms with Latinx students, it nevertheless raises questions about marginalization and undervaluing Latinx students' learning resources in mathematics.

Garcia, Ibarra Johnson, and Seltzer (2017) reminds us that all good education must begin with recognizing students' strengths that come from their own community's linguistic and cultural repertoire. Much of the research with Latinx students has focused on bilingual language learners. The research is often framed from a deficit perspective focusing on the relationship between students' proficiency in their first language and learning mathematics (e.g., Mestre & Gerace, 1986) or the obstacles faced by Latinx bilinguals learning mathematics across languages (English and Spanish) (e.g., Khisty, 1995). Deficit perspectives emerge when bilinguals' linguistic resources are ignored or forbidden in the classroom while only privileging the dominant school language (Langer-Osuna et al. 2016). These are de-humanizing practices with the sole purpose of controlling and dominating students' cultural identity and excluding it from the classroom and school (Gutierrez, 2017). Garcia (2017) argues that this view of language and academic discourse in schools acts as a barrier to knowledge (in our case, mathematical knowledge), only privileging those students whose linguistic repertoire mirrors the dominant school language.

Other studies have also shown that Latinx students use a wide variety of cultural resources to construct, negotiate, and communicate (spoken or written) about mathematics (Chval & Khisty, 2009; Morales Jr., 2012) These resources include cultural knowledge (Gutiérrez 2002), linguistic resources (e.g. mathematics register, mathematical discourse) (Celedon-Pattichis, 2003; Moschkovich, 2000), everyday experiences, life histories, and community funds of knowledge (Moll 2001). Moreover, Razfar, Khisty and Chval (2011) advocates for a social-cultural model for language development in mathematics classrooms. They juxtapose social-cultural theory (SCT) against second language acquisition (SLA) models. In the SLA model, learners are perceived as passive recipients of mathematical knowledge proceeding in a linear developmental path and language is seen as an external tool. In contrast, SCT positions bilingual students as active agents in their language use, capable of working collaboratively, interacting, and communicating while grappling with challenging mathematical tasks. Given this disparity, a rehumanizing perspective, or a counter-narrative to common deficit perspectives, will allow us to illuminate the ways in which Latinx bilinguals encouragingly use their linguistic and other funds of knowledge while engaging their individual and collective agency to assert their identities as perseverant mathematics learners. Gutierrez (2017) discusses such a rehumanizing perspective as one that positions the student as central to the meaning-making process while engaging in the practice of doing mathematics. This perspective refutes the imposing of standardized or normalized practices onto students, such as the routine expectation of students reproducing the teachers' idea of productive mathematical activity. Instead, a rehumanizing perspective fosters respect and dignity through privileging the viewpoint and experiences of the student, and the ways in which they develop personal understandings through their own disciplinary perspective on mathematics.

Translanguaging

Bakhtin (1981) extended Vygotsky's notion of mediating tools (e.g., words and speech) to include dialogue as a mediating artifact that makes a relation between social-cultural factors and the mental functioning of an individual. Vygotsky emphasized language as a mediating factor; however, language here is perceived simply as something static. Conversely, Bakhtin emphasized dialogue or a more dynamic view of language. Bakhtin (1981) explains that:

> …all the languages of heteroglossia (discourse types)… are specific points of view on the world, forms of conceptualizing the world in words, specific worldviews, each characterized by its own objects, meanings, and values. As such they may all be juxtaposed to one another, mutually supplement one another, contradict one another, and be interrelated dialogically. (p. 291-2)

When we speak, we speak with the voice of our community that we create out of the discourse types available to us. In this dialogic process of meaning making, we appropriate or borrow others' words or utterances to speak ourselves. We borrow from what we have heard from others in order to express our own ideas. This process of borrowing and mixing our own ideas as a semiotic act forms the basis of translanguaging.

We draw on translanguaging to reconceptualize bilingualism as a liberating and empowering communicative practice and resource capable of transforming learning the goes beyond students' transition to the dominate school language. Translanguaging is more than just a simple shift between two languages (i.e., English and Spanish). It is a complex and interrelated communicative practice that make up bilinguals' linguistic repertoire (Cenoz, 2017). Garcia (2017) posits, "…speakers use their languaging, bodies, multimodal resources, tools and artifacts in dynamically entangled, interconnected and coordinated ways to make meaning" (p. 258). A translanguaging lens will allow us to better understand holistically how bilingual students use their linguistic, multimodal, and mathematical artifacts repertoire to make meanings as they wrestle with challenging mathematical ideas.

Perseverance in Learning Mathematics for Understanding

Learning something new takes effort and working to overcome obstacles is central to the notions of perseverance and making sense of a new idea. In the context of working on a challenging mathematical task, perseverance is initiating and sustaining in-the-moment productive struggle in the face of one or more obstacles, setbacks, or discouragements (DiNapoli, in press). Perseverance is especially important for learning mathematics because students develop conceptual understanding as they productively struggle with ideas that are not immediately apparent (Hiebert & Grouws, 2007).

Maintaining one's productive struggle through moments of setback may be especially important for meaning making, and thus a learner's willingness to amend their problem-solving plan in the moment in favor of a new approach is key for perseverance practice (Middleton, Tallman, Hatfield, & Davis, 2015). The self-regulatory actions amidst uncertainty as students navigate an obstacle during problem solving helps describe perseverance, and analysis of such engagement should consider the ways in which students first explore an uncertain mathematical situation, and how (if necessary) they amend their initial plan of attack to find a way to continue to make progress toward building understanding. Therefore, it is important for educators to encourage perseverance via their classroom practice. This stance has been adopted and disseminated by NCTM (2014), specifically advising for teaching practices that "consistently provide students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" (p. 48).

It seems translanguaging practice may help support and facilitate perseverance in problem solving for Latinx bilingual students, but this relationship is still empirically unclear. Translanguaging might open a nurturing space for Latinx bilingual students' perseverance on a mathematical task because they can draw on their fluid linguistic repertoires and be an agent of their own engagement (de los Rios & Seltzer, 2017).

Bilinguals employing translanguaging practice to attempt to persevere past a mathematical obstacle are not guaranteed success, but are equipped with more resources to approach the learning opportunity than those choosing not to persevere. In this way, the conception of perseverance described here does not portray a fixed feature of one's character or personality, but instead a deep, nuanced state of continued engagement in the context of challenging mathematical task work.

One important factor in supporting perseverance is the structure of the task itself. IMP tasks have been shown to support perseverance by encouraging initial engagement via multiple entry points while also maintaining cognitive demand enough to warrant students overcoming obstacles to make their own mathematical connections (Clarke, Breed, & Fraser, 2004; DiNapoli, 2016). Such task features compel the use of diverse linguistic resources for bilingual Latinx students (Aguirre et al., 2012). In order for Latinx bilinguals to engage at all levels of language proficiency in their mathematical work, they need challenging mathematical tasks to promote sense making and reasoning (Driscol, Heck, & Malzhan, 2012).

Thus, providing Latinx bilinguals opportunities to engage with inviting tasks with room to explore, coupled with the autonomy to leverage translanguaging to help coordinate their meaning making actions, can be conducive for perseverance.

Recognizing and Analyzing Perseverance

We employ an analytical framework aligned to our conceptualization of perseverance (DiNapoli, in press). We also adopt Sengupta-Irving and Agarwal's (2017) "perseverance as collective enterprise" (p. 116) perspective to study the ways in which peers engage together in productive struggle. Studying perseverance collectively as opposed to individually affords greater insights into the role of translanguaging during deep collaboration with challenging mathematics. Our framework outlines potential perseverance pathways students take during problem solving, and makes explicit the specific moments when perseverance is necessary. We consider three phases of student perseverance with each containing specific components (see Table 1). In our analysis, we consider if and how a group of students working on a mathematics problem persevere, or do not, at specific moments during their work.

Table 1.
Analytical Framework for Perseverance

Phase	Components	Characteristics
Entrance	Perceived goal; Initial obstacle	Identify the goal of the task; determine if they immediately know how to solve the problem
First Attempt	Initial effort; Diligence of first effort; Metacognitive progress	Spend some intellectual effort making a first attempt to solve the problem or not; solve the problem, do not solve but have made metacognitive progress
Additional Attempt(s)	Amending the plan(s); Diligence of next effort(s); Metacognitive progress	Spend more intellectual effort trying something different after their first attempt or not; solve the problem; do not solve but have made metacognitive progress

Our perseverance perspective is designed to combat the deficit perspective of grit (see DiNapoli, 2018). Grit is often interpreted as the innate skill of pulling oneself up by the bootstraps and is self-evident with long-term achievement, while in-the-moment perseverance is a natural engagement state. Perseverance with a challenging task is influenced by a variety of environmental factors, including interest level, goals for engagement, pre-requisite knowledge, empowerment, identity, autonomy to leverage linguistic resources, and task structure. If even one variable is incompatible with a student in a particular situation, a reasonable response to a challenge task may be to not persevere (see Star, 2015). In our study, we did not necessarily expect participants to persevere, nor did we ask them to. Instead, our aim was to capture the ways in which translanguaging helped this group of Latinx bilinguals to persevere naturally and build their understanding of important mathematical ideas. Observed perseverance and the translanguaging practices that support it reveals details about students' identities as bilingual mathematics learners, especially when such perseverance is unexpected or spontaneous.

Methodology

Our observations are part of a larger study of Latinx bilinguals in mathematics (Morales Jr., 2004) and were made in a twelfth-grade mainstream class in a school that has a student population which was nearly 80% Latinx. The school's mathematics curriculum was IMP. All observed mathematics students had taken two to three prior IMP courses and were currently enrolled in their final IMP course focused on vectors, geometric transformations, functions, statistics, and calculus concepts.

Participants

The students – Carina, Jessica, Elena, Ines (pseudonyms) – had worked collaboratively all year and were selected based on their level of bilingualism while doing mathematics in small groups. These students represent fairly typical students in the school with a history of average level performance in their mathematics courses. Carina, Elena, and Jessica were all born in the United States, but their parents are from Mexico. All of them grew up speaking both Spanish and English at home and they all stated that they felt comfortable speaking Spanish but not comfortable reading or writing in Spanish. Ines was the only student who immigrated to the United States from ages 12 to 14, returned to Mexico for ninth grade, and then returned to the United States to complete her high school education. Ines' formal educational experiences included speaking, reading, and writing in Spanish.

The teacher – Ms. Patrick (pseudonym) – was a monolingual English-speaker with 20 years of mathematics teaching experience. Her classroom was chosen because she allowed her students to collaborate around challenging mathematics in any language, and our intent was to learn more about how Latinx bilingual students use translanguaging to help persevere. Prior to any data collection, Ms. Patrick recommended we observe Carina, Jessica, Elena, and Ines' group because they had demonstrated resilience while working together previously.

Data Collection and Analysis

Carina, Jessica, Elena, and Ines were observed for six weeks in Ms. Patrick's IMP classroom. A total of 25 class sessions were attended, accumulating over 20 hours of observation. Observations concentrated on the students' discourse patterns and particular attention was paid to how students used their linguistic resources (linguistic, visual-graphic, or gestures) while negotiating meaning, and how students interacted with available resources as a means for aiding the development of meaning. As such, each class session was video recorded with accompanying field notes, and copies of student generated artifacts were collected.

To select the vignettes to report in this article, we drew from a thematic analysis of the entire data set. We purposely selected an episode of Carina, Jessica, Elena, and Ines' group because it demonstrated how spontaneous translanguaging can facilitate in-the-moment perseverance. Such collaboration was common for this group of students. They were observed working on several challenging tasks over the course of six weeks. In this time, they consistently demonstrated perseverance during collaborative problem-solving. However, in our view, Carina, Jessica, Elena, and Ines' group was not particularly representative of the entire student population in Ms. Patrick's IMP class. This group's translanguaging practice helped to stimulate in-the-moment perseverance which seemed exceptional compared to other groups. This particular episode was analyzed using translanguaging and perseverance as a lens to make explicit how Latinx bilinguals leveraged their linguistic resources to help propel their collective productive struggle with a challenging mathematical task.

Context for the Vignettes

The examined vignettes began in week three of observations. Carina, Jessica, Elena, and Ines were assigned the task of discerning ways in which mathematical functions were helpful (Figure 1). They had to select a past unit and a function they had encountered previously.

Function Analysis

Select a specific function from a past unit.

(1) Describe the problem context in which the function was used, and explain what the input and the output for the function represent in terms of the problem context.

(2) Describe how the function was helpful to you in solving the central unit problem or some other problem in the unit.

(3) If possible, determine what family the function is from.

Figure 1. Function Analysis Task

The students chose All About Alice (Figure 2), a second year IMP unit which starts with a modeling task based on Lewis Carroll's Alice's Adventures in Wonderland. From this situation come the basic principles for working with exponents and an introduction to exponential growth and decay problems. Moreover, this task of analyzing a previously encountered function necessitates perseverance because the Alice problem is familiar, yet the analysis is open to student exploration.

Alice's height changes when she eats the cake. Assume as before that her height doubles for each ounce she eats. (a) Find out what Alice's height is multiplied by when she eats 1, 2,3,4,5, or 6 ounces of cake. (b) Make a graph of this information.

Figure 2. All About Alice (Fendel et al., 2015, p. 385, Year 2)

We share the following vignettes as examples of how a rehumanizing perspective reveals how students activate their agency to leverage translanguaging to affirm their Latinx bilingual mathematical identities, as evidenced by their autonomous perseverance. These episodes demonstrate Carina, Jessica, Elena, and Ines dialogically leveraging their communicative and mathematical practices (spontaneous translanguaging practice) to make meaning and persevere while solving a complex mathematical situation involving Alice's exponential growth. This group collaborated around this task across three days: they were introduced to the task for five minutes on Day 1, they worked without Ms. Patrick for 30 minutes on Day 2, and they worked without Ms. Patrick for 10 minutes on Day 3.

Vignettes

Here we present vignettes from Day 2 and Day 3 of this group's collaboration around the Alice problem. These exchanges exemplify Latinx bilinguals' translanguaging meaning making processes and perseverance while doing advanced mathematics. When students' utterances are spoken in Spanish, we set apart the English translation using italics within parentheses. At this point, the group had already shown evidence of understanding the goal of the problem and did not immediately know how to solve it (during Day 1). Thus, our perseverance analyses begin on Day 2 as the group passes through the Entrance Phase into the First Attempt Phase.

Vignette 1: During Day 2

> JESSICA: ¿Qué era la primera, se hace así? Okay, dice: Alice changes when she eats the cake, assume that her height doubles for each ounce she eats, so that means if she eats one ounce, that means that she grows twice, dos ¿qué? Double, no double, two. (What was the first one, do you do it like this? Okay, it says: Alice changes when she eats the cake, assume that her height doubles for each ounce she eats, so that means if she eats one ounce, that means that she grows twice, two what? Double, no double two.)
> JESSICA: See, so when two is four, and then three is six, and four is eight, y así, y así vamos hacer la graph. Going like that para arriba.(Gesturing) You get it? (See, so when two is four, and then three is six, and four is eight, like this, and this is how we are going to make the graph. Going like that, up. (Gesturing) You get it?)

> ELENA: Um hmm. Pero, how do we times it? (But how do we times it?)
> JESSICA: Porque mira, two, times two. Well no. (Because look, two, times two. Well no.)
> ELENA: But that's what you were telling me yesterday y yo pensé que no. Okay so we. (But that's what you were telling me yesterday and I didn't think so. Okay so we.)
> JESSICA: Double it by, nomas double the number of ounces, so that means if she takes…[Elena interrupts] (Double it by, just double the number of ounces, so that means if she takes…)
> ELENA: Two times two, y luego four times two, y luego six times two, is that what you are saying? (Two times two, and then four times two, and then six times two, is that what you are saying?)
> JESSICA: Más o menos como sumando el mismo número. (More or less like adding the same number.)
> CARINA: Pero es lo mismo de sumando si lo multiplicas por dos. (But it is the same as adding if you multiply by two.)
> INES: [Referencing table from year one] Lo que parece es como hicimos un in/out table y ya lo sacamos. (It looks like we just did an in/out table and that's it.)
> CARINA: Yeah. In times two equal out.

From a translanguaging perspective, the students coordinated their meaning making actions (agency) by deploying a fluid movement between mathematical and everyday speaking across Spanish and English, providing this group with the resources necessary to clarify their mathematical thinking regarding their interpretation of the task. This group was more than just code-switching back and forth across languages; they were translanguaging by deploying everyday linguistic features and mathematics register resources in dialogically entangled ways with the intention to make meaning (Garcia-Mateus & Palmer, 2017). As demonstrated in vignette 1, the concept of doubling was expressed linguistically across Spanish and English by the students in a variety of ways: "twice, two, dos, double, multiply by two, two is four, three is six, four is eight, como sumando el mismo numero". These complex and interrelated discursive practices reaffirm how language and the students' bilingual identities are deeply intertwined, offering the possibility to transform their mathematical practice. The students also explored and questioned how to represent their ideas symbolically, which demonstrates the diligence of their work regarding understanding the nature of doubling. Jessica expressed her equation in terms of x and y and Carina expressed her equation in terms of IN and OUT (IN times two equals OUT). Here, Carina was drawing on her earlier experiences, a resource, from her freshman year when she learned the IN/OUT terms. Translanguaging supported their continued intellectual effort to help persevere and begin to make sense of doubling.

This group entered the First Attempt Phase by expressing differently what doubling meant to them. Drawing on such resources is further evidence of spending diligent effort during their First Attempt Phase of perseverance to make sense of the Alice function. Unfortunately, they did not realize immediately that they were multiplying the number of ounces of cake by two, instead of Alice's height. This is important in perseverance analysis because it provides an opportunity for the group become metacognitively aware of their error, thus empowering them to (a) engage more deeply to recognize their error, and (b) overcome the setback and make a second attempt to make sense of the function. Their equation correctly spanned the table of values, yet these representations did not properly model an exponential function. Not completely agreeing with the other students' mathematical representation, Ines specifically studied the table of values and helped her peers realize that they were doubling the number of ounces of cake instead of Alice's height.

Ines' revelation is a good example of how perseverance as a collective enterprise can emerge from a group dynamic. Ines' individual metacognitive awareness of the mistake during their First Attempt Phase of perseverance helped collectively move the discussion in a different direction that took into account Alice's initial height. As the rest of the group begins to buy into Ines' point of view, they begin to see each other as learners and doers of mathematics, affirming their identities as bilingual Latinx mathematics learners. Developing such agency amidst moments of struggle is essential for perseverance and building equitable and effective learning communities (Sengupta-Irving & Agarwal, 2017). At the end of the day, Ines recalled her prior experience with the Alice problem from her sophomore year and brought up the issue of starting with an initial height. This helped the others rethink about the mathematics, cross out their IN/OUT table (Figure 3), and begin to think about starting a new one.

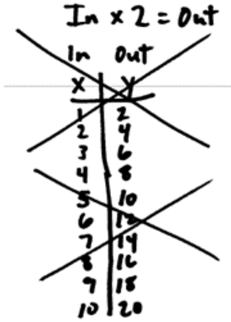

Figure 3. Their IN/OUT table, crossed out (recreated by authors)

Vignette 2: During Day 3

> INES: …empezamos de cuatro pies. (…we start at four feet.)
> INES: Si toma si come un pedacito son ocho, si come un pedacito son dieciseis, el tercer pedazo dieciseis y dieciseis. Treintaidos ¿no? (If she drinks, if she eats one piece it becomes eight, if she eats one piece it becomes sixteen, the third piece, sixteen and sixteen, thirty two, no?)
> JESSICA: Pero, ¿cómo sacastes eso? (But how did you get that?)
> INES: Porque si empezamos con cuatro pies, como yo les digo, si come un pedacito y sale, aumenta de altura de doble. (Because, if we start at four feet, like I'm telling you, if she eats one piece and it comes out to, her height grows double.)
> JESSICA: Ohh, her height doubles.
> ELENA: You know it's the same thing mira. Dos, you multiply one times two is two, two times four is eight, y si pones two times two is four, four times four is sixteen. (You know it's the same thing look. Two, you multiply one times two is two, two times four is eight, and if you put two times two is four, four times four is sixteen.)
> CARINA: In squared times 2 is equal to your out.

At the start of Day 3, Ines tried to refocus the group to understand that Alice's initial height is necessary to compute subsequent heights as she ate each ounce of cake. It is important to note that the problem was written in

English yet Ines used her agency to skillfully leverage her first language to revoice the problem. She modeled mathematically the concept of doubling for the other group members in Spanish, "empezamos de cuatro pies…si come un pedacito son ocho, si come un pedacito son dieciseis, el tercer pedazo dieciseis y dieciseis. Treintaidos, no?" Finally, Ines said in Spanish, "aumenta de altura de doble" and at the same time Ines gestured with her hand demonstrating how Alice's height doubles for each ounce of cake she eats. Immediately after Ines spoke, Jessica had her "a-ha" moment realizing that she needed to double Alice's height not the number of ounces of cake.

This translanguaging exchange exemplified further perseverant diligence of exploring what it means to double, and illustrated how the group became aware about what was flawed in their first attempt to make sense of the function. What makes this so significant is the students' commitment to a collective Latinx bilingual identity where they collaboratively draw on linguistic and mathematical resources fluidly as they continue to try to overcome the conceptual obstacle of doubling. It is the combination of all interactions that affords Jessica her liberating moment of understanding, which lead these students to amend their plan and adapt their mathematical thinking to continue to persevere. These exchanges suggest the continued development of a positive bilingual identity as this group affirmed their excitement about the progress being made and felt empowered to continue their effort toward a solution.

Following this discussion, the students created a new table of values (Figure 4) that correctly modeled Alice's exponential growth. This new approach demonstrated exiting the First Attempt Phase of perseverance and entering the Additional Attempt Phase by the group amending their plan and making a second attempt to make sense of the Alice function. Unfortunately, the equation was not correct and did not span all of their entries. Again, this is an important opportunity for the group to recognize more mistakes and continue to persevere.

Near the end of the Day 3, Carina used a graphing calculator to make a table of values for $y=2x^2$ (Figure 5) and discovered that it did not match their current table. Carina, Elena, and Ines then questioned the source of this inconsistency and ultimately realized and discussed the ways in which their equation was incorrect. The girls recognized the flaws in their attempt to model the situation with the function $y=2x^2$ and were ready to continue to explore ways to change their equation and enter another Additional Attempt Phase of perseverance but they ran out of time.

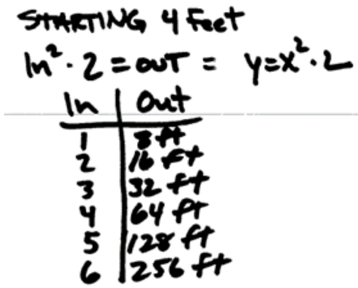

Figure 4. New table of values (recreated by authors)

X	Y_1
0	0
1	2
2	8
3	18
4	32
5	50
6	72

$Y_1 = 2 \times X^2$

Figure 5. Table on graphing calculator (recreated by authors)

The actions above showcased thoughtful diligence and evidence of metacognition as the group drew on their translanguaging repertoire to explore the Alice function more deeply and become aware of their mistakes. Although this group did not yet find the proper equation to span their IN/OUT table in this episode, they used their agency to overcome several obstacles to facilitate more deep mathematical thinking about the Alice function.

Discussion and Implications

We situate the discussion of our findings within the larger context that surrounds schools and Latinx youth. There is a misconception that the linguistic resources that students bring from home and communities have no place in schools and are often categorized from a deficit perspective (Willey, Gatza, & Flessner, 2017). This is particularly true of Latinx bilinguals within current educational setting where they experience a dehumanizing social status of their linguistic repertoire (Garcia, 2009). Our findings indicate that this assumption is false. For Latinx bilinguals Carina, Jessica, Elena, and Ines, the entirety of their twelfth-grade mathematics instruction was in English, and their teacher paid no special attention to nurturing the students' linguistic resources beyond simply allowing them to speak in their first language. Yet, the students on their own exercised agency to engage in a translanguaging practice that played a central role in their learning and asserting their identities as bilingual Latinx mathematics students.

By adopting a rehumanizing perspective that considered the ways in which Latinx bilinguals were agents of their own learning (Gutiérrez, 2017), we position these students as competent problem solvers who leveraged their translanguaging practices to help persevere with challenging mathematical ideas. In this way, this groups' in-the-moment perseverance was the coveted outcome of their translanguaging and not the arbitrary achievement of solving the problem. Learning to leverage translanguaging to persevere in the face of mathematical challenge, all while asserting a positive bilingual identity and sense of empowerment as doers and learners of mathematics, is the achievement of these Latinx bilinguals in this episode. While we acknowledge the happenings in this study may not have addressed every factor of a truly rehumanizing perspective in mathematics education (e.g. Gutierrez 2017), our findings demonstrate the interplay between translanguaging and perseverance practice in the context of developing mathematical understandings and how these practices can be a transformational power in developing academic identities for bilingual learners (Cenoz, 2017).

Reported here was only one example of a small group of Latinx bilinguals naturally persevering with a mathematical task with relatively little external support. Carina, Jessica, Elena, and Ines were not asked nor expected to persevere in this episode. Although their monolingual teacher allowed Spanish speaking during class, and although the structure of the Alice task warranted deep engagement, this group's translanguaging practice was completely spontaneous. As such, we contend Latinx bilinguals need to be better supported in their translanguaging practice. Much has been done to raise the status on resources Latinx students bring into the classroom, but there is a pressing need to develop holistic pedagogical tools that can invite, nurture, and normalize transformative translanguaging practice (Garcia-Mateus & Palmer, 2017). We echo Garcia's (2017) sentiment, "all teachers, whether bilingual or monolingual, are capable of having a translanguaging stance and are able to design translanguaging instruction" (p. 262), and we extend it specifically to the mathematics classroom.

We hope our study inspires other researchers to explore the potential of translanguaging practices to support the development of positive identities for Latinx bilingual mathematics learners. Specifically, we contend the next steps of such research must explore translanguaging instructional design, that is, how lesson plans, classroom spaces, and pedagogies can be purposefully designed to encourage student translanguaging around challenging mathematics (see Garcia, Johnson, & Seltzer, 2017 for design ideas outside of a mathematics context). We can learn from Carina, Jessica, Elena, and Ines' example that translanguaging can free Latinx students from the static notions of school language and privilege their communicative repertoires to help them persevere with challenging mathematics. However, more clear support systems must be developed to help ensure all students have access to the power of translanguaging while grappling with important mathematical ideas.

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