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Computer application for extraocular muscle analysis based on a parallel kinematics model of the eye

Aplicación computacional para el análisis de los músculos extraoculares con base en un modelo de cinemática paralela del ojo

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Abstract

We propose a mechanical model of eyeball movements based on a parallel kinematics mechanism, where the extraocular muscles can be replaced by cables. Based on this framework, we present the anatomical conditions for the eyeball movement, analyze its inverse kinematics to obtain isodeformation curves of extraocular muscles. A computer application was developed that simulates eyeball movement and displays the associated deformation curves in real time. The results are a novel contribution to the modeling of biomechanical systems using kinematics theory; the computer application is a useful resource for medical and clinical training.

Keywords: Eyeball movement; extraocular muscles; inverse kinematics; isodeformation curves; parallel kinematics mechanisms.

Resumen

Se propone un modelo mecánico para los movimientos oculares basado en un mecanismo de cinemática paralela, donde los músculos extraoculares son reemplazados por cables. Basados en este enfoque se presentan las condiciones anatómicas para el movimiento del ojo y se analiza la cinemática inversa para obtener curvas de isodeformación de los músculos extraoculares. Se desarrolló una aplicación computacional que simula el movimiento del ojo y presenta las curvas de deformación asociadas en tiempo real. Los resultados son una contribución novedosa al modelado de sistemas biomecánicos utilizando teoría cinemática; la aplicación computacional es un recurso útil para entrenamiento médico y clínico.

Palabras clave: movimiento ocular; músculos extraoculares; cinemática inversa; curvas de isodeformación; mecanismos de cinemática paralela.

1. Introduction

The eye is a highly specialized organ for capturing radiation from the electromagnetic spectrum between 390 nm and 750 nm, and therefore, information from the environment that surrounds us. Its orientation in 3D space is possible thanks to complex and very precise movements made by the extraocular muscles (EOM), whose coordination obeys well established physiological

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facts, such as, Listing's Law (LL) and Sherrington's Law (SL) [1,2].

On the one hand, several analytical models of the kinematics of the eye have been proposed in the literature; some works are based on a merely geometric description of the movements of the eyeball as [3] and [4]. There are works on the design of robotic eyes that do not recreate the actual movements of the eye or reduce the degrees of freedom to simplify their control [5, 6]. Recently, there has been interest in developing artificial eyes that resemble human eye movements in order to make robotic faces more amenable for interaction with humans [7]. Prototypes reported in literature allow for the orientation of the eyeball by the activation of dielectric elastomer actuators, cam-based or articulated linkages [8, 9, 10]. However, none of these implement physiologically inspired movements that replicate the relationship between orientation and extraocular muscles deformation.

On the other hand, theoretical descriptions have been proposed that predict muscle contraction states such as [11] and computational applications to graph isoinnervation curves such as [12], both based on static equilibrium considerations. The work presented in this paper is based on the theory of parallel kinematics robots [13] and is an improvement to the previous effort, presented in [14], which uses a simplified cable-driven platform to simulate the extraocular muscles and analyze the inverse kinematics of the eye.

This paper is organized as follows. Section 2 discusses the anatomical and physiological considerations involved in the movement of the eyeball relevant to this work. Section 3 describes a kinematical model of the movement of the eye based on a cable-driven parallel kinematics mechanism. Section 4 presents the isodeformation curves obtained from the model proposed, as well as the interface that recreates eye movement and displays the deformation values of the extraocular muscles in real time. Conclusions are presented in section 5.

2. Anatomy and physiological rules of eye movement

The movement of the human eye occurs due to the combination of three pairs of extraocular muscles. The superior rectus (SR) and the inferior rectus (IR) move the eyeball upwards and downwards, this is, supraduction and infraduction, respectively; the lateral rectus (LR) and the medial rectus (MR) move the eye outwards and inwards, this is, abduction and adduction, respectively; the superior oblique (SO) and the inferior oblique (IO) rotate the eyeball torsionally, intorsion and extorsion. In figure 1, these muscles and their associated movements can be seen for a right eye, and in table 1, the principal action of each extraocular muscle is listed as well as the secondary actions in which it is involved. The measurements of the coordinates of the origin and insertion points can be found elsewhere in the literature [15].

When the human eye rotates, some physiological rules are observed. Listing's Law (LL) states that any gaze direction the eye is pointing to, is such as if it was achieved by a rotation starting from a reference position, called the primary position, following a great arc on the sphere of the eyeball. LL implies that all of the vectors about which these rotations occur lie on a single plane, called Listing's plane.

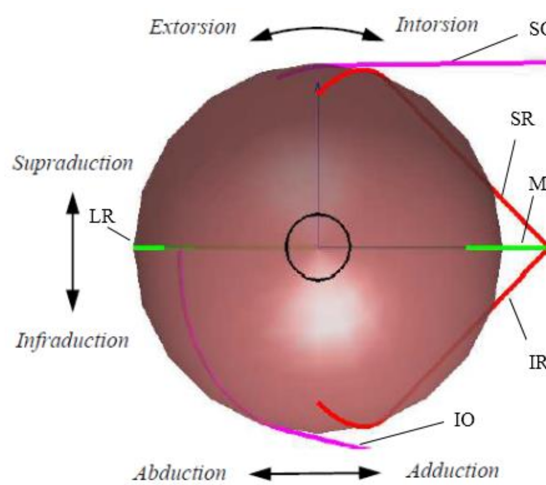

Figure 1. Movements of a right eyeball and extraocular muscles. Source: Authors.

Table 1. Principal and secondary actions of extraocular muscles.

Muscle	Actions
SR	Supraduction Adduction Intorsion
IR	Infraduction Adduction Extorsion
LR	Abduction
MR	Adduction
SO	Intorsion Infraduction Abduction
IO	Extorsion Supraduction Abduction

Source: Authors.

Referring to figure 2, **z** is the direction of the primary position, usually assumed when the eye is looking straight ahead; **s** is the unit vector representing the gaze direction at a secondary position; **n** is the unit rotation vector, perpendicular to **z** and **s**, which lies on Listing's plane; the pupil moves along a great arc, centered at the center point **c** of the eyeball, an angle θ.

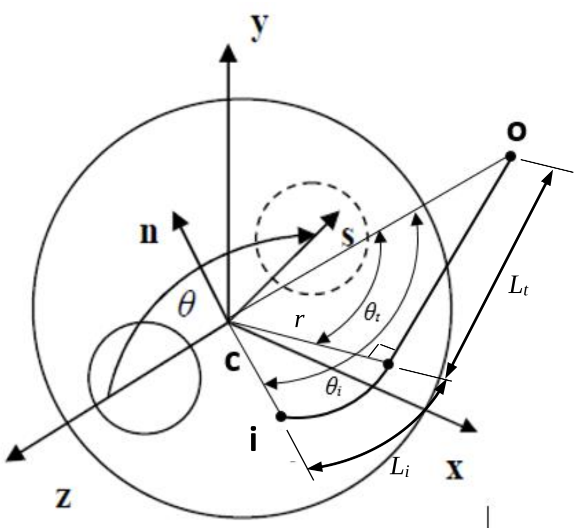

Figure 2. Listing's law and an extraocular muscle. Source: Authors

Meanwhile, Sherrington's law establishes the reciprocal innervation between each pair of extraocular muscles, that is, while on muscle is stimulated, the muscle that opposes the action of the first is relaxed. Even though Sherrington's law does not determine Listing's law, it is a physiological behavior not only observed in the extraocular muscles, but also in other pairs of muscles of the body and has been widely demonstrated by electromyography studies [2].

3. Inverse kinematics of the eyeball

The rotational movements of the human eye can be represented mathematically by means of rotation operators called quaternions. Eyeball rotation an angle θ around the unit vector **n** = (n_1, n_2, n_3), see figure 2, can be encoded in the quaternion **q** = $q_1 i + q_2 j + q_3 k + q_4$, where i, j and k are the quaternion units that follow the multiplication rules $i^2 = j^2 = k^2 = ijk = -1$. The values of the constants q_i are shown in Equation 1.

$$q_1 = n_1 sin\left(\frac{\theta}{2}\right), q_2 = n_2 sin\left(\frac{\theta}{2}\right), q_3 = n_3 sin\left(\frac{\theta}{2}\right), q_4 = cos\left(\frac{\theta}{2}\right) \qquad (1)$$

A vector **x**, with Cartesian coordinates (x_1, x_2, x_3), can be represented as a pure quaternion with no real part as **x** = $x_1 i + x_2 j + x_3 k$. The new coordinates **X** of the vector **x**, after a rotation described by **q**, can be obtained by the quaternion product **X** = **qxq***, where **q*** = $- q_1 i - q_2 j - q_3 k + q_4$ is the conjugate of **q**. A deeper treatment of quaternions can be found in theoretical kinematics textbooks [16, 17].

To carry out the inverse kinematics analysis of the eye, the equivalent cable-driven parallel kinematics mechanism shown in figure 3 is proposed. In this mechanism, the eyeball is articulated to the eye socket by a spherical joint *(S)*. Each of the extraocular muscles is attached, by spherical joints *(S)*, to the eyeball and to the skull, at the insertion and origin points, correspondingly. The contraction or stretching of each extraocular muscle is achieved by a prismatic joint *(P)*.

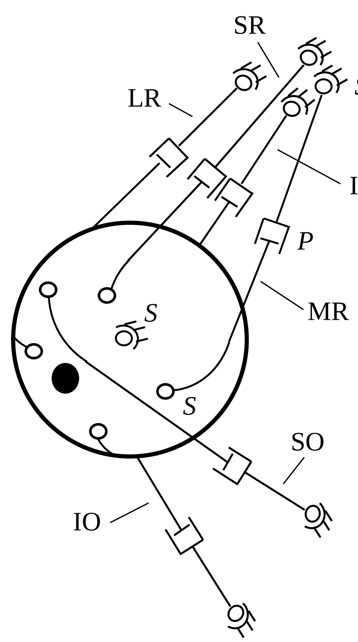

Figure 3. Equivalent cable-driven parallel kinematics mechanism of the right eye. Source: Authors

One can use the Grubler-Kutzbach criterion, Equation 2, to determine the mobility of the equivalent mechanism:

$$M = 6(n-1) - \sum_{i=1}^{5}(6-i)f_i \qquad (2)$$

where M is the mobility or number of degrees of freedom, n is the number of links and f_i is the number of joints with i degrees of freedom [18]. In this case n = 14 (i.e., the eyeball, the eye socket or skull, and each of the links attached by the P joints representing the extraocular muscles); f_3 = 13, corresponding to the S joints; and f_5 =

6, corresponding to the P joints. With these parameters a mobility $M = 9$ is obtained. However, 6 degrees of freedom are passive, which correspond to the twist of each cable, or muscle, about its own axis. Therefore, only 3 degrees of freedom are required to achieve a desired orientation of the eye. From a kinematics standpoint, the three shortest muscles are the ones that should be stimulated in order to position the eyeball.

Considering the sphere of the eyeball as a rigid body, all of its points must perform the same rotation. Therefore, applying the quaternion **q** to the insertion points of the EOMs on the eyeball will result in their position after such a rotation. Referring to Figure 2, the total length of each EOM, at a specific orientation, can be computed as the sum of the arc length of the great arc on the eyeball, L_i, connecting the insertion and tangency points of a given muscle, plus the length from the tangency point to the origin of the muscle, L_t. The latter, L_t, is a side of the right triangle formed with the radius r of the eyeball, whose hypothenuse is the distance between the center of the sphere, **c**, and the origin of the corresponding EOM, **o**. The angle subtended by L_i, whose radius is r, can be computed from the difference between the angle θ_i, obtained from the dot product of the vector representing the position of the insertion point and the origin of the EOM, minus the angle θ_t of the right triangle opposite to L_t. The deformation of each EOM, required to achieve a specific orientation, is the difference between the length, thus computed, and the corresponding length at the reference position.

The algorithm has been implemented, for a right eye of radius $r = 12$ mm and the center of the globe, **c**, as the origin of coordinates, in terms of the horizontal gaze angle θ_h, positive for adduction, negative for abduction, and the vertical gaze angle θ_v, positive for supraduction, negative for infraduction, both relative to the primary position. The procedure can be summarized as follows; refer to Figure 2 for notation used in this algorithm:

- Compute gaze direction vector, $\mathbf{s} = (\cos\theta_v \sin\theta_h, \sin\theta_v, \cos\theta_v \cos\theta_h)$.
- Compute unit axis vector and angle of rotation, $\mathbf{n} = (\mathbf{z} \times \mathbf{s})/\|\mathbf{z} \times \mathbf{s}\|$, $\theta = \arccos(\mathbf{z} \cdot \mathbf{s}/(\|\mathbf{z}\|\,\|\mathbf{s}\|))$.
- Assemble unit quaternion **q**.
- Compute location of insertion points **i** of each EOM after the rotation according to quaternion product.
- Compute angle θ_t between vector from center of the globe to tangency point and vector from the center to the origin of the EOM, $\theta_t = \arccos(r/\|\mathbf{o}\|)$.
- Compute angle θ_i between vector from center of the globe to insertion point and vector from the center to the origin of the EOM, $\theta_i = \arccos(\mathbf{i} \cdot \mathbf{o}/(\|\mathbf{i}\|\,\|\mathbf{o}\|))$.
- Compute arc length of EOM segment wrapped around the globe, $L_i = r\,(\theta_i - \theta_t)$.
- Compute length of EOM segment from tangency point to origin, $L_t = (\|\mathbf{o}\|^2 - r^2)^{1/2}$
- Compute deformation of EOM, $\Delta L = (L_i + L_t) - L_o$, where L_o is the EOM length at primary position.

4. Results and discussion

4.1. Isodeformation curves

Figure 4 shows level plots of equal deformation, i.e. isodeformation curves, of the six extraocular muscles of a right eye, in a range of horizontal and vertical rotation of +/- 20°. The plots were obtained using the command *contour* in MATLAB®. These curves clearly show the agonist-antagonist role of each pair of muscles. It is noted that the superior and inferior recti have a share on adduction whether the eye is looking up or downwards, and that the superior and inferior obliques are contracted during infraduction and supraduction, respectively.

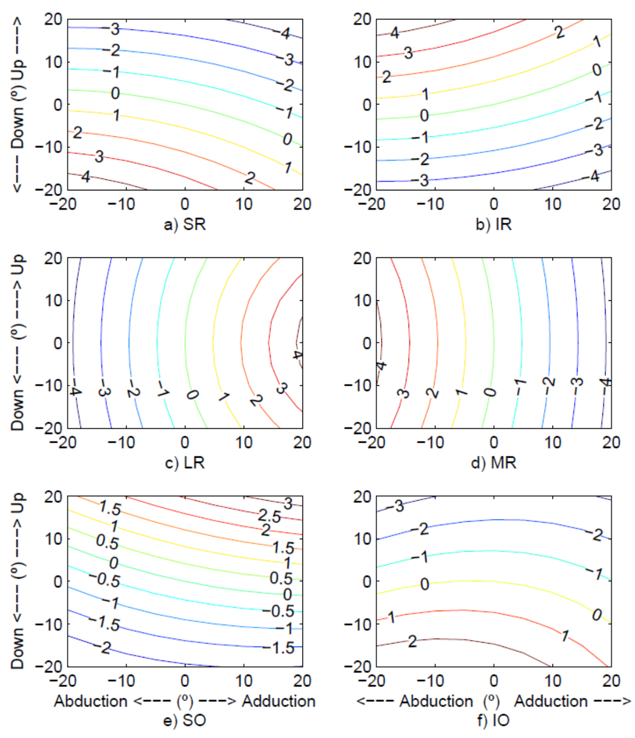

Figure 4. Isodeformation curves, in mm, for extraocular muscles of a right eye. Source: Authors

Moreover, the three actuators for supraduction and abduction are the SR, the LR and the IO, and the three actuators for infraduction and adduction are the IR, the MR and the SO. It is noticeable that the pattern observed in isodeformation curves is similar to that obtained using electromyography for isoinnervation curves in monkeys [19]. This result offers a hint that the brain may position the eye performing an inverse kinematics procedure such as that described above. However, it is not the purpose of this work to claim that this is the case. Nevertheless, it does provide a feasible approach to model the kinematic behavior of the eye.

On the other hand, Listing's law seems to be neurologically rather than mechanically dictated, because it has been incorporated a priori in the inverse kinematics procedure, however, in a human engineered system different criteria could be implemented in order to achieve a desired orientation.

Sherrington's law is also evident in the plots. Whenever the SR is contracted, the IR is not, and vice versa; this reciprocity is also observed between the LR and the MR.

4.2. Computer application

A computer implementation was developed in MATLAB that allows to play with different orientations of the eye and see, in real time, the deformations of the extraocular muscles required to generate this position. As can be observed in figure 5, it includes a solid model representation of the eye and its movements can be controlled by means of two sliders, one for the horizontal orientation and the other for the vertical orientation. The corresponding angles, θ_h and θ_v, respectively, in the previous section, appear at the bottom of this 3D representation.

The isodeformation diagrams for each of the extraocular muscles are displayed on the left side of the interface. As the 3D representation of the eye moves, the deformation level of each muscle is indicated with a marker on each diagram.

This interface can be used as a didactic tool for the teaching of extraocular muscle physiology, as well as a training tool for medical and bioengineering students and technicians in optometry.

5. Conclusions

An equivalent cable-driven parallel kinematics mechanism of the eyeball and its extraocular muscles was presented. The deformation curves for the extraocular muscles were determined based on the inverse kinematics of the equivalent mechanism, in which Listing's law is incorporated. The results are presented for a right eye, however, the described algorithm can be applied to a left eye. It only takes changing the corresponding insertion and origin points of the extraocular muscles; the results would be a mirror of the isodeformation curves presented in this paper.

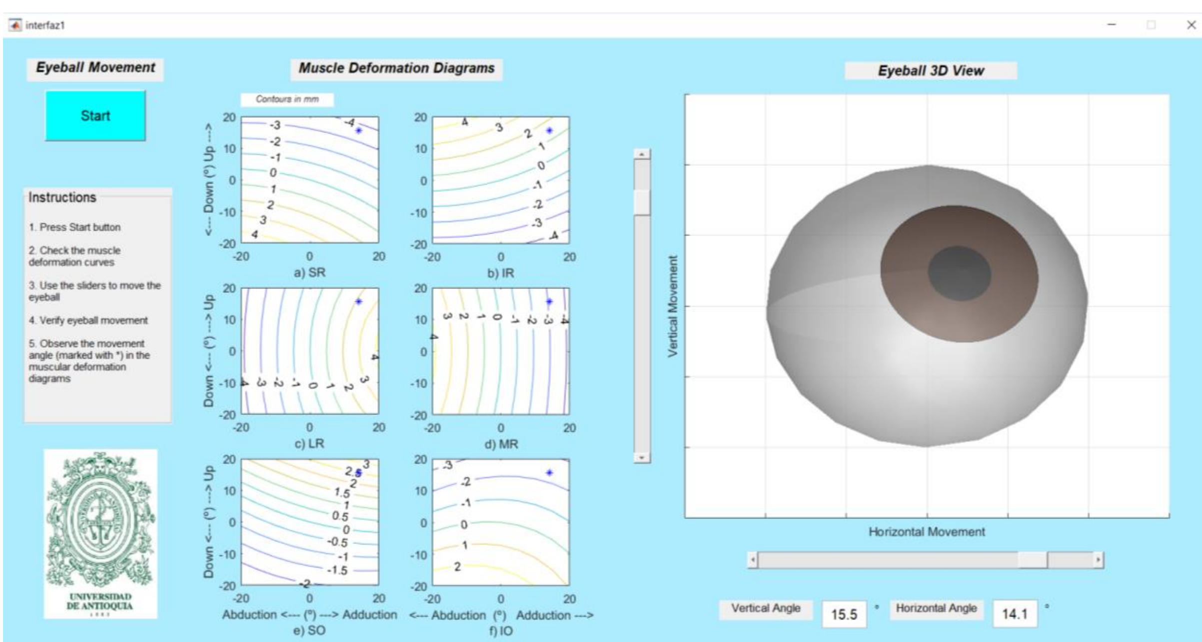

Figure 5. Didactic interface displaying eyeball orientation, isodeformation curves and markers indicating the associated level of deformation of each extraocular muscle. Source: Authors.

It is also interesting to note that Sherrington's law is observed in the isodeformation curves and that they follow the same patterns obtained in electromyography studies of the extraocular muscles in monkeys. This is a feasible modeling approach to the kinematic behavior of the eye which may prove useful for biomechanical modeling.

A computer application with a didactic interface was developed in MATLAB. It allows for visualization of eye movements, deformation curves, and the specific deformation state of the extraocular muscles associated with a desired orientation of the eyeball. This interface is proposed as a didactic tool for medical and clinical training.

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