Properties of faintly ω-continuous functions

Propiedades de las funciones débilmente continuas

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Abstract. In [3] the authors, introduced the notion of faint ω-continuity in topological spaces. In this paper, we investigate new characterizations of this type of continuity.

Keywords: ω-open sets, ω-continuity, faintly ω-continuity.

Resumen. En [3], los autores introdujeron la nocion de ω-continuidad débil en espacios topologicos. En este artículo, investigamos nuevas caracterizaciones de este tipo de continuidad.

Palabras claves: conjuntos ω-abiertos, ω-continuidad, ω-continuidad débil.

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1. Introduction and Preliminaries

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real Analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized closed sets. Recently, as generalization of closed sets, the notion of ω-closed sets were introduced and studied by Hdeib [5]. A point $x \in X$ is called a condensation point of A if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable. A is said to be ω-closed [5], if it contains all its condensation points. The complement of an ω-closed set is said to be an ω-open set. It is well known that a subset W of a space (X, τ) is ω-open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U \setminus W$ is countable. The family of all ω-open subsets of a topological space (X, τ) forms a topology on X finer than τ. The ω-closure and the ω-interior, that can be defined in the same way as $Cl(A)$ and $Int(A)$, respectively, will be denoted by $\omega Cl(A)$ and $\omega Int(A)$, respectively. The family of all ω-open subsets of a topological space

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(X, τ) is denoted by τ_ω. A point $x \in X$ is called a θ-cluster point of A if $Cl(V) \cap A \neq \varnothing$ for every open set V of X containing x. The set of all θ-cluster points of A is called the θ-closure of A and it is denoted by $Cl_\theta(A)$. If $A = Cl_\theta(A)$, then A is said to be θ-closed. The complement of θ-closed set is said to be θ-open. The union of all θ-open sets contained in a subset A is called the θ-interior of A and it is denoted by $Int_\theta(A)$. It follows from [16] that the collection of θ-open sets in a topological space (X, τ) forms a topology τ_θ on X. In [3] the authors, introduced the notion of faint ω-continuity in topological spaces. In this paper, we investigate more properties of this type of continuity. Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f : (X, \tau) \to (Y, \sigma)$ (or simply $f : X \to Y$) denotes a function f of a space (X, τ) into a space (Y, σ).

Definition 1.1. A subset A of a space (X, τ) is said to be regular open [15] if $A = Int(Cl(A))$. The complement of a regular open set is called a regular closed set. The family of all regular open (resp. regular closed) subsets of (X, τ) is denoted by $RO(X)$ (resp. $RC(X)$).

Definition 1.2. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be:

(i) Strongly continuous [8] if $f^{-1}(V)$ is both open and closed in X for each subset V of Y.

(ii) Faintly continuous [9] if $f^{-1}(V)$ is open in X for every θ-open set V of X.

(iii) Slightly ω-continuous functions [11] if $f^{-1}(V)$ is ω-open in X for every clopen set V of X.

(iv) Weakly ω-continuous [3] if for each $x \in X$ and each open set V containing $f(x)$ there exists $U \in \omega O(X, x)$ such that $f(U) \subseteq Cl(V)$.

(v) Almost ω-continuous [12] if $f^{-1}(V)$ is ω-open in X for every regular open subset V of Y.

(vi) ω-irresolute [2] if $f^{-1}(V)$ is ω-closed (resp. ω-open) in X for every ω-closed (resp. ω-open) subset V of Y.

2. Faintly ω-continuous functions

Definition 2.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be faintly ω-continuous [3] (resp. ω-continuous [6]) at a point $x \in X$ if for each θ-open (resp. open) set V of Y containing $f(x)$, there exists $U \in \omega O(X, x)$ such that $f(U) \subset V$. If f has this property at each point of X, then it is said to be faintly ω-continuous (resp. ω-continuous [6]).

Theorem 2.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(i) f is faintly ω-continuous.

(ii) $f^{-1}(V)$ is ω-open in X for every θ-open set V of Y.

(iii) $f^{-1}(F)$ *is* ω*-closed in* X *for every* θ*-closed subset* F *of* Y*.*

(iv) $f : (X, \tau_\omega) \to (Y, \sigma_\theta)$ *is continuous.*

(v) $f : (X, \tau_\omega) \to (Y, \sigma)$ *is faintly continuous.*

(vi) $f : (X, \tau) \to (Y, \sigma_\theta)$ *is* ω*-continuous.*

(vii) $\omega Cl(f^{-1}(B)) \subseteq f^{-1}(Cl_\theta(B))$ *for every subset* B *of* Y*.*

(viii) $f^{-1}(Int_\theta(G)) \subseteq \omega Int(f^{-1}(G))$ *for every subset* G *of* Y*.*

Proof. (i)⇒(ii). Let V be an θ-open set of Y and $x \in f^{-1}(V)$. Since $f(x) \in V$ and f is faintly ω-continuous, there exists $U \in \omega O(X, x)$ such that $f(U) \subset V$. It follows that $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is ω-open in X.
(ii)⇒(i). Let $x \in X$ and V be an θ-open set of Y containing $f(x)$. By (ii), $f^{-1}(V)$ is an ω-open set containing x. Take $U = f^{-1}(V)$. Then $f(U) \subset V$. This shows that f is faintly ω-continuous.
(ii)⇒(iii). Let V be any θ-closed set of Y. Since $Y \setminus V$ is an θ-open set, by (2), it follows that $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is ω-open. This shows that $f^{-1}(V)$ is ω-closed in X.
(iii)⇒(ii). Let V be an θ-open set of Y. Then $Y \setminus V$ is θ-closed in Y. By (iii), $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is ω-closed and thus $f^{-1}(V)$ is ω-open.
(i)⇔(iv)⇔(v)⇔(vi) and (iii)⇔(vii)⇔(viii) are obvious. □

Theorem 2.3. *If a function* $f : (X, \tau) \to (Y, \sigma)$ *is* ω*-continuous, then it is faintly* ω*-continuous.*

Proof. Let $x \in X$ and V be a θ-open set containing $f(x)$. Then, there exists an open set W such that $f(x) \in W \subset Cl(W) \subset V$. Since f is ω-continuous, there exists an ω-open set U containing x such that $f(U) \subset W \subset V$. Therefore, f is faintly ω-continuous. □

The following example shows that the converse of Theorem 2.3 is not true in general.

Example 2.4. Let $X = \Re$ with the topology $\tau = \{\emptyset, \Re, Q\}$, and $Y = \{a, b, c\}$ with the topology $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$. Take a fixed number $e \in \Re - Q$, and define $f : (\Re, \tau) \to (Y, \sigma)$ as follows:

$$f(x) = \begin{cases} a & \text{if } x \in Q \text{ or } x = e \\ b & \text{if } x \in \Re - Q \text{ and } x \neq e. \end{cases}$$

Then f is faintly-ω-continuous but is not ω-continuous.

Definition 2.5. A topological space (X, τ) is said to be an ω-space [1] if every ω-open subset of (X, τ) is open.

Theorem 2.6. *Let* (Y, σ) *be an* ω*-space. Then a function* $f : (X, \tau) \to (Y, \sigma)$ *is faintly* ω*-continuous if and only if it is faintly continuous.*

Proof. Follows from the Definition 2.5. □

Definition 2.7. A topological space (X, τ) is said to be almost-regular [13] if for each regular closed set F of X and each point $x \notin F$, there exist disjoint open sets U and V of X such that $x \in U$ and $F \subset V$.

Theorem 2.8. *If $f : (X, \tau) \to (Y, \sigma)$ is faintly ω-continuous function and (Y, σ) is almost-regular, then f is almost ω-continuous.*

Proof. Let $x \in X$ and V be any regular open set of (Y, σ) containing $f(x)$. Since every regular open set in an almost-regular space is θ-open [9], V is θ-open. Since f is faintly ω-continuous, there exists $U \in \omega O(X, x)$ such that $f(U) \subset V$. It follows from Theorem 2.2 that f is almost ω-continuous. □

Corollary 2.9. *Let (Y, σ) be an almost-regular space. Then for a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(i) f is almost ω-continuous.

(ii) f is weakly ω-continuous.

(iii) f is faintly ω-continuous.

Proof. The proof follows from Theorems 2.3 and 2.8. □

Theorem 2.10. *If $f : (X, \tau) \to (Y, \sigma)$ is faintly ω-continuous function and (Y, σ) is a regular space, then f is ω-continuous.*

Proof. Let V be any open set of Y. Since Y is regular, V is θ-open in Y. Since f is faintly ω-continuous, by Theorem 2.2, we have $f^{-1}(V)$ is ω-open and hence f is ω-continuous. □

Theorem 2.11. *If $f : (X, \tau) \to (Y, \sigma)$ is a faintly ω-continuous function, then it is slightly ω-continuous.*

Proof. Let $x \in X$ and V be any clopen subset of (Y, σ) containing $f(x)$. Then V is θ-open in Y. Since f is faintly ω-continuous, there exists $U \in \omega O(X, x)$ containing x such that $f(U) \subset V$. This shows that f is slightly ω-continuous. □

Example 2.12. Let $X = \Re$ with the topology $\tau = \{\emptyset, \Re, \Re - Q\}$, and $Y = \Re$ with the topology $\sigma = \{\emptyset, \Re, Q\}$. Define $f : (X, \tau) \to (Y, \sigma)$ as the identity function. Then f is slightly ω-continuous but is not faintly ω-continuous.

Definition 2.13. Let (X, τ) be a topological space. Since the intersection of two clopen sets of (X, τ) is clopen, the clopen sets of (X, τ) may be use as a base for a topology for X. This topology is called the ultra-regularization of τ [10] and is denoted by τ_u. A topological space (X, τ) is said to be ultra-regular [4] if $\tau = \tau_u$.

Theorem 2.14. *Let (Y, σ) be an ultra-regular space. Then for a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(i) f is ω-continuous.

(ii) f is almost ω-continuous.

(iii) f is weakly ω-continuous.

(iv) f *is faintly* ω*-continuous.*

(v) f *is slightly* ω*-continuous.*

Proof. The proof follows from definitions and Theorems 2.3, 2.8 and 2.11. $\square$

Definition 2.15 ([1])**.** A ω-frontier of a subset A of (X, τ) is $\omega Fr(A) = \omega Cl(A) \cap \omega Cl(X \setminus A)$.

Theorem 2.16. *The set of all points* $x \in X$ *in which a function* $f : (X, \tau) \to (Y, \sigma)$ *is not faintly* ω*-continuous is the union of* ω*-frontier of the inverse images of* θ*-open sets containing* $f(x)$*.*

Proof. Suppose that f is not faintly ω-continuous at $x \in X$. Then there exists a θ-open set V of Y containing $f(x)$ such that $f(U)$ is not contained in V for each $U \in \tau_\omega$ containing x and hence $x \in Cl_\theta(X \setminus f^{-1}(V))$. On the other hand, $x \in f^{-1}(V) \subset \omega Cl(f^{-1}(V))$ and hence $x \in \omega Fr(f^{-1}(U))$. Conversely, suppose that f is faintly ω-continuous at $x \in X$ and let V be a θ-open set of Y containing $f(x)$. Then there exists $U \in \tau_\omega$ containing x such that $U \subset f^{-1}(V)$. Hence $x \in Int_\theta(f^{-1}(V))$. Therefore, $x \in \omega Fr(f^{-1}(V))$ for each open set V of Y containing $f(x)$. $\square$

Theorem 2.17. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a function and* $g : (X, \tau) \to (X \times Y, \tau \times \sigma)$ *the graph function of* f*, defined by* $g(x) = (x, f(x))$ *for every* $x \in X$*. If* g *is faintly* ω*-continuous, then* f *is faintly* ω*-continuous.*

Proof. Let U be a θ-open set in (Y, σ), then $X \times U$ is a θ-open set in $X \times Y$. It follows that $f^{-1}(U) = g^{-1}(X \times U) \in \tau_\omega$. This shows that f is faintly ω-continuous. $\square$

Definition 2.18. A space (X, τ) is said to be ω-connected [1] if X cannot be written as a union of two nonempty disjoint ω-open sets.

Theorem 2.19. *If* $f : (X, \tau) \to (Y, \sigma)$ *is a faintly* ω*-continuous function and* (X, τ) *is a* ω*-connected space, then* Y *is a connected space.*

Proof. Assume that (Y, σ) is not connected. Then there exist nonempty open sets V_1 and V_2 such that $V_1 \cap V_2 = \varnothing$ and $V_1 \cup V_2 = Y$. Hence we have $f^{-1}(V_1) \cap f^{-1}(V_2) = \varnothing$ and $f^{-1}(V_1) \cup f^{-1}(V_2) = X$. Since f is surjective, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty subsets of X. Since V_i is open and closed, V_i is θ-open for each $i = 1, 2$. Since f is faintly ω-continuous, $f^{-1}(V_i) \in \tau_\omega$. Therefore, (X, τ) is not ω-connected. This is a contradiction and hence (Y, σ) is connected. $\square$

Definition 2.20. A space (X, τ) is said to be ω-compact [1] (resp. θ-compact [7]) if each ω-open (resp. θ-open) cover of X has a finite subcover.

Theorem 2.21. *The surjective faintly* ω*-continuous image of a* ω*-compact space is* θ*-compact.*

***Proof*.** Let $f : (X, \tau) \to (Y, \sigma)$ be a faintly ω-continuous function from a ω-compact space X onto a space Y. Let $\{G_\alpha\colon \alpha \in I\}$ be any θ-open cover of Y. Since f is faintly ω-continuous, $\{f^{-1}(G_\alpha)\colon \alpha \in I\}$ is a ω-open cover of X. Since X is ω-compact, there exists a finite subcover $\{f^{-1}(G_i) : i = 1, 2, \ldots. n\}$ of X. Then it follows that $\{G_i\colon i = 1, 2, \ldots. n\}$ is a finite subfamily which cover Y. Hence Y is θ-compact. □

Definition 2.22. A space (X, τ) is said to be:

(i) Countably ω-compact (resp. countably θ-compact) if every ω-open [1] (resp. θ-open) countable cover of X has a finite subcover.

(ii) ω-Lindelof [1] (resp. θ-Lindelof) if every ω-open (resp. θ-open) cover of X has a countable subcover.

Theorem 2.23. *Let $f : (X, \tau) \to (Y, \sigma)$ be a faintly ω-continuous surjective function. Then the following statements hold:*

(i) If X is ω-Lindelof, then Y is θ-Lindelof.

(ii) If X is countably ω-compact, then Y is countably θ-compact.

***Proof*.** The proof is similar to Theorem 2.21. □

3. Separation Axioms

Definition 3.1. A topological space (X, τ) is said to be:

(i) ω-T_1 [1] (resp. θ-T_1) if for each pair of distinct points x and y of X, there exists ω-open (resp. θ-open) sets U and V containing x and y, respectively such that $y \notin U$ and $x \notin V$.

(ii) ω-T_2 [1] (resp. θ-T_2 [14]) if for each pair of distinct points x and y in X, there exists disjoint ω-open (resp. θ-open) sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 3.2. *If $f : (X, \tau) \to (Y, \sigma)$ is faintly ω-continuous injection and Y is a θ-T_1 space, then X is an ω-T_1 space.*

***Proof*.** Suppose that Y is θ-T_1. For any distinct points x and y in X, there exist $V, W \in \sigma_\theta$ such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is faintly ω-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are ω-open subsets of (X, τ) such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is ω-T_1. □

Theorem 3.3. *If $f : (X, \tau) \to (Y, \sigma)$ is faintly ω-continuous injection and Y is a θ-T_2 space, then X is an ω-T_2 space.*

***Proof*.** Suppose that Y is θ-T_2. For any pair of distinct points x and y in X, there exist disjoint θ-open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is faintly ω-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are ω-open in X containing x and y, respectively. Therefore, $f^{-1}(U) \cap f^{-1}(V) = \varnothing$ because $U \cap V = \varnothing$. This shows that X is ω-T_2. □

Definition 3.4. A space (X, τ) is called strongly θ-regular (resp. strongly ω-regular) if for each θ-closed (resp. ω-closed) set F and each point $x \notin F$, there exist disjoint θ-open (resp. ω-open) sets U and V such that $F \subset U$ and $x \in V$.

Definition 3.5. A space (X, τ) is said to be strongly θ-normal (resp. strongly ω-normal) if for any pair of disjoint θ-closed (resp. ω-closed) subsets F_1 and F_2 of X, there exist disjoint θ-open (resp. ω-open) sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Definition 3.6. A function $f : (X, \tau) \to (Y, \sigma)$ is called:

(i) $\omega\theta$-open if $f(V) \in \sigma_\theta$ for each $V \in \tau_\omega$.

(ii) $\omega\theta$-closed if $f(V)$ is θ-closed in Y for each $V \in \omega C(X)$.

Theorem 3.7. *If f is faintly ω-continuous $\omega\theta$-open injective function from a strongly ω-regular space (X, τ) onto a space (Y, σ), then (Y, σ) is strongly θ-regular.*

Proof. Let F be an θ-closed subset of Y and $y \notin F$. Take $y = f(x)$. Since f is faintly ω-continuous, $f^{-1}(F)$ is ω-closed in X such that $f^{-1}(y) = x \notin f^{-1}(F)$. Take $G = f^{-1}(F)$. We have $x \notin G$. Since X is strongly ω-regular, then there exist disjoint ω-open sets U and V in X such that $G \subset U$ and $x \in V$. We obtain that $F = f(G) \subset f(U)$ and $y = f(x) \in f(U)$ such that $f(U)$ and $f(V)$ are disjoint θ-open sets. This shows that Y is strongly θ-regular. □

Theorem 3.8. *If f is faintly ω-continuous $\omega\theta$-open injective function from a strongly ω-normal space (X, τ) onto a space (Y, σ), then Y is strongly θ-normal.*

Proof. Let F_1 and F_2 be disjoint θ-closed subsets of Y. Since f is faintly ω-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are ω-closed sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \varnothing$. Since X is strongly ω-normal, there exist disjoint ω-open sets A and B such that $U \subset A$ and $V \subset B$. We obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that $f(A)$ and $f(B)$ are disjoint θ-open sets. Thus, Y is strongly θ-normal. □

Recall that for a function $f : (X, \tau) \to (Y, \sigma)$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 3.9. A graph $G(f)$ of a function $f : (X, \tau) \to (Y, \sigma)$ is said to be θ-ω-closed if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \omega O(X, x)$ and $V \in \sigma_\theta$ containing y such that $(U \times V) \cap G(f) = \varnothing$.

Lemma 3.10. *A graph $G(f)$ of a function $f : (X, \tau) \to (Y, \sigma)$ is θ-ω-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \omega O(X, x)$ and $V \in \sigma_\theta$ containing y such that $f(U) \cap V = \varnothing$.*

Proof. It is an immediate consequence of Definition 3.9. □

Theorem 3.11. *If $f : (X, \tau) \to (Y, \sigma)$ is faintly ω-continuous function and (Y, σ) is θ-T_2, then $G(f)$ is θ-ω-closed.*

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$, then $f(x) \neq y$. Since Y is θ-T_2, there exist θ-open sets V and W in Y such that $f(x) \in V, y \in W$ and $V \cap W = \varnothing$. Since f is faintly ω-continuous, $f^{-1}(V) \in \omega O(X, x)$. Take $U = f^{-1}(V)$. We have $f(U) \subset V$. Therefore, we obtain $f(U) \cap V = \varnothing$. This shows that $G(f)$ is θ-ω closed. □

Theorem 3.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function with a θ-ω-closed graph $G(f)$. If f is a faintly ω-continuous injection, then (X, τ) is ω-T_2.*

Proof. Let x and y be any two distinct points of X. Then since f is injective, we have $f(x) \neq f(y)$. Then, we have $(x, f(y)) \in (X \times Y) \setminus G(f)$. By Lemma 3.10, there exist $U \in \tau_\omega$ and $V \in \sigma_\theta$ such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \varnothing$. Hence $U \cap f^{-1}(V) = \varnothing$ and $y \notin U$. Since f is faintly ω-continuous, there exists $W \in \omega O(X, y)$ such that $f(W) \subset V$. Therefore, we have $f(U) \cap f(W) = \varnothing$. Since f is injective, we obtain $U \cap W = \varnothing$. This implies that (X, τ) is ω-T_2. □

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