Hull girder: Forced vibration analysis by propeller excitations

Viga buque: Análisis de vibraciones forzadas por excitaciones de la hélice

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Abstract

The non-uniform wake around the propeller generates fluctuating forces on the propulsion shaft. This article presents a methodology used for the forced vibrations analysis of hull girder due to this propeller excitation. This approach is applied to a research boat considering the propeller working in the operating range using a finite element model including all ship structures, rudder, and propulsion lines with their respective supports. Added mass and damping in all submerged elements were also considered. Vibration levels acting in the vessel structure are compared with the limits proposed by ISO 6954 (2000).

Key words: lateral vibration, finite element model.

Resumen

La estela no uniforme alrededor de la hélice genera fuerzas fluctuantes en el eje de propulsión. Este artículo presenta una metodología usada para el análisis forzado de vibración de la viga buque debida a esta excitación de la hélice. Este procedimiento es aplicado a una lancha de investigación usando el método de elementos finitos incluyendo todas las estructuras de la nave, timón y líneas de propulsión con sus respectivos apoyos, considerando la hélice en el rango de operación. La masa añadida y amortiguamiento de todos los elementos sumergidos también se consideran en el análisis. Los niveles de vibración obtenidos en la estructura de la embarcación se comparan con los límites propuestos por ISO 6954 (2000).

Palabras claves: vibración lateral, modelo de elementos finitos, modos de vibración de viga buque.

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Introduction

Dynamic propeller forces need to be included to accurately verify that the hull girder supports all loads acting on it. These forces are a function of thrust, torque, and propeller frequency. Namely, it is a function of: i) rotational speed for fixed pitch propellers, and ii) pitch angle for controllable pitch propellers.

In the present work, Finite Element Method was used to estimate the deformation due to these forces acting over the hull girder. This method allows modeling hull girder considering all structural elements and the propeller dynamic forces. Numerical results expressed in RMS speed of vibration are compared to the limits proposed by the Classification Societies.

Finite Element Method Applied to the Hull Girder

There are several recommendations for the development of a Finite Element Model (FEM), especially by Classification Societies, in the present work; it was necessary to include mass and inertia of the structural elements of hull and superstructure, incorporating machinery foundations for hull girder vibration analysis.

Shell elements were used in the hull and in primary structures, and frame elements to model the secondary structures and pillars, see Figs. 1 and 2. The mass of the structure, equipment, tank liquid, and the added mass values of 40 and 20 [kg/m^2] on decks and sides respectively were distributed on

Fig. 1. Ship Finite Element Model.

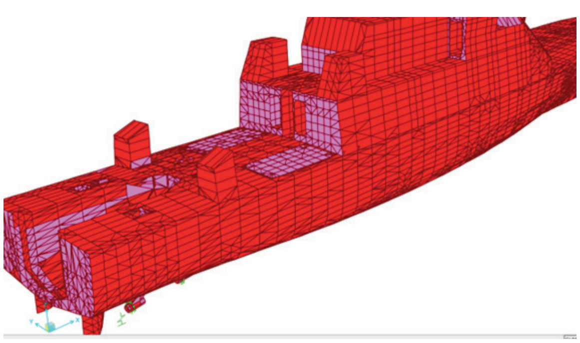
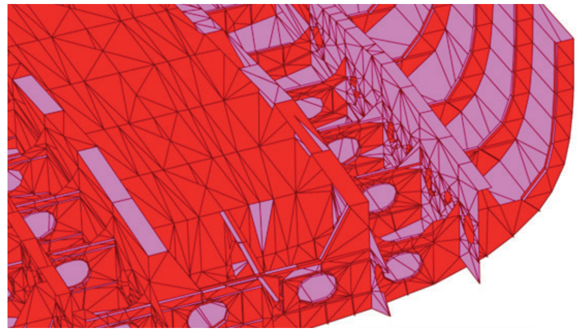


Fig. 2. FEM interior view: Engine room.

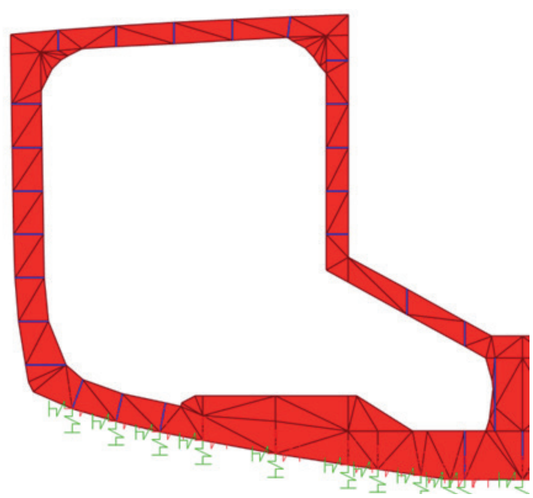


the corresponding nodes following Germanischer Lloyd (GL) recommendation.

Hydroelastic Hull Behavior

Restraints were placed to simulate the ship behavior into the water, using an equivalent spring system placed in the submerged surface of the hull to balance the ship weight. Spring stiffness was calculated using the volume of water displaced within specific sections, see Fig. 3. To verify that the restrictions are conveniently applied, each node deformation was verified using static weight of the vessel.

Fig. 3. Aft frame – Hull bottom restraints.



Hull added mass estimation

On elements that are submerged in water vibration moves a small fluid volume; its mass is called added mass. This mass is added to FEM model as a distributed mass over all submerged elements.

The added mass can be obtained from Seakeeping analysis for each speed and sea state [Lewis, FM 1929]. This mass is a function of the vessel encounter frequency,

$$\omega_e = \omega + (\omega^2/g)(U.\cos\mu) \quad (1)$$

Where ω_e = Encounter frequency (rad/s), ω = Wave frequency (rad/s), g= Gravity acceleration (m/s^2), U = Ship speed (m/s), and μ = Wave incidence angle (rad)

Added mass values for three different ship speeds with the same sea state 3 and following seas are shown in Table 1.

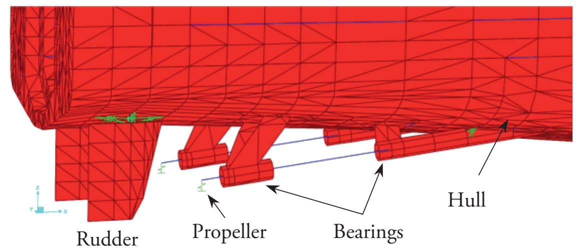
Table 1. Hull added mass.

Vel.	**Sea State**	**ωe**	**Added mass**	**% of displacement**
Knts		rad/s	ton	%
11	3	1.24	483	173%
18	3	1.5	436.9	157%
21	3	1.61	425.1	152%

Propulsion Line Behavior

The propulsion line transmits the thrust of the ship as well as the exciting forces from the propeller. It's lateral, axial, and torsional natural frequencies need to be considered to assess resonance. Figs. 4 and 5 show the FEM propulsion line included in this analysis.

Fig 5. Propulsion line in FEM.



Bearings location

Cutless bearings or roller bearings are included in our FEM model. Usually, bearing center is the support point, except for bearing close to the propeller, which is considered to 1/3 from the aft end of bearing.

The propulsion line natural frequencies depend on bearing position and stiffness. In the present study, 3 bearings were used, as shown in Fig. 6. Two aft supports are bronze – rubber cutless bearings. The propeller shaft had been modeled using beam elements. The propeller and flanges masses are included in their respective locations. The manufacturer provided the bearing stiffness value for accurate results.

Propeller added mass

The propeller accelerates its surrounding water and an added mass is generated, that was estimated using PRAMAD program [U OF MICHIGAN, 1980]. There are several formulas for estimating these masses M. Parsons (1980), Schwanecke (1963) or D. MacPherson (2007).

Fig. 4. Longitudinal cut at propulsion line.

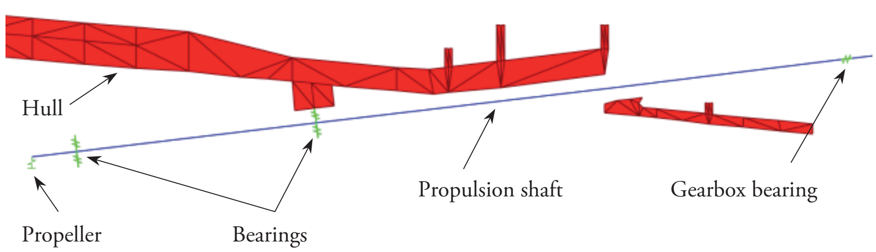


Fig. 6. Propulsion line drawing.

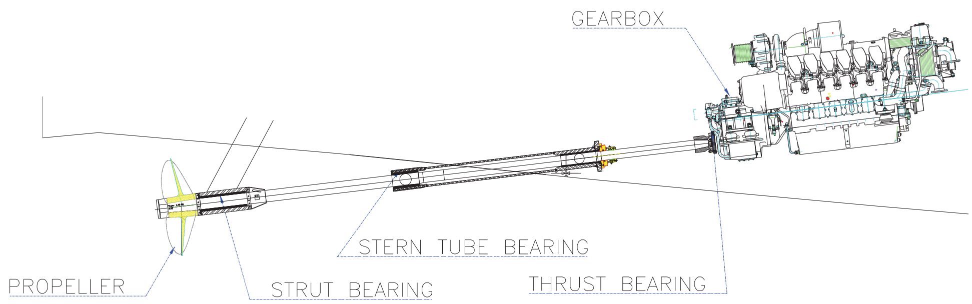


Table 2. Propeller added mass.

Direction	Units	Value
M11	N.s2/m	332.6
M22	N.s2/m	48.9
M52	N.s2	37.2
M55	N.m.s2	40.0

Fig. 7. Added mass nomenclature.

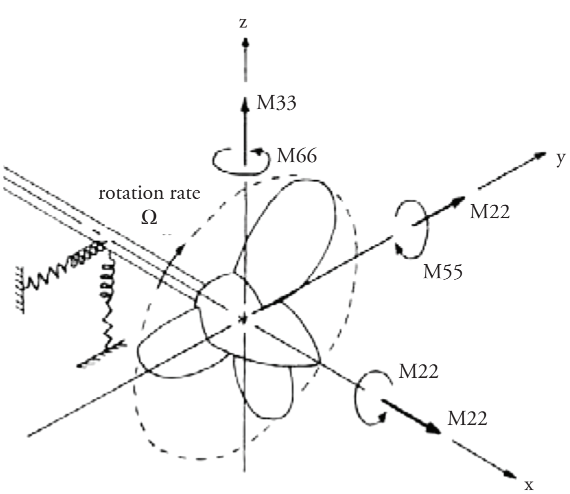
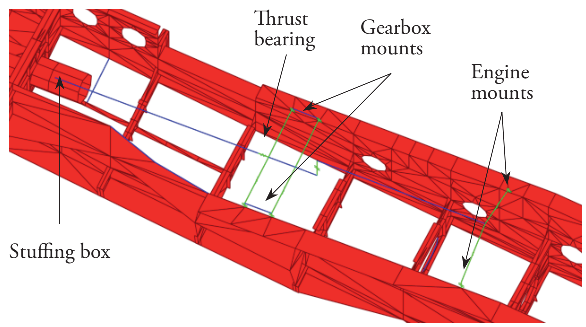


Fig. 8. Thrust bearing and engine/gearbox mounts.



Thrust bearing and engine/gearbox flexible coupling Stiffness

In our model, thrust bearings are placed on the gearbox; and flexible mounts of gearbox & engine with the stiffness provided by manufacturer. Mounts properties were modeled for each direction (x , y , z). Fig. 8 shows the position of the elements used.

Natural vibration analysis

Once the mentioned hull and propulsion line properties are modeled, vibration analysis can be performed for both, hull girder and propulsion line natural frequencies.

The FEM and the eigenvalue matrix method had been used to calculate the propulsion system vibrational modes. The finite element method divides a body in finite elements interconnected by nodes, which are equivalent to the original body; in the elastic zone the equations to find the nodes deformation can be expressed in matrix form as follows:

$$[M]\{Y\} + [K]\{\ddot{Y}\} = \{0\} \quad (2)$$

where:

$[M]$ is the mass matrix of the system
$[K]$ is the stiffness matrix of the system
$\{Y\}$ is the displacement vector
$\{\ddot{Y}\}$ is the second derivative of displacement Y

Nowadays, computers allow this calculation accurately and for several degrees of freedom.

Tables 3 and 4 present the natural frequencies in the system working range.

Table 3. Hull girder natural frequencies.

Vertical direction			
Mode 1	**Mode 2**	**Mode 3**	**Mode 4**
Hz	Hz	Hz	Hz
3.59	7.46	15.16	28.3
Horizontal direction			
Mode 1	**Mode 2**	**Mode 3**	**Mode 4**
Hz	Hz	Hz	Hz
5.59	10.23	16.17	28.27

Table 4. Propulsion line natural frequencies.

Mode 1	**Mode 2**
Hz	Hz
23.49	29.73

Figs. 9 and 10 show the modal shape of natural frequencies.

It is recommended that the working range has to be from 650 RPM to 2000 RPM on engine, due to coincidence between hull girder natural frequencies and engine and propeller excitation range as can be seen in Table 5.

Fig. 9. Vertical direction mode shapes.

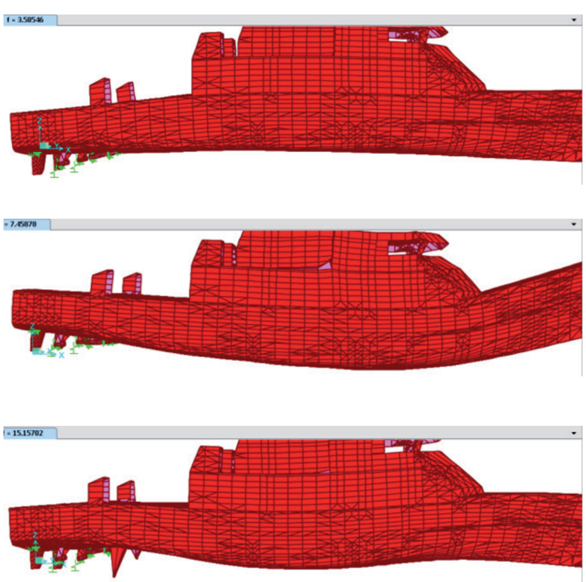


Fig. 10. Horizontal direction mode shapes.

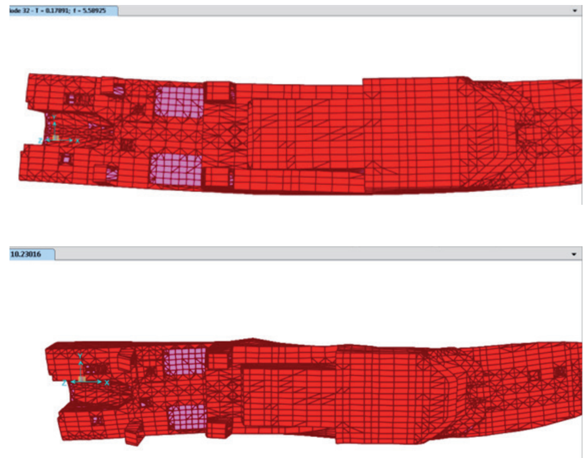


Table 5. Engine and propeller working ranges.

	Engine working range		
RPM	600	650	2000
Hz	10	10.83	33.3
	Propeller working range		
RPM	197	213.4	658
Hz	13.16	14.22	43.86

Figs. 11 and 12 show the natural frequencies mode shape found in the propulsion line. It should be noted that the first vibration mode is at the tunnel between Stern tube bearing and gearbox and the second mode at the propeller end. The first axial natural frequency is 37.77 Hz.

Fig. 11. First vibration mode 23.49 Hz.

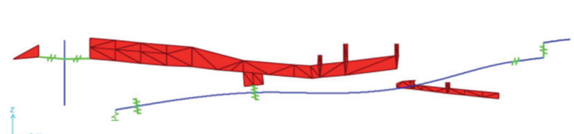
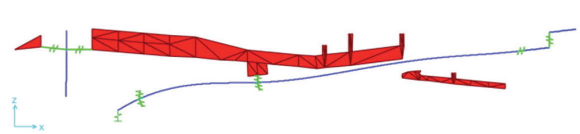


Fig. 12. Second vibration mode 29.73 Hz.



Propulsion line natural frequencies are within the working range and forced analysis should be considered to check the structures resistance and whether the proposed vibration levels standards are met.

Damping

Energy due to vibration on the ship can be dissipated as damping. Vibration analysis should consider three types of damping, namely: the propeller damping, the hysteresis damping and hull damping in water.

Propeller damping
Damping is generated when the propeller rotates in the water, the approximation of these values are shown in Schwanecke (1963) or M. Parsons (1980). The damping depends on the propeller rotation speed, therefore is determined for each operating condition, see Table 6. The damping is placed on the propeller node in the FEM. Structural deformations caused by the propeller excitation forces decreases due to the damping effect.

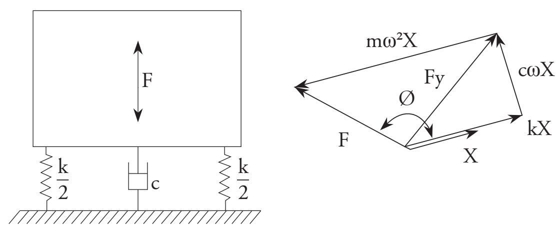
Structure damping
Hysteresis damping is caused by internal molecular friction on vessel structures, and its value is estimated using a coefficient 0.05 proportional to the stiffness.

Hull damping
Ship motion generates a damping (B33) which can be obtained from Seakeeping for each speed and sea state under study. These results are applied to the submerged hull.

Transmissibility

Flexible mounts reduce vibration effect produced by the engine on their foundations. In the case of study, the propulsion system has 2 front flexible rubber mounts for each engine and 2 flexible mounts for each gearbox.

Fig. 13. Perturbing force transmitted by the springs and damper. Thomson W., (1972).



Transmissibility is the relationship between the perturbing force and the transmitted force to the foundation and depends especially of the connection stiffness between the engine and the boat structure. For this study, flexible rubber mounts had been used.

There are several references that provide recommendations to know whether a particular mount is suitable to reduce engine forces transmission to structures. W. Thomson, (1972), shows a graph that has frequency (cpm) and the static deformation produced by the engine on the flexible mount or the connecting element as variables.

For the present study, the engine manufacturer provides several options for flexible mounts,

Table 6. Propeller damping for vibration analysis.

Damping (Schwanecke)					
Engine	RPM	1071	1356	1722	2000
Frec	HZ	23.49	29.73	37.77	43.86
C11	N.s/m	109071	138045	175354	203655
C22	N.s/m	11070	14011	17797	20670
C52	N.s	11119	14073	17876	20761
C55	N.m.s	12197	15436	19608	22773

tables 7 and 8 show the percentage of effectiveness of the system foundation.

According to transmissibility analysis results had been decided to use the 315 mount - 55SH due to the appropriate reduction of excitation forces transmitted by the engine and therefore these forces will not be considered in the analysis of vibration of the boat.

Table 7. Analysis of transmissibility for engine mount at 650 RPM.

Equiv. arrangement Flexible mounts	650 RPM Effectiveness dB	% Isolation
RD314 B-65Sh	-8.390	-590.2%
RD314 B-60Sh	-3.821	-141.0%
RD314 B-55Sh	-1.790	-51.0%
RD314 B-50Sh	0.165	3.7%
RD314 B-45Sh	1.821	34.3%
RD315 HD-65Sh	-1.009	-26.1%
RD315 HD-60Sh	2.819	47.8%
RD315 HD-55Sh	3.785	58.2%
RD315 HD-50Sh	5.221	69.9%
RD315 HD-45Sh	6.919	79.7%

Table 8. Analysis of transmissibility for engine mount at 2000 RPM.

Equiv. arrangement Flexible mounts	2000 RPM Effectiveness dB	% Isolation
RD314 B-65Sh	9.742	89.4%
RD314 B-60Sh	10.877	91.8%
RD314 B-55Sh	11.651	93.2%
RD314 B-50Sh	12.604	94.5%
RD314 B-45Sh	13.578	95.6%
RD315 HD-65Sh	12.006	93.7%
RD315 HD-60Sh	14.236	96.2%
RD315 HD-55Sh	14.919	96.8%
RD315 HD-50Sh	16.011	97.5%
RD315 HD-45Sh	17.403	98.2%

Forces and moments of propeller excitation

Propeller vibration forces are predominant in calculating propulsion line and boat structure vibration. These forces occur due to non-uniform water flow in the propeller creating periodic forces depending on the number of blades called propeller excitation forces. These forces are generated in the vertical, transverse and longitudinal directions.

Forces transmitted to the propulsion shaft (bearing forces)

For lateral vibration analysis (bending) should be considered vertical forces F_{33} and transverse F_{22} and their moments M_{33} and M_{22}, while for the axial analysis the longitudinal force F_{11} is considered, following same nomenclature shown in Fig. 7.

Exciting forces are decomposed into harmonic components using the Fourier analysis [Kumai, 1961]. Currently, Classification Societies recommend excitation values for each order based on the number of blades and thrust or torque on the propeller, as appropriate. For the study boat, the excitation values recommended by ABS (2006) had been used. The values of the forces applied to the study boat in 4 different working conditions are shown in Table 9.

Table 9. Propeller excitation forces for vibration analysis.

Frequency (HZ)	23.49	29.73	37.77	43.86
Order	1Z	1Z	1Z	1Z
Engine RPM	1071	1356	1722	2000
Total thrust	20503	35677	65830	91605
Total torque	4974	8476	15141	20936
Propeller excitation forces				
Axial F11 (N)	2358	4103	7571	10535
Vertical F33 (N)	246	428	790	1,099
Transv. F22 (N)	472	821	1514	2107
Moment M11 (N.m)	433	737	1317	1821
Moment M33 (N.m)	622	1059	1893	2617
Moment M22 (N.m)	1134	1932	3453	4773

Hull pressure forces (pressure fluctuation)

There are several causes that produce fluctuating pressures on the hull in the area of the propeller. These pressures fluctuate proportional to the propeller rotation speed, its number of blades (blade rate frequency), and cavitation.

Pressures can be obtained by experimentation, by numerical approximation (CFD) or by empirical formulas (*Holden, 1980*). For the current analysis, Holden formulas were used. These pressures vary according to working condition. Table 10 shows pressures values applied in an area of 1 m2 of each propeller, in all working conditions analyzed.

Table 10. Hull fluctuating pressures applied.

Frec. (HZ)	**15.2**	**23.5**	**28.3**	**29.7**	**37.8**	**43.9**
Order	**1Z**	**1Z**	**1Z**	**1Z**	**1Z**	**1Z**
Engine RPM	691	1071	1290	1356	1722	2000
Pressure PT (N/m^2)	1108	3114	4962	5667	9847	14335

Working conditions to evaluate

For this study case, the reduction ratio is R = 3.04, and a 4 blades propeller was used. Therefore, the excitation will occur at a frequency:

$$f_{exc} = (RPM_{engine})(N_{blades})/R \quad (3)$$

Generally, the two first orders of the propeller excitation are considered: 1Z and 2Z, due to lower excitation magnitudes presented by higher orders.

Resonance conditions between the excitation frequency and propulsion line natural frequencies had been analyzed. Additionally maximum working condition (MCR) had been analyzed, which in this case is 2000 RPM.

The following table shows the resonances to consider

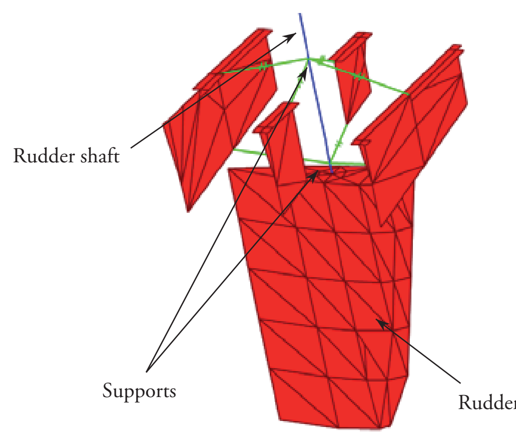
Table 11. Resonances table.

Natural Frec. (HZ)	**15.2**	**23.5**	**28.3**	**29.7**	**37.8**	**43.9**
Order	**1Z**	**1Z**	**1Z**	**1Z**	**1Z**	**1Z**
Engine RPM	691	1071	1290	1356	1722	2000

Rudder Line Behavior

Vibration analysis must consider the rudder behavior, to check if there is any resonance in the working range. Additionally it is important to know if the rudder holds up propeller fluctuating stress loads.

Fig. 14. Rudder finite element model



Rudder supports location

Rudder supports are usually cutless bearings. The FEM represents these supports in the corresponding directions. In the present study, rudder has two supports, upper one restricting rudder shaft axial movement and allowing only rotation and lower support. Due to the rudder shaft is modeled with frame element; constraints simulating the contact between the flanged shaft and the rudder shell had been included.

Rudder added mass

Rudder is also immersed in water and its added mass is considered in the FEM. Rudder is considered as a plate to find the rudder added mass. Mukundan (2002) method was used and its values are presented in Table 12. The rudder added mass was evenly distributed at nodes on rudder surface in their respective directions.

Table 12. Calculated rudder added mass.

Transversal added mass		
C	0.73	
psw	1025	Kg/m^3
B	0.824	m
L	1.44	m
B/L	0.572	
M* y	574.6	Kg
Longitudinal added mass		
C	0.21	
psw	1025	Kg/m^3
B	0.824	m
A	0.182	m
B/L	4.53	
M* x	20.89	Kg
Added inertia		
C	0.73	
psw	1025	Kg/m3
B	0.824	m
L	1.44	m
B/L	0.572	
MI*	12.2	$Kg.m^2$

Classification Societies Acceptance criteria

Classification Societies recommend limits for vibration velocity for crew, passengers, structures and machinery areas.

These limits are recommended to ensure people comfort in the accommodation areas and to prevent fatigue failure in local structures.

Living areas limits

Classification Societies recommend limits depending on craft type and accommodation or work sectors.

ISO 6954 (2000) proposed by Classification Societies such as ABS and Germanischer Lloyd had been taken as acceptance criteria for the evaluation of the study boat.

On Table 13, the classification refers to the area of application:

- A Class: is for passenger cabins,
- B Class for accommodation of crews and
- C Class for workspaces

Table 13. RMS vibration limits from ISO 6954 (2000) of 1-80 Hz.

RMS values of global vibration			
Classification	A	B	C
	mm/s	mm/s	mm/s
Values on which adverse comments are probable	4	6	8
Values below which adverse comments are not probable	2	3	4

There are voluntary limits known as comfort notations, which are limit values proposed by the Classification Societies to grant class notations, especially for passenger vessels.

Structure limits

There are vibration limits for not accommodating areas as tanks, mast, lazaretto structures, engine room, etc. These limits seek to avoid structural damage due to fatigue and the cracks occurrence due to vibration. Fig. 15, taken from ABS (2006), shows vibration peak limits for structures bellow which the risk to fatigue crack is expected to below. From Fig. 15 can be seen that for frequencies between 5 Hz and 10 Hz vibration peak limit recommended is 30 mm/s.

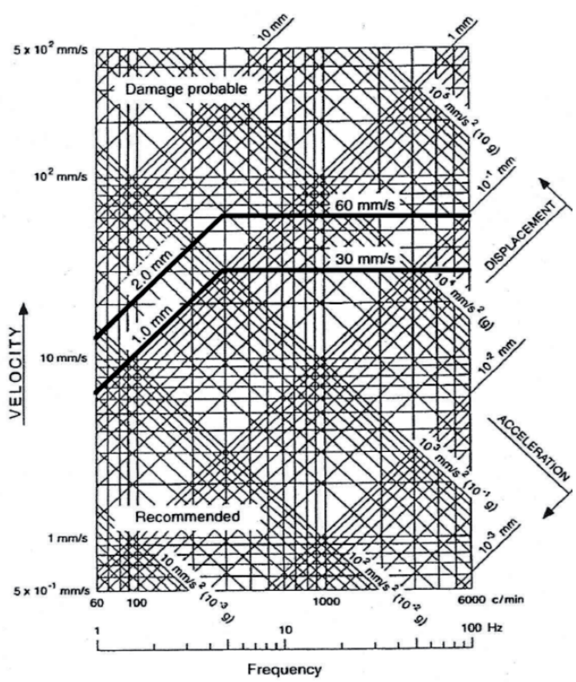
Hull structure and deck house forced analysis evaluation

Finite element method was used to perform a forced vibration analysis of the hull structure, using the following equation:

$$[M]\{\ddot{Y}\} + [C]\{\dot{Y}\} + [K]\{Y\} = \{F\} \qquad (4)$$

where $[M]$ is the mass matrix of the system, $[K]$ is the stiffness matrix of the system, $[C]$ is the damping matrix system, $\{Y\}$: is the displacement vector, $\{\dot{Y}\}$the first derivative of displacement Y, $\{\ddot{Y}\}$ is is the second derivative of displacement Y, $\{F\}$: is the excitation force vector.

Fig. 15. Peak vibration limits for local structures.



Deformation was estimated at all nodes in the model, for each load condition. Figs. 16, 17, 18 and 19 graphically show deformation results.

Harmonic motion deformation as the case of vibration can be represented as follows, at any time t:

$$\delta = A * \sin(\omega_v . t) \qquad (5)$$

where:

A = deformation amplitude (m)
ω_v= Vibration frequency (rad/s)
t = time (s)

Since the speed is the relationship between the deformation and the time, the vibration speed magnitude (V) can be obtained by the following equation:

$$V = A * \omega_v \qquad (6)$$

Figs. 16, 17, 18 and 19 show as color-sectors the deformation that is proportional to vibration speed. There is greater deformation on aft bulkhead of upper deck house at 43.86 Hz condition.

Fig. 16. Deformation (mm) at 23.49 Hz condition.

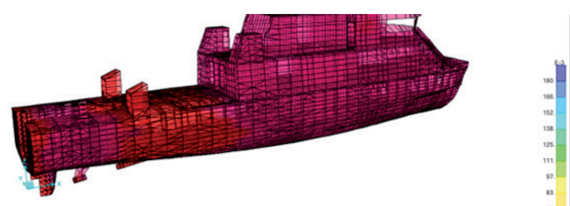


Fig. 17. Deformation (mm) at 29.73Hz condition.

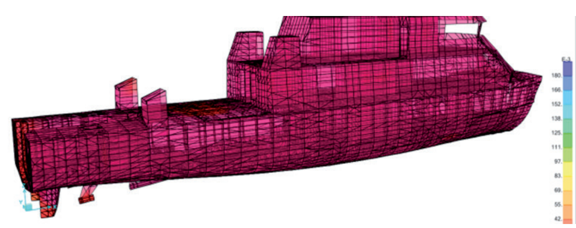


Fig. 18. Deformation (mm) at 37.77 Hz condition.

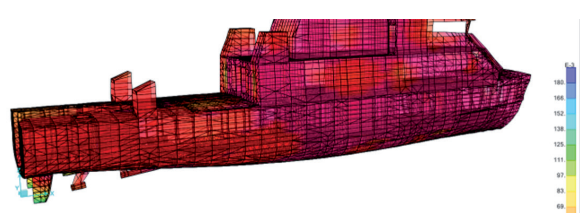


Fig. 19. Deformation (mm) at 43.86 Hz condition.

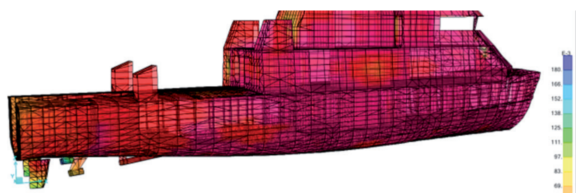
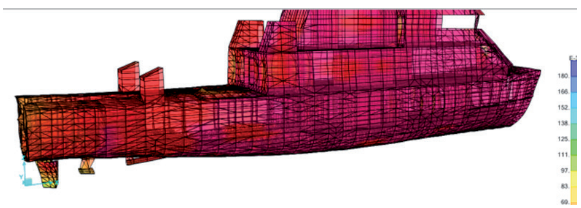


Fig. 20. Deformation (mm) at 43.86 Hz condition (reinforced bulkhead at upper deck)



Tables 14 and 15 show vibration speed values by sector for each condition, calculated from the deformation found with the FEM. Most vibration levels do not exceed the limits set by the rules, except the aft bulkhead on upper deck at 2000 RPM, where the limit is 6 mm/s.

Local Reinforcements to reduce higher vibration levels

The FEM allow us to carry out structural modifications to comply with the recommended limits.

In this case, it is requested to increase the section of the vertical reinforcements to the aft bulkhead on upper deck house.

Table 14. Vibration speed at accommodation.

	RMS vibration velocity (mm/s)					
Frec, (HZ) / Sector	**15.2**	**23.5**	**28.3**	**29.7**	**37.8**	**43.9**
	ACCOMODATION AREAS					
Inner main deck	0.44	0.32	0.25	0.33	0.84	0.52
Fore exterior main deck	0.16	0.15	0.09	0.24	0.99	0.51
Exterior deck 200	0.22	0.37	0.09	0.37	1.38	0.71
Interior deck 200	0.3	2.27	0.77	0.46	0.59	0.73
Upper side deckhouse	0.38	0.17	0.13	0.38	2.58	2.05
Lower side deckhouse	0.55	0.12	0.54	0.22	4.24	2.65
Aft bulkhead at deck 200	0.27	0.48	0.26	0.10	5.13	20.9

Table 15. Vibration speed at structure.

	RMS vibration velocity (mm/s)					
RPM	**691**	**1071**	**1290**	**1356**	**1722**	**2000**
Frec, (HZ) / Sector	**15.2**	**23.5**	**28.3**	**29.7**	**37.8**	**43.9**
	STRUCTURE					
Long. Beam over strut	0.25	0.25	0.65	0.42	0.80	4.99
Hull at stuffing box	0.65	0.94	1.82	1.44	2.60	2.70
Long. Beam over tunnel bearing	0.39	2.32	1.28	1.84	5.62	5.77
Pilot house roof	0.07	0.49	0.04	0.25	5.98	5.84
Transom/side intersect.	1.05	2.14	3.98	2.26	5.64	10.0
Aft bulkhead at deck 200	0.27	0.48	0.26	0.10	5.13	20.9

Fig. 20 shows the deformation in the same scale as the previous figures and shows the deformation decrease on aft bulkhead at upper deck, with respect to Fig. 19. This improvement can be seen in Table 16.

Table 16. Vibration speed after upper deck bulkhead reinforcement.

RMS vibration velocity (mm/s)			
RPM	**2000**		
Frec, (HZ) / Sector	**43.86**	**Limit**	**Direction**
ACCOMODATION AREAS			
Aft bulkhead at deck 200	4.42	6	X

Conclusions

- Acceptance criteria are effective, so the best way to avoid resonance problems is configuring the propulsion system to keep vibration below criteria.
- Natural frequencies of the propulsion line need to avoid the working range to prevent resonances.
- Natural frequencies of ship panels and structure need to avoid the working range of propeller excitation forces.
- Forced vibration analyses on hull girder including propeller excitation forces should be performed to identify sectors that do not meet standards.
- The results obtained in the design stage allow identifying possible failures, especially when there is resonance risk in the propulsion line.

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Appex

Boat main particulars		
Ship type	Investigation boat	
Overall length:	46 m	
Bea:	7 m	
Depth:	4m	
Draft:	1,9 m	
Engine data		
Strokes	4	
Cylinders Nº	12	
V angle	90	Grades
Bore	165	mm
Stroke	190	mm
Connecting rod l.	354	mm
Weight	6800	kg
Minimum rpm	500	
Maximum rpm	2000	
Max power x rpm	1680KW x 2000 RPM	
Torque @ 1680 kw	8.02	KN.m
Propeller data		
Type	Fixed pitch	
Blades Nº	4	
Diameter	1.397	m
Pitch	1.283	m
D.A.R	0.91	
Mass	363.7	kg
Polar inertia	41.18	$kg.m^2$
Gearbox data		
Ratio	3.04	