Analysis of Dynamic Stability of Planing Craft on the Vertical Plane

Análisis de estabilidad dinámica de embarcaciones de planeo en el plano vertical

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Abstract

A dynamic model for the motion of planing craft on the vertical plane was developed; the motions of surge, heave, and pitch are coupled. Critical conditions that produce the inception of instability are evaluated. The Wagner model (1932) for 2D impact is extended for section with knuckles. Planing hulls were analyzed through the application of slender body theory. The results are compared with Tveitnes (2001), Peterson (1997), Savitsky (1964), Troesch (1992) and Celano (1998).

Key words: Slamming, Porpoising, planing boats, dynamic stability.

Resumen

Se desarrolla un modelo dinámico del movimiento de embarcaciones de planeo en el plano vertical acoplando los movimientos de avance y levantamiento, cabeceo. Se determinan las condiciones críticas que dan origen de la inestabilidad. El modelo de Wagner (1932) para el impacto 2D es extendido para secciones con codillos. Mediante la aplicación de la teoría de cuerpos esbeltos se analizan las embarcaciones de planeo. Los resultados obtenidos son comparados con Tveitnes (2001), Peterson (1997), Savitsky (1964), Troesch (1992), Celano (1998).

Palabras claves: impacto en el agua, botes de planeo, estabilidad dinámica.

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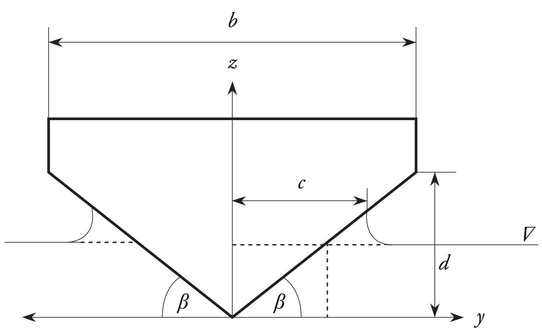
Introduction

Wagner (1932) developed an analytic model to predict pressure distribution in the impact with symmetry entry. Fig. 1 shows a wedge section with symmetry entry, where b is the beam of the section, d is the keel-knuckle distance, β is the dead rise angle, y and z are the horizontal and vertical axis, respectively.

Fig. 1. Wedge section impact with symmetry entry



For this kind of section, pressure distribution is evaluated as:

$$\frac{P}{\rho} = \dot{w}\sqrt{c^2 - y^2} + \frac{\dot{w}cc}{\sqrt{c^2 - y^2}} - \frac{w^2}{2}\frac{y^2}{c^2 - y^2} \tag{1}$$

Where ρ is the fluid density, P is the pressure, c is the wetted half-beam, w is the vertical velocity, $\dot{c}$ is the time variation of the half-beam, which is evaluated as:

$$\dot{c} = \frac{\pi}{2}\frac{w}{tan\beta} \tag{2}$$

2D Impact

The Wagner model (1932) does not work after the flow separation from the knuckle, for this case the boundary condition P=0 on y=b/2 is applied, thus:

$$\frac{P}{\rho} = \dot{w}\sqrt{c^2 - \left(\frac{b}{2}\right)^2} + \frac{wc\dot{c}}{\sqrt{c^2 - \left(\frac{b}{2}\right)^2}} - \frac{w^2}{2}\frac{\left(\frac{b}{2}\right)^2}{c^2 - \left(\frac{b}{2}\right)^2} = 0 \tag{3}$$

When the section is moved with constant velocity $\dot{w} = 0$, replacing

$$\dot{c} = \frac{w^2}{2}\frac{\left(\frac{b}{2}\right)^2}{c\sqrt{c^2 - \left(\frac{b}{2}\right)^2}} \tag{4}$$

The variables are separated and integrated, the virtual half-beam wetted after the flow separation is:

$$c = \sqrt{\left(\frac{b}{2}\right)^2 + \left[\frac{3}{2}\left[w\left(\frac{b}{2}\right)^2 (t - t_0)\right]\right]^{2/3}} \tag{5}$$

Where c is the half-beam in the instant, t, and t_0 is the time when the flow is on the knuckle.

2D Dynamic Impact

The total force in the section is:

$$f_z = f_{HD} + f_{Hs} + f_v \tag{6}$$

Where f_{HD} is the hydrodynamic force, f_{HS} is the hydrostatic force and f_v is the sectional drag force. By integrating equation 1 the hydrodynamic force in the section is obtained:

$$\begin{aligned} f_{HD} = {} & \rho\dot{w}\int_{-y1}^{y1}\sqrt{c^2 - y^2}\,dy \\ & + \rho\dot{w}\int_{-y1}^{y1}\frac{c\dot{c}}{\sqrt{c^2 - y^2}}\,dy \\ & - \rho\frac{w^2}{2}\int_{-y1}^{y1}\frac{y^2}{c^2 - y^2}\,dy \end{aligned} \tag{7}$$

$$f_{HD} = -\dot{w}m_a + f_{HD} \tag{8}$$

Where

$$m_a = \rho\int_{-y1}^{y1}\sqrt{c^2 - y^2}\,dy \tag{9}$$

$$\begin{aligned} f'_{HD} = {} & \rho\dot{w}\int_{-y1}^{y1}\frac{c\dot{c}}{\sqrt{c^2 - y^2}}\,dy \\ & - \rho\frac{w^2}{2}\int_{-y1}^{y1}\frac{y^2}{c^2 - y^2}\,dy \end{aligned} \tag{10}$$

Fig. 2. Forces in the section

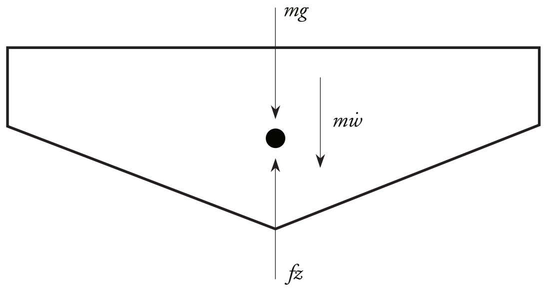


Fig. 2 shows the forced in the section. The equilibrium of forces in the section is:

$$\sum f_z = \dot{w} m_a + f'_{HD} + f_{HS} + f_v - mg = -m\dot{w} \quad (11)$$

Replacing

$$f'_z = f'_{HD} + f_{HS} + f_v \quad (12)$$

The hydrostatic force is calculated as:

$$f_{HS} = \rho g A_0 \quad (13)$$

Where g is the gravity constant and A_0 is the immersed area, the drag force is calculated as:

$$f_v = \frac{1}{2} \rho C_a (2y_1) w^2 \quad (14)$$

Where C_d is the sectional drag coefficient, which takes values of:

$$C_d = \begin{Bmatrix} w \geq 0, C_d = 0.5 \\ w < 0, C_d = -1.0 \end{Bmatrix} \quad (15)$$

The acceleration in the section is:

$$\dot{w} = \frac{mg - f'_z}{m + m_a} \quad (16)$$

2D Impact Results

The sectional force coefficient is defined as:

$$C_f = \frac{f_z}{\frac{1}{2} \rho b w^2} \quad (17)$$

Figs. 3, 4, and 5 show the results of force coefficient variation with immersion for different wedge sections; the results are compared with Tveitnes (2001) with a good agreement, the peak of force is over predicted.

Fig. 3. C_F vs. z/d, β=10°

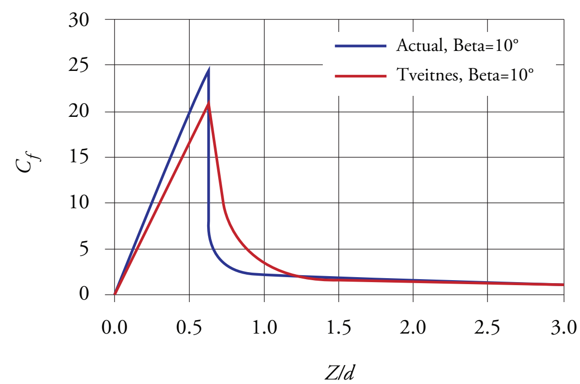


Fig. 4. C_F vs. z/d, β=20°

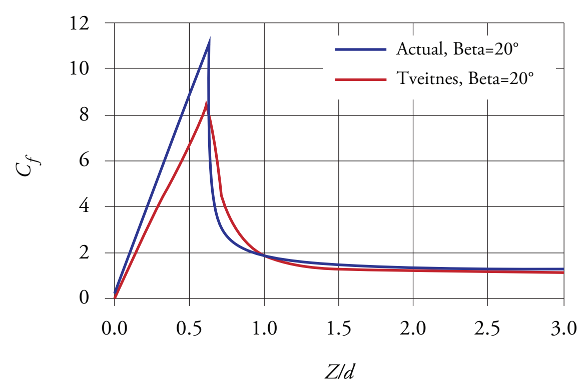
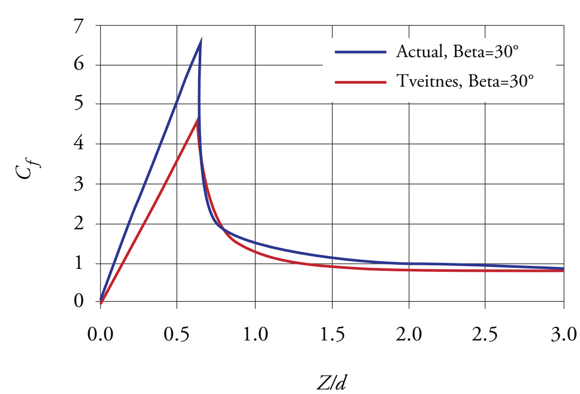


Fig. 5. C_F vs. z/d, β=30°



Figs. 6, 7, 8, 9, 10, and 11 show the results of acceleration with free drop; the section has the following properties: b = 2 ft, β = 20°, width = 8 ft. The initial height was varied by 2, 4, and 6 ft, while the mass of the wedge took values of 269 and 641lb. The results obtained are compared with Peterson (1997) with good agreement.

Fig. 6. Acceleration vs. time, h = 2 ft, m = 269 lb

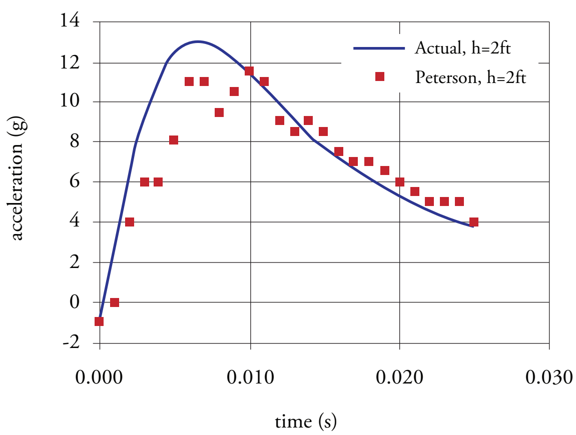


Fig. 7. Acceleration vs time, h = 4ft, m = 269lb

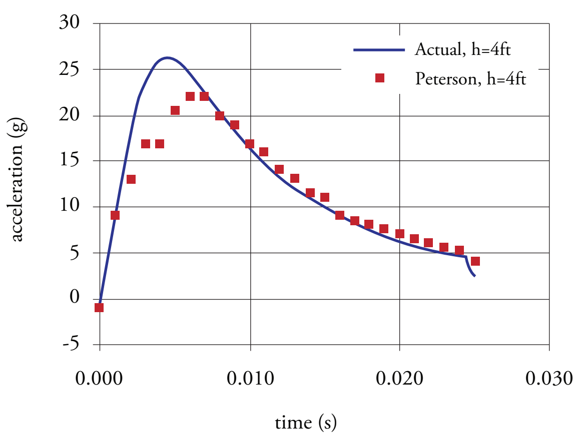


Fig. 8. Acceleration vs time, h = 6ft, m = 269lb

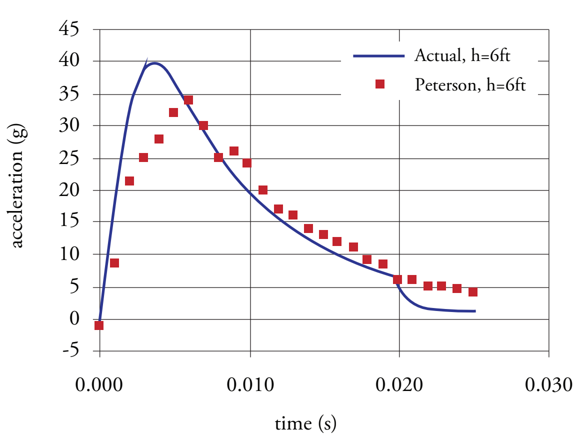


Fig. 9. Acceleration vs. time, h = 2 ft, m = 641 lb

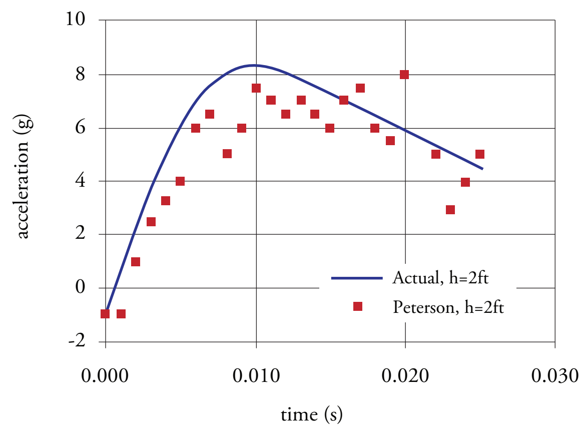


Fig. 10. Acceleration vs time, h =4 ft, m = 641 lb

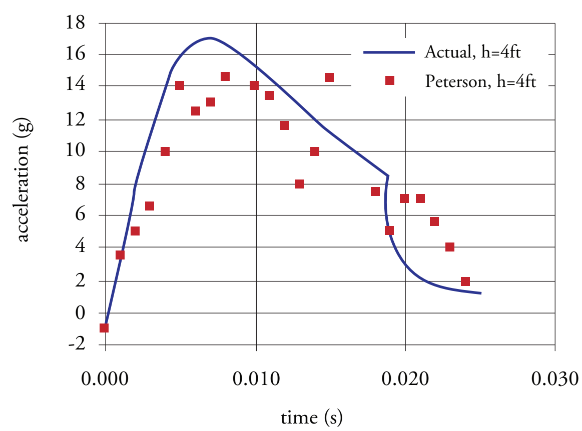
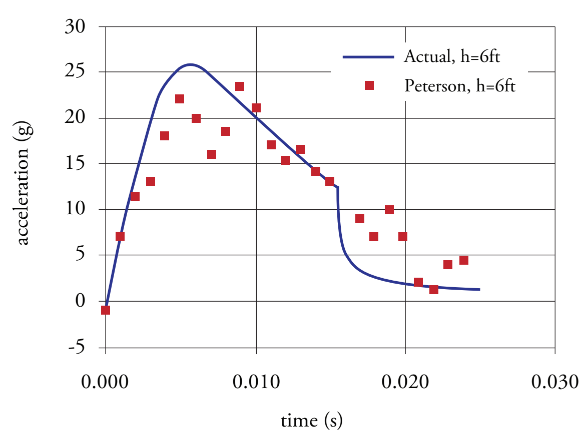


Fig. 11. Acceleration vs. time, h = 6 ft, m = 641 lb



Dynamic Model of Motion on the Vertical Plane of Planing Boats

Fig. 12 shows a planing boat on calm water, D is the draft in the transom, L_{cdg} and V_{cdg} are the horizontal and vertical position of the gravity center, θ is the trim angle, z_{cdg} is the height of the center of gravity with respect to the water line, x and z are fixed coordinates in the boat.

$$D = L_{cdg}\sin\theta + V_{cdg}\cos\theta - z_{cdg} \tag{18}$$

$$T_0 = D - (x\sin\theta + Z_1\cos\theta) \tag{19}$$

Replacing

$$T_0 = -(X - L_{cdg})\sin\theta + (V_{cdg} - Z_1)\cos\theta - z_{cdg} \tag{20}$$

If $\cos\theta \approx 1$

$$T_0 = -(x - L_{cdg})\sin\theta + (V_{cdg} - z_1) - z_{cdg} \tag{21}$$

The velocity impact of each section is evaluated as:

$$w = \frac{DT_0}{Dt} = \frac{dT_0}{dt} - U\,\frac{dT_0}{dx} \tag{22}$$

$$w = -(x - L_{cdg})\cos\theta\,(\dot{\theta}) - \dot{z}_{cdg} + U\left(\sin\theta + \frac{dz_1}{dx}\right) \tag{23}$$

By simplification

$$w = -(x - L_{cdg})\,(\dot{\theta}) - \dot{z}_{cdg} + U\left(\sin\theta + \frac{dz_1}{dx}\right) \tag{24}$$

Where $\dot{\theta}$ is the pitch velocity, $\dot{z}_{cdg}$ is the vertical velocity in the *cg*. The acceleration impact of each section is evaluated as:

$$\dot{w} = \frac{Dw}{Dt} = \frac{dw}{dt} - U\,\frac{dw}{dx} \tag{25}$$

$$\dot{w} = -(x - L_{cdg})\,(\ddot{\theta}) - \ddot{z}_{cdg} + 2U\cos\theta\,(\dot{\theta}) + \dot{U}\left(\sin\theta + \frac{dz_1}{dx}\right) - U^2\,\frac{d^2 z_1}{dx^2} \tag{26}$$

By simplification

$$\dot{w} = -(x - L_{cdg})\,\ddot{\theta} - \ddot{z}_{cdg} + 2U\dot{\theta} + \dot{U}\left(\sin\theta + \frac{dz_1}{dx}\right) - U^2\,\frac{d^2 z_1}{dx^2} \tag{27}$$

Calculation of Forces in the hull

Fig. 13 (pag. 30) shows the forces in the hull, where T is the thrust, F_N is the normal hydrodynamic force, M_{HD} is the hydrodynamic moment, mg is the weight, F_v is the viscous drag, δ is the angle of the propulsion shaft, d_v is the perpendicular distance between F_v and *cg*, d_T is the perpendicular distance between T and *cg*.

The normal force to the keel is evaluated as:

$$F_N = \int_0^l C_{tr}(x) f dx \tag{28}$$

Where l is the length of the boat and $c_{tr}(x)$ is the suction pressure coefficient of the transom, which is calculated as:

Fig. 12. Geometry properties in planing hull in calm water

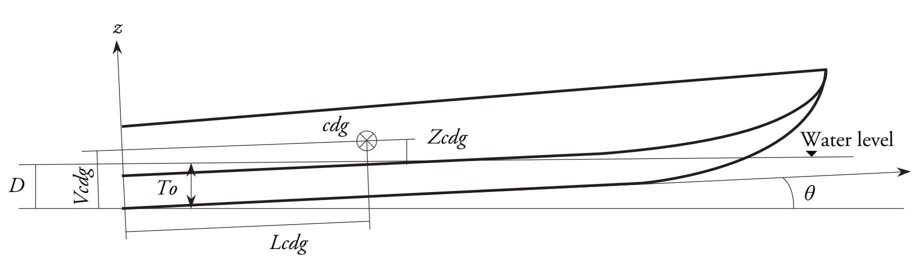
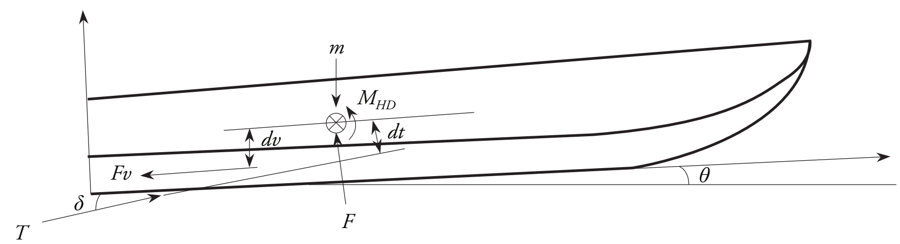


Fig. 13. Forces in the planing hull



$$c_{tr}(x) = \tanh\left(\frac{2.5}{0.34 B\, C_v}\, x\right) \tag{29}$$

Replacing the normal force is:

$$F_N = \int_0^l \dot{w} c_{tr}(x) m_a\, dx + \int_0^l c_{tr}(x) f_z' dx \tag{30}$$

The solution of the first integral is:

$$\int_0^l \dot{w} c_{tr}(x) m_a\, dx = \int_0^l \Big[-(x - L_{cdg})\,\ddot{\theta} - \ddot{z}_{cdg} + 2U\dot{\theta} + \dot{U}\left(\sin\theta + \frac{dz_1}{dx}\right) - U^2\, \frac{d^2 z_1}{dx^2} \Big]\, c_{tr}(x) m_a\, dx \tag{31}$$

$$\int_0^l \dot{w} c_{tr}(x) m_a\, dx = (M_1 \sin\theta)\,\dot{U} - M_1\,\ddot{z}_{cdg} + 2M_1 U\dot{\theta} - M_2\ddot{\theta} - \int_0^l U^2\, \frac{d^2 z_1}{dx^2}\, m_a\, dx \tag{32}$$

Where

$$M_1 = \int_0^l c_{tr}(x) m_a\, dx \tag{33}$$

$$M_2 = \int_0^l (x - L_{cdg})\, c_{tr}(x) m_a\, dx \tag{34}$$

By substitution

$$F_N = (M_1 \sin\theta)\,\dot{U} - M_1\,\ddot{z}_{cdg} + 2M_1 U\dot{\theta} - M_2\ddot{\theta} + F_N' \tag{35}$$

Where

$$F_N' = \int_0^l - U^2\, \frac{d^2 z_1}{dx^2}\, m_a\, dx + \int_0^l f' dx \tag{36}$$

The hydrodynamic moment is calculated as:

$$M_{HD} = \int_0^l (x - L_{cdg})\, c_{tr}(x) f dx \tag{37}$$

$$M_{HD} = \int_0^l \dot{w}\,(x - L_{cdg})\, c_{tr}(x)\, m_a dx + \int_0^l (x - L_{cdg})\, c_{tr}(x) f_z' dx \tag{38}$$

$$\int_0^l \dot{w}\,(x - L_{cdg})\, c_{tr}(x)\, m_a dx = (M_2 \sin\theta)\,\dot{U} - M_2\,\ddot{z}_{cdg} + 2M_2 U\dot{\theta} - M_3\ddot{\theta} - \int_0^l U^2 (x - L_{cdg})\, \frac{d^2 z_1}{dx^2}\, c_{tr}(x)\, m_a dx \tag{39}$$

Where M_3 is:

$$M_3 = \int_0^l (x - L_{cdg})^2\, c_{tr}(x)\, m_a dx \tag{40}$$

By substitution

$$M_{HD} = (M_2 \sin\theta)\,\dot{U} - M_2\,\ddot{z}_{cdg} + 2M_2 U\dot{\theta} - M_3\ddot{\theta} + M_{HD}' \tag{41}$$

Where

$$M_{HD}' = -\int_0^l U^2 (x - L_{cdg})\, \frac{d^2 z_1}{dx^2}\, c_{tr}(x)\, m_a dx + \int_0^l (x - L_{cdg})\, c_{tr}(x) f_z' dx \tag{42}$$

The viscous drag force is calculated as:

$$f_v = \frac{1}{2} \rho C_a A U^2 \tag{43}$$

C_f is the friction coefficient, A is the wetted area of the hull. The equilibrium of forces in X is:

$$\sum F_x = T\sin(\theta+\delta) - F_N \sin\theta - F_v \cos\theta = M\ddot{x}_{cdg} \quad (44)$$

By replacing

$$T\cos(\theta+\delta) - (F_N' + 2M_1 U\dot{\theta})\sin\theta - F_v \sin\theta = (M + M_2 \sin^2\theta)\dot{U} - (M_1 \sin\theta)\ddot{z}_{cdg} - (M_2 \sin\theta)\ddot{\theta} \quad (45)$$

The equilibrium of forces in Z is:

$$\sum F_z = F_N \cos\theta - F_v \sin\theta + T\sin(\theta+\delta) - Mg = M\ddot{z}_{cdg} \quad (46)$$

By replacing

$$(F_N' + 2M_1 U\dot{\theta})\cos\theta - F_v \sin\theta + T\sin(\theta+\delta) - Mg = -(M_2 \sin\theta\cos\theta)\dot{U} + (M + M_1\cos\theta)\ddot{z}_{cdg} + (M_2\cos\theta)\ddot{\theta} \quad (47)$$

The equilibrium of momentum in y is:

$$\sum M_y = M_{HD} - d_v F_v + d_t T = l_{yy}\ddot{\theta} \quad (48)$$

By replacing

$$M_{HD}' - d_v F_v + d_t T = -(M_2 \sin\theta)\dot{U} + M_2 \ddot{z}_{cdg} + (l_{yy} + M_3)\ddot{\theta} \quad (49)$$

The equations of motion are coupled obtaining the following expression:

$$\begin{bmatrix} M + M_2\sin^2\theta & M - M_1\sin\theta & M_2\sin\theta \\ -M_1\sin\theta\cos\theta & M + M_1\cos\theta & M_2\cos\theta \\ -M_2\sin\theta & M_2 & l_{yy} + M_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_{cdg} \\ \ddot{z}_{cdg} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} T\cos(\theta+\delta) - (F_N' + 2M_1U\dot{\theta})\sin\theta - F_v\sin\theta \\ (F_N' + 2M_1U\dot{\theta})\cos\theta - F_v\sin\theta + T\sin(\theta+\delta) - Mg \\ = M_{HD}' - d_v a F_v + d_t T \end{bmatrix} \quad (50)$$

When the velocity of the boat is constant:

$$\begin{bmatrix} M + M_1\cos\theta & M_2\cos\theta \\ M_2 & l_{yy} + M_3 \end{bmatrix} \begin{bmatrix} \ddot{z}_{cdg} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} (F_N' + 2M_1U\dot{\theta})\cos\theta - F_v\sin\theta + T\sin(\theta+\delta) - Mg \\ = M_{HD}' - d_v a F_v + d_t T \end{bmatrix} \quad (51)$$

Results of the Application on Planing Boats

For analysis, the following parameters are defined:

$$c_v = \frac{U}{\sqrt{gB}} \quad (52)$$

$$c_\Delta = \frac{M}{\rho B^3} \quad (53)$$

$$c_L = \frac{F_z}{\frac{1}{2}\rho U^2 B^2} \quad (54)$$

$$\lambda = \frac{L_k + L_c}{2B} \quad (55)$$

Where c_v, c_Δ, c_L are the coefficients of velocity, load, and lift; λ is the mean wetted length, M is the displacement, B is the beam of the boat, L_k and L_c are the wetted length of the keel and the wetted length of the chine. Figs. 14, 15, 16, and 17 show the results of forces on steady condition for a constant forward velocity, the boat has the following properties: $V_{cdg}/B = 0.65$, $L_{cdg}/B = 1.47$ $\theta = 4°$ and $\lambda = 3$. η_3 and η_4 are the vertical elevation and rotation of the cg from the equilibrium position. The results are compared with Troesch (1992) and Savitsky (1964).

Fig. 14. F_z vs. η_3/B

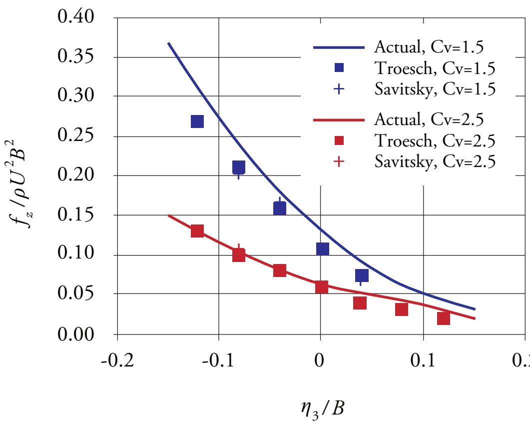


Fig. 15. M_y vs. η_3/B

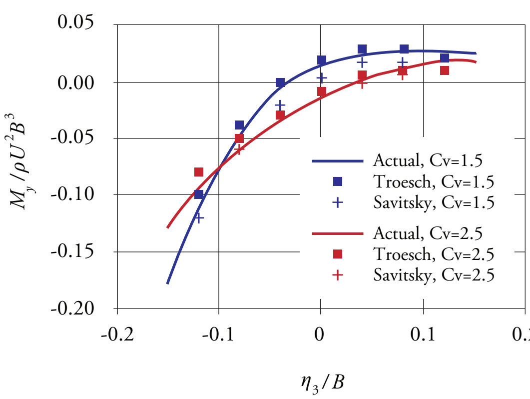


Fig. 16. F_z vs. η_5/B

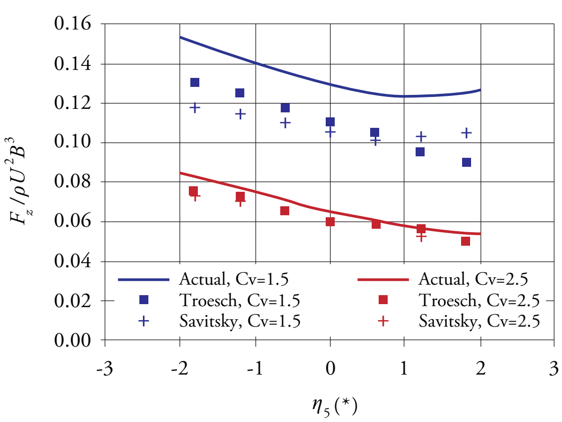


Fig. 17. M_y vs. η_5/B

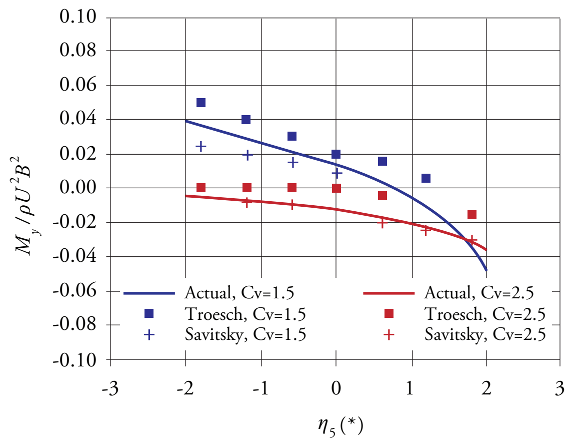


Fig. 18. $\theta_{critical}$ vs. CL, ß = 10°

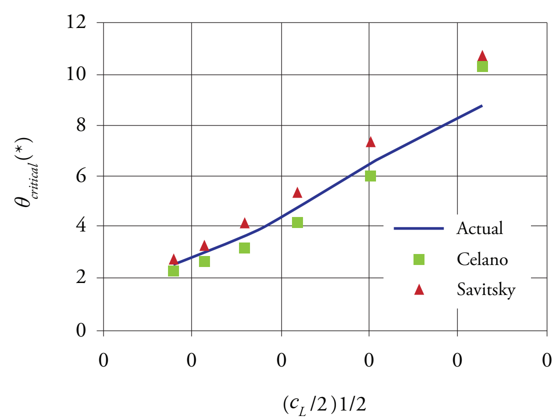
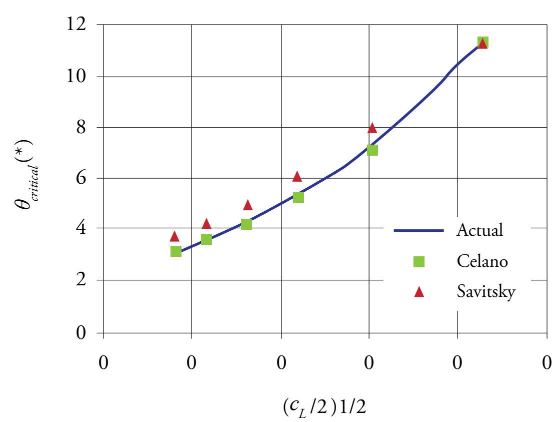


Fig. 19. $\theta_{critical}$ vs. CL, ß = 20°



Figs. 18 and 19 show the critical condition that produces the inception of instability in the vertical plane for boats with β = 10° and β = 20°; the results are compared with Celano (1998) and Savitsky (1964). The criteria for the instability is the pitch amplitude oscillation, η_{50} > 1°. The boat has the following conditions: C_Δ = 0.394, *kyy/B* = 1.25 V_{cdg}/B = 0.4 and *l/B* = 5, where *kyy* is the gyration radius. The value of Lcdg was changed to find the critical condition for each configuration.

Conclusions

The Wagner model (1932) was extended for section with knuckle; force and pressure distribution were evaluated after the flow separation from the knuckle. The 2D dynamic impact was simulated; the results were compared with Peterson (1997) obtaining good agreement. The 2D impact was applied to a planing boat by the slender body theory and a dynamic model for planing hull in calm water was developed. The lift force and trim moment were calculated for ships in steady condition; the results obtained were compared with Savitsky (1964) and Troesch (1992). Critical conditions that cause the inception of porpoising were determined; the results are compared with Savitsky (1964) and Celano (1998) with good agreement.

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