

Optimal consumption of polluting and non-polluting goods: The role of routines

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Abstract

We analyse the optimal consumption of a clean and a dirty consumption good, taking an interdisciplinary perspective. Thus, we posit that individuals follow economic rationality when determining consumption but, on the other hand, we take into account findings from social psychology as regards human behaviour by allowing for the formation of behavioural routines as a result of consuming the polluting good. We show under which conditions routine formation raises the consumption of the clean good relative to the dirty one in the competitive economy and we demonstrate when the formation of routines generates a lower steady-state pollution stock compared to the situation without routines. Finally, we determine the Pigou tax rates and we illustrate that the social optimum may imply a higher steady-state pollution than the competitive economy if the effect of routine formation is sufficiently strong.

Keywords:

Environmental pollution, Clean and dirty consumption, Routines, Optimal control

JEL classification:

Q50, H23, C61.

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Consumo óptimo de bienes contaminantes y no contaminantes: El papel de las rutinas

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Resumen

En este artículo se analiza el consumo óptimo de un bien de consumo limpio y otro sucio desde una perspectiva interdisciplinaria. Se postula que los individuos actúan con racionalidad económica a la hora de consumir pero, por otra parte, se tienen en cuenta los resultados obtenidos desde la óptica de la psicología social relativos al comportamiento humano, permitiendo la formación de rutinas comportamentales como consecuencia del consumo del bien contaminante. Se muestra bajo qué condiciones, en una economía competitiva, la formación de rutinas eleva el consumo del bien limpio en términos relativos al del bien sucio, y se demuestra cuándo la formación de dichas rutinas genera un stock de contaminación estable menor que el que tendría lugar en una situación sin rutinas. Finalmente, se determinan los tipos de gravamen pigouvianos y se muestra cómo el óptimo social puede implicar un estado estable de contaminación más elevado que el de la economía competitiva si el impacto de la formación de rutinas es suficientemente fuerte.

Palabras clave:

Contaminación medioambiental, consumo limpio y consumo sucio, rutinas, control óptimo.

■ 1. Introduction

An important aspect in solving problems caused by environmental pollution, such as global warming for example, is technical progress. A major step towards a cleaner environment will have been achieved once polluting methods of production have been successfully replaced by non-polluting ones. In analysing that topic, economists often consider energy production and study the question of how a less polluting production process can be implemented without leading to output and welfare losses.

Thus, switching from dirty to clean energy is the subject of a great many studies. For example, Hoel and Kverndokk (1996) present a resource model where the use of the resource generates negative externalities. There exists a non-polluting perfect substitute for the polluting resource, with the non-polluting backstop being available at a constant unit cost. Hoel and Kverndokk show, among other things, that it is optimal to extract the polluting resource even when its price is equal to the price of the non-polluting resource. In a more recent contribution, van der Ploeg and Withagen (2014) adopt the model by Hoel and Kverndokk and change two assumptions: They do not allow for a decline in greenhouse gases and they assume that capital must be built up to produce the final output. However, capital is not a perfect substitute for the energy input; rather, there is a backstop that can perfectly substitute the non-renewable energy source. Energy is produced using a polluting non-renewable resource and a non-polluting renewable energy source that is available at a constant unit cost, as in Hoel and Kverndokk. They show that it is optimal to use only the polluting resource initially and, later on, only renewables, when the initial stock of the polluting resource is small. The lower the cost of the renewable, the greater the amount of the polluting energy source left in situ and the sooner the renewable-only phase starts. The models by Hoel and Kverndokk (1996) and by van der Ploeg and Withagen (2014) have the same structure: There is a polluting resource that can be perfectly substituted by a non-polluting one at a given cost, with both variables being control variables. Hence, their models belong to the general class of models analysed by Krautkraemer (1998).

Greiner *et al.* (2014) present a more elaborate framework with regard to the backstop technology. They assume that a capital stock must be built up first in order to produce the renewable energy. The model considers the decentralized market economy and derives optimal taxes and subsidies such that the market economy replicates the social optimum. It turns out that it can be optimal to not completely exploit the non-renewable resource that is used to generate energy, but rather to leave a certain part in situ. The outcome depends on the efficiency of the backstop technology, i.e. of the renewable energy source, and on the initial stock of the non-renewable resource.

What all those contributions have in common is that they adopt a supply side view. Indeed, there are very few studies that analyse the role of preference with respect to the choice between a clean and a dirty consumption good. One exception is the contribution by Scalera (1996), who assumes that the stock of pollution depends on aggregate production and on the amount of the dirty consumption good, whereas clean consumption does not pollute the environment. The paper derives the Pigouvian taxes that make the decentralized competitive economy replicate the social optimum. Further, the paper shows that the trade-off between economic activity and the environment is mitigated by the presence of the non-polluting consumption good. Orecchia and Tessitore (2011) also present an intertemporal model where they distinguish between clean and dirty consumption. Those authors analyse an endogenous growth model where they allow for a clean and a dirty consumption good and demonstrate that the substitution of the dirty consumption good by the clean consumption good is not sufficient to reduce the pollution of the environment. They also show that an environmental Kuznets curve may arise in their framework. Another paper that distinguishes between a polluting and a non-polluting consumption good is the contribution by Mittnik *et al.* (2013). There, the period utility function of the household contains both polluting and non-polluting goods as arguments. The goal of the paper, then, is to analyse employment effects of different environmental policies as well as effects resulting from a change in preferences towards non-polluting goods.

One aspect that is neglected in those contributions is that they do not allow for imperfections in the economic model under consideration. However, it goes without saying that the real world is far from ideal in the sense of economic agents having full information at their disposal or acting perfectly rationally, to mention just two examples. Bondarev *et al.* (2014) analyse the effects of informational constraints as concerns the role of technical progress with respect to climate change. In that context, informational constraints mean that the agents do not optimize over an infinite time horizon but rather over a finite time horizon and, then, re-optimize after the final period has been reached. It could be demonstrated that this assumption has considerable effects both in terms of the evolution of the economy as well as climate change.

In this paper, we intend to analyse the effects that individuals forming routines has on the optimal consumption of a polluting and of a non-polluting good. Thus, we take into account the fact that economic agents are human beings, which implies that their behaviour can be constrained by their cognitive abilities, by routines or by social representations, to mention just a few examples that we focus on in this study. Nevertheless, we retain the assumption that agents try to follow economic rationality when taking economic decisions, thus, adopting social systems theory as developed by the Bielefeld sociologist Luhmann (1984, 1989). According to that theory, economic phenomena such as consumption and investment, for example, can be best

explained by the analysis of the function and mode they have within the economic sub-system. According to Luhmann (1989), the mode of operation of the economy as a sub-system is to ensure the ability to pay. In other words, the actors within the sub-system economy at least try to behave rationally from an economic point of view even if their behaviour may be constrained as pointed out above. Therefore, we study the optimal allocation between a clean and a dirty consumption good allowing for routines that arise from the consumption of the dirty good. We assume that, by the time a clean consumption good becomes available, the dirty consumption good has previously been consumed over a certain time period. The consumer then solves a new intertemporal optimization problem where he can choose between the clean and the dirty good. We analyse both the competitive economy and the social optimum.

The rest of the paper is organized as follows. Section 2 presents the general model with a clean and a dirty consumption good, where the dirty consumption good leads to routine formation. Section 3 first analyses the competitive economy in which the planner does not take into account the externalities in solving the optimization problem and Section 4 then studies the social optimum. Section 5, finally, concludes the paper.

■ 2. The model with clean and dirty consumption

We consider an economy with one homogenous clean and one homogenous dirty consumption good, where the consumption of the dirty good leads to routines. Routines are generally built up as a by-product of human behaviour and, in our model, they result from consuming the dirty consumption good in the past. Routines positively affect the utility of the individual since they help the individual act without having to devote mental resources to achieve a certain goal. Routines thus simplify everyday life because new situations are dealt with automatically that, otherwise, would require awareness and attention. The latter entail efforts – or in economic terms, costs – that can thus be avoided. In this respect, psychologists speak of automatic thinking that is based on schemas, i.e. on cognitive structures, which are the result of unintended learning processes (see Aronson *et al.*, 2004). Therefore, the routines are beneficial for individuals and raise their utility. In this context, we should like to point to the so-called perseverance effect that makes individuals stick to certain routines, even if they have been proven to be wrong, which reinforces the effects of routines.

The household sector is represented by a continuum of infinitely-lived homogenous households with household production. Each individual household has measure zero and the household sector has mass one. As regards the utility function U of the rep-

representative household, we follow Scalera (1996) and Orcechchia and Tessitore (2011) and assume that it is linearly separable in clean and dirty consumption and in pollution. Thus, it is given by¹

$$U(\cdot) = \frac{(C_d(1+H)^\chi)^{1-\sigma} - 1}{1-\sigma} + \frac{C_c^{1-\sigma} - 1}{1-\sigma} - D(P), \quad (1)$$

with C_d (C_c) dirty (clean) consumption, H the stock of routines and $1/\sigma > 0$ the intertemporal elasticity of substitution of consumption. The parameter $\chi = \{0, 1\}$ determines whether routine formation occurs (for $\chi=1$) or whether routines do not arise (for $\chi=0$). It should be noted that the term $(1+H)$ guarantees that routines exert a positive effect on utility along with the consumption of the dirty good even for $H < 1$ (in the case of for $\chi=1$). Further, the function specified in equation (1) implies that clean and dirty consumption are substitutes and that each good yields positive utility even if the consumption of the other good equals zero. Environmental pollution is denoted by P and $D(P)$ denotes the damages resulting from the stock of pollution that reduce the utility of the household.

We should like to point out that the household has consumed only polluting goods in the past up to $t=0$. This consumption behaviour has led to the formation of routines. At $t=0$, a non-polluting consumption good becomes available and the household has to choose between the polluting good and the new, non-polluting good. Since it is new, we assume that it is not subject to routine formation, even though that phenomenon may subsequently occur if the non-polluting good is consumed over a sufficiently long time horizon. We make this assumption because our goal with this paper is to find the effects of routines accompanying a certain consumption behaviour over a long time span in the past.²

Routines in our model arise as a side effect of cumulated past consumption of the dirty consumption good. Thus, routine formation is described by the following equation

$$H(t) = \gamma \int_{-\infty}^t e^{\gamma(t-s)} C_d(s) ds, \text{ for } \chi=1. \quad (2)$$

The parameter $\gamma > 0$ gives the weight attributed to more recent levels of dirty consumption in the process of routine formation. The higher γ is, the larger the weight given to more recent levels of dirty consumption compared to consumption flows further back in time. Differentiating equation (2) with respect to time leads to

$$\dot{H} = \gamma(C_d - H), \quad H(0) = H_0. \quad (3)$$

¹ We neglect the time argument t as long as no ambiguity arises.

² Analysing the model allowing for the gradual building-up of routines as a result of consuming the non-polluting good is left for future research.

Equation (2) shows that, from a technical point of view, the process of routine formation is equivalent to that of habit formation as introduced by Ryder and Heal (1973). In economics, habit formation is often used to describe addictive behaviour (see e.g. Becker and Murphy, 1986, or Iannaccone, 1986). Addictive behaviour, however, is seen as a defect from a psychological point of view, whereas we want to emphasize the positive effects of routines on individuals' well-being, resulting from certain kinds of behaviour in the past, such as consuming a certain type of good over a long time period. Therefore, we refer to the variable in equation (2) as routines rather than habits.

It must be pointed out that in the case of routine formation ($\chi=1$), preferences are no longer intertemporally independent. In our model, the constant intertemporal elasticity of substitution $1/\sigma$ determines whether the utility function displays distant or adjacent complementarity. The condition $1/\sigma < 1$ is sufficient for distant complementarity and the condition $1/\sigma > 1$ is necessary for adjacent complementarity. Even if neither of those conditions is necessary and sufficient at the same time, we speak of distant (adjacent) complementarity if $1/\sigma < (>)1$ holds. From an economic point of view, distant (adjacent) complementarity or, more specifically, complementarity between distant (adjacent) dates, means that a small increase in consumption at t_3 shifts consumption from t_1 to t_2 (from t_1 to t_2), with $t_1 < t_2 < t_3$.³ Loosely speaking, distant complementarity means that the household prefers to have consumption smoothed over time, while adjacent complementarity implies that the consumer prefers to have bundles of the consumption good at nearby dates.

As mentioned above, environmental pollution negatively affects the utility of the household, with $D(\cdot)$ giving the damages. The function $D(\cdot)$ is specified as

$$D(P) = P^2/2. \tag{4}$$

The environmental pollution is modelled as a stock and evolves according to

$$\dot{P} = \omega_1 C_d + \omega_2 C_c - \xi P, P(0) = P_0. \tag{5}$$

The constant parameters $\omega_1 > \omega_2 \geq 0$ give the contribution of one unit of the dirty and of the clean consumption good to environmental pollution, respectively, and $\xi > 0$ reflects the ability of the environment to recover.

Production in our economy takes place with physical capital, K , as the input factor. The evolution of the capital stock can be written as

³ See Ryder and Heal (1973) or Greiner (1998), p. 72-74, for a more detailed discussion.

$$\dot{K} = AK^\beta - p_d C_d - p_c C_c - \delta K, K(0) = K_0. \quad (6)$$

The term $Y = AK^\beta$ gives output as a function of the capital stock. Output can be used for investment and for the consumption of the clean and of the dirty good, where p_d and p_c denote the constant price of the dirty consumption good and of the clean consumption good in terms of output, the price of which is set equal to one. The parameter δ is the depreciation rate, A is a constant technology parameter and β gives the elasticity of output with respect to capital.

■ 3. The competitive economy

When we talk of competitive economy, we refer to the situation where the household maximizes the discounted stream of utility subject to the resource constraint (6), but neglects the two externalities, (3) and (5). The justification here is that there are a great many households and the pollution of each individual household is small, so the representative household does not take it into account when formulating its optimization problem. In equilibrium, however, where all households behave identically, the aggregate stock of pollution is built up as a by-product of consuming the polluting consumption good and affects the utility of each household. As concerns routine formation, this process happens unconsciously, so the household is not explicitly aware of it and, therefore, does not take account of it in setting up its optimization problem.

Denoting the rate of time preference by ρ , the optimization problem of the representative household can then be written as

$$\max_{C_d, C_c} \int_0^\infty e^{\rho t} \left(\frac{(C_d(1+H)^\chi)^{1-\sigma} - 1}{1-\sigma} + \frac{C_c^{1-\sigma} - 1}{1-\sigma} - P^2/2 \right) dt, \quad (7)$$

subject to (6). To solve the optimization problem, we set up the current-value Hamiltonian as

$$\mathcal{H}^s = U(\cdot) + \lambda_1 (AK^\beta - p_d C_d - p_c C_c - \delta K), \quad (8)$$

with the utility function $U(\cdot)$ given by (1) and λ_1 denoting the shadow price or co-state variable of physical capital. The necessary optimality conditions for an optimum are given by

$$\frac{\partial \mathcal{H}}{\partial C_d} = 0 \leftrightarrow C_d = \left(\frac{(1+H)^\chi}{\lambda_1 p_d} \right)^{1/\sigma} (1+H)^{-\chi} \quad (9)$$

$$\frac{\partial \mathcal{H}}{\partial C_c} = 0 \leftrightarrow C_c = (\lambda_1 p_c)^{-1/\sigma} \quad (10)$$

$$\dot{\lambda}_1 = (\rho + \delta)\lambda_1 - \lambda_1 \beta AK^{\beta-1} \quad (11)$$

In addition, we require that the usual transversality condition $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_1 K = 0$ must hold.

In equilibrium, environmental pollution (5) and routine formation (3) occur and influence the evolution of the economy. Thus, the economy is completely described by the following four-dimensional system of differential equations, in case of routine formation ($\chi = 1$), and by the three-dimensional system if routines do not occur ($\chi = 0$),

$$\dot{\lambda}_1 = (\rho + \delta)\lambda_1 - \lambda_1 \beta K^{\beta-1} \quad (12)$$

$$\dot{K} = AK^{\beta} - p_d \left(\frac{(1+H)^{\chi(1-\sigma)}}{\lambda_1 p_d} \right)^{1/\sigma} - p_c (\lambda_1 p_c)^{-1/\sigma} - \delta K, K(0) = K_0 \quad (13)$$

$$\dot{P} = \omega_1 \left(\frac{(1+H)^{\chi(1-\sigma)}}{\lambda_1 p_d} \right)^{1/\sigma} + \omega_2 (\lambda_1 p_c)^{-1/\sigma} - \xi P, P(0) = P_0 \quad (14)$$

$$\dot{H} = \chi \gamma \left(\left(\frac{(1+H)^{\chi}}{\lambda_1 p_d} \right)^{1/\sigma} (1+H)^{-\chi} - H \right), H(0) = H_0 \quad (15)$$

Before we analyse the asymptotic behaviour of this system, we study the effect of routine formation on clean and dirty consumption by looking at the consumption of clean goods relative to dirty goods, defined as $C_{rel} = C_c / C_d$. The share of the clean consumption good relative to the dirty one is given by

$$C_{rel} := \left(\frac{p_d}{p_c} \right)^{1/\sigma} (1+H)^{\chi(\sigma-1)/\sigma} \quad (16)$$

From equation (16) we can derive our first result regarding the effect of routine formation in proposition 1.

Proposition 1 *In the competitive economy, the presence of routines raises (reduces) the consumption of the clean good relative to the dirty good if the preferences of the consumer are characterized by distant (adjacent) complementarity.*

Proof: Follows immediately from (16) with $1/\sigma < (>) 1$ characterizing distant (adjacent) complementarity. □

Proposition 1 shows that the ratio of the clean consumption good to the dirty one will be higher in the situation where routine formation occurs than in the situation without routine formation, when there is distant complementarity. The reason for that outcome is that in the case of distant complementarity, the marginal product of the dirty consumption good is smaller when routines are present compared to the

situation without routine formation. This holds because with distant complementarity the consumer tends to smooth consumption over time. A lower marginal product of consumption reduces the ratio of dirty to clean consumption or, equivalently, raises clean relative to dirty consumption. In case of adjacent complementarity, the reverse holds. Then, routine formation raises the marginal product of the dirty consumption good so that the ratio of clean consumption relative to the dirty one becomes smaller. Finally, we should also like to point out that in the situation without routine formation, the ratio of clean to dirty consumption is completely determined by the price ratio of these two goods.

Before we study the transitional dynamics, we analyse the effect of routine formation on the steady-state stock of pollution. Proposition 2 gives the result.

Proposition 2 *For $\omega_1 > \omega_2(p_d/p_c)$, routines reduce (raise) the steady-state pollution if the preferences are characterized by distant (adjacent) complementarity. In the case of $\omega_1 < \omega_2(p_d/p_c)$ routines raise (reduce) the steady-state pollution if the preferences are characterized by distant (adjacent) complementarity.*

Proof: See appendix A. □

In order to understand and interpret proposition 2, we note that routines raise the optimal value of the clean consumption good relative to the dirty one in the case of distant complementarity (see proposition 1). Since total aggregate output at the steady-state is given and independent of the presence of routines, more clean consumption implies less dirty consumption. This tends to reduce pollution. However, the overall effect depends on the amount by which clean and dirty consumption are increased and reduced, respectively, and on the contribution to pollution of one unit of the dirty consumption good and of one unit of the clean consumption good, i.e. on ω_1 and on ω_2 . From the budget constraint of the household at the steady-state, we know that the relative price p_c/p_d determines the amount by which the dirty good declines when the clean good rises by one unit. If that ratio is larger than one or, equivalently, if p_d/p_c is smaller than one, a rise in the clean consumption good always reduces pollution because $\omega_1 > \omega_2$. If, however, p_c/p_d is small so that a one-unit increase in the clean consumption good leads to only a small decline in dirty consumption, the latter may not be large enough to cause a rise in pollution at the steady-state, due to the higher consumption of the clean good that also contributes to pollution, although to a smaller degree. This is particularly relevant when the difference between ω_1 and ω_2 is small, i.e. when the clean consumption good is only slightly less polluting than the dirty good. Note that in the extreme case of $\omega_2 = 0$, i.e. when the clean consumption good is not polluting at all, the presence of routines will always reduce steady-state pollution in the case of distant complementarity. The same holds when

the clean consumption good is more expensive than the dirty good, which is expected to be more commonly the case in real-world economies.

In the case of adjacent complementarity, the argument is the exact opposite to the one in the situation with distant complementarity. With adjacent complementarity, the presence of routines reduces the ratio of clean consumption to dirty consumption at the steady-state. This effect will raise steady-state pollution, unless the increase in the dirty consumption is very small, as clean consumption declines, due to a very high price of the clean consumption good relative to the dirty good.

Before we analyse the social optimum, we will study the question of the uniqueness and stability of the steady-state. Proposition 3 gives the result for the competitive economy without routine formation.

Proposition 3 *In the competitive economy without routine formation, there exists a unique saddle point stable steady-state.*

Proof: See appendix B. □

Proposition 3 shows that our model is characterized by the usual long-run behaviour of this type of growth model, even in the presence of an environmental externality. Hence, there exists a unique value of the initial shadow price, $\lambda_1(0)$, giving unique values of initial clean and dirty consumption such that the economy converges to the long-run steady-state.

When we allow for routine formation the economy is described by a four-dimensional differential equation system. In this case, the economy is also expected to be characterized by a unique steady-state for plausible values of the intertemporal elasticity of substitution. Proposition 4 shows the result.

Proposition 4 *In the competitive economy with routine formation, a sufficient condition for the existence of a unique steady-state is $1/\sigma \leq 2$ and distant complementarity of the preferences, i.e. $1/\sigma < 1$, is sufficient for saddle point stability of the steady-state.*

Proof: See appendix C. □

Proposition 4 demonstrates that the model is not necessarily characterized by a unique saddle point stable steady-state when routines are allowed for. However, it must be pointed out that the conditions in the proposition are sufficient but not necessary for uniqueness and saddle point stability, so we can expect a unique saddle point stable steady-state even if they are not fulfilled. As regards the intertemporal elasticity of sub-

stitution, most of the empirical studies that estimate it find small values. However, there are substantial cross-country differences in this parameter and rich countries and economies with high stock market participation substitute a larger fraction of consumption intertemporally in response to changes in expected asset returns. Havranek *et al.* (2015) perform a meta-analysis and find a mean of 1/2 for the intertemporal elasticity, but they also point out that the estimates vary greatly between economies. In particular, there are also countries for which the point estimate clearly exceeds 1, so high values for that parameter cannot be considered as purely academic.

In order to illustrate our results so far, we resort to a numerical example, where we use the following parameter values. We set the capital share to 30 percent, $\beta = 0.3$, and the technology parameter to one, $A = 1$. The prices of both consumer goods are set equal to one, $p_c = p_d = 1$, and one unit of the dirty consumption good raises the stock of pollution by 0.1, $\omega_1 = 0.1$, which is ten times as high as the contribution of one unit of the clean good, $\omega_2 = 0.01$. The depreciation rate of physical capital is 7.5 percent, $\delta = 0.075$, and pollution recovers at a rate of $\xi = 0.1$. The parameter γ is set to 10 percent, $\gamma = 0.1$, implying that the contribution of one unit of the dirty consumption good to current routines is $e^{-0.1 \cdot 1} = 0.905$ and the contribution of dirty consumption five years back is $e^{-0.1 \cdot 5} = 0.606$. Finally, the discount rate is set to 3.5 percent, $\rho = 0.035$, and $1/\sigma$ takes the value 1/2 or 4/3. Table 1 shows the steady-state values of the model without and with routine formation.⁴

● **Table 1. Steady-state values of the competitive economy ($K^* = 4.192$)**

	No routines ($\chi=0$)		With routines ($\chi=1$)		
	C_c^*/C_p^*	P^*	C_c^*/C_p^*	P^*	H^*
$1/\sigma = 1/2$	1	0.673	1.243	0.613	0.545
$1/\sigma = 4/3$	1	0.673	0.844	0.719	0.663

Table 1 demonstrates that the model with routines leads to a higher steady-state ratio of clean to dirty consumption in the case of distant complementarity ($1/\sigma < 1$) and to a lower value of that ratio for adjacent complementarity ($1/\sigma > 1$). Further, with distant complementarity, the stock of pollution at the steady-state is lower than for the model without routine formation because with both prices set equal to one, i.e. $p_c = p_d = 1$, the condition $\omega_1 > \omega_2(p_c/p_d)$ in proposition 2 always holds. With adjacent complementarity of the preferences, the presence of routines raises the steady-state stock of pollution. Finally, we should like to point out that for the parameter values underlying the outcome in Table 1, the two model versions are saddle point stable with two and three negative real eigenvalues, respectively.

⁴ The * denotes steady-state values.

In the next section, we present the social optimum and compare it to the competitive economy.

■ 4. The social optimum

For the social optimum, the maximization problem is written as

$$\max_{C_d, C_c} \int_0^{\infty} e^{-\rho t} \left(\frac{(C_d (1+H)^\chi)^{1-\sigma} - 1}{1-\sigma} + \frac{C_c^{1-\sigma} - 1}{1-\sigma} - P^2/2 \right) dt, \quad (17)$$

subject to (3), (5), (6). The difference to the competitive economy is that now both the pollution externality and the externality of consuming the dirty good, i.e. the routine formation, are taken into account in setting up the intertemporal optimization problem. To solve the optimization problem, we set up the current-value Hamiltonian, which is written as

$$\mathcal{H}^s = U(\cdot) + \lambda_1^s (K^\beta - p_d C_d - p_c C_c - \delta K) + \lambda_2^s (\omega_1 C_d + \omega_2 C_c - \xi P) + \lambda_3^s \chi \gamma (C_d - H), \quad (18)$$

with the utility function $U(\cdot)$ given by (1) and λ_i^s , $i = 1, 2, 3$, the shadow prices or co-state variables of capital, pollution and routines, respectively.

The necessary optimality conditions for an optimum are as follows

$$\frac{\partial \mathcal{H}}{\partial C_d} = 0 \leftrightarrow C_d = \left(\frac{(1+H)^\chi}{\lambda_1 p_d - \lambda_3 \gamma - \lambda_2 \omega} \right)^{1/\sigma} (1+H)^{-\chi} \quad (19)$$

$$\frac{\partial \mathcal{H}}{\partial C_c} = 0 \leftrightarrow C_c = (\lambda_1 p_c - \lambda_2 \omega_2)^{-1/\sigma} \quad (20)$$

$$\dot{\lambda}_1^s = (\rho + \delta) \lambda_1^s - \lambda_1^s \beta AK^{\beta-1} \quad (21)$$

$$\dot{\lambda}_2^s = (\rho + \xi) \lambda_2^s + P \quad (22)$$

$$\dot{\lambda}_3^s = \chi ((\rho + \gamma) \lambda_3^s - C_d^{1-\sigma} (1+H)^{-\sigma}) \quad (23)$$

In addition, we require that the usual transversality condition must hold, which can be written as $\lim_{t \rightarrow \infty} e^{-\rho t} (\lambda_1^s K + \lambda_2^s P + \lambda_3^s H) = 0$. Thus, in the case of routine formation, i.e. for $\chi = 1$, the economy is completely described by the differential equations (3), (5), (6), (21), (22), (23), with C_c and C_d determined by (19)-(20) respectively, and

where $\lambda_i^s(0)$, $i = 1, 2, 3$, are free.⁵ When routine formation does not occur, i.e. for $\chi = 0$, the economy is described by the differential equations (5), (6), (21), (22) and the corresponding transversality condition.

Since the competitive economy and the social optimum do not coincide, we first derive the tax rates on the polluting and on the non-polluting consumption good such that the competitive economy replicates the social optimum. To do so, we denote by τ_d the Pigou tax rate on the dirty consumption good and by τ_c the Pigou tax rate on the clean good.⁶ The budget constraint of the household in the competitive economy then becomes $\dot{K} = AK^\beta - (p_d + \tau_d)Cd - (p_c + \tau_c)C_c - \delta K + \Gamma$, with Γ lump-sum transfers or a lump-sum tax that are adjusted such that the budget of the government is balanced at each point in time. Proposition 5 gives the Pigou tax rates.

Proposition 5 *The competitive economy replicates the social optimum if the tax rates are set such that $\tau_c = (-\lambda_2^s)\omega_2/\lambda_1$ and $\tau_d = (-\lambda_2^s)\omega_1/\lambda_1 - \chi\lambda_3^s\gamma/\lambda_1$ holds, with $\lambda_1(0) = \lambda_1^s(0)$.*

Proof: See appendix D. □

Proposition 5 demonstrates that the Pigou tax rate on the clean consumption good is positive since the clean consumption good also contributes to environmental pollution. It depends on the absolute value of the shadow price of pollution multiplied by the contribution of one unit of clean consumption to pollution growth, relative to the shadow price of physical capital. Only when the clean consumption good does not lead to environmental pollution, i.e. when $\omega_2 = 0$, does the optimal tax rate equal zero. In that case, clean consumption is not accompanied by externalities, so there is no need to impose a tax on that good.

The Pigou tax rate on the dirty consumption good also depends on the absolute value of the shadow price of pollution multiplied by the contribution of one unit of dirty consumption to pollution growth, relative to the shadow price of physical capital. However, in contrast to the optimal tax rate on the clean consumption good, the Pigou tax on the dirty consumption good is also a function of routines. The stronger the routine formation and the higher the shadow price, the lower the optimal tax rate on the dirty good. If routine formation is very strong, the tax rate on the dirty consumption good may even be negative meaning that the government pays a subsidy for the polluting good. In that case, the consumption of the dirty good in the competitive economy is too low compared to the social optimum. This also implies that

⁵ For $1/\sigma \leq 2$, the utility function and, thus, the Hamiltonian is jointly concave in the control and state variables so that the necessary optimality conditions are also sufficient.

⁶ We consider a quantity tax but an ad-valorem tax would lead to the same result.

pollution at the steady-state in the competitive economy without government intervention is lower than in the social optimum. The reason for that outcome is to be seen in the second externality in this model, namely in routine formation, which has a positive effect on utility. Since that effect is not taken into account in the competitive economy, the consumption of the dirty good in the social optimum can be higher than in the competitive economy. This will occur when routine formation is strong and its shadow value is large.

If, on the other hand, the consumption of the dirty good in the competitive economy is too high, the Pigou tax rate on the polluting good will be strictly positive. This is the case when routine formation is not very strong so the social planner does not attach great importance to that phenomenon but instead puts more emphasis on the negative externality accompanying consumption, i.e. on pollution control. More concretely, this will always hold when the cost of consuming one unit of the dirty good exceeds its external benefits, which consists in the formation of routines. Where there is no routine formation, we get the usual result wherein the steady-state pollution in the competitive economy is higher than in the social optimum. We summarize these results in the following proposition 6.

Proposition 6 *Without routine formation, the steady-state pollution in the competitive economy exceeds the steady-state pollution in the social optimum. With routine formation, the steady-state pollution in the competitive economy falls short of the steady-state pollution in the social optimum if and only if $(\lambda_3^s)^* \gamma > (-\lambda_2^s)^* \omega_1$ holds.*

Proof: See appendix E. □

As concerns the existence, uniqueness and stability of the steady-state, it is difficult to make statements for the analytical model due to its complexity. However, for the model without routine formation it is possible to show that the steady-state of the social optimum is a saddle point, as indicated by proposition 7.

Proposition 7 *In the social optimum without routine formation, the steady-state is saddle point stable.*

Proof: See appendix F. □

To gain additional insight and in order to illustrate our analytical results, we turn to the numerical example presented above. Table 2 presents the steady-state values for the social optimum without and with routines.⁷ As in the competitive economy, the

⁷ Again, there exists a unique saddle point stable steady-state with two and three negative real eigenvalues, respectively.

presence of routines raises the ratio of clean to dirty consumption and thus reduces steady-state pollution if the preferences are characterized by distant complementarity. In the case of adjacent complementarity, the reverse holds and routines reduce clean consumption relative to dirty consumption and lead to higher pollution.

● **Table 2. Steady-state values of the social optimum ($K^* = 4.192$)**

	No routines ($\chi=0$)		With routines ($\chi=1$)		
	C_c^*/C_p^*	P^*	C_c^*/C_p^*	P^*	H^*
$1/\sigma = 1/2$	1.084	0.650	1.207	0.621	0.554
$1/\sigma = 4/3$	1.419	0.577	0.859	0.714	0.663

Comparing the competitive economy with the social optimum when routine formation is present, it can be seen that in the case of distant complementarity, the ratio of clean consumption to dirty in the social optimum is smaller than in the competitive economy leading to a higher steady-state pollution and to a higher stock of routines. The reason for that outcome lies in the benefits of routine formation that exceed the costs of pollution. The optimal steady-state tax rates for the economy in this case are $\tau_c = 0.021$ and $\tau_d = -0.043$, showing that the government in the competitive economy should subsidize the dirty consumption good. In the case of adjacent complementarity, the steady-state pollution and the stock of routines in the social optimum are lower than in the competitive economy and the ratio of clean to dirty consumption is higher. The Pigou tax rates at the steady-state for this case are $\tau_c = 0.036$ and $\tau_d = 0.049$, showing that both goods should be taxed in the competitive economy.

Our considerations have shown that it may be necessary to pay subsidies so that the competitive market economy replicates the social optimum. This raises the question of how the subsidies should be financed. One possibility is to use the tax revenue from that good on which a Pigou tax is levied and to use the revenue for the subsidy. If the tax revenue falls short of the required amount for the subsidy, the policy maker should levy an additional non-distortionary tax in order to attain a balanced government budget. If the planner wants to encourage the use of the clean good and to influence the preferences of the individual so that the consumption of the clean good leads to routine formation, substantial subsidies should be provided for the clean good. If the latter is consumed over a sufficiently long time period, the bias towards the established dirty good will vanish. To achieve that goal, the clean good must be heavily subsidized so that its net price is clearly below that of the dirty good. That subsidy can be financed by a tax on the dirty good. Over time, the clean good will lead to routine formation, and so the subsidy can be reduced. A rigorous analysis of this policy, however, is beyond the scope of this paper and is left for future research.

5. Conclusion

While we believe that individuals try to follow economic rationality when making economic decisions, as predicted by sociological systems theory, we understand that human behaviour is constrained by cognitive abilities, by routines or by social representations. This paper presents an approach to study the role of imperfections resulting from constrained human behaviour with regard to the consumption of polluting and of non-polluting goods. Here, we have analysed the effects of routine formation on pollution for an otherwise standard exogenous growth model, where the consumer can choose between a conventional dirty good and a good that contributes less to pollution. Routines result from certain kinds of behaviour in the past – in our model, from past consumption of the polluting good – and can be seen as unintended learning processes that simplify everyday life.

The analysis of the competitive model economy has shown that the presence of routines raises the consumption of the clean good relative to the dirty good if the preferences are characterized by distant complementarity but reduces that ratio if there is adjacent complementarity. Loosely speaking, when individuals prefer consumption smoothing rather than bundles of consumption at nearby dates, routine formation is more likely to raise clean relative to dirty consumption. Further, the effects of routines on the steady-state pollution stock depend on whether distant or adjacent complementarity is given and on the contribution of the goods to pollution as well as on their relative price. The Pigou tax rates have been derived and it turns out that pollution in the social optimum can be higher than in the competitive economy if routine formation is strong enough that the Pigou tax on the dirty good is negative, i.e. the optimal policy is to subsidize the dirty consumption good.

As regards the policy implications, it must be underlined that the outcome that subsidizing the polluting good may be optimal, crucially depends on the structure of the model. More concretely, it is due to the fact that the non-polluting good does not lead to routine formation. Hence, taking into account the fact that the consumption of the non-polluting good can also lead to routine formation, if this good were to be consumed over a sufficiently long time horizon, it would change the outcome. Thus, if policy makers want to foster the consumption of non-polluting goods they have to give incentives, such as subsidies, over a certain time period. Once the good has been consumed over a sufficiently long time period, the incentives can be reduced or completely abandoned.

The model is very simple and it could be extended in several directions. For example, one could resort to a more general CES utility function and analyse the role of substitutability between the two goods. Further, the prices of the two types of goods have

been assumed to be constant parameters. Thus, it would be interesting to allow for learning effects in the production of the less polluting good, for example, that reduce its price over time. Finally, it would also be interesting to analyse that model for the case of ongoing growth that is generated endogenously.

A Proof of proposition 2

At the steady-state, we have $p_c C_c^* + p_d C_d^* = A(K^*)^\beta - \delta K^* = \text{constant}$, with $K^* = (A\beta/(\rho + \delta))^{1/(1-\beta)}$ obtained from $\dot{\lambda}_1 = 0$. This gives $C_d^* = (A(K^*)^\beta - \delta K^* - p_c C_c^*)/p_d$.

Thus, the pollution in steady-state is obtained from $\dot{P} = 0$ as

$$P^* = \frac{\omega_1(A(K^*)^\beta - \delta K^*)}{\xi p_d} + \frac{\omega_1 p_c C_c^*}{\xi p_d} - \frac{\omega_2 C_c^*}{\xi}$$

giving

$$\frac{dP^*}{dC_c^*} = \frac{1}{\xi} \left(\omega_2 - \frac{p_c \omega_1}{p_d} \right) > (<) 0 \leftrightarrow \omega_2 p_d > (<) \omega_1 p_c$$

From $\dot{K} = 0$, one obtains for the model with routines ($\chi = 1$)

$$\lambda_1^* = \left(\frac{(1+H)^{(1-\sigma)/\sigma} p_d^{1-1/\sigma} + p_c^{1-1/\sigma}}{A(K^*)^\beta - \delta K^*} \right)^{-\sigma}$$

and

$$\frac{d\lambda_1^*}{dH} = \frac{(1-\sigma)(\lambda_1^*)^{\sigma-1} (1+H)^{(1/\sigma)-2} p_d^{1-1/\sigma}}{A(K^*)^\beta - \delta K^*} > (<) 0 \leftrightarrow 1/\sigma > (<) 1$$

Since $dC_c/d\lambda_1 < 0$, this shows that $dC_c/dH > (<) 0$ for $1/\sigma < (>) 1$. Noting that for $1/\sigma < (>) 1$ the preferences are characterized by distant (adjacent) complementarity, this proves the proposition. \square

B Proof of proposition 3

Setting $\dot{\lambda}_1 = 0$ gives $K^* = (A\beta/(\rho + \delta))^{1/(1-\beta)}$. From $\dot{K} = 0$, one obtains for the model without routines ($\chi = 0$)

$$\lambda_1^* = \left(\frac{A(K^*)^\beta - \delta K^*}{p_d^{1-1/\sigma} + p_c^{1-1/\sigma}} \right)^{-\sigma}$$

Note that $A(K^*)^\beta > \delta K^*$ holds because $A(K^*)^\beta \leq \delta K^*$ would imply $\delta \geq (\rho + \delta)/\beta$, which follows from $K^* = (A\beta/(\rho + \delta))^{1/(1-\beta)}$. That, however, is excluded because of $\beta \in (0, 1)$ and $\rho > 0$.

Finally, $\dot{P}=0$ leads to

$$P^* = \xi^{-1} \left(\frac{\omega_1}{(\lambda_1^* p_d)^{1/\sigma}} + \frac{\omega_2}{(\lambda_1^* p_c)^{1/\sigma}} \right)$$

For $\chi=0$, the Jacobian matrix of (12)-(14) is given by

$$J = \begin{pmatrix} \rho + \delta - A\beta K^{\beta-1} & -\lambda_1 A\beta(\beta-1)K^{\beta-2} & 0 \\ \sigma^{-1}\lambda_1^{-1-1/\sigma}(p_d^{1-1/\sigma} + p_c^{1-1/\sigma}) & A\beta K^{\beta-1} - \delta & 0 \\ -\sigma^{-1}\lambda_1^{-1-1/\sigma} \left(\frac{\omega_1}{(\lambda_1^* p_d)^{1/\sigma}} + \frac{\omega_2}{(\lambda_1^* p_c)^{1/\sigma}} \right) & 0 & -\xi \end{pmatrix},$$

with K and λ_1 evaluated at the rest point $\{K^*, \lambda_1^*\}$.

One eigenvalue of that matrix is $\mu_1 = -\xi$. The other two are given by the eigenvalues of

$$J^p = \begin{pmatrix} \rho + \delta - A\beta K^{\beta-1} & -\lambda_1 A\beta(\beta-1)K^{\beta-2} \\ \sigma^{-1}\lambda_1^{-1-1/\sigma}(p_d^{1-1/\sigma} + p_c^{1-1/\sigma}) & A\beta K^{\beta-1} - \delta \end{pmatrix}$$

Using that $\rho + \delta - A\beta K^{\beta-1} = 0$ holds at the steady-state, the determinant of J^p can be computed as

$$\det J^p = -\lambda_1 A\beta(\beta-1)K^{\beta-2} \sigma^{-1} \lambda_1^{-1-1/\sigma} (p_d^{1-1/\sigma} + p_c^{1-1/\sigma}) < 0$$

Hence, one eigenvalue of J^p is negative and one is positive so that J has two negative eigenvalues and one positive yielding saddle point stability. \square

C Proof of proposition 4

Setting $\lambda_1 = 0$ again yields $K^* = (A\beta/(\rho + \delta))^{1/(1-\beta)}$. Solving $\dot{H} = 0$ gives $\lambda_1^* = p_d^{-1}(\gamma H(1+H)^{1-1/\sigma})^{-\sigma}$. Inserting K^* and λ_1^* in \dot{K} leads to

$$\dot{K} = A(K^*)^\beta - \delta K^* - H - p_c \left(\frac{p_d}{p_c} \right)^{1/\sigma} H(1+H)^{1-1/\sigma}$$

The derivative of \dot{K} is

$$\frac{\partial \dot{K}}{\partial H} = -1 - p_c \left(\frac{p_d}{p_c} \right)^{1/\sigma} (1+H)^{-1/\sigma} (1+2H-H/\sigma)$$

Thus, the function \dot{K} starts at a positive value for $H = 0$, is monotonically declining in H , for $1/\sigma \leq 2$, and converges to $-\infty$ for $H \rightarrow \infty$. Consequently, there exists a unique positive finite H^* that solves $\dot{K} = 0$. Finally, solving $\dot{P} = 0$ one obtains

$$P^* = (\omega_1/\xi) \left(\frac{(1+H)^{1-\sigma}}{\lambda_1^* p_d} \right)^{1/\sigma} + (\omega_2/\xi) (\lambda_1^* p_c)^{-1/\sigma}$$

For $\chi=1$, the Jacobian matrix of (12)-(15) is given by

$$J_1 = \begin{pmatrix} -A\beta K^{\beta-1} + \delta + \rho & -A(\beta-1)\beta K^{\beta-2}\lambda_1 & 0 & 0 \\ \frac{p_c^2(\lambda_1 p_c)^{-1-\frac{1}{\sigma}}}{\sigma} + \frac{(H+1)^{\frac{1}{\sigma}-1} p_d^2(\lambda_1 p_d)^{-1-\frac{1}{\sigma}}}{\sigma} & A\beta K^{\beta-1} - \delta & 0 & -C_1 \\ -\frac{p_c \omega_2 (\lambda_1 p_c)^{-1-\frac{1}{\sigma}}}{\sigma} - \frac{(H+1)^{\frac{1}{\sigma}-1} p_d (\lambda_1 p_d)^{-1-\frac{1}{\sigma}} \omega_1}{\sigma} & 0 & -\xi & -\omega_1 C_1 \\ -\frac{\gamma(H+1)^{\frac{1}{\sigma}-1} p_d (\lambda_1 p_d)^{-1-\frac{1}{\sigma}}}{\sigma} & 0 & 0 & \gamma(C_1-1) \end{pmatrix},$$

with $C_1 = (H+1)^{(1/\sigma)-2} p_d (\lambda_1 p_d)^{-1/\sigma} ((1/\sigma)-1)$ and K, λ_1, P and H evaluated at the rest point $\{K^*, \lambda_1^*, P^*, H^*\}$.

One eigenvalue of that matrix is $\mu_1 = -\xi$. The other three eigenvalues are the eigenvalues of

$$J_1^p = \begin{pmatrix} A\beta K^{\beta-1} - \delta & -C_1 & \frac{p_c^2(\lambda_1 p_c)}{\sigma} + \frac{(H+1)^{\frac{1}{\sigma}-1} p_d^2(\lambda_1 p_d)^{-1-\frac{1}{\sigma}}}{\sigma} \\ 0 & \gamma(C_1-1) & -\frac{\gamma(H+1)^{\frac{1}{\sigma}-1} p_d (\lambda_1 p_d)^{-1-\frac{1}{\sigma}}}{\sigma} \\ -A(\beta-1)\beta K^{\beta-2}\lambda_1 & 0 & -A\beta K^{\beta-1} + \delta + \rho \end{pmatrix},$$

Using that $\rho + \delta - A\beta K^{\beta-1} = 0$ holds at the steady-state, the determinant of J_1^p can be computed as

$$\det J_1^p = \frac{A(1-\beta)\beta K^{\beta-2}(\lambda_1 p_c)^{-1/\sigma}(\lambda_1 p_d)^{-1/\sigma}(\gamma p_c(\sigma-1)(H+1)^{\frac{1}{\sigma}})}{(H+1)^{2\sigma^2}} + \frac{A(1-\beta)\beta K^{\beta-2}(\gamma(p_d(H+1)^{\frac{1}{\sigma}}(\lambda_1 p_c)^{\frac{1}{\sigma}} + (H+1)p_c(\lambda_1 p_d)^{\frac{1}{\sigma}}))}{(\lambda_1 p_c)^{1/\sigma}(\lambda_1 p_d)^{1/\sigma}(H+1)\sigma},$$

so that $1/\sigma \leq 1$ is sufficient but not necessary for $\det J_1^p > 0$. Next, we define

$$W_1 = a_{11}a_{22} - a_{12}a_{21} + a_{22}a_{33} - a_{23}a_{32} + a_{11}a_{33} - a_{13}a_{31},$$

with a_{ij} the element of the i -th row and j -th column of the matrix J_1^p . In our case, we get

$$W_1 = (A\beta K^{\beta-1} - \delta)\gamma(C_1-1) - A\beta(1-\beta)K^{\beta-2}\lambda_1^c a_{13}$$

From $\rho + \delta - A\beta K^{\beta-1} = 0$ we know that $A\beta K^{\beta-1} - \delta > 0$ holds. Recalling that $C_1 = (H+1)^{(1/\sigma)-2} p_d (\lambda_1 p_d)^{-1/\sigma} ((1/\sigma)-1)$ and using $a_{13} > 0$ shows that $1/\sigma \leq 1$ is a sufficient but not necessary condition for $W_1 < 0$. Since $\det J_1^p > 0$ and $W_1 < 0$ are sufficient for two negative and one positive eigenvalue of J_1^p (see Wirl, 1997), this demonstrates that $1/\sigma \leq 1$ is a sufficient but not a necessary condition for J_1 to have one positive and three negative eigenvalues.

D Proof of proposition 5

We equate (9) with (19) to obtain the optimal tax rate on the dirty good and (10) with (20) to obtain the optimal tax rate on the clean consumption good, with $p_d + \tau_d$ the price of the dirty good in the competitive economy and $p_c + \tau_c$ the price of the clean good in the competitive economy. Further, setting the initial shadow prices of physical capital to the same value, i.e. $\lambda_1(0) = \lambda_1^s(0)$, ensures that the shadow price of capital in the competitive economy is equal to that in the social optimum for all $t \in [0, \infty)$. \square

E Proof of proposition 6

To prove this proposition, we note that in the economy without routine formation the relative steady-state consumption is given by $C_{rel} = p_c / p_d$ for the competitive economy.

In the social optimum, it is given by

$$C_{rel}^s = \frac{p_c - \omega_2 \lambda_2^s / \lambda_1^s}{p_d - \omega_1 \lambda_2^s / \lambda_1^s}$$

Thus, steady-state consumption of the clean good relative to the dirty good in the competitive economy is always lower than in the social optimum because $\omega_2 < \omega_1$ and $\lambda_2 < 0$. Since total consumption is fixed at the steady-state, this implies that dirty consumption is higher and clean consumption is lower so that the steady-state stock of pollution in the competitive economy is larger.

With routine formation, the tax rate on the dirty good is negative if the condition in the proposition holds. A negative tax implies that consumption of the dirty good in the competitive economy without government intervention is lower than in the social optimum. Together with a fixed total steady-state consumption this implies that the steady-state pollution in the competitive economy is lower than in the social optimum. \square

F Proof of proposition 7

To prove this proposition, we note that the dynamics of the social optimum without routine formation is described by the equations (5), (6), (21) and (22). The Jacobian

for that dynamic system is

$$J_2 = \begin{pmatrix} A\beta K^{\beta-1} - \delta & 0 & \frac{p_c^2 C_c^{-1}}{\sigma} + \frac{p_d^2 C_d^{-1}}{\sigma} & -\frac{p_c \omega_2 C_c^{-1}}{\sigma} - \frac{p_d \omega_1 C_d^{-1}}{\sigma} \\ 0 & -\xi & \frac{p_c \omega_2 C_c^{-1}}{\sigma} - \frac{p_d \omega_1 C_d^{-1}}{\sigma} & \frac{\omega_2^2 C_c^{-1}}{\sigma} + \frac{\omega_1^2 C_d^{-1}}{\sigma} \\ -A(\beta - 1)\beta \lambda_1 K^{\beta-2} & 0 & -A\beta K^{\beta-1} + \delta + \rho & 0 \\ 0 & 1 & 0 & \rho + \xi \end{pmatrix},$$

with $C_c = (\lambda_1 p_c - \lambda_2 \omega_2)^{-1/\sigma}$ and $C_d = (\lambda_1 p_d - \lambda_2 \omega_1)^{-1/\sigma}$. Using $\rho + \delta = A\beta K^{\beta-1}$ at the steady-state that follows from $\dot{\lambda}_1 = 0$, the determinant can be computed as

$$\det J_2 = A(1 - \beta)\beta \lambda_1 K^{\beta-2} \left(\frac{\xi(\rho + \xi)(p_c^2 C_c^{-1} + p_d^2 C_d^{-1})}{\sigma} + \frac{(p_c \omega_1 - p_d \omega_2)^2 C_c^{-1} C_d^{-1}}{\sigma^2} \right) > 0$$

Next, we define W_2 as

$$W_2 = a_{11}a_{33} - a_{13}a_{31} + a_{22}a_{44} - a_{24}a_{42} + 2(a_{12}a_{34} - a_{14}a_{32})$$

For our model, W_2 is computed as

$$W_2 = A(\beta - 1)\beta \lambda_1 K^{\beta-2} \left(\frac{p_c^2}{C_c \sigma} + \frac{p_d^2}{C_d \sigma} \right) - \xi(\rho + \xi) - \frac{\omega_2^2}{C_c \sigma} - \frac{\omega_1^2}{C_d \sigma} < 0$$

According to lemma 2 in Dockner and Feichtinger (1991), a positive determinant and a negative W_2 implies two negative real eigenvalues or two complex conjugate eigenvalues with negative real parts. This proves the proposition.

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