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Mathematics achievement and the science of mathematics education: evidence from different nations

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A SYMBOLIC KNOWLEDGE OF NUMBERS: A WINDOW INTO THE EARLY UNDERSTANDING OF NUMERIC STRUCTURE[1]

Conocimiento simbólico de los números: una ventana hacia la comprensión temprana de la estructura numérica

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INTRODUCTION. Accumulating evidence, primarily from English-speaking children, indicates that acquisition of multi-digit numbers begins prior to formal math instruction. The present study replicated this phenomenon in a novel cultural/linguistic context and extended the current knowledge of early symbolic numeric development. **METHOD**. The study involved a sample of Russian preschoolers who took part in two testing sessions. In one session, children completed symbolic numeric tasks: writing and reading of multi-digit numbers. In another session, they completed a non-verbal intelligence task. Children's performance on the two numeric tasks was compared, controlling for their general intelligence level. **RESULTS**. Russian preschoolers found the reading task more challenging than the corresponding writing task. In particular, when reading numbers that included two or three digits, children were more likely to make conceptual errors that revealed the difficulty of understanding the hierarchical structure of multi-digit numbers. In contrast, the frequency of errors in which the structure of the multi-digit number was preserved (for example, substituting one of the digits) was similar across the writing and reading tasks. **DISCUSSION**. Consistent with prior work, preschoolers in the present study revealed a partial knowledge of multi-digit numbers that emerges prior to formal instruction and is likely based on informal learning. The relative difficulty of the reading task -compared to the writing task- suggests that at the early stages of learning symbolic numbers children may require additional cues about numeric structure, which may be provided by spoken number names. The written numerals do not provide linguistic cues about numeric structure, making the reading task more challenging. Implications of these findings for early educational practice are discussed.

Keywords: *Symbolic number, Numeric knowledge, Early mathematics, Cognitive development.*

Introduction

A key feature of the conventional (decimal) number system is its hierarchical organization that reflects the base-10 structure of numbers. In this system, any multi-digit number can be thought of as a combination of increasing powers of ten: ones (10^0), tens (10^1), hundreds (10^2), etc. This structural feature is captured by the concept of place value. In written multi-digit numerals, place value is conveyed by the relative position of the digit; in spoken number words, the place value is conveyed by the order in which digits are named and by linguistic markers that indicate each digit's value. Learning how to read or write multi-digit numbers requires figuring out the rules of organizing individual digits into a hierarchical structure. Thus, the extent to which the child can do these tasks can be used as an early indicator of the child's understanding of numeric structure.

Consistent with this argument, empirical evidence from recent studies shows that early symbolic number knowledge – in particular, the knowledge of written and spoken number symbols – serves as a powerful predictor of subsequent math performance. It has been shown, for example, that the kindergartners' mastery of the numeral system, including the ability to identify multi-digit numbers, predicted arithmetic accuracy a year later, whereas non-symbolic magnitude comparison skills did not predict any additional variance in arithmetic development (Göbel, Watson, Lervåg & Hulme, 2014). In another study, children's knowledge of numerals fully mediated the relation between preschoolers' informal math knowledge (e.g., cardinality principle) and formal math skills (e.g., computation) assessed a year later (Purpura, Baroody & Lonigan, 2013). Finally, a meta-analysis of 45 articles examining symbolic and non-symbolic quantitative processing in relation to a wide range of math outcomes showed that symbolic skills had a stronger association with broader math competence than non-symbolic skills (Schneider *et al.*, 2016).

Symbolic Number Knowledge Prior to Formal Instruction

The predictive power of early symbolic number knowledge has led developmental and educational researchers to a closer examination of this aspect of children's quantitative development. A key finding emerging from this research is that children begin acquiring the multi-digit number system well before they receive formal school instruction on this topic. In one study, children between 4 and 6 years were given a task of writing three-digit numbers (Byrge, Smith & Mix, 2014). In their responses, almost all 6-year-olds, the majority of 5-year-olds, and some 4-year-olds demonstrated at least a partial understanding of the structure of multi-digit numbers. In another study, children between 3 and 7 years had to identify which of the two written multi-digit numbers corresponded to a spoken number word (Mix, Prather, Smith & Stockton, 2014). The results showed that even 3-year-olds performed significantly above chance on this task. The authors suggested that the initial knowledge of multi-digit numbers develops through informal exposure. Everyday experiences may allow children to extract structural patterns from the number names they hear and map them onto the written numerals they see, leading at least to partial knowledge of the number system.

Evidence indicating that children begin acquiring the knowledge of multi-digit numbers before formal instruction raises further questions about the nature of this early symbolic knowledge. The question addressed in the present study concerns the relation between different manifestations of the symbolic numeric skills – namely, number writing and number reading. None of the extant studies, to our knowledge, directly compared how young children write and read multi-digit numbers. Yet, examining differences and commonalities between these tasks may lead to a better understanding of the nature of conceptual difficulties children face as they master the symbolic number system.

Measuring Early Knowledge of Multi-Digit Numbers

In the past, investigators used three types of experimental tasks to assess children's knowledge of number symbols. One of them is a choice task, where the child has to match a number name spoken by the experimenter to one of several written numerals presented (Göbel *et al.,* 2014). While this task can provide information about the extent of familiarity with multi-digit numbers, it does not require the child to generate either a written numeral or an oral number name, which limits the ability to examine the types of errors children make in interpreting number symbols. The other two tasks used to evaluate children's knowledge of number symbols – writing and reading numbers – provide an opportunity to assess error patterns and, thus, were used in the current study. Both of these tasks rely on a conceptual understanding of the hierarchical organization of multi-digit numbers; yet, each task has its own unique requirements.

A reading task requires decoding a sequence of written digits, where the only indicator of the digit's place value is its position within the sequence. In this task, the child has to generate proper linguistic markers to convey the place value of each digit. A number writing task, on the other hand, requires decoding a spoken number name that provides information about the sequence of digits, as well as additional linguistic cues about the value of each digit. For example, the tens place in double-digit numbers is typically marked by a short morpheme, such as -***ty*** in English (64 = six***ty*** four, 74 = seven***ty*** four), -***desyat*** in Russian (64 = shest***desyat*** chetyre, 74 = sem***desyat*** chetyre), or ***shi*** in Chinese (64 = liu ***shi*** si, 74 = qi ***shi*** si). Hearing these linguistic markers may have both a specific and a general effect on children's understanding of the number structure. The specific effect – i.e., providing information about the place value of particular digits – may vary across languages as a function of "transparency" of language-specific place-value markers. For example, in Chinese, the tens marker (***shi***) sounds exactly like "ten", making the tens value more explicit than in English where the tens marker (***ty***) is less transparent. Regardless of the degree of transparency of a given numeric language, hearing distinct linguistic markers after each digit may have a more general effect of activating the child's thinking about the number structure and drawing the child's attention to place value.

Based on the task analysis, which indicated that spoken number names provide more cues about the structure of multi-digit numbers than written numerals, we hypothesized that the number reading task may be more challenging for young children than the writing task. This hypothesis refers specifically to reading and writing multi-digit numbers. When learning individual symbols for single digits, the reading task may be initially easier – children may be able to recognize and name a single-digit number before learning the mechanics of writing it. Yet, once they master the skill of writing individual digits, they may find it easier to translate the structure of a multi-digit number from its spoken name (which contains cues about the place value of each digit) into its written form than vice versa. The present study tested this hypothesis.

Examining the Nature of Errors in Symbolic Number Tasks

To better understand the relation between the number writing and reading tasks, we examined not only accuracy, but also errors children make on these tasks. Error analysis offers a unique opportunity to understand the type and extent of misconceptions at the early stages of developing a new skill. In particular, errors in reading and writing multi-digit numbers can reveal difficulties in understanding the place value – a key principle governing the structure of multi-digit numbers. Yet, extant research offers a limited analysis of errors in this area of numeric knowledge. Many studies that examined early symbolic number skills analyzed the accuracy with which children identified multi-digit numbers rather than the nature of their errors (Göbel *et al*., 2014; Purpura *et al*., 2013). Some studies of number writing skills did investigate children's errors, but most of these studies focused on specific error types (e.g., expansion, Byrge *et al*. [2014], or reversal, Zuber, Pixner, Moeller & Nuerk [2009]) rather than providing a comprehensive error analysis.

Our examination of specific errors reported in prior work suggested that they vary in the extent to which they reflect children's understanding of the number structure. Certain errors clearly convey a failure to capture the structure of the multi-digit number and, as such, can be categorized as conceptual errors. For example, when children read "51" as *five-one*, they treat the double-digit number as a collection of individual digits without any attempt at integration based on place value. In writing, a child may translate the number name *fifty one* literally as "501" or add a wrong number of zeroes (e.g., "5001" or "5010"), indicating a lack of understanding of how place value is expressed through the structure of the written number. Further, in both writing and reading tasks involving multi-digit numbers, children have been observed to make partial responses (e.g., writing *fifty one* as "1" or reading "51" as *five*). These partial errors also appear to reflect a challenge of integrating the tens and the ones within a single number such that one of the digits is ignored.

In addition to conceptual errors, children make mistakes on writing and reading tasks that do not violate the structure of the multi-digit number and, as such, can be categorized as mechanical errors. For example, a child may confuse written symbols "2" and "5" , as a result making a substitution error, such as reading "51" as *twenty-one* or writing *fifty-one* as "21". Even though this response is incorrect, it preserves the base-10 structure of the number, with the correct order and place-value markers. Another type of mechanical error involves the reversal of digits in a multi-digit number. For example, a child may be confused about the left-right orientation of the written text (a phenomenon observed in early stages of word writing e.g., Sidman & Kirk, 1974), as a result making a reversal error, such as writing *fifty one* as "15". Even though this response is incorrect, it preserves – from the child's perspective – the place value notation whereby the tens are written before the ones.

A recent study provided empirical evidence supporting this theoretical distinction between conceptual and mechanical errors (Authors, 2017). It showed that the frequency of conceptual errors in number reading was negatively associated with children's performance on a number representation task. In this task, children had to "construct" double-digit numbers using different types of blocks, including single-unit and ten-unit blocks. The ability to represent numeric structure was measured by the frequency of constructing base-10 representations that utilized ten-unit blocks to represent the tens and single-unit blocks to represent the ones (e.g., "34"=3 ten-unit blocks+4 single-unit blocks). The results showed that children who had a poor ability to create base-10 representations were more likely to make conceptual errors in number reading. In contrast, the frequency of mechanical errors, such as substitution and reversal, was not related to children's ability to use base-10 representations. In other words, the likelihood of making a mechanical error in number reading did not vary as a function of conceptual understanding of number structure. In the present study, we used the distinction between conceptual and mechanical errors to better understand the difference in children's performance on the number reading and writing tasks.

Present Study

The current study was part of a larger project on math learning conducted in Russia. All participating children were in the last year of preschool. Our examination of curricular materials indicated that math instruction received by our participants was limited to single-digit numbers, suggesting that any knowledge of multi-digit numbers that children may reveal in the present study must be based on informal learning outside of classroom instruction. To examine children's knowledge of multi-digit numbers, we administered parallel number writing and reading tasks. To test our hypothesis about the relative difficulty of the two tasks, we compared accuracy of children's performance on each task. To examine the nature of children's errors, we coded incorrect responses into one of two categories described above – *conceptual* and *mechanical* – and compared the frequency of these errors in number writing and reading tasks.

Method

Participants

The study included 173 children (49% females) with the mean age of 69 months (range: 59 – 77 months), recruited from municipal preschools in Moscow, Russia. It should be noted that Russian children enter kindergarten at around six years of age, making the preschoolers in the present study about a year older than their counterparts in some other countries, such as the US. The children were primarily from middle-class educated families: parental reports indicated that 76% of primary caregivers obtained a college degree, 14% had some college education and 10% completed vocational training beyond high school. All participants were monolingual native speakers of Russian.

Materials and Procedure

There were two sessions, in which children were tested individually at their school. In Session 1, they completed Raven's matrices – a common measure of non-verbal intelligence that served as a control variable in analyses. In Session 2, children completed two symbolic number tasks that were the main focus of analysis – number writing and number reading.

Raven's test. We used the *Raven's Colored Progressive Matrices* that have been normed for children aged 5 through 11 years (Raven, Court & Raven, 1990). The task included 3 sets of matrices, 12 items per set. Each item presented a pattern of geometric designs with a missing piece. The task was to pick the missing piece from six available options. Children were tested individually with no time limit. They were told: "Look at this picture. A mouse ate a piece of it and now there is a hole. We need to fix it. One of these (pointing to the options) is the missing piece. Which one is it?". The procedure was stopped when the child responded incorrectly on four items in a row. Accuracy scores were calculated as the percent correct out of 36 items.

Number writing task. The task included 12 items: six double-digit (DD) numbers (24, 38, 47, 53, 69, 85); and six triple-digit (TD) numbers (173, 349, 436, 581, 754, 962). The DD numbers were presented in the first block, followed by TD numbers; within each block, numbers were presented in random order. At the start of testing, the child received a booklet with blank pages. On each trial, the tester asked: "Can you write number X here?". If the child did not produce a response within 10 seconds following the initial question, the tester provided a second prompt: "Go ahead, try to write number X". If the child did not produce a response in another 10 seconds, the tester said: "OK, let's try another one". Ambiguous responses were clarified: when the child scribbled a form that resembled a number but was not properly connected, the tester asked: "What is it?" and if the child named a number, the answer was taken as the written digit. If the child produced no response for three trials in a row, the testing stopped.

Number reading task. The task included 14 items: seven DD numbers (25, 37, 46, 58, 63, 74, 82); and seven TD numbers (192, 259, 341, 427, 634, 803, 968). The child received a booklet with target numbers printed one per page. The DD numbers were presented in the first block, followed by TD numbers; within each block, numbers were presented in a predetermined random order. On each trial, the tester turned a page and said: "Can you read this number?". When the child produced a response, the tester wrote it down verbatim in the answer sheet. If the child spontaneously corrected the initial response, the last response was used for coding. The rules of task administration (e.g., providing a second prompt and stopping the procedure after three consecutive non-responses) were the same as for the writing task.

Data coding for number writing and reading tasks. Children's responses on both tasks were coded for accuracy. On the writing task, in order to be coded as correct, the child's written response had to include all digits depicted correctly and in the right order. On the reading task, in order to be coded as correct, the child's oral response had to include all the digits in the right order with proper verbal markers of their place value. It should be noted that the linguistic features of the Russian number system are similar to those in English. In terms of order, the component of a multi-digit number with the highest place value is named first. In terms of place-value markers, Russian utilizes short morphemes whose meaning is not transparent. For example, a marker for the tens' value (equivalent to "*ty*" in "twen*ty*") is an archaic word for "ten" that does not sound like "ten" to a contemporary speaker. In the present study, each response was coded by a native speaker of Russian to determine whether it included all the required elements of the multi-digit number name. The total accuracy score was computed separately for the writing and reading tasks as proportion of correct responses out of all trials.

In addition to accuracy coding, children's responses on both tasks were coded for error type. Specifically, each incorrect response was further examined and assigned to one of three categories: conceptual error, mechanical error, and other. The "other" category included mostly non-responses and trials which were not administered due to the stopping rule; it also included uninterpretable responses, such as drawing a wavy line on the writing task. The other two categories were each comprised of several types of errors. Table 1 presents examples of different types of writing errors that were categorized as conceptual or mechanical and table 2 presents parallel information for reading errors. Two separate error scores were computed on each task, reflecting the frequency of conceptual and mechanical errors out of all responses.

Reliability of child assessment. We have examined the reliability of our measures in two ways. First, for each task, we analyzed accuracy scores on individual trials to determine the internal consistency of children's performance across items. This analysis involved computing the Kuder-Richardson statistics (KR-20), which is a version of Cronbach's alpha that is used for dichotomous responses (correct or incorrect). KR-20 values above .70 are considered good indicators of reliability for test instruments with fewer than 50 items (Salkind, 2010). The

TABLE 1. Examples of Conceptual and Mechanical Errors in Number Writing

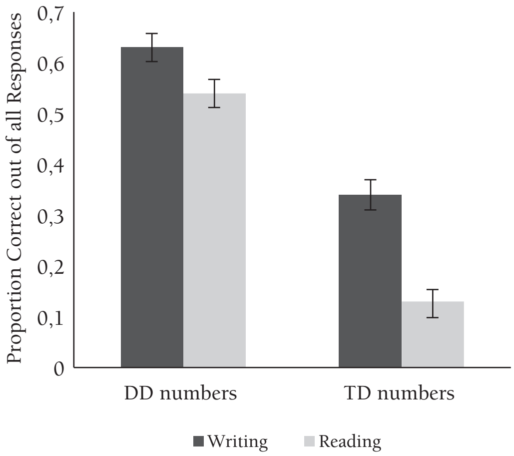
Error Type	Description	Examples	Percentage
Category: Conceptual Errors			Out of all conceptual errors
Expanded number	Response contains all required digits in order, with extra zeroes added between them to show place values explicitly	*twenty four*→204 *six hundred fifty three*→600503	35%
Unit confusion	A version of expanded notation that has indications of unit confusion (using wrong place-value markers)	*fifty three*→ 153 *four hundred thirty six*→ 4036	37%
Partial number	Response contains at least one required digit; the other one(s) are missing	*twenty four*→ 4 *three hundred forty eight* → 34	28%
Category: Mechanical Errors			Out of all mechanical errors
Substitution	One of the digits is wrong. Response contains one (for DD) or two (for TD) required digits in the right place	*twenty four*→ 54 *three hundred forty eight*→ 349	63%
Reversal	Response contains all required digits, but in the wrong order	*twenty four*→ 42 *three hundred forty eight*→ 843	37%

TABLE 2. Examples of Conceptual and Mechanical Errors in Number Reading

Error Type	Description	Examples	Percentage
Category: Conceptual Errors			Out of all conceptual errors
Concatenation	Response contains required digits in order, but at least one of them is read as a separate number	24 →*two-four* 348→*three-four-eight*	43%
Unit confusion	Response contains required digits in order, but at least one of them is read with a wrong place-value marker	97 →*nineteen seven* 97 →*nine hundred seven*	33%
Partial number	Response contains at least one required digit with the other one(s) missing	24→*four* 348→*three* or *thirty four*	24%
Category: Mechanical Errors			Out of all mechanical errors
Substitution	One of the digits is named incorrectly. Response contains one (for DD) or two (for TD) required digits in the right place	24→*fifty four* 342→*three hundred forty five*	55%
Reversal	Response contains all required digits, but in the wrong order	24→*forty two* 342→*two hundred forty three*	45%

results showed excellent internal consistency for each task used in the present study: Raven's KR20 = .94; writing task KR-20 = .92; reading task KR-20 = .91. In addition to examining the internal consistency of each task based on the accuracy scores, we determined the inter-rater reliability of error coding. Toward this goal, a second rater independently coded 30% of responses on each task. The two raters were in agreement on 96% of trials on the writing task and 94% of trials on the reading task. It should be noted that the few cases of disagreement were discussed with the research team and eventually resolved by consensus.

Figure 1. Accuracy of Number Writing and Reading



Results

Accuracy of Number Reading and Writing

Descriptive analysis. Accuracy scores on the two numeric tasks are presented in Figure 1. As shown in the figure, accuracy on TD numbers was lower than on DD numbers for both tasks. Within each number type, accuracy appeared lower on the reading, compared to the writing, task, with the difference being particularly pronounced on TD numbers. The mean accuracy on the Raven's task was .50 (18 out of 36 items correct), which is the 65th percentile for the corresponding age group in the norming sample (Raven *et al.*, 1990). Individual scores on all three tasks varied widely. On Raven's test, they varied from chance level (1 out of 6, or .17 correct) to the level of accuracy corresponding to the 99th percentile in the norming sample. On both numeric tasks, there were children who were unable to read or write any numbers, as well as those who were correct on all trials.

Next, we computed bivariate correlations between the variables examined in the study. As could be expected, accuracy scores on the two numeric tasks were correlated with Raven's scores, and with each other. The fact that the scores on all three tasks were positively correlated raised the possibility that the relation between the two numeric tasks was driven by general intelligence. Thus, we computed partial correlations between the reading and writing accuracy controlling for Raven's scores. The results, presented in table 3, indicate that the

Table 3. Partial Correlations between Writing and Reading Tasks (Controlling for Raven's Scores)

Measures	Writing DD	Writing TD	Reading DD	Reading TD
Writing DD	1			
Writing TD	.63	1		
Reading DD	.70	.54	1	
Reading TD	.36	.47	.52	1

Note. All correlations reported in the table are significant at $p < .001$.

relations between the two numeric tasks remained highly significant, indicating that these tasks share unique variance not accounted for by general intelligence.

Inferential analysis. To examine statistical differences in number reading and writing skills, we conducted a repeated-measures ANCOVA on accuracy scores with Raven's scores as a covariate. The model included two within-subject factors: task (reading vs. writing) and problem type (DD vs. TD). In order to examine potential developmental changes in number reading and writing, we included child's age in the model as a between-subject factor. Age was entered as a categorical variable – for the purposes of analysis, participants were divided into two age groups: younger preschoolers (N = 91, M = 66 months, range: 59-68) and older preschoolers (N = 83, M = 72 months, range: 69-77). The groups were formed by a median split with an adjustment to allow all the children of the median age (68 months) to be in the same group.

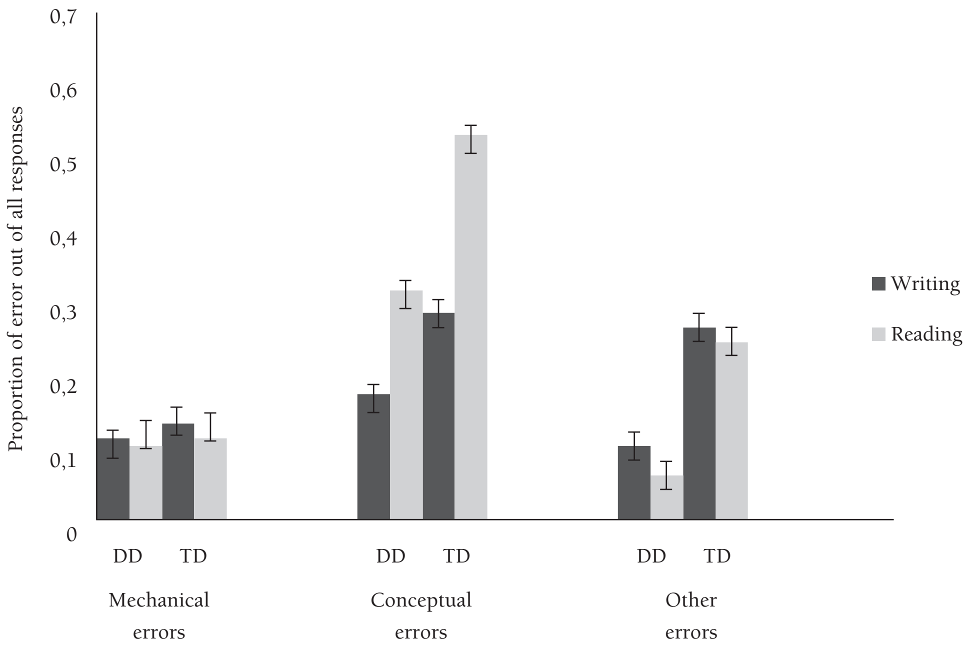
The results of ANCOVA showed a main effect of task, $F(1,168)=58.07$, $p<.001$, $\eta_p^2=.27$, with higher accuracy in writing ($M=.49$), compared to reading ($M=.34$). There was also a main effect of number type, $F(1,168)=219.83$, $p<.001$, $\eta_p^2=.67$, with higher accuracy on problems involving DD ($M=.59$), compared to TD numbers ($M=.24$). These effects were qualified by an interaction, $F(1,168)=12.84$, $p<.001$, $\eta_p^2=.07$, indicating that the difference between the two tasks was greater for TD problems ($M_{writing}=.33$ vs. $M_{reading}=.13$) than for DD ($M_{writing}=.62$ vs. $M_{reading}=.53$). Further, children's performance differed across the two age groups, $F(1,168)=5.6$, $p=.02$, $\eta_p^2=.03$ ($M_{younger}=.36$; $M_{older}=.46$). Age did not interact with either task or problem type (both *p*'s>.05) – the older group performed better than the younger group, demonstrating a similar advantage across the two tasks and both problem types.

Errors in Number Reading and Writing

Descriptive results. As noted earlier, incorrect responses were divided into three categories: conceptual errors, mechanical errors and other. The frequency of errors from each category is reported in Figure 2. As shown in the figure, the pattern of errors varied across the three categories. (1) Errors coded as *other* were more frequent on TD than DD numbers on both tasks. In other words, when presented with a TD number, children were more likely to refuse either to write or to read it, compared to trials where they were presented with DD numbers. Interestingly, within each number type, the frequency of refusals was comparable on the reading and writing task. (2) Errors coded as *mechanical* showed comparable frequency across both tasks and number types. In other words, children were as likely to substitute an individual digit or reverse the order of digits within a multi-digit number on the reading task, as on the writing task. (3) Errors coded as *conceptual* appeared to vary both as a function of task and number type. Unlike mechanical errors, they were more frequent on the reading task than on the writing task. Further, on both tasks, children committed these errors more frequently when presented with TD than DD numbers. Below we examine these patterns statistically, focusing on the relative frequency of conceptual versus mechanical errors in the number reading and writing tasks.

Inferential analysis. We conducted a repeated-measures ANOVA with task (reading *vs.* writing), problem type (DD vs. TD), and error type (conceptual *vs.* mechanical) as within-subject factors. Similar to the analysis of accuracy, age group was added to the model as a between-subject factor. We tested two parallel models with the same independent variables – in the first one, the dependent variable was computed as a proportion of errors of each type out of all trials; in the second one, the dependent variable was computed as a proportion of errors of each type out of all attempted responses. In the second

Figure 2. Proportion of Errors in Number Writing and Reading



model, the trials that were not administered due to the stopping rule were excluded. The two models produced the same pattern of results. Here, we report the findings with error frequency computed out of all trials.

Parallel to accuracy findings, age had a main effect on error frequency, $F(1,169)=13.87$, $p<.001, \eta_p^2=.08$, and did not interact with other variables, such that older children made fewer errors of both types across tasks and problem types $M_{older}=.37$; $M_{younger}=.49$). Further, there were main effects of task, $F(1,169)=174.53$, $p<.001, \eta_p^2=.51$ and problem type, $F(1,169)=18.76$, $p<.001, \eta_p^2=.10$, mirroring the findings of the accuracy analysis – children made significantly more errors on the reading than the writing task ($M_{reading}=.50$; $M_{writing}=.36$) and on TD, compared to DD, numbers ($M_{TD}=.52$; $M_{DD}=.34$). There was also a main effect of error type, $F(1,169)=163.05$, $p<.001, \eta_p^2=.50$ ($M_{mech.er.}=.12$; $M_{concep.er.}=.33$), which was qualified by two interactions – with task, $F(1,169)=172.69$, $p<.001$, $\eta_p^2=.51$, and problem type, $F(1, 169)=19.11$, $p<.001, \eta_p^2=.10$. To examine the nature of these interactions, we conducted follow-up analyses using pair-wise LSD comparisons, leading to the following findings. *Error type x task*: there was no difference between tasks in the frequency of mechanical errors ($M_{reading}=.10$; $M_{writing}=.11$), but conceptual errors were more frequent in reading ($M_{reading}=.41$) than in writing ($M_{writing}=.23$). *Error type x problem type*: there was no difference between DD and TD problems in the frequency of mechanical errors ($M_{TD}=.12$; $M_{DD}=.11$), but conceptual errors were more frequent on TD than DD problems ($M_{TD}=.39$; $M_{DD}=.24$).

Discussion

There is accumulating evidence, primarily from English-speaking children in the US, that the acquisition of multi-digit numbers begins prior to formal instruction (Byrge *et al.*, 2014; Mix *et al.*, 2014). The present study, conducted with a sample of Russian preschoolers, provided an opportunity to investigate this phenomenon in a novel context. In the process of this investigation, we pursued two interrelated goals. First, we aimed to compare the accuracy of children's performance on the number writing and reading tasks to determine which of these two aspects of symbolic number knowledge presents a greater challenge at the early stages of learning multi-digit numbers. Second, we aimed to examine children's errors on the writing and reading tasks to determine to what extent the relative difficulty of each task is driven by the conceptual challenge of understanding the hierarchical organization of multi-digit numbers.

Early Knowledge of Multi-Digit Numbers

Consistent with prior work, many preschoolers in the present study were able to read and write multi-digit numbers. In both tasks, more than a half of responses on trials involving DD numbers were accurate. As could be expected, the accuracy was much lower on TD numbers, but even on this challenging task, preschoolers produced some accurate responses. This is especially impressive because our examination of curricular materials showed that math instruction received by our participants in preschools was limited to single digits. Thus, their knowledge of multi-digit numbers is likely based on informal learning outside of classroom instruction.

All children participating in the present study were attending the last year of preschool, which allowed us to control for the amount of formal school instruction they received. At the same time, a relatively wide age range provided an opportunity to divide participants into the younger and older groups, which, in turn, allowed us to examine possible developments over the age period studied. The results showed that older children performed better on both tasks. In part, this age-related improvement may be due to the general growth of cognitive skills, including memory and executive functions, which occurs rapidly in preschoolers (e.g., Anderson & Reidy, 2012). Yet, given the cultural origins of the symbolic number knowledge, its acquisition must be mediated by interactions with more knowledgeable social partners (Vygotsky, 1978). Thus, the observed age-related improvement in symbolic number knowledge is likely to reflect not only increasing cognitive maturity, but also accumulating experience that involves activities and interactions with adults and older peers around numbers.

Mix and colleagues (2014) suggested that the early knowledge of multi-digit numbers develops largely through informal exposure: as children encounter such numbers in their environment, they may extract structural patterns using a statistical learning mechanism similar to that implicated in early language learning (Aslin & Newport, 2012). Children's ability to extract numeric patterns from the environment may be supported by parental input. There is growing evidence that parents of preschoolers engage their children in interactions that include incidental numerical components, such as reading street signs with numbers or discussing prices while shopping (LeFevre, Skwarchuk, Smith-Chant, Fast, Kamawar & Bisanz, 2009; Skwarchuk, Sowinski & LeFevre, 2014). These types of interactions draw children's attention to numeric stimuli and provide clues about the mapping between a written number structure and a corresponding number name. As a result, when multi-digit numbers are introduced in the elementary school math curriculum, many children may already have at least a partial understanding of the numeric system. Knowing this could help teachers calibrate their instruction so as to capitalize on children's early skills.

Commonalities and Differences between Number Reading and Writing Tasks

Learning how to read and write represents two complementary aspects of acquiring a symbolic written system. There is a large body of work in the domain of early literacy investigating these processes with respect to children's mastery of letters and words (e.g., Diamond, Gerde & Powell, 2008; Fitzgerald & Shanahan, 2000; Skibbe, Bindman, Hindman, Aram & Morrison, 2013). Yet, there have been no studies, to our knowledge, investigating the relation between reading and writing in the context of acquiring a symbolic number system – the key issue addressed in the present study. Similar to results reported in literacy research (Fitzgerald & Shanahan, 2000), we found that the reading and writing tasks involving multi-digit numbers were closely related. Even after controlling for general intelligence, there was a high correlation between children's accuracy scores on these two symbolic number tasks. At the same time, the average level of accuracy on the reading task was lower than on the writing task. To better understand this phenomenon, we turned to error analysis comparing the frequency of mechanical and conceptual errors in number reading and writing.

Mechanical errors. Our analysis showed that the frequency of mechanical errors was similar across the two tasks. For example, children were as likely to make a substitution error in reading a multi-digit number as they were in writing such a number. This finding suggests that if children have not yet formed a strong visual representation of a digit, they may have a similar difficulty recognizing that digit in the reading task or depicting its shape in the writing task. It should be noted that mistakes similar to mechanical errors observed in the present study have been reported in reading research (e.g., Blackburne, Eddy, Kalra, Yee & Sinha, 2014). Just as children at the early stages of word reading/writing replace individual letters (e.g., confusing *d*, *b*, and *p*) and reverse the direction of reading (e.g., reading *on* as *no*), they make parallel errors at the early stages of numeral reading/writing, including substitution (e.g., reading "5" as *two*) and reversal (e.g., writing *thirty four* as "43"). These errors may reflect a lack of reading and writing practice. Importantly, these types of errors in number reading or writing are not necessarily indicative of the lack of understanding of the numeric structure. In fact, when children read 356 as *three hundred twenty six*, they show the understanding of the structure of a multi-digit number while making an error at the level of single digit.

Conceptual errors. In contrast to mechanical errors, conceptual errors reveal difficulty in understanding how individual digits are integrated within a multi-digit number. Our task analysis suggested that the writing task, in which the child is presented with a number name, offers more cues regarding the organization of multi-digit numbers than the reading task. This led to a hypothesis that reading multi-digit numbers may be conceptually harder for preschool children than writing these numbers. Indeed, we found a significantly higher frequency of conceptual errors in reading than in writing multi-digit numbers.

In interpreting this finding, we considered a possibility that a relative ease of the writing task could be due to some children circumventing the challenge of translating the number structure from a spoken to a written form. That is, children who do not understand the meaning of place value markers may ignore them and simply write down the sequence of digits in the order in which they were listed. Such a response would look like a proper multi-digit number. In contrast, on the reading task, children who have a similar conceptual difficulty understanding the number structure would reveal it by naming each digit separately (e.g., reading "352" as *three-five-two*). To address this possibility, we conducted additional analysis examining whether participants successful in number writing tended to make such errors in reading. We found that this was not the case: none of the children who were accurate on the writing task made this type of reading error, suggesting that these children did not represent multi-digit numbers as collections of unrelated single digits. Thus, a lower frequency of conceptual errors on the writing task is not likely due to a seemingly accurate performance of children who have no understanding of number structure.

We also considered a possibility that a greater difficulty of the reading task was due to the requirement to generate specific linguistic markers, which was not required in the writing task. In other words, children with a fragile knowledge of place value markers may fail a reading task because of choosing the wrong linguistic term (e.g., reading "81" as *eighteen one* or *eight hundred one*). Although this factor may indeed have contributed to the difficulty of multi-digit number reading, we believe it cannot completely explain the difference in performance between reading and writing. This is because failing to choose the correct marker would most likely result in a unit confusion error. While we observed this type of error on the reading task, the most frequent error type was concatenation: reading all digits separately. This error reflects not just a child's confusion about the use of a specific linguistic term, but rather a lack of any attempt at integrating separate digits within a multi-digit number.

We suggest that at the early stages of symbolic numeric development when the knowledge of the number system can be characterized as partial or fragile, children may need additional support to activate that knowledge and apply it to novel numeric stimuli. When children hear a name of a multi-digit number, the linguistic cues (e.g., six***ty*** four) may facilitate that knowledge by highlighting the difference between the place values of digits comprising the number. Given the lack of such cues in a written numeral, children may be more likely to interpret multi-digit numbers presented in the reading task as a collection of separate digits. To help them overcome the difficulty of "seeing" the structure of the number, it may be even useful to begin the instruction of multi-digit numbers with writing tasks. This may highlight the numeric structure helping children to form distinct categories representing different place values.

We acknowledge that because the present findings were obtained on a sample of children speaking a particular language (Russian), it would be important to establish whether the observed patterns can be reproduced in children speaking other languages. The extent of transparency of a particular numeric language may influence how early children acquire the skills of reading and writing multi-digit numbers, as well as the relative difficulty of the two tasks in the early stages of learning these numbers. Yet, as noted earlier, even if a given language is not particularly transparent with respect to place value markers, the fact that each digit is followed by a different linguistic marker may have a general effect of raising the child's awareness of the number structure. Future research will do well to examine this effect across different languages to uncover potential language-specific effects on the acquisition of symbolic number skills.

Another issue that should be addressed in future research concerns the relation between children's acquisition of symbolic number knowledge and their language skills. Addressing this issue may illuminate the nature of individual differences observed in the present study. On the one hand, the numeric tasks used in this study posed minimal verbal demands and it is unlikely that the differences in number reading and writing were related to individual variability in language comprehension. On the other hand, the process of learning how to translate between the written symbol of a multi-digit number and its spoken name may rely, in part, on mechanisms similar to those involved in learning how to read and write words (Hecht, Torgeson, Wagner & Rashotte, 2001). A better understanding of the common underlying processes may benefit both literacy and numeracy instruction. For example, improving verbal working memory, which has been implicated in processing numeric and non-numeric symbolic information (e.g., Geary, Bow-Thomas, Liu & Siegler, 1996; Hecht *et al.*, 2001) may affect the acquisition of both number and word reading skills.

Several educational implications emerge from the findings of the current study. In particular, our results highlight the importance of paying attention not only to the accuracy of children's performance on symbolic numerical tasks but also to the errors they make. Examining the types of errors children make in number reading and writing may help teachers target specific areas of numeric learning, such as practicing the shapes of individual digits versus improving conceptual understanding of number structure. Conceptual weaknesses could be addressed by using tasks and materials that highlight the hierarchical organization of multi-digit numbers. For example, using base-10 blocks to represent numbers could help children visualize the numeric structure, thus facilitating the growth of knowledge that is critical for subsequent math learning (Geary, 2006). Furthermore, the present results suggest that writing numbers from dictation provides children with extra cues that may benefit their understanding of the numerical structure of multi-digit numbers. Thus, it may be helpful to include more exercises that require translating a spoken name of a multi-digit number into a written numeral, as this practice might make the numerical structure more salient to children.

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Resumen

Conocimiento simbólico de los números: una ventana hacia la comprensión temprana de la estructura numérica

INTRODUCCIÓN. La acumulación de evidencia, principalmente de niños de habla inglesa, indica que la adquisición de números de dígitos múltiples comienza antes de la instrucción formal de matemáticas. El presente estudio reprodujo este fenómeno en un nuevo contexto cultural / lingüístico y amplió el conocimiento actual del desarrollo numérico simbólico temprano. MÉTODO. El estudio

incluyó una muestra de preescolares rusos que participaron en dos sesiones de prueba. En una sesión, los niños completaron tareas numéricas simbólicas: escribir y leer números de dígitos múltiples. En otra sesión, completaron una tarea de inteligencia no verbal. Se comparó el rendimiento de los niños en las dos tareas numéricas, controlando su nivel general de inteligencia. **RESULTADOS**. Los preescolares rusos encontraron la tarea de lectura más desafiante que la tarea de escritura correspondiente. En particular, cuando se leen números que incluyen dos o tres dígitos, los niños eran más propensos a cometer errores conceptuales que revelaron la dificultad de comprender la estructura jerárquica de los números de dígitos múltiples. Por el contrario, la frecuencia de errores en los que se conservó la estructura del número de dígitos múltiples (por ejemplo, la sustitución de uno de los dígitos) fue similar en todas las tareas de escritura y lectura. **DISCUSIÓN**. De manera consistente con estudios previos, los preescolares en el presente estudio revelaron un conocimiento parcial de los números de dígitos múltiples que surge antes de la instrucción formal y que probablemente se basa en el aprendizaje informal. La dificultad relativa de la lectura, en comparación con la tarea de escribir, sugiere que en las primeras etapas de aprendizaje de números simbólicos los niños pueden necesitar instrucciones adicionales sobre la estructura numérica, que pueden ser proporcionadas por los nombres de los números hablados. Los números escritos no proporcionan pistas lingüísticas sobre la estructura numérica, por lo que la tarea de lectura es más desafiante. Se discuten las implicaciones de estos hallazgos para la práctica educativa temprana.

Palabras clave: *Número simbólico, Conocimiento numérico, Matemáticas tempranas, Desarrollo cognitivo.*

Rèsumè

La connaissance symbolique des nombres: une fenêtre sur la comprehension prècoce de la structure numerique

INTRODUCTION. De nombreux travaux de recherche, réalisés principalement avec des enfants anglophones, indiquent que la compréhension des nombres de plusieurs chiffres commence avant l'enseignement formel des mathématiques. Cette étude réplique ces résultats dans un nouveau contexte culturel/linguistique et accroît la recherche sur le développement de la connaissance symbolique des nombres en enfants d'âge préscolaire. **MÉTHODE**. Un échantillon d'enfants russes d'âge préscolaire participants dans deux sessions de recherche. Lors de la première session, les enfants ont écrit et lu des nombres de plusieurs chiffres, deux tâches permettant d'évaluer leur connaissance symbolique des nombres. Lors de la deuxième session, ils ont complété une tâche évaluant leur intelligence non-verbale. Les performances en lecture et en écriture des nombres ont été comparés, en tenant pleinement compte le niveau d'intelligence général des enfants. **RÉSULTATS**. Les enfants russes d'âge préscolaire ont trouvé la tâche de lecture plus difficile que la tâche d'écriture. En particulier, quand la lecture comprenait des nombres de deux ou trois chiffres, les enfants étaient plus susceptibles de faire des erreurs révélant leur difficulté à comprendre la structure hiérarchique des nombres à plusieurs chiffres. En comparaison, la fréquence des erreurs où la structure d'un nombre de plusieurs chiffres a été retenue (par exemple, le remplacement d'une des chiffres) était similaire pour l'écriture que pour la lecture des nombres. **DISCUSSION**. Conformément aux travaux précédents, les enfants d'âge préscolaire participant dans cette étude ont montré une connaissance partielle des nombres de plusieurs chiffres avant leur rentrée dans le système d'éducation formel, ce qui est probablement la conséquence directe de leur apprentissage informel.

Les difficultés de lecture que ces enfants ont trouvé par rapport à l'écriture suggèrent que les enfants ont besoin d'informations supplémentaires sur la structure des nombres, leur être fournies en les dictant, pendant les premiers stages d'apprentissage des nombres symboliques. qui peuvent. Les chiffres écrits n'offrent pas de repères linguistiques sur la structure des nombres et, en conséquence, la lecture se révèle comme une tâche plus ardue que l'écriture. Les implications de ces résultats pour les pratiques éducatives préscolaires sont discutées.

Mots clés: *nombre symbolique, connaissance des nombres, mathématique préscolaire, développement cognitif.*

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