# Green`s function formalism for the study of the role of 2d-magnetoplasmons on magneto infra-red absorption in high electronic density quantum wells*

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## Resumen

El presente trabajo desarrolla un formalismo de funciones de Green con el fin de calcular las componentes del propagador fotónico en un pozo cuántico con alta densidad electrónica en un campo magnético transversal. Se discuten el poder espectral y las curvas de dispersión de los conjuntos de modos colectivos que aparecen en tal sistema cuando la frecuencia ciclotrónica toma valores en la región *reststrahlen* del espectro. Se discuten las resonancias entre la frecuencia ciclotrónica y los modos *TO* y *LO*.

Palabras clave: Magnetoplasma, pozo cuántico, resonancia ciclotrónica, fonones ópticos.

## Abstract

We *have* develope*d* Green*'s* function formalism in order to calculate the components of the photon propagator in a quantum well with high electronic densities in a transversal magnetic field. The power spectra and the dispersion curves of the sets of collective modes which arise in such *a* system are discussed for cyclotron frequencies lying in the *reststrahlen* region of the spectra. The resonance between the cyclotron frequency and the *TO* and *LO* modes are *also* discussed.

Key words: Magnetoplasmons, Quantum well, Cyclotron resonance, Optical phonons.

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## Introduction

The study of the dynamics of electrons under the action of magnetic fields is an important tool for understanding the processes which take place in physical systems. The cyclotron resonances of high density and high mobility electron space-charge layers in thin GaAs quantum wells in Faraday geometry at frequencies covering the GaAs reststrahlen regime were considered experimentally by Poulter *et al.* [1,2]. It was observed that under resonant conditions, when the cyclotron resonance reaches energies close to the longitudinal optical phonon energy, there is no interaction between the cyclotron and phonon modes. Instead, an interaction is observed with a mode which has energy close to the transverse optic (TO) phonon energy. The results were interpreted with reference to a model appropriate only for bulk systems as a coupling to a collective magnetoplasmon-phonon mode. This model was criticized by B. Zhang *et al.* [3], and some qualitative considerations on the character of the spectra in the long-wavelength limit were included. It is necessary to emphasize that in the system considered in [1,2] it is possible to have two layers of two-dimensional (2D) electrons, which have a collective excitation spectrum with dispersions depending on a wave vector $\boldsymbol{k}$ parallel to the layer plane. It is the aim of this communication to discuss the character of the coupled magnetoplasmon-phonon modes which can arise in the considered system.

## Methodology

We apply the formalism of Theory Lineal Response. It is considered a quantum well of width $d$ occupying the region $0 < z < d$ and sandwiched between two infinite media. Homogeneity and isotropy are assumed in the *xy* plane. An external magnetic field $\boldsymbol{H}$ is applied perpendicular to the well plane (Faraday geometry). The dielectric function of such system is $\varepsilon(z) = \varepsilon_1[\theta(-z) + \theta(z-d)] + \varepsilon_2(\omega)[\theta(z)\theta(d-z)]$, with $\varepsilon_2(\omega) = \varepsilon_\infty(\omega^2 - \omega_{LO}{}^2)/[\omega(\omega + i\gamma_{TO}) - \omega_{TO}{}^2]$, $\varepsilon_\infty$ being the high-frequency dielectric permittivity of the well; $\omega_{TO}$, $\omega_{LO}$ and $\gamma_{TO}$ are, respectively, the frequencies of the transversal (*TO*) and longitudinal (*LO*) optical modes and the damping of *TO* modes. Two-dimensional (*2D*) electron layers with carrier concentration $n_s$ are located at the surfaces $z = 0, d$. The dynamics of such electron system induces a *2D* current density $j_i(r, t) = \sigma_{ij}[E_j(r_{||}, 0, t)\,\delta(z) + E_j(r_{||}, d, t)\,\delta(z-d)]$, where $r = (r_{||}, z)$, $\sigma_{ij}$ are the components of the magneto conductivity tensor of the 2D electron gas and $E_j(r_{||}, z, t)$ is the dynamic electric field at $z$. In a gauge in which the scalar potential vanishes and in the presence of an external current $j^{ext}(r_{||}, z, \omega)$ the vector potential $A_j(r_{||}, z, \omega)$ can be written in the form $A_j(r_{||}, z, \omega) = -(1/c)\int d^2r_{||}dz D_{jk}(r_{||} - r'_{||}, z, z'; \omega)\, j_k^{ext}(r_{||}, z, \omega)$, where the dependence on $r_{||} - r'_{||}$ accounts for translational invariance in the *xy* plane. The components of the photon-Green tensor $D_{jk}$ satisfy the set of differential equations

$$
\begin{aligned}
&\{\varepsilon(z)\omega^2/c^2\delta_{ij} - \partial^2/\partial x_i \partial x_j + \nabla^2\delta_{ij}\}\, D_{jk}(r_{||} - r'_{||}, z, z'; \omega) + \\
&\qquad (4\pi i\omega\sigma_{ij}/c^2)\,[D_{jk}(r_{||} - r'_{||}, 0, z'; \omega)\,\delta(z) + D_{jk}(r_{||} - r'_{||}, d, z'; \omega)\,\delta(z-d)] = \\
&\qquad\qquad 4\pi\delta_{ik}\delta(z - z').
\end{aligned}
$$

The homogeneity of the system allows us to assume

$D_{jk}(r_{||}\text{-}r'_{||}, z, z'; \omega) = [1/(2\pi)^2] \int d^2 r_{||} d_{jk}(k, \omega; z, z') \exp[i\, \boldsymbol{k} \bullet (\boldsymbol{r}_{||}\text{-}\boldsymbol{r}'_{||})]$.

On the other hand, the isotropy in the *xy* plane can be exploited by introducing the tensor $g_{ik}(k, \omega; z, z') = S_{il}(k)\, S_{mk}(k)\, d_{lm}(k, \omega; z, z')$, where $S_{xx}(k) = S_{yy}(k) = k_x/k$, $S_{xy}(k) = -S_{yx}(k) = k_y/k$, $S_{zi}(k) = S_{iz}(k) = \delta_{iz}$. We obtain the following set of differential equations for the components of the tensor $g_{ik}$:

$$\begin{pmatrix} \varepsilon\kappa^{-2} \\ 1 \end{pmatrix}\left(\frac{d^2}{dz^2}-\kappa^2\right)\begin{pmatrix} g_{xk}(z,z') \\ g_{yk}(z,z') \end{pmatrix}-\frac{4\pi i}{\omega}\begin{pmatrix} \sigma_{xi} \\ \omega^2\sigma_{yi}/c^2 \end{pmatrix} \mathrm{X}$$

$$[g_{ik}(0, z')\delta(z) + g_{ik}(d, z')\delta(z - d)] = 4\pi\ [(c^2/\omega^2)\ \delta_{xk} + \delta_{yk}]\ \delta(z - z') \tag{1a}$$

$$g_{zk}(k,\omega,z,z') = -\frac{ik}{\kappa^2}\frac{dg_{xk}(k,\omega,z,z')}{dz}-\frac{4\pi}{\kappa^2}\delta_{zk}\delta(z-z') \tag{1b}$$

Where $\kappa=(k^2\text{-}\varepsilon(z)\omega^2/c^2)^{1/2}$. Here we have omitted for brevity the dependence on $k$ and $\omega$. Let us denote by $g_{ij}^{\mathrm{I}}(z, z')$, $g_{ij}^{\mathrm{II}}(z, z')$ and $g_{ij}^{\mathrm{III}}(z, z')$ the components of $g_{ij}^{\mathrm{I}}$ in the regions I ($z<0$), II ($0<z<d$) and III ($z>d$) respectively. At $z = 0, d$ we have the following boundary conditions: $[g_{ik}^{\mathrm{I}}\text{-}g_{ik}^{\mathrm{II}}]_{z=0}=0$, $[g_{ik}^{\mathrm{III}}\text{-}g_{ik}^{\mathrm{II}}]_{z=d}=0$, $[(\varepsilon_2/\kappa_2^2)dg_{xx}^{\mathrm{II}}/dz\text{-}(\varepsilon_1/\kappa_1^2)dg_{xk}^{\mathrm{I}}/dz\text{-}(4\pi i\sigma_{xi}/\omega)g_{ik}^{\mathrm{I}}]_{z=0}=0$, $[(\varepsilon_1/\kappa_1^2)dg_{xk}^{\mathrm{III}}/dz\text{-}(\varepsilon_2/\kappa_2^2)dg_{xk}^{\mathrm{II}}/dz\text{-}(4\pi i\sigma_{xi}/\omega)\ g_{ik}^{\mathrm{III}}]_{z=d}=0$, $[dg_{yk}^{\mathrm{II}}/dz\text{-}dg_{yk}^{\mathrm{I}}/dz+(4\pi i\omega\sigma_{yi}/c^2)g_{ik}^{\mathrm{I}}]_{z=0}=0$, $[dg_{xk}^{\mathrm{III}}/dz\text{-}dg_{xk}^{\mathrm{II}}/dz+(4\pi i\omega\sigma_{ix}/c^2)\ g_{ik}^{\mathrm{III}}]_{z=d}=0$, where $\kappa_a=(k^2\text{-}\varepsilon_a\omega^2/c^2)^{1/2}$, $(a = 1,2)$. The solutions of (1a-1b) for the above boundary conditions and for $0<z,z'<d$ are

$$\begin{pmatrix} g_{xx}^{\mathrm{II}}(z,z') \\ g_{yy}^{\mathrm{II}}(z,z') \end{pmatrix} = \frac{2\pi}{\Delta}\begin{pmatrix} c^2\kappa_2/(\varepsilon_2\omega^2) \\ 1/\kappa_2 \end{pmatrix}\left\{\begin{pmatrix} \alpha_{xx} \\ \alpha_{yy} \end{pmatrix} e^{\kappa_2(z-z')}+\begin{pmatrix} \beta_{xx} \\ \beta_{yy} \end{pmatrix} e^{-\kappa_2(z+z')}\right\} \tag{2a}$$

$$g^{\mathrm{II}}_{zz}(z, z') = \frac{4\pi}{\kappa^2}e^{-\kappa_2|z-z'|} \tag{2b}$$

where

$$\Delta=\{f_-^2(g_-^2\text{-}e^2g_+^2)\text{-}2f_-f_H(g_-+e^2g_-\text{-}2e^2g_+)g_H+f_H^2g_H^2+e^4(f_+g_+\text{-}f_Hg_H)^2$$
$$-e^2[f_+^2g_-^2+2f_+f_H(\text{-}2g_-+g_+)g_H+2f_H^2g_H^2]\}/e^2 \tag{3}$$

$$\alpha_{xx}(k,\omega,z')=\{e_zf_-[f_-(\text{-}g_-^2+e^2g_+^2)+f_H(g_-+e^2g_-\text{-}2e^2g_+)g_H]+ef_Hg_H[f_-(\text{-}g_-+e^2g_+)+$$
$$f_Hg_H+e^2(f_+g_-\text{-}f_+g_+\text{-}f_Hg_H)]+ee_z^2[f_Hg_H[f_+(g_-\text{-}e^2g_+)+(\text{-}1+e^2)f_Hg_H]+$$
$$f_-[f_+(g_-^2\text{-}e^2g_+^2)+f_H[\text{-}3g_-+(2+e^2)g_+]g_H]\}/(e^3e_z), \tag{4a}$$

$$\beta_{xx}(k,\omega,z')=\{e^2f_Hg_H[f_-(\text{-}g_-+g_+)+f_+(g_-\text{-}e^2g_+)+(e^2\text{-}1)f_Hg_H]+$$
$$ee_zf_-[f_+(g_-^2\text{-}e^2g_+^2)+f_H(\text{-}2g+g_++e^2g_+)g_H]+e_z^2[f_-^2(\text{-}g_-^2+e^2g_+^2)+e^2f_-f_H(g_-\text{-}g_+)g_H+$$
$$f_Hg_H[f_Hg_H+e^2(f_+g_-\text{-}f_+g_+\text{-}f_Hg_H)]]\}/(e^2e_z) \tag{4b}$$

$$\alpha_{yy}(k,\omega,z')=\text{-}f_H\{e^2\ e_z\ g_-\ (f_+\ g_-\text{-}g_++f_-g_+\text{-}f_H\ g_H)+e_z\ g_-(f_H\ g_H\text{-}f_-g_-)+$$
$$e^3[f_-g_+^2+f_H(g_-\text{-}2\ g_+)g_H\text{-}e_z^2(g_-+g_+)(f_+\ g_+\text{-}f_H\ g_H)]+e[\text{-}f_-\ g_-\ (g_-+e_z^2g_-\text{-}e_z^2g_+)+$$
$$f_Hg_-g_H+e_z^2\ (2\ f_+\ g_-^2+f_H\ (\text{-}3\ g_-+g_+)\ g_H)]\}/(e^3\ e_z) \tag{4c}$$

$$\beta_{yy}(k,\omega,z')=-f_H\{ee_z g_-[f_+g_-+f_-(-g_-+g_+)-f_H g_H]+2e_z^2 g_-(f_H g_H-f_-g_-)+$$
$$e^3 e_z g_-(-f_+ g_++f_H g_H)+e^4 g_+(f_H g_H-f_+ g_+)+e^2 f_+ g_-(g_-+e_z^2 g_--e_z^2 g_+)+$$
$$f_H(g_+-2g_-)g_H+e_z^2 g_+[f_-(g_-+g_+)-2f_H g_H)]\}/(e^2 e_z) \qquad (4d)$$

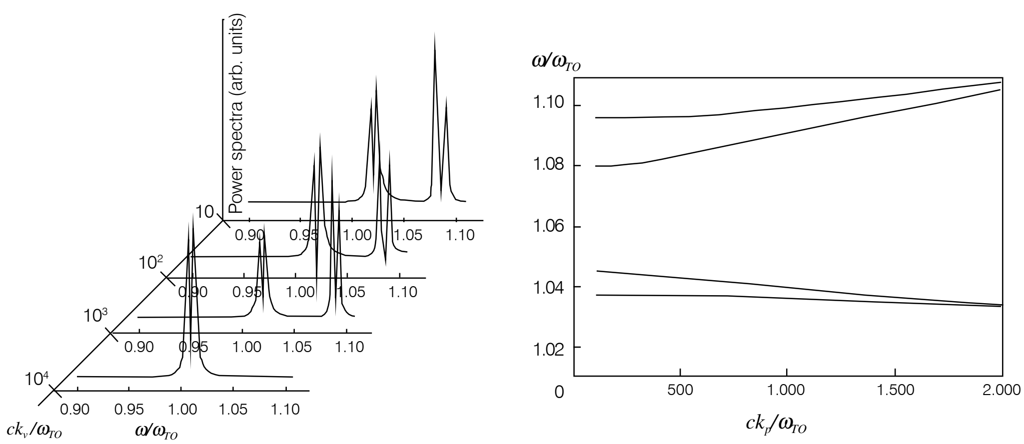
where $f\pm=\varepsilon_1/\kappa_1\pm\varepsilon_2/\kappa_2+4\pi i\sigma_{xx}/\omega$, $g\pm=\kappa_1\pm\kappa_2-4\pi i\omega\sigma_{yy}/c^2$, $f_H=4\pi i\sigma_{xy}/\omega$, $g_H=4\pi i\omega\sigma_{yx}/c^2$.

The dispersion laws of the collective modes arising in the considered system corresponds to the roots of the equation $\Delta=0$. This dispersion relation contains as particular cases the dispersion relations of confined polariton modes in a slab of ionic crystal in the non-retarded region of spectra ($\sigma_{ij}=0$) [4]) and the collective modes of a double layer sheet[5].

## Results

Let us discuss our results in the reststrahlen region for magnetic fields corresponding to cyclotron frequencies obeying $\omega_{TO}\leq\omega_c\leq\omega_{LO}$. We use the following set of parameters [1]: $d$=10 nm, $n_s=1.28´10^{12}$ cm$^{-2}$, $h\omega_{TO}$=33.6 meV, $\omega_{LO}=1.08\omega_{TO}$, $\varepsilon_\infty=10.6$, $\gamma_{TO}$=0.25 meV, $\gamma_e$=0.1 meV, $m=0.77m_0$ (where $m_0$ is the free electron mass). For frequencies in the considered region non-local effects on the magneto conductivity tensor can be neglected and Drude like expressions for $\sigma_{ij}$ can be used.

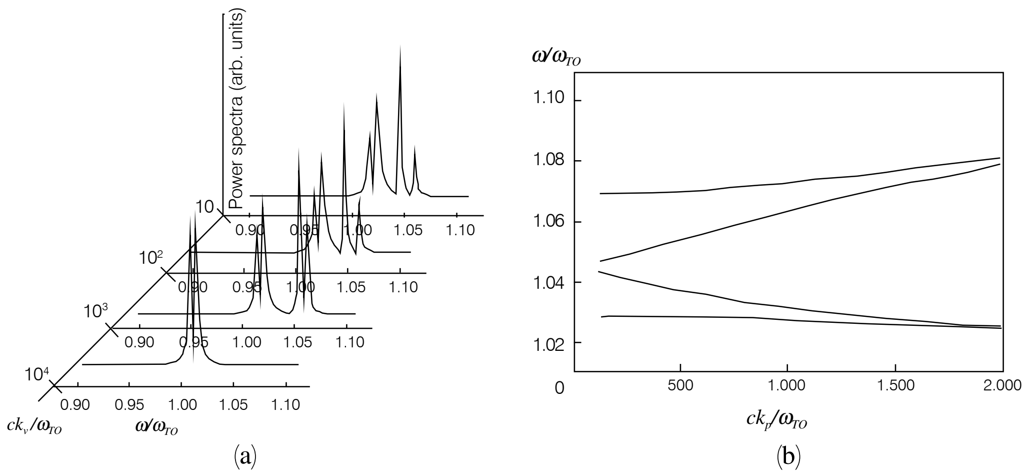
**Figura 1**




(a) (b)

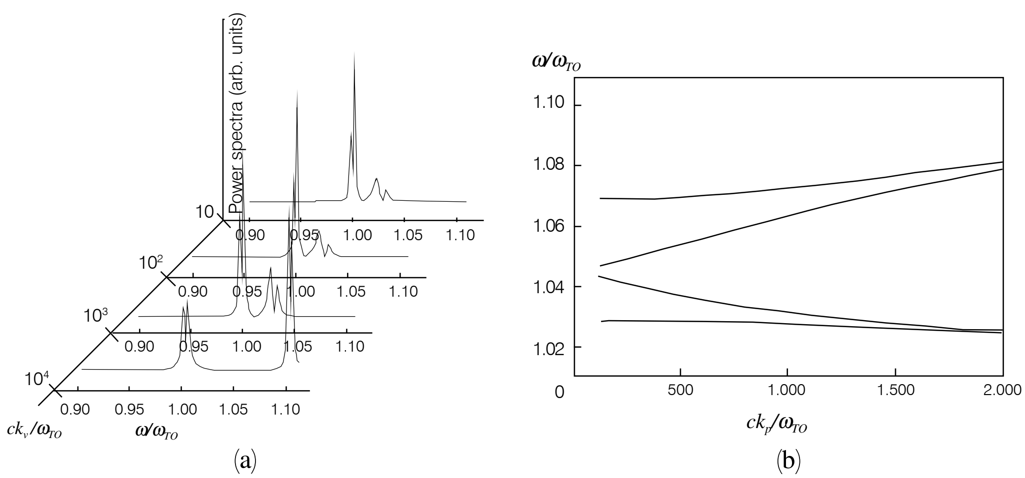
In Fig. 1 we show some power spectra ($a$) and the dispersion curves ($b$) for the modes arising in the described system for the case when $\omega_c=\omega_{LO}$. It can be seen that in the long-wavelength region of the spectra, when the fields induced by the dynamics of the electron system are coupled at $z$=0 and $z=d$, there are two sets of modes: the low-frequency set have negative group velocity ($\partial\omega/\partial k_p<0$) and shifts toward the frequency of the *TO* phonons; the high-frequency set increases with the in-plane wave vector $k_p$ above the frequency of the longitudinal optical phonons. With increasing $k_p$ the coupling between the fields at the well boundaries becomes marginal; in this case the low-frequency set approaches $\omega_{TO}$ and the high-frequency set acquires frequencies which are far above the reststrahlen region of the spectra considered in [1]. From this discussion we can state that when the cyclotron resonance reaches energies close to the longitudinal optical phonon energy there is no interaction between the cyclotron and phonon modes when can be neglected the coupling between the fields at the well boundaries. In the opposite (long-wavelength) limit it is seen that at $\omega_c = \omega_{LO}$ the *TO* mode splits in a doublet.

**Figura 2**




In Figure 2 we have illustrated the case $\omega_{TO}<\omega_c<\omega_{LO}$ ($\omega_c$=1.044$\omega_{TO}$). In the long-wavelength limit the high-frequency set of collective modes start at frequencies below $\omega_{LO}$ and only at moderate wave-vectors ($k_p$>2x10$^3\omega_{TO}/c$) the frequency of the doublet becomes comparable with that of the *TL* modes. The low-frequency set, on the other hand, shows a similar qualitative behavior than that of Figure 1.

**Figura 3**



In Figure 3 it is illustrated the case of resonance between the cyclotron frequency and the *TO* optical mode ($\omega_c=\omega_{TO}$). Here we observe two modes increasing slowly below $\omega_{LO}$ with $k_p$ and a small splitting of the *TO* frequency for all values of the in-plane wave-vector. This agrees with the results reported in [1].

## Conclusions

In conclusion, on the basis of a Green function calculation we have discussed the role of *2D*-magnetoplasmons on magneto infra-red absorption in high electronic density quantum wells. It was shown that the resonance between the *LO* mode and the cyclotron frequency can split the *LO* mode in two modes with frequencies lying far above the reststrahlen region if the coupling between electromagnetic fields at the boundaries can be neglected. Resonance with *TO* leads to a small splitting of the *TO* mode which takes place for all values of the in-plane wave vector.

## References

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