# Probabilistic semihyperrings

**Semihiperanillos probabilísticos**

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**Abstract.** In this paper, we study the concept of fuzzy $T$-sub-semihyperrings of a semihyperring. We define a probabilistic version of semihyperrings using random sets. We show that fuzzy sub-semihyperrings defined by triangular norms are consequences of probabilistic semihyperrings under certain conditions.

**Keywords:** semihyperrings, fuzzy set, probability space.

**Resumen.** En este trabajo, estudiamos el concepto de $T$-sub-semihiperanillo difuso de un semihiperanillo. Definimos una versión probabilística de semihiperanillos usando conjuntos aleatorios. Se muestra que los sub-semihiperanillos difusos definidos por normas triangulares son consecuencias de los semihiperanillos probabilísticos bajo ciertas condiciones.

**Palabras claves:** Semihiperanillos, conjunto difuso, espacio de probabilidad.

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## 1. Semihyperrings and fuzzy sets

A *semiring* is a system consisting of a non-empty set $S$ together with two binary operations on $S$ called addition and multiplication (denoted in the usual manner) such that (1) $S$ together with the addition is a (commutative) monoid with identity element 0; (2) $S$ together with the multiplication is a semigroup; (3) $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$, for all $a, b, c \in S$; (4) The element $0 \in S$ is an absorbing element, i.e., $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$. In the following table we present some examples of semirings which occur in combinatorics [4].

| $S$ | addition | multiplication | zero element |
|---|---|---|---|
| $\mathbb{R}^+$ | $+$ | $\cdot$ | 0 |
| $\mathbb{R}^+$ | max | $+$ | 0 |
| $\mathbb{R}^+$ | $(a^m + b^m)^{1/m}$ | $\cdot$ | 0 |
| [a, b] | max | min | $a$ |
| $\mathbb{R} \cup \{+\infty\}$ | and | or | 0 |

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Semihyperrings are a generalization of semirings. The concept of semihyperrings is studied by Vougiouklis [12], Davvaz [3], Ameri and Hedayati [1], and many others. In what follows, we summarize some basic definitions about algebraic hyperstructures and semihyperrings.

A mapping $\circ : H \times H \to \rho^*(H)$, where $\rho^*(H)$ denotes the family of all non-empty subsets of $H$, is called a *hyperoperation* on $H$. The couple $(H, \circ)$ is called a *hypergroupoid*. In the above definition, if $A$ and $B$ are two non-empty subsets of $H$ and $x \in H$, then we denote

$$A \circ B = \bigcup_{\substack{a\in A\\ b\in B}} a \circ b,\ A \circ x = A \circ \{x\} \quad \text{and} \quad x \circ B = \{x\} \circ B.$$

A hypergroupoid $(H, \circ)$ is called a *semihypergroup* if for every $x, y, z \in H$, $x \circ (y \circ z) = (x \circ y) \circ z$, that is

$$\bigcup_{u\in y\circ z} x \circ u = \bigcup_{v\in x\circ y} v \circ z.$$

**Definition 1.1.** A *semihyperring* is an algebraic hypersructure $(R, +, \cdot)$ which satisfies the following axioms:

(1) $(R, +)$ is a commutative semihypergroup with a zero element 0 satisfying $x + 0 = 0 + x = \{x\}$, i.e., (i) For all $x, y, z \in R, x + (y + z) = (x + y) + z$, (ii) For all $x, y \in R, x + y = y + x$, (iii) There exists $0 \in R$ such that $x + 0 = 0 + x = \{x\}$ for all $x \in R$;

(2) $(R, \cdot)$ is a semihypergroup;

(3) The multiplication $\cdot$ is distributive with respect to the hyperoperation $+$, that is, $x \cdot (y + z) = x \cdot y + x \cdot z$ and $(x + y) \cdot z = x \cdot z + y \cdot z$ for all $x, y, z \in R$;

(4) The element $0 \in R$ is an absorbing element, i.e., $x \cdot 0 = 0 \cdot x = 0$ for all $x \in R$.

A semihyperring $R$ is called *commutative* if $(R, \cdot)$ is a commutative semihypergroup. A non-empty subset $A$ of a semihyperring $(R, +, \cdot)$ is called a *subsemihyperring* of $R$ if for all $x, y \in A$, $x+y \subseteq A$ and $x\cdot y \subseteq A$. A non-empty subset $I$ of a semihyperring $(R, +, \cdot)$ is called a *left* (resp. *right*) *hyperideal* of $(R, +, \cdot)$ if for all $x, y \in I$, $x + y \subseteq I$ and $r \cdot x \subseteq I$ for all $x \in I$ and $r \in R$ (resp. $x \cdot r \subseteq I$). A non-empty subset $I$ of $R$ is called a *hyperideal* of $R$ if it is both left and right hyperideal of $R$, that is, $x + y \subseteq I$, for all $x, y \in I$ and $x \cdot r, r \cdot x \subseteq I$, for all $x \in I$ and $r \in R$.

**Example 1.2.** Let $(S, +, \cdot, 0)$ be a semiring. We define

$$\begin{aligned} &x \oplus y = \langle x, y\rangle, \text{ the ideal generated by } x, y,\\ &x \odot y = x \cdot y. \end{aligned}$$

Then, $(S, \oplus, \odot, 0)$ is a semihyperring.

**Example 1.3.** Let $R = \{0, a, b, c\}$ be a set with two hyperoperations $\oplus$ and $\odot$ as follows:

| $\oplus$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $0$ | $0$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $a$ | $\{0, a, b\}$ | $\{0, a, c\}$ |
| $b$ | $b$ | $\{0, a, b\}$ | $\{0, b\}$ | $\{0, b, c\}$ |
| $c$ | $c$ | $\{0, a, c\}$ | $\{0, b, c\}$ | $\{0, c\}$ |

| $\odot$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $0$ | $0$ | $0$ | $0$ | $0$ |
| $a$ | $0$ | $0$ | $0$ | $0$ |
| $b$ | $0$ | $0$ | $0$ | $\{0, a\}$ |
| $c$ | $0$ | $0$ | $\{0, a\}$ | $\{0, b\}$ |

Then, $(R, \oplus, \odot)$ is a semihyperring [6].

Zadeh [14] introduced the concept of a fuzzy set. Let $X$ be a non-empty set. A map $\mu : X \to [0, 1]$ is called a *fuzzy subset* of $X$. Let $\mu$ and $\nu$ be two fuzzy subsets of $X$. Then, $\mu \cap \nu$ and $\mu \cup \nu$ are defined as follows: $(\mu \cap \nu)(x) = \min\{\mu(x), \nu(x)\}$ and $(\mu \cup \nu)(x) = \max\{\mu(x), \nu(x)\}$, for all $x \in X$. Rosenfeld [9] applied this concept to the theory of groups. If $S$ is a semigroup and $\mu$ be a fuzzy subset of $S$, then $\mu$ is called a *fuzzy subsemigroup* if it satisfies $\min\{\mu(x), \mu(y)\} \leq \mu(xy)$ for all $x, y \in S$. Since then many papers concerning various fuzzy algebraic structures have appeared in literature. A *fuzzy ideal* of a semiring $(S, +, \cdot, 0)$ is a fuzzy subset $\mu$ satisfying the following conditions: $\min\{\mu(x), \mu(y)\} \leq \mu(x + y)$ and $\min\{\mu(x), \mu(y)\} \leq \mu(xy)$, for all $x, y \in S$. In what follows let $R$ denote a semihyperring $(R, +, \cdot, 0)$.

**Definition 1.4.** A fuzzy subset $\mu$ of a semihyperring $R$ is called a *fuzzy subsemihyperring* if

(1) $\min\{\mu(x), \mu(y)\} \leq \inf_{z \in x+y}\{\mu(z)\}$,

(2) $\min\{\mu(x), \mu(y)\} \leq \inf_{z \in x \cdot y}\{\mu(z)\}$,

for all $x, y \in R$.

# 2. Triangular norms

In mathematics, a t-norm (or, triangular norm) is a kind of binary operation used in the framework of probabilistic metric spaces and in multi-valued logic, specifically in fuzzy logic. A t-norm generalizes intersection in a lattice and conjunction in logic. The name triangular norm refers to the fact that in the framework of probabilistic metric spaces t-norms are used to generalize triangle inequality of ordinary metric spaces. The concept of a triangular norm was introduced by Menger [8] in order to generalize the triangular inequality of a metric. The current notion of a t-norm and its dual operation is due to Schweizer and Sklar [10]. Anthony and Sherwood [2] redefined a fuzzy subgroup of a group by using the notion of t-norm.

A *t-norm* is a mapping $T : [0, 1] \times [0, 1] \to [0, 1]$ satisfying, for all $x, y, z \in [0, 1]$,

(1) $T(x, 1) = x$,

(2) $T(x, y) = T(y, x)$,

(3) $T(x, T(y, z)) = T(T(x, y), z))$,

(4) $T(x, y) \leq T(x, z)$ whenever $y \leq z$.

These four axioms are independent in the sense that none of them can be deduced from the other three. Let $T$ be a t-norm on $[0, 1]$. The following are the four basic t-norms $T_M, T_P, T_L$, and $T_D$ given by, respectively:

$$
\begin{array}{ll}
T_M(x, y) = \min(x, y), & \text{(minimum)} \\
T_P(x, y) = x \cdot y, & \text{(product)} \\
T_L(x, y) = \max(x + y - 1, 0), & \text{(Lukasiewicz t-norm)} \\
T_D(x, y) = \begin{cases} 0 & \text{if } (x, y) \in [0, 1)^2 \\ \min(x, y) & \text{otherwise.} \end{cases} & \text{(drastic product)}
\end{array}
$$

Let $T_1$ and $T_2$ be two t-norms. $T_2$ is said to be *dominate* $T_1$ and write $T_1 \ll T_2$ if for all $a, b, c, d \in [0, 1]$,

$$T_1(T_2(a, c), T_2(b, d)) \leq T_2(T_1(a, b), T_1(c, d))$$

and $T_1$ is said *weaker* then $T_2$ or $T_2$ is *stronger* then $T_1$ and write $T_1 \leq T_2$ if for all $x, y \in [0, 1]$, $T_1(x, y) \leq T_2(x, y)$. Since a triangular norm $T$ is a generalization of the minimum function, Anthony and Sherwood in [2] replaced the axiom $\min\{\mu(x), \mu(y)\} \leq \mu(xy)$ occurring in the definition of a fuzzy subgroup by the inequality $T(\mu(x), \mu(y)) \leq \mu(xy)$.

## 3. Fuzzy T-sub-semihyperrings

Let $I_T = \{x \in [0, 1] \mid T(x, x) = x\}$, i.e., the set of all $T$-idempotent elements of $[0, 1]$.

**Definition 3.1.** Let $T$ be a $t$-norm. A fuzzy subset $\mu$ of semihyperring $R$ is a $T$-sub-semihyperring of $R$ if the following axioms hold.

(1) $\mathrm{Im}(\mu) \subseteq I_T$,

(2) $T(\mu(x), \mu(y)) \leq \inf_{z \in x+y}\{\mu(z)\}$, for all $x, y \in R$,

(3) $T(\mu(x), \mu(y)) \leq \inf_{z \in x \cdot y}\{\mu(z)\}$, for all $x, y \in R$.

**Theorem 3.2.** *Let $T$ be a $t$-norm and $\mu$ be a fuzzy subset of $R$ such that $Im(\mu) \subseteq I_T$ and $b =sup\ Im(\mu)$. Then, the following conditions are equivalent.*

*(1) $\mu$ is a $T$-sub-semihyperring of $R$,*

*(2) $\mu^{-1}[a, b]$ is a sub-semihyperring of $R$ whenever $a \in I_T$ and $0 < a \leq b$.*

***Proof***. (1⇒2): Suppose that $a \in I_T$ and $0 < a \leq b$. If $x, y \in \mu^{-1}[a, b]$, then $\inf_{z\in x+y}\{\mu(z)\} \geq T(\mu(x), \mu(y)) \geq T(a, a) = a$ and $\inf_{z\in x\cdot y}\{\mu(z)\} \geq T(\mu(x), \mu(y)) \geq T(a, a) = a$. Thus, we obtain $x + y \subseteq \mu^{-1}[a, b]$ and $x \cdot y \subseteq \mu^{-1}[a, b]$. Therefore, $\mu^{-1}[a, b]$ is a sub-semihyperring of $R$.

(2⇒1): Let $x, y \in R$. Since $\mathrm{Im}(\mu) \subseteq I_T$, it follows that $\mu(x)$ and $\mu(y)$ are in $I_T$. We have

$$\begin{aligned} T(T(\mu(x), \mu(y)), T(\mu(x), \mu(y))) &= T(T(\mu(x), T(\mu(y), \mu(x))), \mu(y)) \\ &= T(T(\mu(x), T(\mu(x), \mu(y))), \mu(y)) \\ &= T(T(\mu(x), \mu(x)), T(\mu(y), \mu(y))) \\ &= T(\mu(x), \mu(y)), \end{aligned}$$

and so $T(\mu(x), \mu(y)) \in I_T$. Assume that $a = T(\mu(x), \mu(y))$. If $a = 0$, then $T(\mu(x), \mu(y)) = 0 \leq \inf_{z\in x+y}\{\mu(z)\}$. So, let $0 < a = T(\mu(x), \mu(y)) \leq \min\{\mu(x), \mu(y)\} \leq \mu(x) \leq b$. Hence, $x, y \in \mu^{-1}[a, b]$, which implies $x + y \subseteq \mu^{-1}[a, b]$. Therefore $T(\mu(x), \mu(y)) \leq \inf_{z\in x+y}\{\mu(z)\}$. Similarly, we obtain $T(\mu(x), \mu(y)) \leq \inf_{z\in x\cdot y}\{\mu(z)\}$. □

**Corollary 3.3.** *Let $A$ be a non-empty subset of $R$. Then, the characteristic function $\chi_A$ is a $T$-sub-semihyperring of $R$ if and only if $A$ is a sub-semihyperring of $R$.*

**Corollary 3.4.** *Let $T$ be a t-norm and $\{\mu_i\}_{i\in I}$ be a family of $T$-sub-semihyperrings of $R$. Then $\bigcap\limits_{i\in I} \mu_i$ is a $T$-sub-semihyperring of $R$.*

**Definition 3.5.** Let $R, R'$ be two semihyperrings and $\mu, \nu$ be $T$-sub-semihyperrings of $R, R'$, respectively. The product of $\mu, \nu$ is defined to be the fuzzy subset $\mu \times \nu$ of $R \times R'$ with $(\mu \times \lambda)(x, y) = T(\mu(x), \lambda(x))$, for all $(x, y) \in R \times R'$.

**Lemma 3.6.** *By the above definition, $\mu \times \nu$ is a $T$-sub-semihyperring of $R \times R'$.*

***Proof***. Suppose that $(x_1, x_2)$, $(y_1, y_2) \in R \times R'$. For every $(z_1, z_2) \in (x_1, x_2) + (y_1, y_2)$ we have

$$\begin{aligned} (\mu \times \lambda)(\alpha_1, \alpha_2) &= T(\mu(\alpha_1), \lambda(\alpha_2)) \\ &\geq T(T(\mu(x_1), \mu(y_1)), T(\lambda(x_2), \lambda(y_2)) \\ &= T(T(T(\mu(x_1), \mu(y_1)), \lambda(x_2), \lambda(y_2))) \\ &= T(T(\lambda(x_2), T(\mu(x_1), \mu(y_1)), \lambda(y_2))) \\ &= T(T(T(\lambda(x_2), \mu(x_1)), \mu(y_1), \lambda(y_2))) \\ &= T(\lambda(y_2), T(\mu(y_1), T(\lambda(x_2), \mu(x_1))) \\ &= T(T(\mu(x_1), \lambda(x_2)), T(\mu(y_1), \lambda(y_2)) \\ &= T((\mu \times \lambda)(x_1, x_2), (\mu \times \lambda)(y_1, y_2)). \end{aligned}$$

Taking the infimum over all $(z_1, z_2) \in (x_1, x_2) + (y_1, y_2)$ we have

$$\inf_{(z_1,z_2)\in(x_1,x_2)+(y_1,y_2)} \{(\mu \times \lambda)(z_1, z_2)\} \geq T((\mu \times \lambda)(x_1, x_2), (\mu \times \lambda)(y_1, y_2)).$$

Similarly, we obtain

$$\inf_{(z_1,z_2)\in(x_1,x_2)\cdot(y_1,y_2)}\{(\mu\times\lambda)(z_1,z_2)\}\geq T((\mu\times\lambda)(x_1,x_2),(\mu\times\lambda)(y_1,y_2)).$$

□

## 4. Probabilistic fuzzy semihyperrings

If $\mu$ is a fuzzy subset of $R$, then for any $t\in[0,1]$, the set $\mu_t=\{x\in R\mid\mu(x)\geq t\}$ is called a *level subset* of $\mu$.

**Theorem 4.1.** *Let $R$ be a semihyperring and $\mu$ a fuzzy subset of $R$. Then $\mu$ is a fuzzy sub-semihyperring of $R$ if and only if for any $t\in[0,1]$, $\mu_t$ (when it is non-empty), is a sub-semihyperring of $R$.*

***Proof.*** In Theorem 3.2, take $T=\min$. □

In the theory of probability we start by $(\Omega,\mathbb{A},P)$, with $\Omega$ set of elementary events and $\mathbb{A}$, $\sigma$-algebra of subsets of $\omega$ called *events*. A *probability* on $\mathbb{A}$ is defined as a countable additive and positive function $P$ such that $P(\Omega)=1$.

The following definition is an extract from [11, 13].

Given a universe of discourse $U$, for each arbitrary $u\in U$, let

$$\dot{u}:=\{A\mid u\in A\text{ and }A\subseteq U\}.$$

For each $A$ in the power set of $U$, let

$$\dot{A}:=\{\dot{u}\mid u\in A\}.$$

An ordered pair $(\rho(U),\mathbb{B})$ is said to be an *hyper-measurable structure* on $U$ if $\mathbb{B}$ is a $\sigma$-field in $\rho(U)$ and satisfies the following condition:

$$\dot{U}\subseteq\mathbb{B}.$$

**Definition 4.2.** Given a probability space $(\Omega,\mathbb{A},P)$ and hyper-measurable structure $(\rho(U),\mathbb{B})$ on $U$, a random set on $U$ is defined to be a mapping $r:\Omega\to\rho(U)$ that is $\mathbb{A}-\mathbb{B}$ measurable, that is,

$$\forall C\in\mathbb{B},\ r^{-1}(C)=\{\omega\mid\omega\in\Omega\text{ and }r(\omega)\in C\}\in\mathbb{A}.$$

**Definition 4.3.** Let $R$ be a semihyperring and $(\Omega,\mathbb{A},P)$ be a probability space. Let $r:\Omega\to\rho(R)$ be a random set, where $\rho(R)$ is the set of all subsets of $R$. If for any $\omega\in\Omega$, $r(\omega)$ is a sub-semihyperring of $R$, then the falling shadow $S$ of the random set $r$, i.e., $S(x)=P(\{\omega|x\in r(\omega)\})$ is called a $\pi$-*fuzzy sub-semihyperring* of $R$.

Based on the concept of a falling shadow, we establish a theoretical approach of the fuzzy sub-semihyperrings.

**Theorem 4.4.** *Let $S$ be a $\pi$-fuzzy sub-semihyperring of semihyperring $R$. Then, for all $x, y \in R$, we have*

(1) $\inf_{z\in x+y}\{S(z)\} \geq T_L(S(x), S(y))$,

(2) $\inf_{z\in x\cdot y}\{S(z)\} \geq T_L(S(x), S(y))$.

***Proof***. (1) We know $r(\omega)$ is a sub-semihyperring of $R$. Now, let $x, y \in r(\omega)$, then $x + y \subseteq r(\omega)$. So, for every $z \in x + y$ we have

$$\{\omega|\ z \in r(\omega)\} \supseteq \{\omega|\ x \in r(\omega)\} \cap \{\omega|\ y \in r(\omega)\}.$$

Then, we obtain

$$\begin{aligned} S(z) &= P(\omega|\ z \in r(\omega)) \\ &\geq P(\{\omega|x \in r(\omega)\} \cap \{\omega|\ y \in r(\omega)\}) \\ &\geq P(\omega|\ x \in r(\omega)) + P(\omega|\ y \in r(\omega)) - P(\omega|\ x \in r(\omega) \text{ or } y \in r(\omega)) \\ &\geq S(x) + S(y) - 1. \end{aligned}$$

Hence, we have $\inf_{z\in x+y}\{S(z)\} \geq T_L(S(x), S(y))$.

(2) The proof is similar to (1). □

**Theorem 4.5.** *(1) Let $\mathbb{H}$ denote the set of all sub-semihyperrings of a semihyperring $R$. Let $H_x = \{A \mid A \in \mathbb{H},\ x \in A\}$ for each $x \in R$. Let $(\mathbb{H}, \sigma)$ be a measurable space where $\sigma$ is a $\sigma$-algebra that contains $\{H_x|\ x \in R\}$ and $P$ a probability measure on $(\mathbb{H}, \sigma)$. We define $\mu : H \to [0, 1]$ as follows: $\mu(x) = P(H_x)$ for $x \in R$. Then, $\mu$ is a fuzzy $T_L$-sub-semihyperring of $R$.*

*(2) Suppose that there exists $\mathbb{A} \in \sigma$ such that $\mathbb{A}$ is a chain with respect to the set inclusion relation and $P(\mathbb{A}) = 1$. Then, $\mu$ is a fuzzy sub-semihyperring of $R$.*

***Proof***. (1) If $x, y \in R$, then $H_z \supseteq H_x \cup H_y$ for all $z \in x + y$. Then, we have

$$\mu(z) = P(H_z) \geq P(H_x \cap H_y) \geq \max\{P(H_x) + P(H_y) - 1, 0\} = T_L(\mu(x), \mu(y)).$$

Therefore, $\inf_{z\in x+y}\{\mu(z)\} \geq T_L(\mu(x), \mu(y))$. In a similar way, we obtain $\inf_{z\in x\cdot y}\{\mu(z)\} \geq T_L(\mu(x), \mu(y))$.

(2) Since $P$ is a probability measure and $P(\mathcal{A}) = 1$ we have $P(H_x \cap \mathcal{A}) = P(H_x)$ for all $x \in H$. Therefore for every $z \in x + y$ we have

$$\mu(z) = P(H_z) \geq P(H_x \cap H_y) = P(H_x \cap \mathcal{A}) \cap (H_y \cap \mathcal{A}).$$

Since $\mathcal{A}$ with the set inclusion forms a chain, it follows that either $H_x \cap \mathcal{A} \subseteq H_y \cap \mathcal{A}$ or $H_y \cap \mathcal{A} \subseteq H_x \cap \mathcal{A}$. Therefore, we obtain

$$\mu(z) \geq \min\{P(H_x \cap \mathcal{A}), P(H_y \cap \mathcal{A})\} = \min\{\mu(x), \mu(y)\},$$

and so $\inf_{z\in x+y}\{\mu(z)\} \geq \min\{\mu(x), \mu(y)\}$. Similarly, we obtain $\inf_{z\in x\cdot y}\{\mu(z)\} \geq \min\{\mu(x), \mu(y)\}$. □

Let $(\Omega, \sigma, P) = ([0,1], \sigma, m)$, where $\sigma$ is a Borel field on $[0,1]$ and $m$ the usual Lebesgue measure. Let $\mu$ be a fuzzy subset of $R$ and $\mu_t = \{x \in R \mid \mu(x) \geq t\}$ be a level subset of $\mu$. Then,

$$\begin{aligned} r : [0,1] &\to \rho(R) \\ t &\longmapsto \mu_t \end{aligned}$$

is a measurable function. This notion was firstly investigated by Goodman in [5], also see [11, 7].

**Theorem 4.6.** *Let $R$ be a semihyperring and $\mu$ be a fuzzy subsemihyperring of $R$. Then, there exists a probability space $(\Omega, \mathbb{A}, P)$ such that for some $A \in \mathbb{A}$, $\mu(x) = P(A)$.*

***Proof***. Suppose $\Omega = \mathbb{H}$, the set of all sub-semihyperring of $R$. Consider $r : [0,1] \to \mathbb{H}$ given by $t \longmapsto \mu_t$. Then, $r$ is a measurable function and so $r$ is a random set. Let

$$\mathbb{A} = \{A \mid A \in \mathbb{H},\ r^{-1}(A) \in \sigma\}$$

and $P = m \circ r^{-1}$. It is easy to see that $(\mathbb{H}, \mathbb{A}, P)$ is a probability space. If we put $H_x = \{A \mid A \in \mathbb{H},\ x \in A\}$, then for $x \in R$ we have $\mu_t \in H_x$ for all $t \in [0, \mu(x)]$ and $\mu_s \notin H_x$ for all $s \in (\mu(x), 1]$. So, $r^{-1}(H_x) = [0, \mu(x)]$ and so $H_x \in \mathbb{A}$. Now we obtain $P(H_x) = m \circ R^{-1}(H_x) = m([0, \mu(x)]) = \mu(x)$. □

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