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Comentarios sobre "Puntos de equilibrio asintóticamente estables en nuevos sistemas caóticos"
Comments on "Asymptotically stable equilibrium points in new chaotic systems"

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Resumen

En el trabajo comentado, los autores presentan diez sistemas autónomos no lineales caóticos, de los que afirman que no tienen caos en el sentido de Shilnikov. Desgraciadamente, esta afirmación carece de fundamento pues utilizan un teorema erróneo de la literatura.

Palabras clave: sistemas caóticos; equilibrio asintóticamente estable; no existencia del caos de Shilnikov; exponentes de Lyapunov

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Abstract

In the commented paper ten nonlinear chaotic systems are presented. Authors state that these systems do not exhibit Shilnikov chaos. Unfortunately, this assertion is not correctly proved because they use an erroneous theorem from the literature.

Keywords: chaotic systems, asymptotically stable equilibrium, non-existence of Shilnikov chaos, Lyapunov exponents

In the commented paper (Casas-García *et al*., 2016) the authors analyze ten nonlinear chaotic systems. As an important feature, they affirm that these systems do not exhibit Shilnikov chaos (see Abstract). To demonstrate this fact, first they copy, in Sect. 2, Definitions 1-3 and Theorem 1 from (Elhadj and Sprott, 2012) and state: *Theorem 1 characterizes the conditions under which a system does not present homoclinic and heteroclinic orbits. From this information we can identify systems that present Smale's horseshoe behavior.* Second, they assert in Sect. 3: *From Definitions 1-3 and Theorem 1 we know that the chaos presented is not of the Smale horseshoe type due to the fact that the systems does not contain homoclinic or heteroclinic orbits.*

However, as we clearly demonstrate in (Algaba *et al*., 2013a), Theorem 1 is erroneous. Consequently, some of the ten systems considered might have homoclinic or heteroclinic orbits and then they might exhibit Shilnikov chaos. Therefore, the sentence stated in the Conclusions, *The chaos behaviour of the studied systems is not of the class of Smale's horseshoe type, due to their orbits are not either homoclinic nor heteroclinic in the sense Shilnikov*, has not scientific basis.

We would like to add a last comment. Chen's and Lü's systems, cited in the commented paper, are only particular cases of the Lorenz system as it is demonstrated in (Algaba *et al*., 2013b, 2013c), by using a linear scaling in time and state variables. This fact is illustrated in (Algaba et al., 2014, 2015, 2016).

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