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RESEARCH ARTICLE

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Input point distribution for regular stem form spline modeling

Karel Kuželka* and Róbert Marušák

Fakulta lesnická a dřevařská, Česká zemědělská univerzita v Praze, Kamýcká 1176, Praha 6 – Suchdol, 165 21

Abstract

Aim of study: To optimize an interpolation method and distribution of measured diameters to represent regular stem form of coniferous trees using a set of discrete points.

Area of study: Central-Bohemian highlands, Czech Republic; a region that represents average stand conditions of production forests of Norway spruce (*Picea abies* [L.] Karst.) in central Europe.

Material and methods: The accuracy of stem curves modeled using natural cubic splines from a set of measured diameters was evaluated for 85 closely measured stems of Norway spruce using five statistical indicators and compared to the accuracy of three additional models based on different spline types selected for their ability to represent stem curves. The optimal positions to measure diameters were identified using an aggregate objective function approach.

Main results: The optimal positions of the input points vary depending on the properties of each spline type. If the optimal input points for each spline are used, then all spline types are able to give reasonable results with higher numbers of input points. The commonly used natural cubic spline was outperformed by other spline types. The lowest errors occur by interpolating the points using the Catmull-Rom spline, which gives accurate and unbiased volume estimates, even with only five input points.

Research highlights: The study contributes to more accurate representation of stem form and therefore more accurate estimation of stem volume using data obtained from terrestrial imagery or other close-range remote sensing methods.

Key words: spline; stem curve; taper model; diameter position; Norway spruce.

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Correspondence should be addressed to Karel Kuželka: kuzelka@fld.czu.cz

Introduction

Stem curve models are of great importance in forest management and planning. They allow for the prediction of the diameter at any location along the stem, provide estimation of both total and partial stem volume and also serve for estimating assortment structure (Sharma and Parton, 2009).

A number of simple taper models of polynomial (e.g. Kozak *et al.*, 1969), logarithmic (Demaerschalk, 1972), trigonometric (Thomas & Parresol, 1991), and other forms (e.g. Biging, 1984) have been developed for trees from a wide range of species and geographical areas. Later segmented taper models (Max & Burkhart, 1976), that describe the stem as several geometrical shapes, were developed. Variable exponent models (e.g. Lee *et al.*, 2003) are based on a continuous change between several geometrical forms and are considered as the most accurate models (Rojo *et al.*, 2005).

Both scientific and practical applications in recent forestry often require reconstruction of stem curve of an individual stem, especially within the context of exploitation of recently evolving techniques of data acquisition, as terrestrial laser scanners (Lovell *et al.*, 2011), terrestrial close-range photogrammetry (Hapca *et al.*, 2007), and other close-range remote sensing techniques. Splines are a useful tool allowing for interpolation or approximation of a series of discrete points representing stem diameters at given heights in order to obtain the model the stem curve.

The term spline is a general expression for a wide class of functions defined piecewise usually by polynomial functions. The most commonly used spline is the cubic interpolation spline. It is frequently used to describe stem form (Lahtinen & Laasasenaho, 1979) or e.g. bark thickness (Laasasenaho *et al.*, 2005). Lahtinen (1988) used monotony-preserving quadratic splines. The smoothing spline was tested by Liu (1980) with lesser success, but utilized successfully by Nummi & Möttönen (2004) and later by Koskela *et al.* (2006) for stem profile predictions. Regression models utilizing splines were introduced by Sloboda *et al.* (1998) and later refined by Lappi (2006) and Kublin *et al.* (2008); mixed effect regression taper model based on B-spline was developed by Kublin *et al.* (2013).

To enable accurate representation of the stem form, Smaltschinski (1983) states that six measured diameters is the minimum number of input points required. Lahtinen (1988) modeled the taper curve using five points, which provided a satisfactory approximation to the taper curve with good total volume estimation, but with high differences of diameter. Figueiredo-Filho *et al.* (1996) state that for seven input points or fewer, their placement along the stem is very important.

This work reports the results of an investigation regarding the use of different spline types to represent regular stem forms using different numbers of input points.

The selection of spline types used in this study is based on results of preliminary analyses, where several splines were compared regarding their suitability to represent the stem profile. The natural cubic spline (NCS) is a widely used interpolation curve which has minimal curvature among twice continuously differentiable interpolating curves. For B-splines (Piegl & Tiller, 1996) of both approximation (BS) and interpolation (IBS) form, the accuracy of a curve declines with rising degree, and adding weights to the B-splines does not improve the accuracy; therefore, second degree B-splines with uniform weights was selected. The Catmull-Rom spline (CRS) (Kochanek & Bartels, 1984) is a flexible cubic interpolation curve with first degree continuity. Using nine input points achieves a near maximum accuracy (Smaltschinski, 1983) and with more input points, the accuracy is not significantly improved.

Materials and methods

The study used data from 85 Norway spruce trees. The trees were selected from three 50- to 100-year-old stands located in the School Forest Enterprise Kostelec nad Černými lesy, Czech Republic. In order to cover the shape variability in the stands, dominant trees as well as suppressed trees were selected for analysis. The diameter at breast height (DBH) of the trees ranged from 88 to 438 mm (mean 204 mm), and tree heights ranged from 10.6 to 37.1 m (mean 21.3 m). Diameters outside

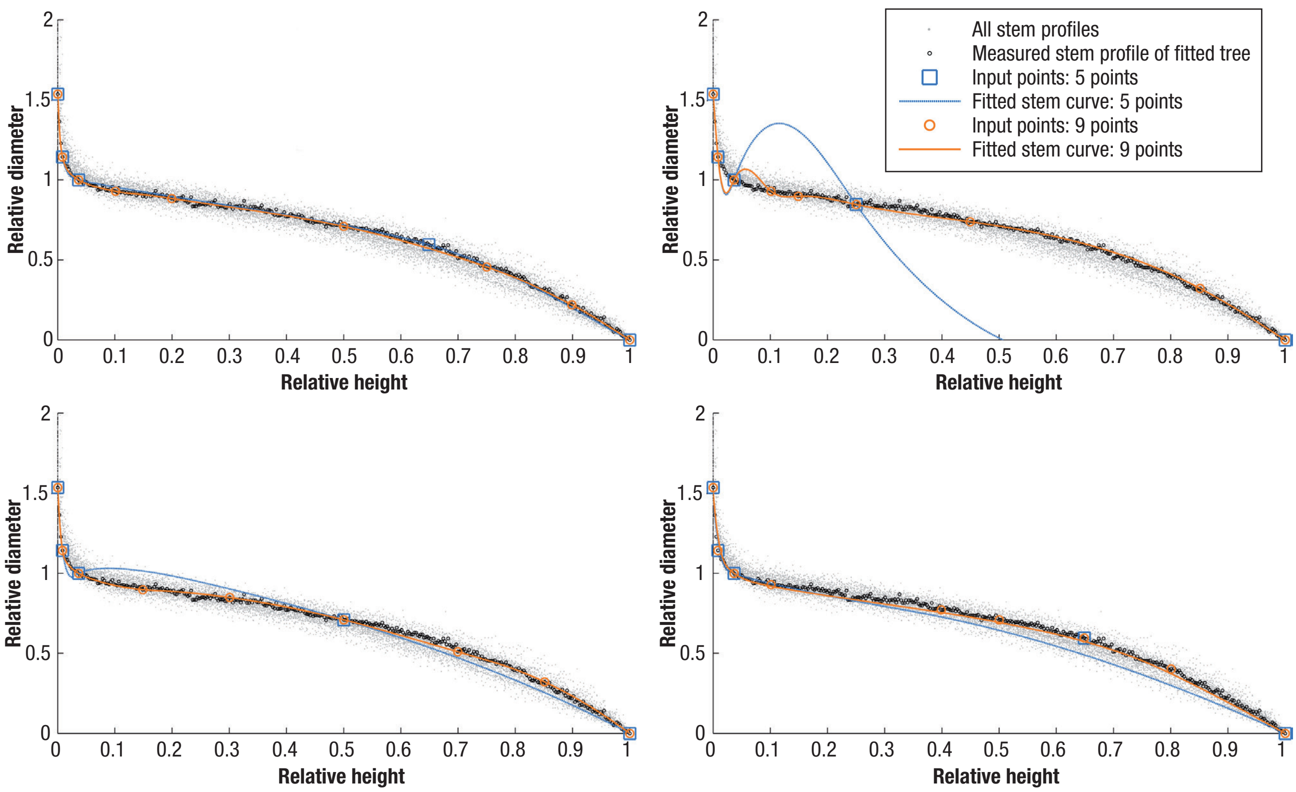

Figure 1. Stem curves modeled by Catmull-Rom spline (a), natural cubic spline (b), interpolation B-spline (c) and B-spline (d) with 5 (the leas number investigated) and 9 (the highest number investigated) input points in optimal positions. Stem curves are expressed as dependence of relative diameter (d/DBH) on relative height (h/H). Stem profiles of all trees are also shown.

bark were measured on the felled trees from the tree base to the top at 0.1-m intervals. Distance from tree base was measured using a steel tape with 0.01-m precision, and the diameters were measured and recorded with an electronic caliper with 0.001-m precision.

Spline curves were computed from sets of input points containing a subset of four fixed input points and a subset of 1–5 additional input points. Positions of the four fixed input points are determined by stem foot (h = 0 m), stump height (h = 0.3 m), breast height (h = 1.3 m), and the stem top. Both the stem foot and the top must be involved in order to obtain the curve of the entire stem. The stump diameter is required for the proper description of the butt swell. DBH is included because DBH is a conventional parameter and its value is always measured. Positions of the additional input points were selected from the set of relative heights 10%, 15% … 95%, and were optimized for each spline type and each point number individually.

The residuals between the predicted and measured diameters were assessed for each position of the measured diameters. The accuracy of the predicted curves was evaluated using five criteria: bias (B) computed as mean residual indicates whether a modeled curve systematically under- or over-estimates stem thickness; mean absolute residual (MAR) reflects the average distance between the predicted and the original diameters; standard deviation of residuals (SDR) detects heterogeneity in residual values; mean squared residual (MSR) value reveals locally high deviations in the curve; and total volume difference (TVD) expresses the difference between the predicted and the real volume. The volumes of the spline models were calculated as the sum of the volumes of very short sections using Smalian's equation. All statistics were calculated both for the entire stem and for ten uniformly spaced height sections (0%–10%, 10%–20%, etc.).

The positions of the additional input point were optimized using a multi-criteria method of aggregate objective function. The weights (Table 1) were chosen so that the average accuracy of the curves (minimization of means) is well balanced with their reliability (minimization of variances). A third of the total weight is given to criteria controlling the shape of the curve (MAR, SDR and MSR); a third is given to criteria signalizing systematic shift of the curves (DB, TVD); the last third penalizes statistical significance of systematic shift of the whole curves and sections. Statistical evaluation was carried out using MATLAB Statistics Toolbox (The MathWorks, Inc. 2012). Due to the variances of the criteria not being equal in all cases, the Kruskal-Wallis test was used to test the equality of means of the criteria among different diameter distributions and subsequently among taper models.

Results

The pronounced curvature of the lower stem is located at approximately 10% of the stem height. From input point optimization for individual trees results, that it is crucial to place an input point at a location corresponding to approximately 10% of the stem height, so that the lower stem curvature is fitted properly. For smaller trees, this is satisfied by the point at breast height. Therefore, the data set was split into two height classes using a threshold value of 20 m and the input point placement was optimized separately for each class.

The combinations considered best in terms of the aggregate objective function, were evaluated for stability. An input point combination was selected as optimal if a small shift of the point positions (up to 5% of the stem height) did not significantly affect the accuracy of the curve. Owing to the different behavior of individual splines, the optimal input point positions vary. With natural cubic splines, the input points are added

Table 1. Criteria and their weights used in aggregate objective function for optimizing input point positions.

Criterion			Weight (%)
Shape	MAR	median	4.17
		variance	4.17
	SDR	median	4.17
		variance	4.17
	MSR	median	8.33
		variance	8.33
Systematic error	DB	median	8.33
		variance	8.33
	TVD	median	8.33
		variance	8.33
Significance of prediction errors	DB	number of sections with significant error	5.56
		presence of significant error in total	11.11
	TVD	number of sections with significant error	5.56
		presence of significant error in total	11.11

preferably to the lower third of the stem in order to reduce oscillations mainly emerging in the lower third. With the B-spline, the points are placed preferably proximal to 70% of the height, such that the approximation spline is able to describe the major change of direction of the upper tree profile. With the Catmull-Rom spline and interpolation B-spline, the points are distributed more evenly along the stem (Table 2).

Using optimized positions, a reliable curve with well-balanced error is produced by the Catmull-Rom spline. For all input point numbers, the Catmull-Rom spline gives unbiased estimates of total volume with a mean total volume difference of less than 1% .The overall diameter prediction is slightly underestimated (less than 2 mm) for five input points; for more input points the prediction is unbiased (Table 3). The low values of SDR and MSR for all input point numbers illustrate the evenness of the error distribution along the stem. When only five input points are used, the spline does not well represent the two major direction changes of the stem (Table 4). With six input points, only the second section is biased (Table 5) and with additional input points, the spline gives predictions without any sectional or total systematic deviations.

The oscillations of the natural cubic spline are more pronounced with lower numbers of input points. With a rising number of input points, the oscillation is reduced; however, it is not completely eliminated even with nine

Table 2. Optimal positions of additional measured diameters; relative heights (%).

Tree height	No. of points	CRS	NCS	IBS	BS
Under 20 m	1	60	35	55	70
	2	50, 85	15, 45	20, 60	65, 85
	3	15, 50, 85	15, 40, 80	20, 40, 75	25, 65, 85
	4	15, 35, 55, 85	15, 25, 35, 95	20, 40, 65, 85	10, 50, 60, 85
	5	15, 25, 35, 55, 85	10, 15, 20, 65, 95	20, 40, 50, 70, 90	10, 15, 50, 60, 85
Over 20 m	1	65	25	50	65
	2	50, 80	15, 45	25, 65	60, 85
	3	10, 50, 80	10, 35, 80	15, 30, 65	30, 50, 80
	4	10, 35, 55, 85	10, 15, 35, 80	15, 30, 55, 75	15, 50, 65, 80
	5	10, 20, 50, 75, 90	10, 15, 25, 45, 85	15, 30, 50, 70, 85	10, 40, 50, 65, 80

Table 3. Accuracy evaluation of the four spline types (CRS – Catmull-Rom spline, NCS – natural cubic spline, IBS – interpolation B-spline, BS – B-spline) for 5 to 9 input points. If means in a column are followed by the same letter, the values are not significantly different. Stars in columns B and TVD indicate values significantly different from zero.

	Spline	DB (cm)		MAR (cm)		SDR (cm)		MSR (10-3 m2)		TVD (%)	
		Mean	StD	Mean	StD	Mean	StD	Mean	StD	Mean	StD
5 points	CRS	–0.18 a*	0.32	0.58 a	0.20	0.75 a	0.26	0.06 a	0.05	–0.91 a,b	4.86
	NCS	–5.35 b*	5.06	8.36 b	6.45	9.49 b	7.32	14.43 b	20.23	4.45 a,b	25.49
	IBS	0.03 a	0.30	0.72 a	0.26	1.34 c	0.58	0.21 c	0.18	1.32 a*	3.95
	BS	–0.49 c*	0.37	1.09 c	0.36	1.36 c	0.43	0.20 c	0.13	–2.36 b*	5.02
6 points	CRS	0.02 a	0.25	0.48 a	0.16	0.63 a	0.22	0.04a	0.03	0.20 a	3.67
	NCS	–0.11 a*	0.41	2.03 b	1.42	2.49 b	1.66	0.89 b	1.18	–1.31 b*	5.25
	IBS	–0.01 a	0.26	0.51 a	0.18	0.76 a	0.29	0.07 a	0.06	–0.07 a,b	3.75
	BS	–0.08 a*	0.29	0.73 c	0.22	1.02 c	0.30	0.11 c	0.07	0.92 a	4.64
7 points	CRS	–0.01 a	0.18	0.43 a	0.14	0.58 a	0.20	0.04 a	0.03	–0.24 a	2.57
	NCS	–0.11 b*	0.38	1.59 b	0.91	2.02 b	1.14	0.53 b	0.60	–1.47 a*	5.37
	IBS	–0.01 a,b	0.20	0.44 a	0.14	0.63 a	0.21	0.04 a	0.03	–0.29 a	2.70
	BS	–0.05 a,b	0.20	0.63 c	0.17	0.94 c	0.26	0.10 c	0.05	1.11 b*	2.67
8 points	CRS	–0.03 a	0.15	0.40 a	0.12	0.56 a	0.18	0.03 a	0.03	–0.16 a	2.06
	NCS	–0.03 a	0.41	0.81 b	0.30	1.19 b	0.47	0.16 b	0.13	0.29 a,b	5.02
	IBS	–0.03 a	0.20	0.44 a	0.14	0.63 a	0.22	0.04 a	0.03	–0.07 a	2.56
	BS	–0.03 a	0.17	0.54 c	0.12	0.86 c	0.24	0.08 c	0.04	0.84 b*	2.47
9 points	CRS	–0.03 a	0.15	0.39 a	0.11	0.54 a	0.18	0.03 a	0.03	–0.26 a,b	1.91
	NCS	–0.13 b*	0.39	0.75 b	0.33	1.16 b	0.54	0.16 b	0.20	–1.04 a	8.60
	IBS	0.01 a	0.12	0.40 a	0.13	0.60 a	0.22	0.04 a	0.03	–0.12 b	1.95
	BS	–0.01 a	0.16	0.53 c	0.12	0.85 c	0.24	0.08 a	0.04	1.18 c*	2.25

Table 4. Sectional Diameter Bias (DB, cm) and relative Volume Difference (VD, %) for 5-point splines. CRS = Catmull-Rom Spline, . NCS = natural cubic spline, IBS = interpolation B-spline, BS = B-spline. Section 1 = 0-10% of stem height, section 2 = 10-20% of stem height etc. Stars indicate values significantly different from zero.

	Spline	Section 1	Section 2	Section 3	Section 4	Section 5	Section 6	Section 7	Section 8	Section 9	Section 10
DB	CRS	–0.01	0.19*	–0.05	–0.12	–0.10	–0.03	–0.02	–0.33*	–0.73*	–0.66*
	NCS	0.46	8.22*	2.75*	–5.21*	–10.69*	–13.39*	–13.61*	–11.83*	–8.35*	–3.35*
	IBS	1.18*	0.23*	0.02	0.04	0.05	–0.04	–0.17*	–0.36*	–0.53*	–0.42*
	BS	1.52*	0.57*	–0.11	–0.48*	–0.75*	–1.08*	–1.35*	–1.54*	–1.49*	–0.84*
VD	CRS	–0.20	2.33*	–0.09	–0.95	–1.08	0.05	0.08	–6.13*	–19.73*	–36.37*
	NCS	7.45*	105.40*	46.41*	–31.80*	–61.38*	–60.63*	–50.60*	–44.17*	–43.29*	–48.48*
	IBS	7.81*	2.58*	0.23	0.22	0.11	–0.70	–2.14	–5.51*	–12.36*	–20.75*
	BS	13.92*	6.07*	–1.00	–5.46*	–9.58*	–14.70*	–20.85*	–28.76*	–38.91*	–48.52*

Table 5. Sectional Diameter Bias (cm) and relative Volume Difference (%) for 6-point splines. CRS = Catmull-Rom Spline, . NCS = natural cubic spline, IBS = interpolation B-spline, BS = B-spline. Section 1 = 0-10% of stem height, section 2 = 10-20% of stem height etc.

	Spline	Section 1	Section 2	Section 3	Section 4	Section 5	Section 6	Section 7	Section 8	Section 9	Section 10
DB	CRS	–0.01	0.22*	0.03	0.01	0.03	–0.01	–0.01	0.03	0.03	–0.18*
	NCS	–0.04	0.17*	–3.86*	–2.82*	–0.07	1.68*	2.08*	1.53*	0.55*	–0.14
	IBS	0.08	0.19*	0.01	–0.15*	–0.19*	–0.09	0.06	0.09	–0.02	–0.13*
	BS	1.51*	0.57*	–0.07	–0.39*	–0.55*	–0.59*	–0.47*	–0.37*	–0.51*	–0.44*
VD	CRS	–0.21	2.49*	0.40	–0.09	–0.19	–0.55	–0.28	0.40	0.27	–5.31
	NCS	1.98	3.06*	–33.92*	–27.12*	0.14	26.56*	40.96*	39.64*	25.89*	6.52
	IBS	0.14	2.17*	0.24	–1.44	–2.30*	–0.77	1.67	2.82	0.58	–1.68
	BS	13.86*	6.06*	–0.56	–4.44*	–7.17*	–8.26*	–7.37*	–6.74*	–14.17*	–25.22*

input points. Although the total volume estimation and overall diameter prediction are not significantly biased, the high sectional diameter and volume errors show the unsuitability of this spline for the given purpose.

A reasonable representation of the stem profile produced by interpolation B-spline is evident by the low values of MAR, SDR and MSR for all numbers of input points. Approximation B-spline is limited by systematic errors in both main curvatures. For all numbers of input points, and overestimation is recorded in the lower part and underestimation in the topmost sections (Tables 4 and 5). With an increasing number of input points, the accuracy improves which is evident in the decreasing values of MAR, SDR and MSR.

Discussion

The optimal input point positions for natural cubic spline found in this study differ from those stated by both Smaltschinski (1983) and Figueiredo-Filho *et al.* (1996) for the following reasons: neither study included the stem foot in the spline; Smaltschinski (1983) avoided the demanding curvature of the stem butt by starting the spline at 1.3 m; Figueiredo-Filho *et al.* (1996) started the stem profile at the height of 0.1 m; they both use only two fixed points at 1.3 m (0.1 m, respectively) and the top of the stem; the other point positions were optimized to minimize errors. In this study, the stem is modeled from the very bottom to the top of the stem; and in addition fewer positions were optimized. As stated by Smaltschinski (1983), the conventional measuring height of 1.3 m is not favorable concerning the accuracy of the resulting curve, but the model is expected to reflect the conventional measuring point. The pronounced butt swell of spruce trees causes a higher propensity for the curves to oscillate. All the constraints mentioned generate greater errors for the natural cubic spline, as found in this study compared with that of Figueiredo-Filho *et al* (1996).

With the exception of the natural cubic spline, all the splines selected for this study have first-degree continuity only. Therefore, they do not suffer from oscillations and as a consequance their errors are lower than the errors of the natural cubic spline. This is in agreement with Lahtinen (1988), who reported that the quadratic spline, which is only once continuously differentiable, was superior to the cubic spline. The results are also concurrent with Goulding (1979), who recommended infracting the second-degree continuity in order to avoid oscillations. Workable cubic segments and interruption of second-order continuity in knots are two important properties of the Catmull-Rom spline, which give it the ability to represent the stem accurately, without the risk of oscillations.

Conclusions

Contrary to previous studies, the entire stem is involved and apart from both the stem foot and the stem top, the conventional measuring points are also included. The rapid curvature of the butt swell and the uneven point distribution along the stem caused by these restrictions, disallow the usage of the natural cubic spline, which has been used previously by many authors. There is no reason to assume that the stem curve should be twice continuously differentiable; thus, splines with first-degree continuity can be a suitable tool for fitting stem profiles. A simply defined and calculated representative of such splines, the Catmull-Rom spline, is proven to produce a reasonable model of the entire stem profile and volume estimation with average volume error of 0.9% with five points (Table 3). The six-point spline slightly overestimates the second section (10–20% of stem height), whereas the volume error is 2.5% (Table 5); volume predictions in other sections are unbiased as are both the total volume and diameter prediction. With seven points or more, the Catmull-Rom spline produces unbiased diameter predictions throughout the profile and unbiased estimations for both total and sectional volume for all sections.

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