Incentives to give up resource extraction and avoid the tragedy of the commons

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Abstract

This paper develops a general model of common resource extraction where we introduce compensations to encourage resource users to give up extraction. The goal is to reach a balance between resource use and conservation. As the essence of conservation is dynamic, we use a dynamic model to study the implementation of the compensation scheme. A stable heterogeneous equilibrium can be reached where both extractors and non-extractors live together. We analyze how the success of the compensation depends on factors such as the elasticity of demand and the biological characteristics of the resource.

Keywords: common resource, overcapacity, compensation, replicator dynamics, elasticity of demand.

JEL classification: Q20; Q29; C61

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1 Introduction

Natural resource conservation seeks the protection, maintenance or rehabilitation of native biota, habitats and life-supporting systems to ensure ecosystem sustainability and biodiversity (Olver *et al.*, 1995). Although conservation often warrants the limitation of use to ensure ecosystem sustainability, conservation should not be confused with non-use (Clark, 2010). Natural resource conservation recognizes roles for both use and preservation but adopts neither as its central premise (Olver *et al.*, 1995). Human uses must be reconciled with intrinsic and necessary ecosystem functions and structures. Compensations for an agreed-upon inaction or for actively improving environmental services are part of a new and more direct conservation paradigm, explicitly recognizing the need to bridge the interests of landowners and outsiders (Wunder, 2007). Typically, landowners want to exploit the resource while outsiders want to preserve the resource. The idea is that outsiders or a central authority can compensate landowners and resource users in return for adopting practices that secure ecosystem conservation and restoration.

Compensations for agreed-upon inaction or to change appropriation behavior have been implemented in different countries. One of the most popular compensation schemes is Payments for Environmental Services. These are voluntary agreements involving environmental service providers who have real land use choices. These providers receive payments from environmental service buyers if they secure the provision of an environmental service such as carbon sequestration, biodiversity conservation, or landscape beauty (Wunder, 2005). These programs have been implemented in Costa Rica (Chomitz *et al.*, 1999; Pagiola, 2008), Ecuador (Wunder and Albán, 2008), France (Perrot-Maître, 2006) and many other countries. In other countries such as the USA agreements to protect ecosystems are based on conservation easements that pay landowners through tax exemptions (Katila and Puustjärvi, 2004). Each of these schemes is different but the common pattern is that users receive compensations in exchange from some change in extraction practice. Providers who can produce the requested ecosystem service at or below the offered price have an incentive to enroll in the program whereas providers who have a higher opportunity cost of enrolling do not (Jack *et al.*, 2008).

Compensation schemes try to change the incentives related to resource use by offering a set of payments for resource conservation and/or service provision. However some schemes have been more successful than others (Wunder, Engel and Pagiola, 2008). Various factors have been found to affect the effectiveness of a compensation scheme to enhance natural resource conservation: what to pay for, how much to pay, how to pay, whom to pay, among others (Wunder, 2005; Wunder 2006; Engel *et al.*, 2008; Jack *et al.*, 2008). Concerning what to pay for, compensations can be directed to change exploitation practices or to promote inaction (Wunder, 2007), depending on the conservation goal and the land-use scenario. The amount of compensation remains an open question, as it should be high enough to motivate participants but not so high that it debilitates pre-existing social markets and lowers the intrinsic motivation to preserve natural resources voluntarily (Wunder, 2005). Concerning who to pay, in order to be cost-effective one would want to compensate those who have credible site-specific claims and can make decisions about resource use. Problems arise when land ownership or rights of use are not well defined. As Wunder (2005, 2006) points out, the more open the access, the less adequate the scenario is for compensations. Nevertheless, compensation schemes are often focused on nonpoint sources or on many individual landowners whose collective activities alter the levels of a given ecosystem service (Jack *et al.*, 2008).

In fact, compensation programs can help to prevent the well-known *tragedy of the commons*: the situation in which natural resources are used or exploited by a large number of individuals, leading to over-exploitation and degradation (Hardin, 1968; Ostrom, 1990). In this paper we develop a general model to study how the introduction of monetary compensations affects the use of common property resources[1] focusing on compensations for giving-up exploitation, that is, compensations for inaction. The main question is whether the entire community of users accepts the compensation or, instead,

[1]Under common property, a well-defined community of users has the right to exploit a resource and the right to exclude from resource use those who are not members of the community. Under open-access, resource extraction is open to all (Ostrom, 1990; Baland and Platteau, 1996).

achieves a balance between use and preservation in which some users accept the compensation and others continue with resource extraction.

As the essence of conservation is dynamic (Clark, 2010), we use a dynamic model to study the implementation of the compensation scheme. The dynamics of the natural resource play a key role as the resource stock level reached with a certain scheme will determine the success of the program. We also focus our attention on the incentives introduced by the compensation program and the decision of users concerning whether to enroll in the program. These decisions are modeled using a replicator dynamics for which users compare extraction profits with compensation for non-extraction. A deep understanding of both dynamics (resource and users) can help to design compensation schemes that lead to stable situations with sustainable stock levels. Resource and users dynamics play a central role in other studies of resource exploitation such as Smith (1968) which develops a general model for exploitation of (renewable and non-renewable) natural resources in a context of perfect competition; Sethi and Somanathan (1996), which analyzes the role of social norms and sanctions in preserving natural resources under common property regimes; Brander and Taylor (1998), which analyzes the exploitation of natural resources in Easter Island and the dynamics that led to its collapse; and Osés-Eraso and Viladrich-Grau (2007), which analyzes how social approval of cooperators works as a reward mechanism that fosters pervasive cooperative behavior and the sustainable management of natural resources. In our paper, we show that not all resource users accept the compensation and that a stable heterogeneous equilibrium can be reached were both extractors and non-extractors live together, maintaining a greater resource stock level.

For the success of compensation scheme, that is, for the dynamics to reach a stable heterogeneous equilibrium,two factors play a key role: the demand for the natural resource and its biological characteristics. In this paper we analyze both. With respect to the first factor, we find that the elasticity of resource demand determines the basins of attraction for the stable heterogeneous equilibrium. When resource demand is inelastic, there are incentives to join the compensation scheme if the resource level is high; when the resource demand is elastic, there are incentives to join if the resource level is low. This means that responses to a certain compensation scheme, including the tendency of dynamics towards equilibrium, differ depending on resource demand. This is a new approach that departs from previous studies of common resource exploitation that only consider perfectly elastic demand (competitive markets). In recent decades, increased product differentiation has led to oligopolistic competition policies for almost all goods, and natural resources are no exception. Quality labels such as a green card, local advertising for the resources, and even some cultural preferences, have made it so that many natural resources are difficult to replace in both consumption and production. Accodingly, it is not entirely realistic to assume constant prices for these resources in local markets, regional or even international markets. As finding substitutes for these resources becomes increasingly difficult, resource prices can vary depending on various factors such as demand, extraction or perhaps scarcity (see Stiglitz, 1979).

As for the second factor, we show that biological characteristics of the natural resource (such as the potential natural growth rate, carrying capacity or biotic potential) also condition the success of the compensation programs. We find that slower-growing species need higher compensations in order to reach the same conservation status. Similarly, we determine that species with greater carrying capacity and species with higher potential biota need lower compensations to reach the same conservation status.

The paper is organized as follows. The next section describes the exploitation of common resources. The Section 3 introduces incentives to give up exploitation and replicator dynamics. Sections 4 and 5 analyze the dynamic model and Section 6 describes the equilibria. Finally, Section 7 develops some policy implications and Section 8 concludes.

2 Exploitation of natural resources

In order to investigate the economics of a renewable resource (Schaefer, 1954; Gordon, 1954), it is first necessary to describe its pattern of biological growth. Most populations of renewable resources exist in a particular environmental setting with a finite carrying capacity that sets bounds on population growth possibilities. Additionally, for very small population sizes, the chance that the species will become extinct is rather large since mating encounters are low (Dasgupta and Heal, 1979). A simple way of representing these effects is to make the actual growth rate depend on the stock size using what is called density-dependent growth. A commonly used functional form that captures these features is a generalized logistic function,

$$G(X) = r(X - T)\left(1 - \frac{X}{K}\right) \tag{1}$$

where X is the resource stock. The positive parameter r is the intrinsic or potential growth rate of the resource. K represents the carrying capacity, that is, the maximum amount of resource that can be supported by the environment and T is a positive stock threshold level such that the resource population would inevitably and permanently decline to zero if the actual resource level were ever to fall below that threshold. Following Taylor (2009) we refer to this threshold level as the tipping point.[2] In addition, we define X_{MSY} (maximum sustainable yield) as the population level at which the growth of the resource is maximized. Notice that $T < X_{MSY} < K$.[3] A generalized logistic growth function $G(X)$ is represented in figure 1.

The rate of change of the resource stock depends on the prevailing stock size but it also depends on harvesting. Many factors determine the size of the harvest H in any given period. The most relevant of these are following. First, the harvest depends

[2]Dasgupta and Heal (1979) point out that for various land animals the biotic potential is low. For such species T would be large. Unlike these, various species of fish have a high biotic potential. For them T would be low and indeed for all practical purposes it may be convenient to regard $T = 0$ for them. When $T = 0$ we get the simple logistic function (Perman et al., 2003; Taylor, 2009).

[3]For a generalized logistic growth function like (1), $X_{MSY} = \frac{K+T}{2}$.

on the means devoted to resource extraction. For the sake of simplicity, we aggregate the different dimensions of harvesting activity into one magnitude called total effort E. Second, the harvest also depends on the resource stock size X.[4] The easiest way to capture these factors is through the following linear harvest function (Schaefer, 1954),

$$H(E, X) = qEX \tag{2}$$

where q is a parameter that represents the state of the harvest technology. If the natural resource is exploited under a common property regime in which n individuals have exploitation rights, the total effort level is the sum of the individual effort level, e_i, of each of the resource users, $i = 1, ..., n$, that is, $E = \sum_{i=1}^{n} e_i$. We refer to n as the community of users and we assume, without loss of generality, that individual effort $e_i \in \{0, 1\}$. This means that agents can choose either to not exploit the resource, $e_i = 0$, or to exploit the resource, $e_i = 1$. For those who exploit the resource, we treat them as homogeneous with respect to effort; then for the sake of simplicity we can normalize individual effort equal to one, $e_i = 1$. Taking this into account, the total effort of a community would be given by the number of individuals of that community who go to harvest n_h, i.e. $E = n_h$.[5] Given G and H the following motion equation describes the resource stock dynamics.

$$\dot{X} = G(X) - H(n_h, X) = r(X - T)\left(1 - \frac{X}{K}\right) - qn_hX \tag{3}$$

where $\dot{X} = \frac{dX}{dt}$. The equilibrium will be achieved whenever $\dot{X} = 0$, that is, whenever the growth rate equals the exploitation level. For the evaluation of exploitation strategies

[4]For a more extensive explanation of the harvest's determinants see Clark (1990), Dasgupta & Heal (1979).

[5]Notice that if the individual effort is equal to one and if all individuals execute their right to exploit the resource, the total effort exerted for resource extraction is driven by the number of individuals entitled to exploit it, and thus the linear harvest function of equation 2 becomes to $H(n_h, X) = qnX$. Now if only a fraction of all potential users n_h exploits the resource, then the total effort will be determined by $H(n_h, X) = qn_hX$.

and resource preservation, we are interested in the resource level that exist at the equilibrium. Following Clark (2010), an exploitation strategy specifies the rate of removals (harvest) over time, as well as the remaining stock over time. A strategy that retains a higher future stock level than some other strategy would be considered to be the more conservational of the two.

In figure 1, we have depicted the *equilibrium stock levels*, $\widehat{X}$, for three different exploitation efforts, $E = n_h^1$, $E = n_h^2$ and $E = n_h^c$ such that $n_h^1 < n_h^c < n_h^2$. Observe that any total effort level above n_h^c, such as n_h^2 in figure 1, leads to extinction as the harvest H is above the resource growth G, for any resource stock level. Meanwhile, any total effort below n_h^c, such as n_h^1 in figure 1, describes two different equilibrium stock levels, $\widehat{X}_1$ and $\widehat{X}_2$.[6] We refer to n_h^c as the *critical extinction level* and the corresponding equilibrium stock level is $\widehat{X}^c$.[7] The equilibrium points $\widehat{X}_1$ are asymptotically locally stable while equilibrium points $\widehat{X}_2$ are asymptotically locally unstable.[8]

Following Copeland and Taylor (2009), we define the users' capacity to harvest as $\Omega = n/n_h^c$. When $\Omega \geq 1$, there is overcapacity; that is, if all the users execute their right to harvest the resource, extinction will occur. When $\Omega < 1$, extinction can be avoided even if all users execute their rights.

[6] We can obtain mathematically the equilibrium stock levels, $\widehat{X}$, that correspond to any exploitation effort, $n_h < n_h^c$, as $\widehat{X} = \frac{K}{2r}\left[r\left(1+\frac{T}{K}\right) - qn_h \pm \sqrt{\left(qn_h - r\left(1+\frac{T}{K}\right)\right)^2 - 4r^2\frac{T}{K}}\right]$.

[7] The critical extinction level is $n_h^c = \frac{r}{q}\left(1-\sqrt{\frac{T}{K}}\right)^2$. This critical level increases with r, the intrinsic growth rate, showing that, other things being equal, fast growing species withstand higher exploitation pressure. Conversely, improvements in harvest technology, q, reduce the critical exploitation level supported by the resource.

[8] Let $\widehat{X}$ be an isolated equilibrium point of the resource stock dynamics described in (3). Following Takayama (1994), this point is asymptotically locally stable (unstable) if $\frac{\partial \dot{X}}{\partial X} < 0$ ($\frac{\partial \dot{X}}{\partial X} > 0$). That is, $\widehat{X}$ is asymptotically locally stable (unstable) if $\frac{\partial G}{\partial X} < \frac{\partial H}{\partial X}$ that is, when $\widehat{X} > \widehat{X}^c$ ($\frac{\partial G}{\partial X} > \frac{\partial H}{\partial X}$, that is, when $\widehat{X} < \widehat{X}^c$).

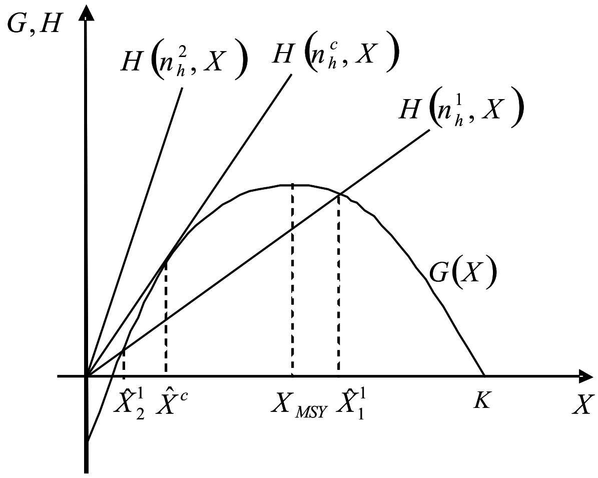

Figure 1: Preservation levels

3 Incentives to give up exploitation

The number of potential users who exploit the common resource depends on the extraction incentives. If an agent exploits the resource and executes an effort level $e_i = 1$ she gets, according to the harvest function (2), an individual harvest $h = qX$. Taking into account the opportunity cost of effort,[9] c, and the resource price, $P(H)$, the extraction payoff of agent i can be defined as follows,

$$\pi_h = P(H)qX - c \tag{4}$$

We assume that the resource price depends negatively on total harvest, $\frac{dP}{dH} < 0$. Whenever $\pi_h > 0$, agents have incentives to exploit the common resource.

If $\pi_h > 0$ for a wide range of resource levels X, a continuous exploitation of the natural resource can threaten resource preservation and even resource survival. As wilderness and natural habitats shrink, environmental services previously provided freely by nature become increasingly threatened (Wunder, 2005). Nervertheless, there is a clear lack of incentives to preserve natural resources and enhance environmental services, as

[9] We assume a constant opportunity effort cost which implies that the total cost of harvest H is an increasing function of H and a decreasing function of X, as stated by Smith (1968).

users typically receive no compensation for the services their resources generate for others. That is, they have no economic reason to take these services into account in making decisions about resource use (Pagiola and Platais, 2002).

Recognition of this problem has led to efforts to develop systems in which resource users are paid for improving resource preservation, thus aligning their incentives with those of society as a whole (Pagiola and Platais, 2002, Jack *et al.*,2008).[10] The implementation of these programs modifies the incentives of resource users with the purpose of achieving improvements in resource conservation. The success of these programs depends on issues such as what type of actions or activities are compensated (compensations should be oriented to incentivize activities that secure conservation), how much is paid (compensations should be high enough to alter the incentives of exploitation), and who receives payment (compensations should be received by those that have rights of use and can decide about resource exploitation, that is, landowners) (Wunder, 2005, Jack *et al., 2008*).

In the context of natural resource exploitation by a community of n users, we can introduce a compensation w that can be received by any of the potential resource users. The condition to receive such a compensation is not to exploit the resource. That is, a resource user that chooses $e_i = 0$ receives a compensation w while a resource user that chooses $e_i = 1$ gets the corresponding extraction payoff, π_h. Any resource user can decide whether she wants to continue with resource extraction, $e_i = 1$, or take the compensation and give up resource extraction, $e_i = 0$. Let s_h be the proportion of owners that decide to extract (or harvest) from the natural resource in a certain period t. $(1 - s_h)$ is the proportion of owners that do not extract and take the compensation instead. The owners shift gradually towards the group (extractors or non-extractors) whose payoff is above the average. The gradual change from one group to another can be captured using the replicator dynamics (Taylor and Jonker, 1978). Recall that the

[10]This reasoning is the core idea of payment for environmental services, which presents a contractual and conditional payment to local resource holders and users in return for adopting practices that secure ecosystem conservation and restoration (Wunder, 2005).

general expression of the replicator dynamics is $\dot{s}_i = s_i(\pi_i - \overline{\pi})$ where $\dot{s}_i = \frac{ds_i}{dt}$, the subscript i applies to both extractors and non-extractors, and $\overline{\pi}$ is the average payoff of resource owners, $\overline{\pi} = s_h\pi_h + (1 - s_h)w$. Therefore, the replicator dynamics reduces to a single equation:

$$\dot{s}_h = s_h(1 - s_h)(\pi_h - w) \tag{5}$$

where $\dot{s_h} = \frac{ds_h}{dt}$ and π_h is the individual extraction payoff and w is the compensation. Resource owners change their exploitation decision, $e_i \in \{0, 1\}$, depending on the payoffs: the greater the difference in payoffs, the greater is the probability of change. The underlying idea is that not all individuals will make this decision at the same time; some will exploit the resource because they believe future earnings will be positive (or for some other reason), while others will not exploit it.

The expression of the replicator dynamics governs the incentives for potential resource users to exploit the natural resource or accept the monetary compensation for non-extraction. Clearly, these incentives depend on the extraction payoffs, and these are conditioned by the stock of the resource, among other factors. Therefore, the resource stock dynamics influence the incentives for resource extraction.

In a similar way, the proportion of potential users that finally decide to exploit the natural resource despite monetary compensations for non-extraction determine the harvest level, and this harvest level affects the evolution of the resource stock. Therefore, incentives for resource extraction influence resource stock dynamics.

Both variables, the proportion of resource extractors (s_h) and the resource stock level (X), constitute a socio-economic system represented by the following equations:

$$\begin{aligned} \dot{s}_h &= s_h(1 - s_h)(\pi_h - w) \\ \dot{X} &= r(X - T)\left(1 - \frac{X}{K}\right) - qns_hX \end{aligned} \tag{6}$$

Observe that the first equation of the system is the replicator dynamics in equation 5 and the second equation is the resource stock dynamics of equation 3 with $n_h = s_hn$. To analyze this dynamic system, we study each dynamic separetely and then consider the entire system.

4 Resource stock dynamics

The evolution of the resource stock depends on resource natural growth, $G(X)$, and on the harvest rate, $H(s_h, X)$, as stated in the second equation of the dynamic system (6). According to this equation, the resource stock will be in equilibrium whenever $\dot{X} = 0$, that is, whenever the extraction rate equals the natural growth. As the extraction rate depends on the proportion of extractors, the resource stock level at equilibrium, $\widehat{X}$, is a function of the proportion of extractors, s_h. The following lemma summarizes this relationship.

Lemma 4.1 *For the resource stock dynamics, there exists a set of stable equilibrium points, $\widehat{X_1}(s_h)$, and a set of unstable equilibrium points, $\widehat{X_2}(s_h)$. The former is a decreasing function of s_h while the latter is an increasing function of s_h.*

Proof.

Given $s_h^* \in [0, 1]$, let $\widehat{X}^*(s_h^*)$ be an isolated equilibrium point of the resource stock dynamics (second equation in the system (6)). Following Takayama (1994; pp.336), $\widehat{X}^*(s_h^*)$ is asymptotically locally stable (unstable) if $\frac{\partial \dot{X}}{\partial X} < 0$ ($\frac{\partial \dot{X}}{\partial X} > 0$) evaluated at $\widehat{X}^*(s_h^*)$. We calculate $\frac{\partial \dot{X}}{\partial X}$,

$$\frac{\partial \dot{X}}{\partial X} = \frac{dG}{dX} - \frac{\partial H}{\partial X} = r\left(1 - \frac{2X}{K} + \frac{T}{K}\right) - qns_h$$

Applying the implicit function theorem to $\dot{X} = 0$, we get the following expression.

$$\frac{d\widehat{X}}{ds_h} = \frac{qnX}{r\left(1 - \frac{2X}{K} + \frac{T}{K}\right) - qns_h} \tag{7}$$

First, the expression in the denominator is $\frac{\partial \dot{X}}{\partial X}$. Second, the expression in the numerator takes always a positive value. Therefore, $\frac{d\widehat{X}}{ds_h} < 0$ for any stable equilibrium point $\widehat{X}^*(s_h^*)$ while $\frac{d\widehat{X}}{ds_h} > 0$ for any unstable equilibrium point $\widehat{X}^*(s_h^*)$. We refer to the set of stable points as $\widehat{X_1}(s_h)$ and to the set of unstable points as $\widehat{X_2}(s_h)$. ■

We have depicted both sets of equilibrium points in figure 2.[11] This figure can be easily compared with figure 1. The critical extinction level is represented now by $s_h^c = \frac{n_h^c}{n}$. Observe that s_h^c is the inverse of the users' capacity to harvest, Ω; therefore, extinction is a credible threat whenever $s_h^c < 1$ ($\Omega > 1$). We can also analyze the dynamics towards these equilibrium levels and we observe that for any level $s_h < s_h^c$, whenever the resource stock X is such that $X > \widehat{X}_1(s_h)$, the resource stock decreases. Similarly, if $X < \widehat{X}_2(s_h)$ the resource stock also decreases. Inversely, whenever $\widehat{X}_1(s_h) > X > \widehat{X}_2(s_h)$, the resource stock increases. When $s_h > s_h^c$, the resource stock always decreases. These dynamics are represented by the arrows in figure 2.

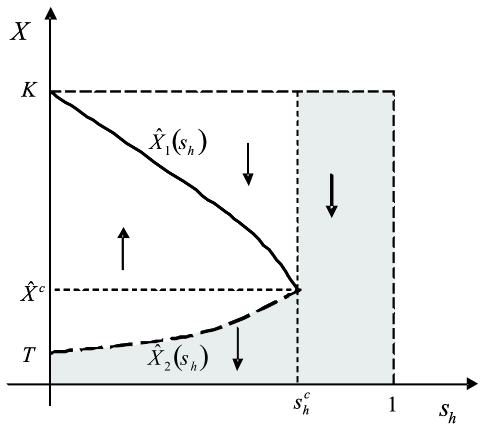

Figure 2: Resource stock dynamics and extinction

In order to understand the resource stock dynamics, assume that the proportion of extractors, s_h, is exogenously given and is not altered through time. Assume also that the resource stock is exogenously given but evolves through time according to the dynamics described above. We can distinguish the set of initial conditions, (s_h^0, X^0) that will end up in resource extinction (shaded area in figure 2) from those initial conditions

[11] The mathematical expressions for these set of equilibrium points are $\widehat{X}_1(s_h) = \frac{K}{2r}\left[r\left(1+\frac{T}{K}\right) - qns_h + \sqrt{\left(qns_h - r\left(1+\frac{T}{K}\right)\right)^2 - 4r^2\frac{T}{K}}\right]$, for the stable ones, and $\widehat{X}_2(s_h) = \frac{K}{2r}\left[r\left(1+\frac{T}{K}\right) - qns_h - \sqrt{\left(qns_h - r\left(1+\frac{T}{K}\right)\right)^2 - 4r^2\frac{T}{K}}\right]$ for the unstable ones. It is straightforward that $\widehat{X}_1(0) = K$, $\widehat{X}_2(0) = T$ and $\widehat{X}_1(s_h^c) = \widehat{X}_2(s_h^c) = \sqrt{KT}$.

that will avoid resource extinction (unshaded area in figure 2). Observe that (i) whenever the proportion of extractors is fixed and above the critical level, $s_h^0 > s_h^c$, the resource will become extinct regardless of the initial resource stock, and (ii) whenever $s_h^0 < s_h^c$ extinction is avoided only if $X^0 > \widehat{X}_2(s_h^0)$.

The sets of equilibrium points of the resource stock dynamics depend on (i) the biological characteristics of the resource (intrinsic growth rate, carrying capacity and tipping point), (ii) the technology of extraction, and (iii) the size of the community of users. Consider first the biological characteristics of the resource and, among those, the intrinsic growth rate, r. Others things being equal, a greater intrinsic growth rate shrinks the set of initial conditions that end up in extinction and therefore increases the set of initial conditions that avoid extinction.[12] In other words, species that grow faster have greater possibility to avoid resource extinction. In figure 3(a) we have depicted in black the equilibrium sets of figure 2 and, over this one, in gray are illustrated the equilibrium sets of a greater intrinsic grow rate, which, as we can see, decreases the area of extinction compared with the one in figure 2. Other biological characteristics that can affect the resource dynamics are the carrying capacity, K, and the tipping point, T. The risk of resource extinction is greater in species with higher tipping point but it is smaller in species with higher carrying capacity, as can be seen in figures 3(b) and 3(c).

Consider now that the resource stock is extracted using a technology that has greater productivity (greater q in our model). Improvements in harvest technology reduce the critical exploitation level supported by the resource, s_h^c, and therefore increase the set of initial conditions that end up in extinction. This can be seen in figure 3(d). More productive technologies for resource extraction may increase the risk of extinction.

A similar result is obtained if we consider resource stocks exploited by larger communities (community size is represented by n). A larger community has the same critical level expressed as number of users, n_c^h, but not when it is expressed as proportion of

[12] Recall that we have already pointed out that the critical exploitation level, n_h^c increases with r and therefore $s_h^c = \frac{n_h^c}{n}$ also increases with r. In addition, $\widehat{X}^c$ does not change as it can be easily demonstrated that it does not depend on r, $\widehat{X}^c = \sqrt{KT}$.

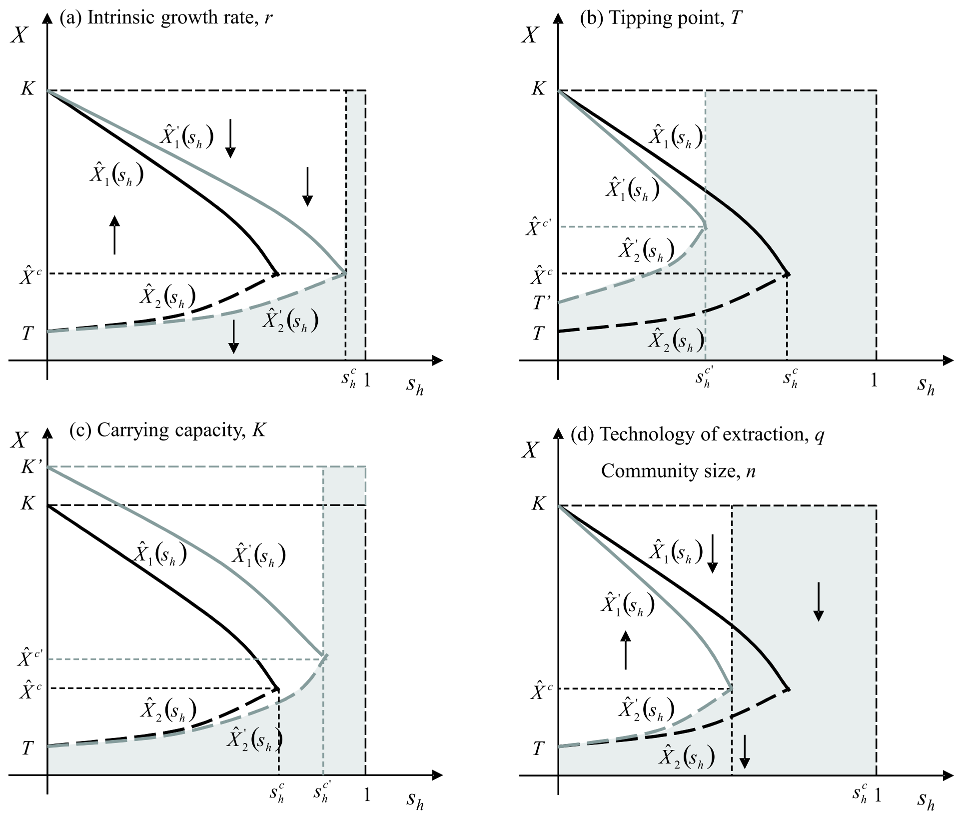

Figure 3: Changes in resource stock dynamics

users, s_h^c, which is smaller. Therefore, other things being equal, larger communities have greater risk of resource extinction. This can also be seen in figure 3(d).

5 Resource users' dynamics

We now consider the decisions of potential resource users concerning whether to extract the resource or give up extraction in exchange for monetary compensation. The incentives for these strategies are summarized in the replicator dynamics (first equation of the dynamic system (6)). This dynamic will reach an equilibrium point whenever $\dot{s_h} = 0$. Observe that, whenever $s_h = 0$ or $s_h = 1$, the users' dynamic is in equilibrium, regardless of the resource stock level. This set of equilibrium points can be considered as trivial. An equilibrium point $s_h \in (0,1)$ will be reached whenever $w = \pi_h$, that is, when the monetary compensation for non-extraction equals the extraction payoff. The following lemma describes the interior equilibrium points of the users' dynamics.

Lemma 5.1 *For the resource users' dynamics there exists a set of equilibrium points $\widetilde{X}(s_h)$ which are asymptotically locally stable whenever $\frac{dP}{dH} < 0$.*

Proof. Given $s_h^* \in (0,1)$, let $\widetilde{X}^*(s_h^*)$ be an isolated equilibrium point of equation 5. Following Takayama (1994; pp.336), $\widetilde{X}^*(s_h^*)$ is asymptotically locally stable (unstable) if $\frac{\partial \dot{s_h}}{\partial s_h} < 0$ ($\frac{\partial \dot{s_h}}{\partial s_h} > 0$) evaluated at $\widetilde{X}^*(s_h^*)$. We calculate $\frac{\partial \dot{s_h}}{\partial s_h}$,

$$\frac{\partial \dot{s_h}}{\partial s_h} = (1-2s_h)(p(H)qX - c - w) + \frac{dP}{dH} q^2 X^2 n s_h (1-s_h)$$

Evaluating this expression in $\widetilde{X}^*(s_h^*)$, we get

$$\frac{\partial \dot{s_h}}{\partial s_h}|_{\widetilde{X}^*(s_h^*)} = \frac{dP}{dH} q^2 X^2 n s_h^* (1-s_h^*)$$

As $s_h^* \in (0,1)$, $\frac{\partial \dot{s_h}}{\partial s_h}|_{\widetilde{X}^*(s_h^*)} < 0$ whenever $\frac{dP}{dH} < 0$ and then $\widetilde{X}^*(s_h^*)$ is asymptotically locally stable. ■

The condition $\frac{dP}{dH} < 0$ means a negative relationship between resource prices and resource extraction. This implies the usual negative slope for the demand function of the natural resource. The shape of this set of equilibrium points depends on the elasticity of demand as stated in the following lemma.

Lemma 5.2 *Let $P(H)$ be the inverse demand function,*

a. If $P(H)$ is elastic, then $\widetilde{X}(s_h)$ is an increasing function of s_h, $\frac{d\widetilde{X}(s_h)}{ds_h} > 0$.

b. If $P(H)$ is inelastic, then $\widetilde{X}(s_h)$ is an decreasing function of s_h, $\frac{d\widetilde{X}(s_h)}{ds_h} < 0$.

c. If $P(H)$ is perfectly elastic, then $\widetilde{X}(s_h)$ is constant, $\frac{d\widetilde{X}(s_h)}{ds_h} = 0$.

Proof. Applying the implicit function theorem to the equilibrium condition to get $\widetilde{X}(s_h)$, $p(H)qX - c - w = 0$, we get

$$\frac{d\widetilde{X}}{ds_h} = -\frac{\frac{dP}{dH} qnX^2}{P\left(1 - \frac{1}{|\epsilon|}\right)}$$

where $|\epsilon|$ is the elasticity of the demand function defined as $|\epsilon| = \left|\frac{dH}{dP}\frac{P}{H}\right|$. The expression in the numerator is always negative as we have assumed $\frac{dP}{dH} < 0$. The sign of the expression in the denominators depends on the elasticity value. It is negative if $|\epsilon| > 1$ (elastic demand function), it is positive if $|\epsilon| < 1$ (inelastic demand function). Therefore, $\frac{d\widetilde{X}}{ds_h} > 0$ if $|\epsilon| > 1$ and $\frac{d\widetilde{X}}{ds_h} < 0$ if $|\epsilon| < 1$ as stated in part (a) and part (b) of lemma 5.2 respectively.

Finally, it is straightforward that if $\frac{dP}{dH} = 0$, that is, if $|\epsilon| = \infty$ (perfect elasticity of the demand function), then $\frac{d\widetilde{X}}{ds_h} = 0$, as stated in part (c) of lemma 5.2. ■

In figure 4 we have depicted the equilibrium points for (a) an elastic demand function, (b) an inelastic demand function and (c) a perfectly elastic demand function. Equilibrium dynamics are represented by the arrows. The shaded area represents situations in which individuals have incentives to accept the compensation. Interestingly, differences in users' behavior can be seen to depend on resource demand elasticity.

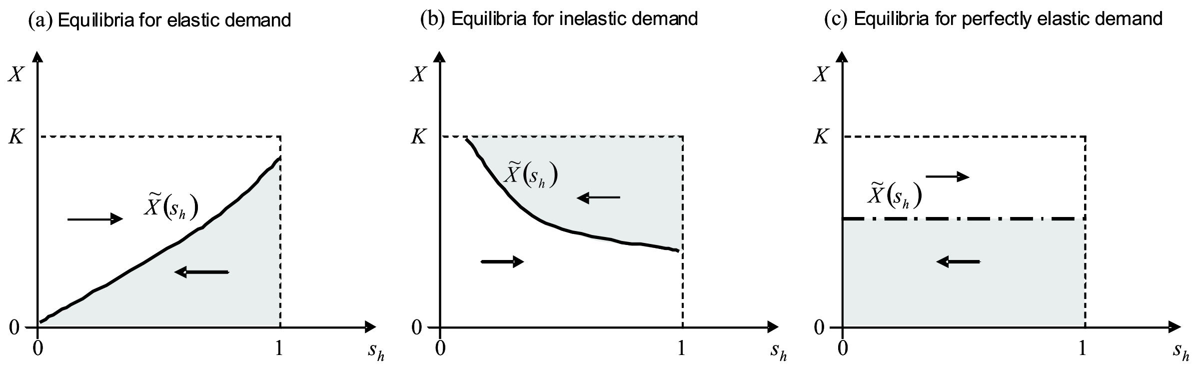

Figure 4: Resource users' dynamics

Consider a situation where the resource stock is above the equilibrium level, $X > \widetilde{X}(s_h)$. As we can see in figure 4, the proportion of extractors increases (decreases) when the resource demand is elastic (inelastic). The individual extraction payoff obtained is greater (lower) than the compensation payment received by a non-extractor. Therefore, some of the resource non-extractors (extractors) choose to exploit (not exploit) the resource, thus increasing (reducing) the proportion of extractors. This suggests that low resource stocks are a greater incentive for resource extraction when the resource demand is inelastic than when it is elastic.[13]

Likewise, consider the case in which the resource stock is below the equilibrium level, $X < \widetilde{X}(s_h)$. In this case, the proportion of extractors decreases (increases) when the resource demand is elastic (inelastic). The individual extraction payoff obtained is lower (greater) than the compensation payment received by a non-extractor. Therefore, some of the resource extractors (non-extractors) choose to not exploit (exploit) the resource thus decreasing (increasing) the proportion of extractors. Analogously, this suggests that high resource stocks are a greater incentive to resource extraction when resource demand is elastic than when it is inelastic.

Finally, it is worth mentioning the special case of perfectly elastic demand function (competitive markets). Whenever the resource stock is below the unique equilibrium level, the proportion of extractors will become zero. Alternatively, whenever the resource stock is above the unique equilibrium level all the users will become extractors.

Apart from the elasticity of demand, the dynamics of resource users is also affected by the quantity of individual compensation for non-extractors as well as by changes in variables that determine individual extraction payoffs (opportunity cost of effort, harvest technology). An increase (decrease) in compensation for non-extraction increases (decreases) the number of situations in which people give up exploitation for any elasticity of demand. This change is reflected by a movement of the equilibrium set curve, $\widetilde{X}$. In the case of elastic demand (figure 5 (a)), $\widetilde{X}$ moves to the northwest (southeast); in the case of inelastic demand (figure 5 (b)), $\widetilde{X}$ moves to the southwest (northeast); for the particular case of perfect elastic demand (figure 5 (c)), $\widetilde{X}$ moves up (down).[14] Notice that any variation of the opportunity cost of effort, c, affects the equilibrium sets

[13]These differences are due to the relationship between harvest payoff and resource stock that depends on the resource demand elasticity. $\frac{\partial \pi_h}{\partial X} = qP\left(1 - \frac{1}{|\epsilon|}\right)$, where $|\epsilon|$ is the absolute value of the resource demand elasticity. $\frac{\partial \pi_h}{\partial X} < 0$ if $|\epsilon| < 1$ (inelastic) and $\frac{\partial \pi_h}{\partial X} > 0$ if $|\epsilon| > 1$ (elastic).

[14]These movements are explained using the already known relationship between harvest payoff and the two variables, resource stock and proportion of extractors.

in the same way as variations in the compensation quantity.

Of special interest is the effect of technology improvements on the resource users' dynamics. Consider an improvement in extraction technology, represented in our model as an increase in parameter q. If the demand for the resource is elastic (inelastic), the improvement in technology increases (decreases) the number of situations in which the resource users have incentives to exploit the resource. This can be seen as a rightward (leftward) shift of the equilibrium set curve, $\widetilde{X}$.[15] (See figure 5 (d), (e) and (f)).

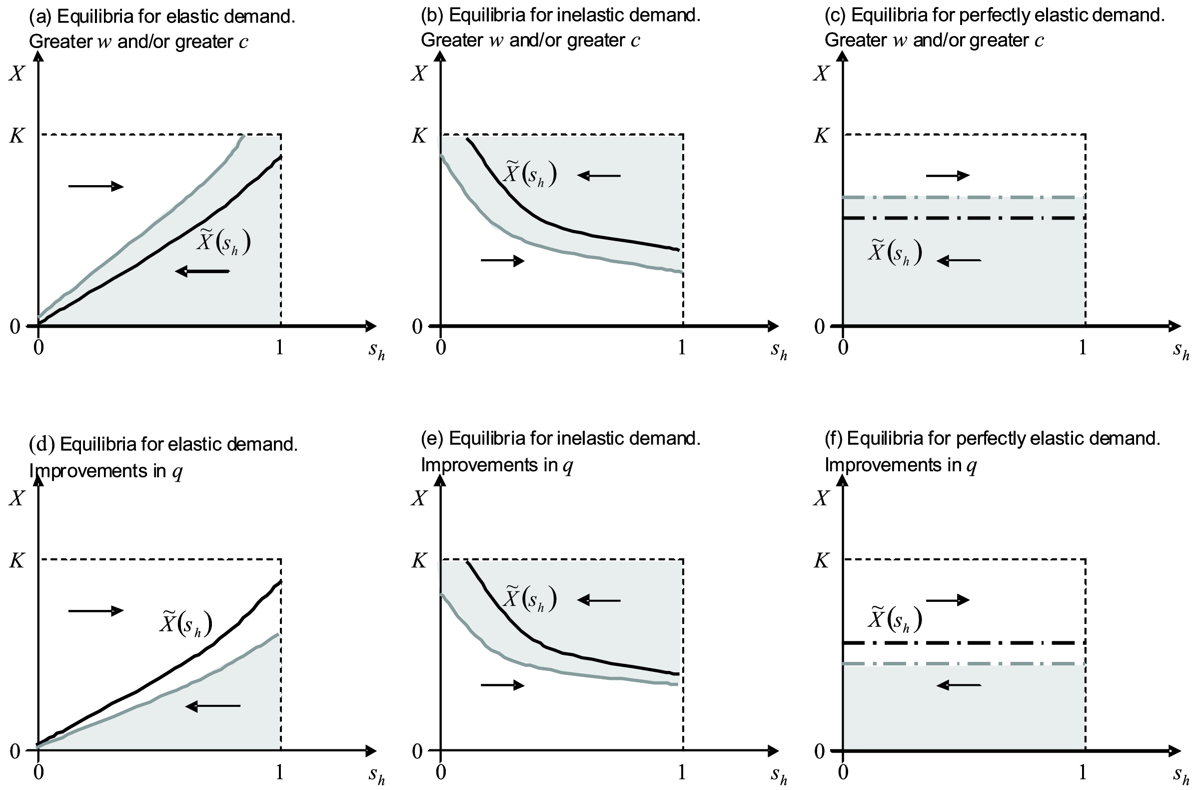

Figure 5: Changes in the users' dynamics

[15]Notice that these differences between elastic and inelastic demand functions are due to the relationship between harvest payoff and technology: $\frac{\partial \pi_h}{\partial q} = XP\left(1 - \frac{1}{|\epsilon|}\right)$, $\frac{\partial \pi_h}{\partial q} < 0$ if $|\epsilon| < 1$ and $\frac{\partial \pi_h}{\partial q} > 0$ if $|\epsilon| > 1$.

6 Complete dynamics and equilibrium points

In the two previous sections we analyzed the resource stock dynamics and the resource users' dynamics on their own. However, these dynamics constitute a unique dynamic system (see equation 6) and must also be considered together. For the system to be in equilibrium we need both variables to be in equilibrium, that is, $\dot{s}_h = 0$ and $\dot{X} = 0$ simultaneously. Taking into account the analyses of the previous sections, there exist six different possibilities for this to occur, as summarized in table 1.

Table 1: Equilibrium points of the socio-economic system

Equilibrium Type	Resource Stock Equilibrium	Users' Dynamics Equilibrium
A	$\widehat{X}_1(s_h)$	$\widetilde{X}(s_h)$
B	$\widehat{X}_1(s_h)$	$s_h = 0$
C	$\widehat{X}_1(s_h)$	$s_h = 1$
D	$\widehat{X}_2(s_h)$	$\widetilde{X}(s_h)$
E	$\widehat{X}_2(s_h)$	$s_h = 0$
F	$\widehat{X}_2(s_h)$	$s_h = 1$

Combining the resource dynamics and the users' dynamics, we obtain figure 6. We have depicted two different cases, one in which there is overcapacity ($\Omega > 1$) and a critical exploitation level; and a second in which there is no overcapacity ($\Omega < 1$). The letters that identify equilibria in the table correspond to the equilibria in figure 6.

In what follows, we analyze whether these equilibrium points are stable and what factors determine the resource stock level that persists in these equilibria. We start with the stability of equilibrium point A in table 1.

Theorem 6.1 (Equilibrium point A) *An equilibrium point of the system* (s_h^*, X^*) *such that* $X^* = \widehat{X}_1(s_h^*) = \widetilde{X}(s_h^*)$ *is asymptotically locally stable,*

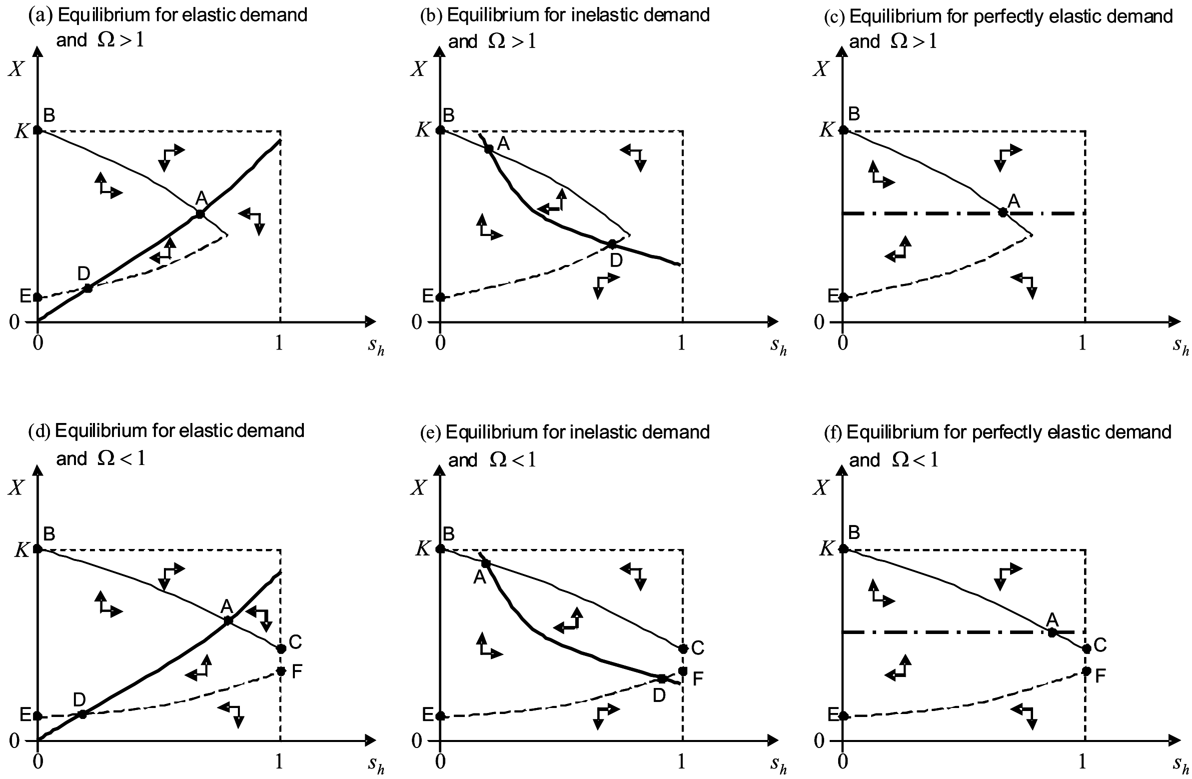

Figure 6: Complete dynamics and equilibrium points

(a) when there exists a perfectly elastic demand for the extracted resource,

(b) when there exists an elastic demand for the extracted resource,

(c) when there exists an inelastic demand for the extracted resource and $\left|\frac{d\widehat{X}_1}{ds_h}\right| < \left|\frac{d\widetilde{X}}{ds_h}\right|$.

Proof. We calculate the Jacobian matrix of the dynamic system (equations 6)

$$J = \begin{pmatrix} r\left(1 - \frac{2X}{K} + \frac{T}{K}\right) - qns_h & -qnX \\ s_h(1-s_h)qP\left(1 - \frac{1}{|\epsilon|}\right) & (1-2s_h)(pqX - c - w) + \frac{dP}{dH}q^2X^2n(1-s_h)s_h \end{pmatrix}$$

We evaluate this Jacobian matrix at the equilibrium point of the system (s_h^*, X^*) such that $X^* = \widehat{X}_1(s_h^*) = \widetilde{X}(s_h^*)$,

$$J\left(s_h^*, X^*\right) = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} r\left(1 - \frac{2X^*}{K} + \frac{T}{K}\right) - qns_h^* & -qnX^* \\ s_h^*(1-s_h^*)qP\left(1 - \frac{1}{|\epsilon|}\right) & \frac{dP}{dH}q^2X^{*2}n(1-s_h^*)s_h^* \end{pmatrix} \quad (8)$$

A sufficient condition for the stability of the equilibrium point is that the Jacobian matrix evaluated at that point has a negative trace and a positive determinant.

We calculate the trace:

$$Tr = J_{11} + J_{22} = r\left(1 - \frac{2X^*}{K} + \frac{T}{K}\right) - qns_h^* + \frac{dP}{dH}q^2X^{*2}n(1-s_h^*)s_h^*$$

The trace condition is $Tr < 0$. As we have assumed $X^* = \widetilde{X}_1$, by lemma 4.1 $J_{11} < 0$. As we have also assumed from the very beginning $\frac{dP}{dH} < 0$, we get $J_{22} < 0$. Therefore, the trace condition is fulfilled for any equilibrium point (s_h^*, X^*) such that $X^* = \widehat{X}_1(s_h^*) = \widetilde{X}(s_h^*)$.

The trace condition is also fulfilled when $\frac{dP}{dH} = 0$ as $Tr = J_{11} < 0$ (observe that $J_{22} = 0$ in such case).

We calculate the determinant:

$$\begin{aligned} Dt = J_{11}J_{22} - J_{12}J_{21} = \ & \left[r\left(1 - \tfrac{2X^*}{K} + \tfrac{T}{K}\right) - qns_h^*\right]\left[\tfrac{dP}{dH}q^2X^{*2}n(1-s_h^*)s_h^*\right] + \\ & + s_h^*(1-s_h^*)q^2nX^*P\left(1 - \tfrac{1}{|\epsilon|}\right) \end{aligned}$$

Observe that $J_{11}J_{22} > 0$. Additionally, we get $J_{12}J_{21} < 0$ whenever $|\epsilon| > 1$ (elastic demand), including the case where $|\epsilon| = \infty$ (perfectly elastic demand, competitive markets). In these cases, the determinant is positive and the determinant condition is also fulfilled.

That is, whenever the demand function for the resource is elastic, the equilibrium point is asymptotically locally stable as stated in parts (a) and (b) of the theorem.

When the demand function for the resource is inelastic, however, $J_{12}J_{21} > 0$ and to get the sign of the determinant we need some more calculations. Applying some algebra to the expression of the determinant we get the following:

$$Dt = q^3n^2X^{*3}\frac{dP}{dH}(1-s_h^*)s_h^*\left[\frac{r\left(1 - \frac{2X^*}{K} + \frac{T}{K}\right) - qns_h^*}{qnX^*} + \frac{P\left(1 - \frac{1}{|\epsilon|}\right)}{\frac{dP}{dH}qnX^{*2}}\right]$$

According to the proofs of lemma 4.1 and 5.2, we can rewrite this expression,

$$Dt = q^3n^2X^{*3}\frac{dP}{dH}(1-s_h^*)s_h^*\left[\frac{1}{\frac{d\widehat{X}_1}{ds_h}} - \frac{1}{\frac{d\widetilde{X}}{ds_h}}\right] \tag{9}$$

The sign of the determinant depends on the slopes of $\widehat{X}_1$ and $\widetilde{X}$. If the demand of extracted resource is inelastic, both functions have negative slopes (see lemma 4.1 and lemma 5.2). Therefore, the determinant will be positive whenever

$$\left[\frac{1}{\frac{d\widehat{X}_1}{ds_h}} - \frac{1}{\frac{d\widetilde{X}}{ds_h}}\right] < 0$$

Using absolute values,

$$\left[\frac{1}{\left|\frac{d\widetilde{X}}{ds_h}\right|} - \frac{1}{\left|\frac{d\widehat{X}_1}{ds_h}\right|}\right] < 0$$

An expression that is equivalent to the following one,

$$\left|\frac{d\widehat{X}_1}{ds_h}\right| < \left|\frac{d\widetilde{X}}{ds_h}\right|$$

■

Now that theorem 6.1 has addressed the issues of stability, the following corollary defines the conditions for uniqueness.

Corollary 6.1

i) If resource demand is elastic and there exists equilibrium A it is unique.

ii) If resource demand is inelastic and there exists equilibrium A uniqueness is not guaranteed.

Theorem 6.1 and corollary 6.1 establish the conditions for the stability and uniqueness of an equilibrium point such as A, showing that there are clear differences between resources with elastic demand and resources with inelastic demand. However, they say nothing about the existence of such equilibrium.

Consider a resource with elastic demand. Theorem 6.1 guarantees stability and corollary 6.1 guarantees uniqueness. However, the existence of such equilibrium depends on the quantity of compensation. This can be easily shown with a look at the phase diagram in figure 6(a). We have already shown that $\widetilde{X}$ moves upwards with higher compensations w.[16] In order for an equilibrium A to exist we need a compensation that guarantees intersection between $\widetilde{X}$ and $\widehat{X}$, that is, a compensation greater than $\pi_h(s_h^c, X^c)$. Notice that similar conclusions can be obtained when the resource has perfectly elastic demand.

On the other hand, stability and uniqueness are not guaranteed in the case of inelastic demand. Since $\widehat{X}_1$ and $\widetilde{X}$ are decreasing functions on s_h they can have one or two intersections. Therefore, equilibrium A could be non-unique. In addition, the stability of the intersection points depends on the slopes of $\widehat{X}_1$ and $\widetilde{X}$ as stated in theorem 6.1. As we have already pointed out in the previous paragraph, the quantity of compensation determines the existence of equilibrium points A. Assume there are two intersection points as shown in figure 7(a). Notice that one of them is stable, A_S, and the other one is unstable, A_U. If we decrease the quantity of compensation, we can reach a situation like the one represented in figure 7(b) with a unique equilibrium point A. The stability of this point A_T is not guaranteed.[17] Observe that for a compensation below the tangent, there is no intersection, that is, no equilibrium point A.[18]

Now we consider equilibrium point B, that is, an equilibrium where nobody extracts the resource. In addition, and according to the description of the equilibrium points in table 1, the resource stock is in the set of stable equilibrium points, $\widehat{X}_1(s_h)$, and it is easy to calculate that $\widehat{X}_1(0) = K$.[19] Therefore, equilibrium point B is $(s_h, X) = (0, K)$.

Theorem 6.2 (Equilibrium point B) *An equilibrium point* $(0, K)$ *is asymptotically locally stable whenever* $w > \pi_h(K)$.

[16]See movements of $\widetilde{X}$ in section 5 and figure 5.

[17]Note that because A_T is a tangency point between $\widehat{X}_1$ and $\widetilde{X}$, the slopes are equal. Observe in the phase diagram that there exists an area that asymptotically leads us to this point while other areas do not.

[18]Very high compensations can also lead to no intersection but in such cases, equilibrium point B takes the leading role (see theorem 6.2).

[19]Taking $s_h = 0$ in the mathematical expression of $\widehat{X}_1(s_h)$ in (11) we get $\widehat{X}_1(0) = K$.

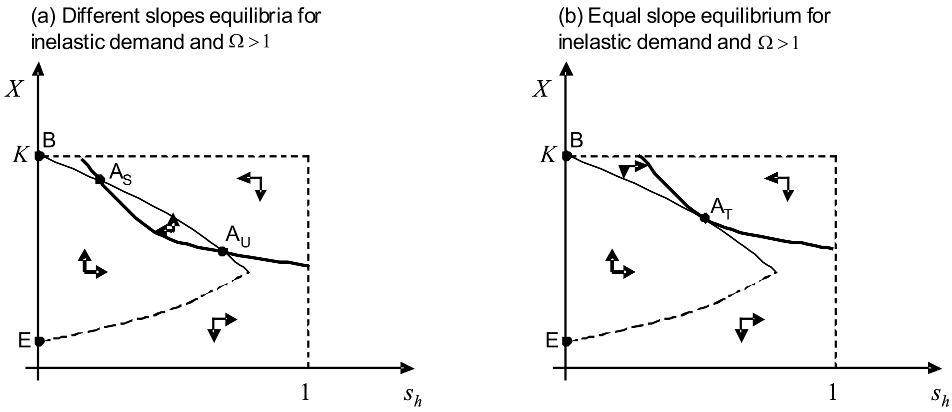

Figure 7: Equilibrium point A and inelastic demand

Proof. We evaluate this Jacobian matrix of the dynamic system (equations 6) at the equilibrium point $(0, K)$.

$$\begin{pmatrix} r\left(\frac{T}{K} - 1\right) & -qnK \\ 0 & pqK - c - w \end{pmatrix}$$

Following Takayama (1994) (Theorem 6.6, pp.341) point $(0, K)$ is asymptotically stable if the Jacobian matrix evaluated at that point has a negative trace and a positive determinant.

We calculate the trace and the determinant.

$$Tr = J_{11} + J_{22} = r\left(\frac{T}{K} - 1\right) + pqK - c - w$$

$$Dt = J_{11}J_{22} - J_{12}J_{21} = \left(r\left(\frac{T}{K} - 1\right)\right)(pqK - c - w)$$

Note that $J_{11} < 0$ because $\frac{T}{K} < 1$. The conditions for local stability will be fulfilled if and only if $J_{22} < 0$, that is, if $pqK - c < w$. Therefore, as $(pqK - c)$ is the extraction payoff evaluated in K, we can write it as $\pi_h(K) < w$. ■

This theorem states that very high compensation can lead to non-exploitation while the resource stock reachs carrying capacity K. The extraction payoff $\pi_h(0, K)$ denotes the minimum compensation that makes this equilibrium stable.

Recall that there is also an equilibrium point of the system where all the users choose resource exploitation, equilibrium point C in table 1.[20] The following theorem establishes the stability conditions of this equilibrium point.

Theorem 6.3 (Equilibrium point C) *An equilibrium point* $(1, \widehat{X_1}(1))$ *is asymptotically locally stable whenever* $w < \pi_h(1, \widehat{X_1}(1))$.

Proof. We evaluate this Jacobian matrix of the dynamic system (equations 6) at the equilibrium point $(1, \widehat{X_1}(1))$.

$$\begin{pmatrix} r\left(1 - \frac{2\widehat{X_1}}{K} + \frac{T}{K}\right) - qn & -qn\widehat{X_1} \\ 0 & -\left(pq\widehat{X_1} - c - w\right) \end{pmatrix}$$

Following Takayama (1994) (Theorem 6.6, pp.341) point $(1, \widehat{X_1}(1))$ is asymptotically stable if the Jacobian matrix evaluated at that point has a negative trace and a positive determinant.

We calculate the trace and the determinant.

$$Tr = J_{11} + J_{22} = r\left(1 - \frac{2\widehat{X_1}}{K} + \frac{T}{K}\right) - qn - \left(pq\widehat{X_1} - c - w\right)$$

$$Dt = J_{11}J_{22} - J_{12}J_{21} = -\left[r\left(1 - \frac{2\widehat{X_1}}{K} + \frac{T}{K}\right) - qn\right]\left[\left(pq\widehat{X_1} - c - w\right)\right]$$

By lemma 4.1, we know that $J_{11} < 0$. The conditions for local stability will be fulfilled if and only if $J_{22} < 0$, that is, if $pq\widehat{X_1} - c > w$. Therefore, as $(pq\widehat{X_1} - c)$ is the extraction payoff evaluated at point C, we can write it as $\pi_h(\widehat{X_1}(1)) > w$. ■

This theorem highlights the fact that compensations below $\pi_h(1, \widehat{X_1})$ lead to the same result that is reached without compensations, making total extraction stable. It is worth noting that this stability condition is incompatible with the existence of equilibrium point A when resource demand is elastic. On the other hand, when resource demand is inelastic, this stability condition does not preclude the existence of point A.

[20] Observe that C is an equilibrium point of the system if and only if $\Omega < 1$ (see figure 6).

Theorem 6.4 (Equilibrium point D) *An equilibrium point of the system* (s_h^*, X^*) *such that* $X^* = \widehat{X}_2(s_h^*) = \widetilde{X}(s_h^*)$ *is asymptotically locally stable when there exists an elastic demand for the extracted resource,* $\frac{1}{\frac{d\widehat{X}_2}{ds_h}} <\mid \frac{dP}{dH} \mid qX(1-s_h)s_h$ *and* $\frac{d\widehat{X}_2}{ds_h} > \frac{d\widetilde{X}}{ds_h}$.

Proof. Equilibrium point D - We evaluate the Jacobian matrix of the dynamic system (equation 6) at the equilibrium point (s_h^*, X^*) such that $X^* = \widehat{X}_2\left(s_h^*\right) = \widetilde{X}\left(s_h^*\right)$. This matrix is equivalent to matrix (8). In this case, $J_{11} > 0$ by lemma 4.1 and $J_{22} < 0$ as $\frac{dP}{dH} < 0$. Therefore, we can not determinate the sign of the trace. Observe that $J_{22} = 0$ when the resource demand is perfectly elastic ($\frac{dP}{dH} = 0$), in which case the trace is positive, $Tr = J_{11} > 0$, implying that equilibrium point D is not asymptotically locally stable.

In order to determine the sign of the determinant we have $J_{12} < 0$ and the sign of J_{21} depends on the resource demand elasticity. For an inelastic resource demand, $J_{21} < 0$ and the determinant, $Dt = J_{11}J_{22} - J_{12}J_{21} < 0$ and therefore equilibrium point D is not asymptotically locally stable. For an elastic resource demand, $J_{21} > 0$ and the sign of the determinant depends on the slopes of $\widetilde{X}(s_h)$ and $\widehat{X}_2(s_h)$ as can be seen in equation 9. The determinant will be positive whenever $\frac{d\widehat{X}_2}{ds_h} > \frac{d\widetilde{X}}{ds_h}$. For stability, we also need a negative trace, which is fulfilled when $\frac{1}{\frac{d\widehat{X}_2}{ds_h}} <\mid \frac{dP}{dH} \mid qX(1-s_h)s_h$. ∎

Theorem 6.4 allows for the possibility of reaching a stable point of the system even when the resource dynamics is within an unstable region. However, this can only be the case when the demand for the resource is elastic and there is a certain relationship between the relevant slopes. Other equilibria in the unstable region of the resource dynamics are not stable as stated in the following theorem.

Theorem 6.5 (Equilibrium points E, F) *Homogeneous equilibria* ($s_h = 0$, $s_h = 1$) *on* $\widehat{X}_2$ *are not asymptotically locally stable.*

Proof. Equilibrium point E - We evaluate the Jacobian matrix of the dynamic system (equation 6) at the equilibrium point $(0, T)$.

$$\begin{pmatrix} r\left(1-\frac{T}{K}\right) & -qnT \\ 0 & pqT-c-w \end{pmatrix}$$

Note that $J_{11} = r\left(1-\frac{T}{K}\right) > 0$. Therefore, in order to have a negative trace we need that $J_{22} < 0$. In such case, the determinant will be negative. Therefore, the stability conditions will never be fulfilled.

Equilibrium point F - We evaluate the Jacobian matrix of the dynamic system (equations 6) at the equilibrium point $(1, \widehat{X}_2(1))$.

$$\begin{pmatrix} r\left(1-\frac{2\widehat{X}_2(1)}{K}+\frac{T}{K}\right)-qn & -qn\widehat{X}_2(1) \\ 0 & -\left(pq\widehat{X}_2(1)-c-w\right) \end{pmatrix}$$

Note that $J_{11} = r\left(1-\frac{2\widehat{X}_2(1)}{K}+\frac{T}{K}\right)-qn > 0$ by lemma 4.1. Therefore, in order to have a negative trace we need that $J_{22} < 0$. In such case, the determinant will be negative. Therefore, the stability conditions will never be fulfilled.

■

Because the goal of a compensation scheme is to mitigate exploitation, some of the potential compensation scheme equilibria are undesirable. This is the case with points C, E and F. The stability analysis rules out the points E and F as they are unstable. While point C can be stable, it should be avoided because it is the point that is achieved without any compensation scheme (in a situation without overcapacity).

Furthermore, the goal of the compesation system is conservation offset, that is, the goal is not to end resource use but to make use and preservation compatible. This approach rules out equilibrium B, as this equilibrium implies that the resource is not exploited and reaches the so-called natural equilibrium (carrying capacity).

Finally, an ambitious compensation system seeks not only to improve the actual resource situation but to avoid biological overexploitation, that is, to reach a resource stock level above that required for maximum sustainable yield (MSY). This approach discards point D because it always leads to a resource level below MSY as it is located on $\widehat{X}_2$.

Therefore, the quest for better conservation involving no biological overexploitation leads us to look for a stable equilibrium A.

7 Policy implications

The objective of the schemes that we have considered is resource conservation. Following Clark (2010) we assume that a scheme that retains a higher future stock level than some other scheme would be consider the more conservationist of the two. The following policy implications relate compensation quantities and future stock levels with different biological and economic factors.

Implication 7.1 *When resource demand is not perfectly elastic, slower-growing species require higher compensations in order to obtain the same resource stock level*

Natural resources have different natural growth rates, a fact captured by the intrinsic growth rate, r; the greater the intrinsic growth rate, the faster the natural growth of the resource. We have already pointed out that the intrinsic growth rate alters the dynamics of the natural resource; therefore it can also affect the results of the compensation scheme. A lower resource stock is achieved at equilibrium point A when intrinsic growth of the natural resource is slower (A_0 vs. A_1 in figure 8)[21]. Therefore, in order to achieve the same resource stock at equilibrium, we need higher compensations (A_0 vs. A_2 in in figure 8), when intrinsic growth is slower.

[21]Observe that in each case, A_0 and A_1 are achieved with the same compensation w but reflect different intrinsic growth rates, different stock levels and different total compensations, $W = n(1-s_h)w$. When the resource demand is elastic, $r(A_1) < r(A_0)$ implies $\widehat{X}_1(A_1) < \widehat{X}_1(A_0)$ and $W(A_1) > W(A_0)$. When the resource demand is inelastic, $r(A_1) < r(A_0)$ implies $\widehat{X}_1(A_1) < \widehat{X}_1(A_0)$ but $W(A_1) < W(A_0)$. That is, for a given compensation w, in both cases slower-growing species obtain lower resource stocks at equilibrium but the total compensation increases with elastic demand and decreases with inelastic demand.

As a conclusion, we can state that, other things being equal, slower-growing species require higher compensations in order to obtain the same resource stock level or, conversely, fater-growing species need lower compensations in order to obtain the same resource stock level. An exception is when the resource market is perfectly competitive: in this case, faster-growing species need the same compensation as slower-growing species to achieve the same resource stock level (see figure 8(c)).

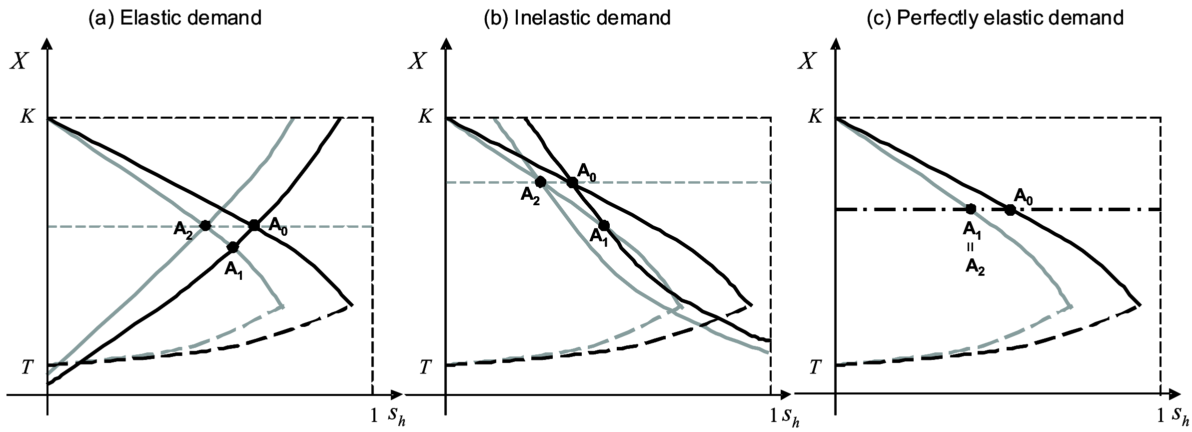

Figure 8: Policy implications

Implication 7.2 *When resource demand is not perfectly elastic, species with lower tipping points need lower compensations in order to obtain the same resource stock level.*

The biotic potential of a species determines what we call the tipping point. The greater the biotic potential, the smaller the tipping point. Following a similar analysis to the one presented above for the intrinsic growth rate, we can see that different tipping points affect the dynamics of the natural resource (see figure 3(b)) but not the users' dynamics. In fact it is easy to see that, for the same compensation, a lower tipping point implies greater resource stock at equilibrium level A. Therefore, in order to obtain the same equilibrium stock level the compensation quantities also depend on the tipping point. Species with lower tipping points require lower compensations to achieve the same conservation goal.

Implication 7.3 *When resource demand is not perfectly elastic, species with greater carrying capacity need lower compensations in order to obtain the same resource stock level.*

Natural species can also differ in their carrying capacity. For those species with greater carrying capacity there is a greater range of resource levels and, as a result, a compensation scheme can reach higher resource stock levels. This implies greater opportunities of conservation as well as lower compensations for the same resource level.

The above three implications are related to the biological parameters of the model. The following two are related to two economic parameters of the model: the extracting technology and the opportunity cost.

Implication 7.4 *When resource demand is competitive or elastic, improvements in harvesting technology should be followed by an increase in the compensation in order to obtain the same resource stock level. When resource markets are inelastic, this increase may not be necessary.*

According to the resource stock dynamics, improvements in harvesting technology mean greater extraction and, as result, the resource stock equilibrium decreases. However, according to the users' dynamics, when resource demand is elastic this effect is reinforced by a greater proportion of extractors (extraction profit increases with an improvement in harvesting technology), whereas,when resource demand is inelastic it can be offset by a lower proportion of extractors (extraction profit decreases with an improvement in harvesting technology). Therefore, in order to obtain the same resource stock level, greater compensations are needed when resource demand is elastic, a conclusion that can not be applied to a resource with inelastic demand.

Implication 7.5 *An increase in the opportunity cost of effort should be followed by a decrease in the compensation in order to obtain the same resource stock level*

The opportunity cost of effort directly affects the incentives for resource extraction through extraction profits. When greater opportunity cost reduces extraction payoff, compensations become more attractive and resource stock increases. Therefore, the same stock level can be obtained by lower compensation when opportunity costs are greater.

Implication 7.6 *Larger communities require higher compensation to achieve the same resource stock level.*

Community size has no effect on the individual extraction payoffs but, as we have already pointed out, it affects the proportion of extractors that sustain any equilibrium resource stock: the larger the community the smaller the proportion. Therefore in order to obtain the same resource stock level we need a lower proportion of extractors so we need greater individual compensations.

8 Conclusions and future research

In this work we have shown that compensation for non-extraction can be sufficient to reach stable equilibria that improve the situation of the commons. Applying this program of compensation, the scheme would introduce incentives for some users to give up exploitation while others would continue with resource extraction. The equilibrium shows that it is not necessary to stop resource extraction completely to achieve resource survival. Likewise, it is possible to reach any resource stock as long as the necessary compensation is implemented.

To determine the necessary compensation, we must take the biological characteristics of the resource into account. The results show that slower-growing species need higher compensations in order to reach the same conservation status. Similarly, species with greater carrying capacity and species with lower potential biota need lower compensations to reach the same conservation status. Other factors such as improvements in harvesting technology or changes in the opportunity costs of effort can also condition the success of a compensation scheme.

In addition, it is important to consider the characteristics of the demand function. The elasticity of demand drives the dynamics towards equilibrium and, as a result, the trajectory of resource exploitation can be completely different: when resource demand is inelastic, there are incentives to join the compensation scheme (less exploitation) if the resource level is high but, when the resource demand is elastic, there are incentives to join the compensation scheme if the resource level is low.

In any case, it is important to realize that the compensation scheme can be introduced when the resource stock is far from overexploitation, as well as when the resource stock is biologically overexploited. However, the open question that arises here is how far-sighted a compensation initiative should be. From an economic or financial point of view the objective is to minimize the social cost of the compensation scheme over time. As this goal can be pursued by different paths, the question is which of these is the path with the lowest cost.

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