New equations for capacitance vs ripple in power supplies

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Abstract
We provide new equations for capacitance vs. ripple in half-wave and full-wave rectifier circuits. We show that these new equations provide more accurate capacitance values than the conventional equations given in textbooks using the linear decay model, especially for supplies with medium to extremely large ripple. We determine the percentage accuracy of these new equations by comparing them to the numerical solution of the exact equations for the capacitance ripple relationship.

Keywords: Half-wave and full-wave rectifiers, Smoothing capacitor, Ripple, Power supplies.

Resumen
Proveemos nuevas ecuaciones para la capacitancia versus la ondulación en rectificadores de media onda y de onda entera. Demostramos que estas nuevas ecuaciones dan valores de capacitancia más precisos que las ecuaciones convencionales presentadas en libros de texto que usan el modelo de decaimiento lineal, especialmente para fuentes con ondulación media a muy grande. Determinamos el porcentaje de precisión de nuestras ecuaciones comparándolas a la solución numérica de las ecuaciones exactas de la relación entre ondulación y capacitancia.

Palabras Clave: Rectificadores de media onda y de onda entera, Condensador de alisado, Ondulación, Fuentes de potencia.

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I. INTRODUCTION

The half-wave rectifier of Fig. 1 and the full-wave rectifier of Fig. 2 have been studied by many authors [1-7]. One of the problems studied by these authors is how to select the smoothing capacitor so that a given ripple can be obtained, where ripple r is defined by Eq. (1).

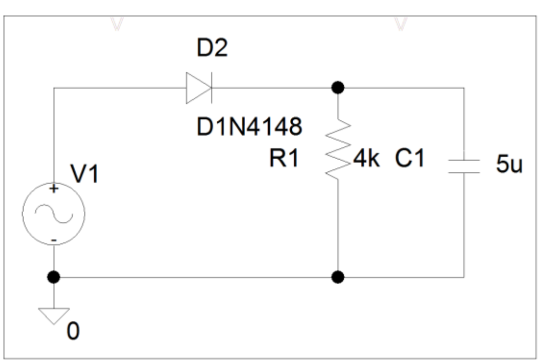

FIGURE 1. PSpice circuit diagram of a half-wave rectifier.

$$r=\frac{V_p-V_{\min}}{V_p}=\frac{\Delta V}{V_p}, \qquad (1)$$

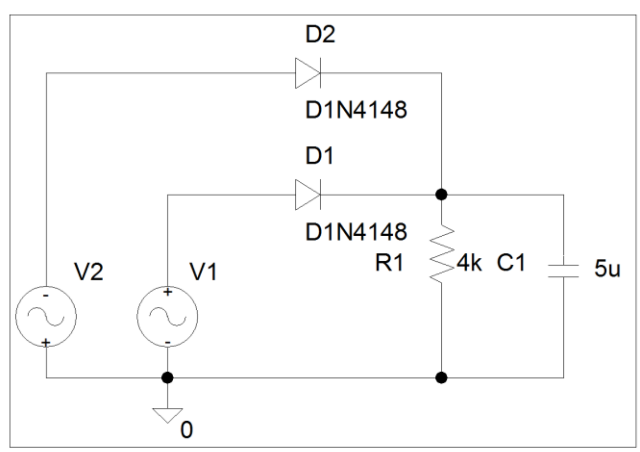

FIGURE 2. PSpice circuit diagram of a full-wave rectifier.

where V_p is the maximum voltage and $V_{\min}$ is the minimum voltage across the capacitor, *i.e*., the voltage at time t_2 in the graphs of Fig. 3 and Figs. 5-7.

Using the linear decay approximation, most authors give, for the half-wave rectifier, the approximation

$$\frac{1}{fRC}=\frac{2\pi}{\omega RC}=r, \tag{2}$$

where f is the frequency (*Hz*) and ω is the angular frequency (*rad/s*) of the applied ac voltage.

For the full-wave rectifier, the usual approximation is

$$\frac{1}{2\,fRC}=\frac{\pi}{\omega RC}=r. \tag{3}$$

Unfortunately, Eq. (2) and Eq. (3) are only valid for small values of ripple or, equivalently, large values of capacitance.

In fact, it is known [2] that these equations overestimate the amount of capacitance needed for a given ripple level. Indeed, the higher the ripple level, the more error in the selection of the capacitor, as we show in Fig. 8 and Fig. 9 in Section V.

In this paper, we derive new expressions for capacitance vs. ripple for both types of rectifiers. These new expressions are compared to the optimum solutions, which are obtained numerically.

We begin with a review of the exact relationship between capacitance and ripple for both types of rectifiers.

II. REVIEW OF THE EXACT RELATIONSHIPS BETWEEN CAPACITANCE AND RIPPLE

A. Exact expression for the half-wave rectifier

Consider Fig. 3, which shows the simulated voltage across the capacitor $v_C(t)$ of Fig. 1; however, in this case, $C=1\,\mu$F. Also shown is the simulated ac input voltage. In this impractical case, the simulated ripple is $r_{sim}=(10-0.5237)/10=0.9476.$

Please, note that all the simulations in this paper are performed using Orcad PSpice [8]. Furthermore, the diodes in Fig. 1 and Fig. 2 are simulated to be "ideal", as explained in [9]. Also, the series resistance of the diode is assumed to be very small, *i.e.,* $1\text{m}\Omega$.

Notice from Fig. 3 that the diode turns off at $t=t_1$ and turns back on at $t=t_2$. Hence, the voltage across the capacitor can be written as

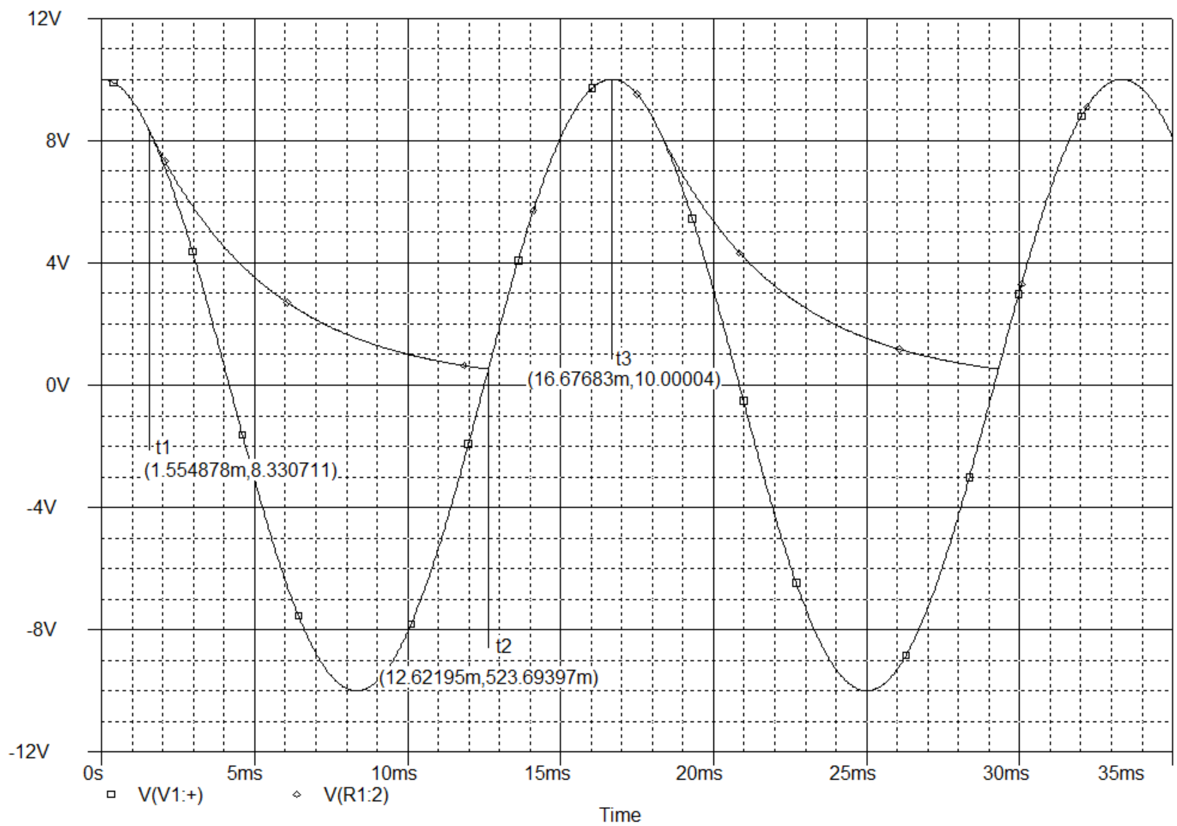

FIGURE 3. Simulated input ac voltage (V($V1$:+)) and voltage across the capacitor (V($R1$:2)) of Fig. 1 for the extremely high ripple case, *i.e.*, $r=0.95$.

$$\begin{aligned} v_C(t) &= V_p \cos(\omega t) && \text{for } 0 \le t \le t_1, \\ &= V_p \cos(\omega t_1)\, e^{-(t-t_1)/(RC)} && \text{for } t_1 \le t \le t_2, \\ &= V_p \cos(\omega t) && \text{for } t_2 \le t \le T, \end{aligned} \quad (4)$$

where $T = 2\pi/\omega$ is the period of the input ac voltage.

Alternatively, we can write

$$\begin{aligned} v_C(\theta) &= V_p \cos(\theta) && \text{for } 0 \le \theta \le \theta_1, \\ &= V_p \cos(\theta_1)\, e^{-(\theta-\theta_1)/(\omega RC)} && \text{for } \theta_1 \le \theta \le \theta_2, \\ &= V_p \cos(\theta) && \text{for } \theta_2 \le \theta \le 2\pi, \end{aligned} \quad (5)$$

where $\theta = \omega t$.

From Fig. 3, it is clear that

$$V_p \cos(\theta_1)\, e^{-(\theta_2-\theta_1)/(\omega RC)} = V_{\min} = V_p - \triangle V. \quad (6)$$

When the diode is on, the current through it is the sum of the current through the resistor and the current through the capacitor, *i.e.*, $V_p\left[\cos(\omega t)/R - \omega C \sin(\omega t)\right]$. At time $t = t_1$ or angle $\theta = \theta_1$, the diode current becomes zero. Hence,

$$\tan\theta_1 = \frac{1}{\omega RC}. \quad (7)$$

Substituting Eq. (7) into Eq.(6) and using Eq. (1) gives

$$\cos(\theta_1)\, e^{-(\theta_2-\theta_1)\tan\theta_1} = 1 - r. \quad (8)$$

Also from Fig. 3, it is clear that

$$V_p \cos(\theta_2) = V_{\min} = V_p - \triangle V,$$

or

$$\cos(\theta_2) = 1 - r. \quad (9)$$

Substituting Eq. (9) into Eq. (8) produces the exact equation that the ripple and capacitance must satisfy, *i.e.*,

$$\cos(\theta_1)\, e^{-(\theta_2-\theta_1)\tan\theta_1} - \cos\theta_2 = 0. \quad (10)$$

Notice that θ_2 is close to 2π; hence, we can write

$$\theta_2 = 2\pi - \delta, \quad (11)$$

where $0 \le \delta \le \pi/2$, by observation of Fig. 3.

From Eq. (9),

$$\cos(\theta_2) = \cos(2\pi - \delta) = \cos(\delta) = 1 - r. \quad (12)$$

Hence, from Eq. (11) and Eq. (12),

$$\theta_2 = 2\pi - \cos^{-1}(1 - r). \quad (13)$$

Finally, substituting Eq. (13) into Eq. (10) and using Eq. (7), gives another version of the exact equation that the ripple and the capacitance (more correctly, ωRC) must satisfy, *i.e.*,

$$\cos\left[\tan^{-1}\left(\frac{1}{\omega RC}\right)\right] e^{-\left[2\pi - \cos^{-1}(1-r) - \tan^{-1}(1/\omega RC)\right]/(\omega RC)} = 1 - r. \quad (14)$$

To design the half-wave rectifier power supply, a suitable ripple is chosen. Eq. (14) is then solved numerically for ωRC. Indeed, when this is done, a plot of ωRC as a function of ripple can be obtained, as given in Fig. 4.

B. Exact expression for the full-wave rectifier

Consider Fig. 5, which shows the simulated voltage across the capacitor $v_C(t)$ of Fig. 2; however, in this case, $C = 1\mu$F. Also shown is the simulated ac input voltage. In this impractical case, the simulated ripple is $r_{sim} = (10 - 3.4328)/10 = 0.6567$.

Notice from Fig. 5 that the diode turns off at $t = t_1$ and turns back on at $t = t_2$. Hence, the voltage across the capacitor can be written as

$$\begin{aligned} v_C(t) &= V_p \cos\omega t && \text{for } 0 \le t \le t_1 \\ &= V_p \cos(\omega t_1)\, e^{-(t-t_1)/(RC)} && \text{for } t_1 \le t \le t_2 \\ &= -V_p \cos\omega t && \text{for } t_2 \le t \le T/2, \end{aligned} \quad (15)$$

where $T = 2\pi/\omega$ is the period of the input ac voltage.

Alternatively, we can write

$$\begin{aligned} v_C(\theta) &= V_p \cos\theta && \text{for } 0 \le \theta \le \theta_1 \\ &= V_p \cos(\theta_1)\, e^{-(\theta-\theta_1)/(\omega RC)} && \text{for } \theta_1 \le \theta \le \theta_2 \\ &= -V_p \cos\theta && \text{for } \theta_2 \le \theta \le \pi, \end{aligned} \quad (16)$$

where $\theta = \omega t$.

We note that Eq. (6), Eq. (7) and Eq. (8) also apply to the full-wave rectifier. However, Eq. (9) is now modified to

$$-V_p \cos\theta_2 = V_{\min} = V_p - \triangle V,$$

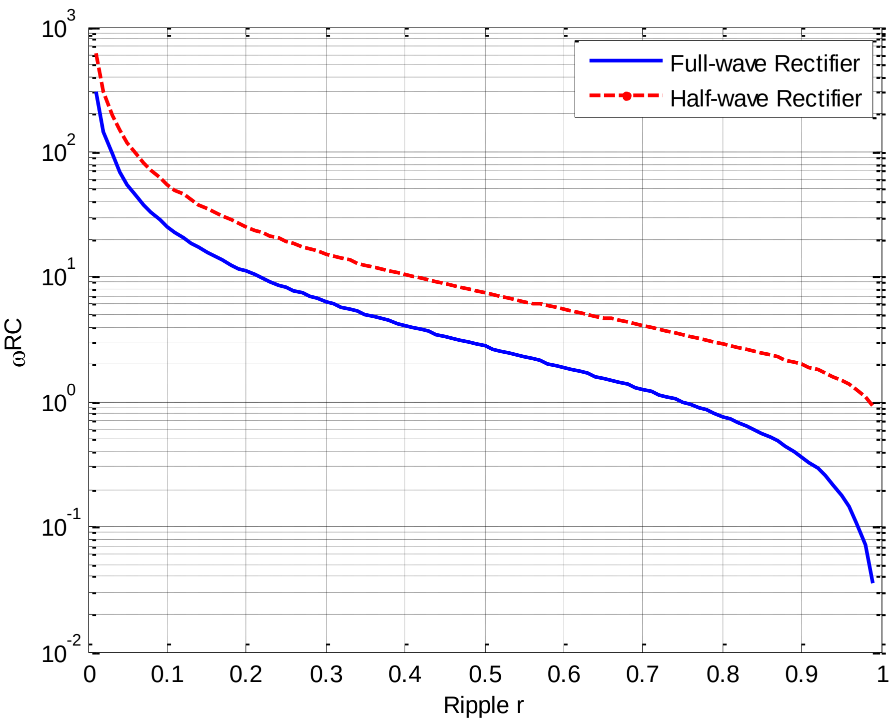

FIGURE 4. Numerical solution of Eq. (14) for the half-wave rectifier and Eq. (22) for the full-wave rectifier.

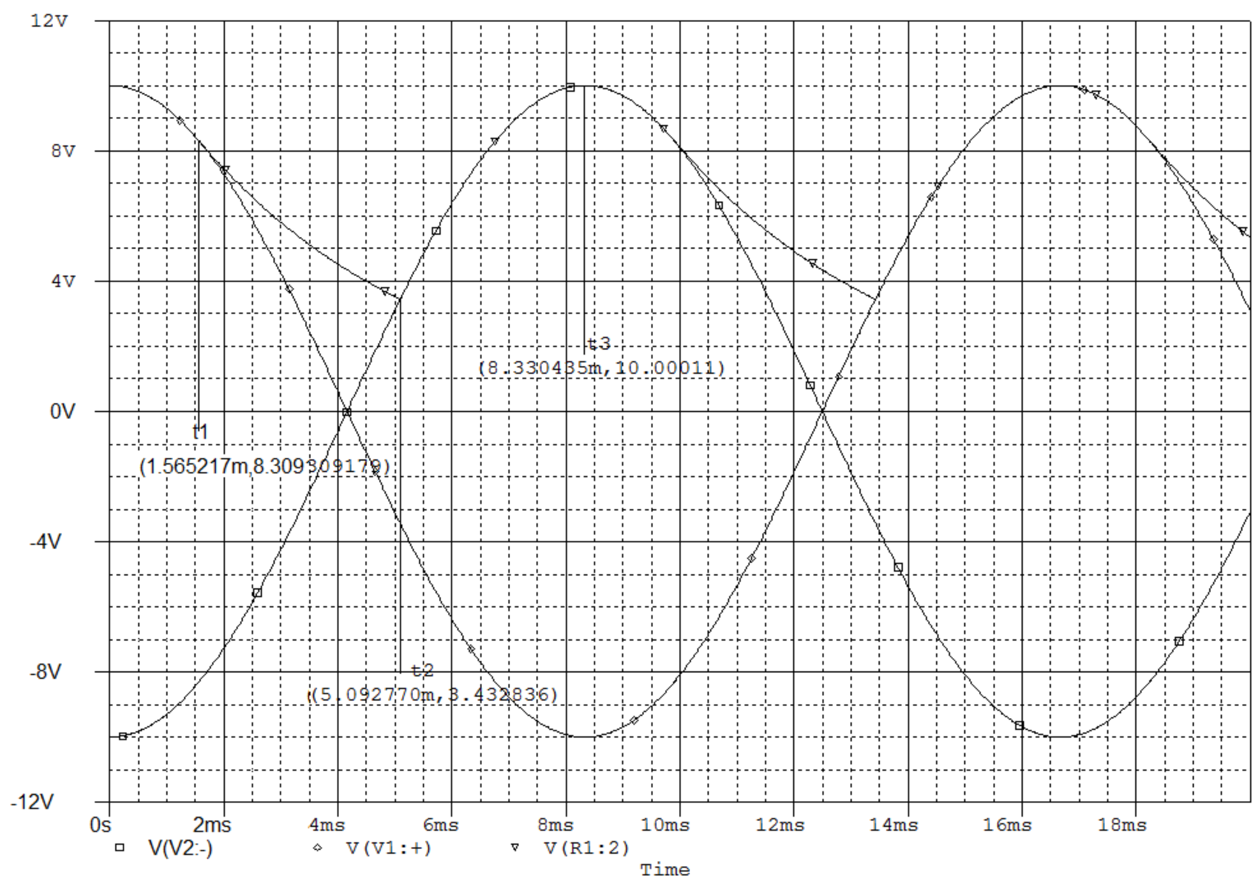

FIGURE 5. Simulated input ac voltages (V(V1:+)) and (V(V2:-)), and voltage across the capacitor (V(R1:2)) of Fig. 2, for the extremely high ripple case, *i.e.*, $r = 0.66$.

or

$$-\cos(\theta_2) = 1 - r. \quad (17)$$

Substituting Eq. (17) into Eq. (8) produces the exact equation that the ripple and capacitance must satisfy, *i.e.*,

$$\cos(\theta_1)\, e^{-(\theta_2-\theta_1)\tan\theta_1} + \cos\theta_2 = 0. \quad (18)$$

Notice that θ_2 is close to π; hence, we can write

$$\theta_2 = \pi - \delta, \quad (19)$$

where $0 \le \delta \le \pi/2$, by observation of Fig. 5.

From Eq. (17),

$$-\cos(\theta_2) = -\cos(\pi-\delta) = \cos(\delta) = 1 - r. \quad (20)$$

Hence, from Eq. (19) and Eq. (20),

$$\theta_2 = \pi - \cos^{-1}(1-r). \quad (21)$$

Finally, substituting Eq. (21) into Eq. (18) and using Eq. (7), gives another version of the exact equation that the ripple and the capacitance (more correctly, ωRC) must satisfy, *i.e.*,

$$\cos\left[\tan^{-1}\left(\frac{1}{\omega RC}\right)\right] e^{-\left[\pi-\cos^{-1}(1-r)-\tan^{-1}(1/\omega RC)\right]/(\omega RC)} = 1 - r. \quad (22)$$

To design the full-wave rectifier power supply, a suitable ripple is chosen. Eq. (22) is then solved numerically for ωRC. Indeed, when this is done, a plot of ωRC as a function of ripple can be obtained, as given in Fig. 4, above.

As can be seen from Fig. 4 for small ripple, the half-wave rectifier requires the capacitor to be twice as large as that needed by the full-wave rectifier, in agreement with Eq. (2) and Eq. (3). However, for very large ripple, $r \ge 0.9$, the capacitor required for the half-wave rectifier is more than five times as large as that needed for the full-wave rectifier.

III. APPROXIMATE EXPRESSIONS FOR THE CAPACITOR VOLTAGES

For small to high ripple, the capacitor voltages given by Eq. (4), Eq. (5), Eq. (15) and Eq. (16) can be simplified as we show in this Section. This simplification has been used by many authors, including [2,3,5], and assumes that the capacitor voltage begins to decay at $\theta_1 = 0$. Hence, we call this simplification the zero-delay discharge approximation. Also, from this simplification, we will derive new relationships between capacitance and ripple.

A. Zero-delay discharge approximate capacitor voltage for the half-wave rectifier

Consider Fig. 6 which shows the simulated voltage across the capacitor $v_C(t)$ of Fig. 1, with $C = 5\,\mu\text{F}$, as shown. Also plotted are the simulated ac input voltage and the exponential voltage $10e^{-50t}$ V which approximates the voltage across the capacitor quite well, for $0 \le t \le t_2$. For this case, the simulated ripple is $r_{sim} = (10-5.0249)/10 = 0.4975$. Notice from Fig. 6, we assume that the diode turns off at $t = 0$ and turns back on at $t = t_2$. Hence, the voltage across the capacitor can be written as

$$\begin{aligned} v_C(t) &= V_p e^{-t/(RC)} \text{ for } 0 \le t \le t_2, \\ &= V_p \cos(\omega t) \text{ for } t_2 \le t \le T. \end{aligned} \quad (23)$$

Alternatively, we can write

$$\begin{aligned} v_C(\theta) &= V_p e^{-\theta/(\omega RC)} \text{ for } 0 \le \theta \le \theta_2, \\ &= V_p \cos(\theta) \quad \text{for } \theta_2 \le \theta \le 2\pi. \end{aligned} \quad (24)$$

B. Zero-delay discharge approximate capacitor voltage for the full-wave rectifier

Consider Fig. 7 which shows the simulated voltage across the capacitor $v_C(t)$ of Fig. 2 with $C = 5\,\mu F$, as shown. Also plotted are the simulated ac input voltage and the exponential voltage $10e^{-50t}$ V which approximates the voltage across the capacitor quite well, for $0 \le t \le t_2$. For this case, the simulated ripple is $r_{sim} = (10-7.3359)/10 = 0.2664$, approximately half of that of the half-wave rectifier.

Notice from Fig. 7, we assume that the diode turns off at $t = 0$ and turns back on at $t = t_2$. Hence, the voltage across the capacitor can be written as

$$\begin{aligned} v_C(t) &= V_p e^{-t/(RC)} \quad \text{for } 0 \le t \le t_2, \\ &= -V_p \cos(\omega t) \quad \text{for } t_2 \le t \le T/2. \end{aligned} \quad (25)$$

Alternatively, we can write

$$\begin{aligned} v_C(\theta) &= V_p e^{-\theta/(\omega RC)} \quad \text{for } 0 \le \theta \le \theta_2, \\ &= -V_p \cos(\theta) \quad \text{for } \theta_2 \le \theta \le \pi. \end{aligned} \quad (26)$$

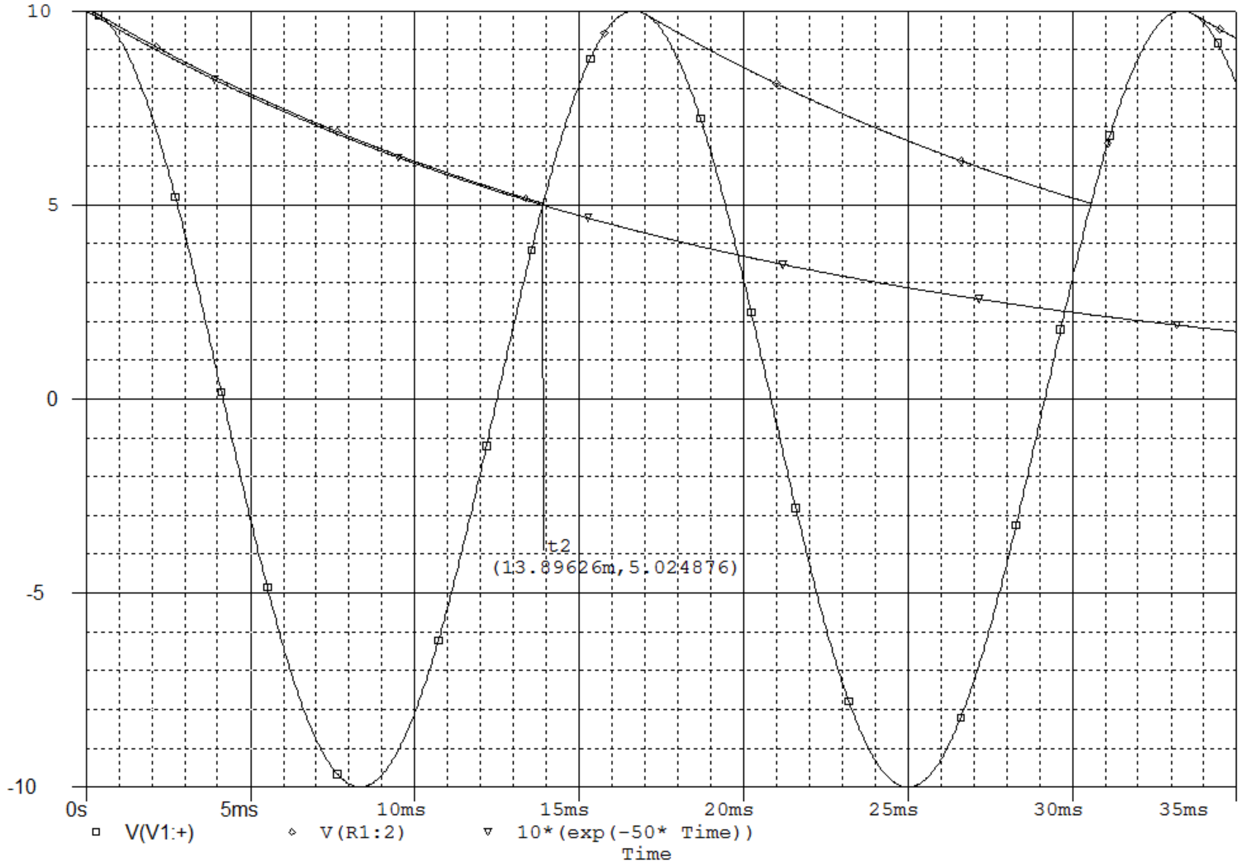

FIGURE 6. Simulated input ac voltage ($V(V1{:}+)$) and voltage across the capacitor ($V(R1{:}2)$) of Fig. 1 for high ripple case, *i.e.*, $r = 0.5$. Also shown is the exponential voltage $10e^{-50t}$ V which approximates the voltage across the capacitor quite well, for $0 \le t \le t_2$.

IV. APPROXIMATE EXPRESSIONS FOR THE CAPACITANCE VS. RIPPLE

From Fig. 6, Fig. 7, Eq. (24) and Eq. (26), it is clear that

$$V_p e^{-\frac{\theta_2}{\omega RC}} = V_{\min} = V_p - \Delta V, \tag{27}$$

or

$$e^{-\frac{\theta_2}{\omega RC}} = 1 - r. \tag{28}$$

Solving Eq. (28) gives an estimate for the capacitance as a function of ripple, *i.e.*,

$$\frac{\theta_2}{\ln(1-r)} = -\omega RC. \tag{29}$$

Eq. (29) can be rewritten as

$$\omega RC = \frac{-P + \cos^{-1}(1-r)}{\ln(1-r)}, \tag{30}$$

where $P = 2\pi$ for the half-wave rectifier, or $P = \pi$ for the full-wave rectifier. Note that ω is the same for both types of rectifiers.

As far as the authors know, Eq. (30) has not appeared in the literature. However, Eq. (30) can be derived from the method of Ludeman [2], as we show in Appendix A.

For small r, as we show in Appendix B, $\cos^{-1}(1-r) \approx 0$, $\cos^{-1}(1-r) \approx \sqrt{2r}$, or $\cos^{-1}(1-r) \approx \sqrt{\frac{4r}{2-r}}$. On the other hand, as we show in Appendix C, $\ln(1-r) \approx -r$, or $\ln(1-r) \approx -\frac{2r}{2-r}$. Hence, for small r, Eq. (30) can be approximated by the following six equations:

$$\omega RC = \frac{P}{r}, \tag{31}$$

$$\omega RC = \frac{(2-r)P}{2r}, \tag{32}$$

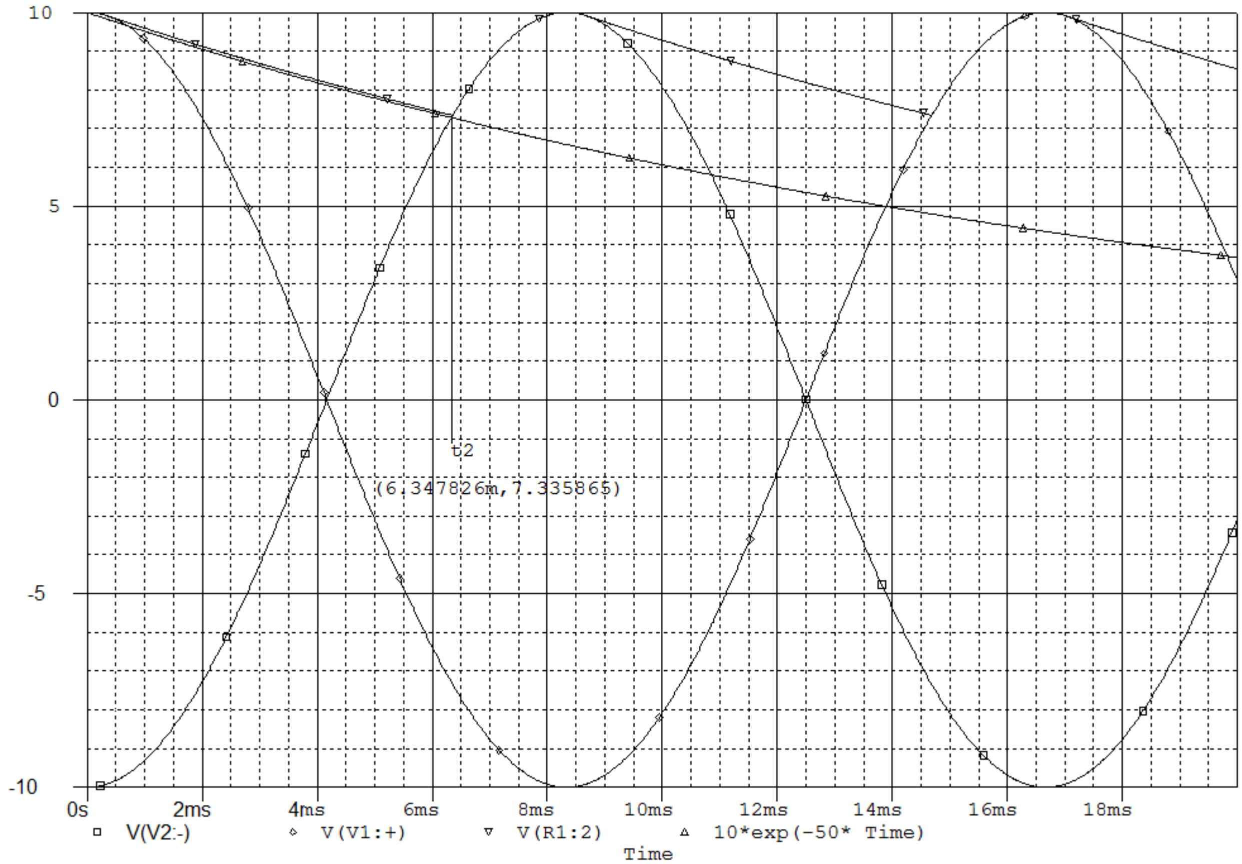

FIGURE 7. Simulated input ac voltages (V(V1:+)) and (V(V2:-)), and the voltage across the capacitor (V(R1:2)) of Fig. 2 for high ripple case, *i.e.*, $r = 0.27$. Also shown is the exponential voltage $10e^{-50t}$ V which approximates the voltage across the capacitor quite well, for $0 \le t \le t_2$.

$$\omega RC = \frac{P - \sqrt{2r}}{r}, \tag{33}$$

$$\omega RC = \frac{(2-r)\left(P - \sqrt{2r}\right)}{2r}, \tag{34}$$

$$\omega RC = \frac{P - \sqrt{\frac{4r}{2-r}}}{r}, \tag{35}$$

and

$$\omega RC = \frac{-P + \sqrt{\frac{4r}{2-r}}}{-\frac{2r}{2-r}} = \frac{P(2-r) - \sqrt{4r(2-r)}}{2r}. \tag{36}$$

Note that Eq. (31) is actually Eq. (2) or Eq. (3).

For completeness, we also mention that Sherman and Hamacher [6] give (without derivation) a very accurate approximation to the solution for the *full*-wave rectifier, *i.e.*, Eq. (22). To make explicit the role played by θ_2, this solution was rewritten by Cartwright [7] as

$$\frac{1}{\omega RC} = \tan\theta_1 \approx \frac{r}{\theta_2\left(1 - 0.586r - 0.358r^3\right)}, \tag{37}$$

where θ_2 is given by Eq. (21).

Of course,

$$\omega RC \approx \frac{\theta_2\left(1 - 0.586r - 0.358r^3\right)}{r}. \tag{38}$$

In the same spirit as that of Sherman and Hamacher, we have found the following approximation for the half-wave rectifier:

$$\omega RC \approx \frac{\theta_2\left(1.002 - 0.5539r + 0.0763r^2 - 0.2495r^3\right)}{r}, \tag{39}$$

where θ_2 is given by Eq. (13).

Note that Eq. (39) was determined by linear regression on the function $\omega RCr/\theta_2$, thereby producing the cubic expression on the right hand side of Eq. (39).

As stated before, Eq. (30) to Eq. (36) are based upon the zero-delay discharge approximation. We can also get a small ripple approximation by solving the exact equations Eq. (14) and Eq. (22), as we do in Appendix D, to get

$$\omega RC = \frac{1}{\tan\left(\frac{(2-r)\theta_2 - \sqrt{(2-r)^2\theta_2^2 - 8r(1-r)}}{2(1-r)}\right)}. \quad (40)$$

V. PERCENTAGE ERRORS OF APPROXIMATIONS FOR CAPACITANCE *vs* RIPPLE

In this Section, we compute percentage errors between the numerical answer and the approximations which relate capacitance to ripple for both types of rectifiers.

A. Percentage errors of approximate expressions for the half-wave rectifier

The percentage errors of Eq. (30) to Eq. (36), Eq. (39) and Eq. (40) relative to the numerical solution of Eq. (14) were computed and are shown plotted in Fig. 8.

As can be seen, the percentage error for Eq. (31) (the widely used Eq. (2)) is quite poor except for very small ripple. On the other hand, Eq. (30) is quite accurate even for very high ripple. Eq. (36) can be used for high ripple values not exceeding $r = 0.7$.

Notice that the percentage errors for all the approximations are positive, meaning that the value of capacitance calculated provides more than what is theoretically needed, *i.e.,* the ripple will be less than the designed value. The exception to this is Eq. (39) which can underestimate the needed capacitance by as much as 0.25%.

B. Percentage errors of approximate expressions for the full-wave rectifier

The percentage errors of Eq. (30) to Eq. (38) relative to the numerical solution of Eq. (22) were computed and are plotted in Fig. 9.

As can be seen, the percentage errors for the approximations for the full-wave rectifier are quite less accurate that those for the half-wave rectifier. The exceptions to this are Eq. (33) and Eq. (35) which are roughly the same for both rectifiers. However, Eq. (40) is considerably more accurate for the full-wave rectifier.

Notice too that the approximation error for Eq. (36) is less than that of Eq. (30). This is unlike the case of the half-wave rectifier. Furthermore, Eq. (36) can easily be inverted to give an expression for ripple vs. capacitance.

The equation for the Sherman and Hamacher approximation [6] (*i.e.,* Eq. (38)) gives the best accuracy, but can underestimate the needed capacitance by as much as 0.34%.

VI. CONCLUSIONS

We have derived new equations which relate capacitance vs. ripple for half-wave and full-wave rectifiers. We have provided percentage errors of these new equations compared with the capacitance that is determined using the exact equations, which were solved numerically.

In the future, we intend to search for more accurate equations to determine ripple as a function of capacitance, which should be helpful in the analysis of power supplies that have already been designed.

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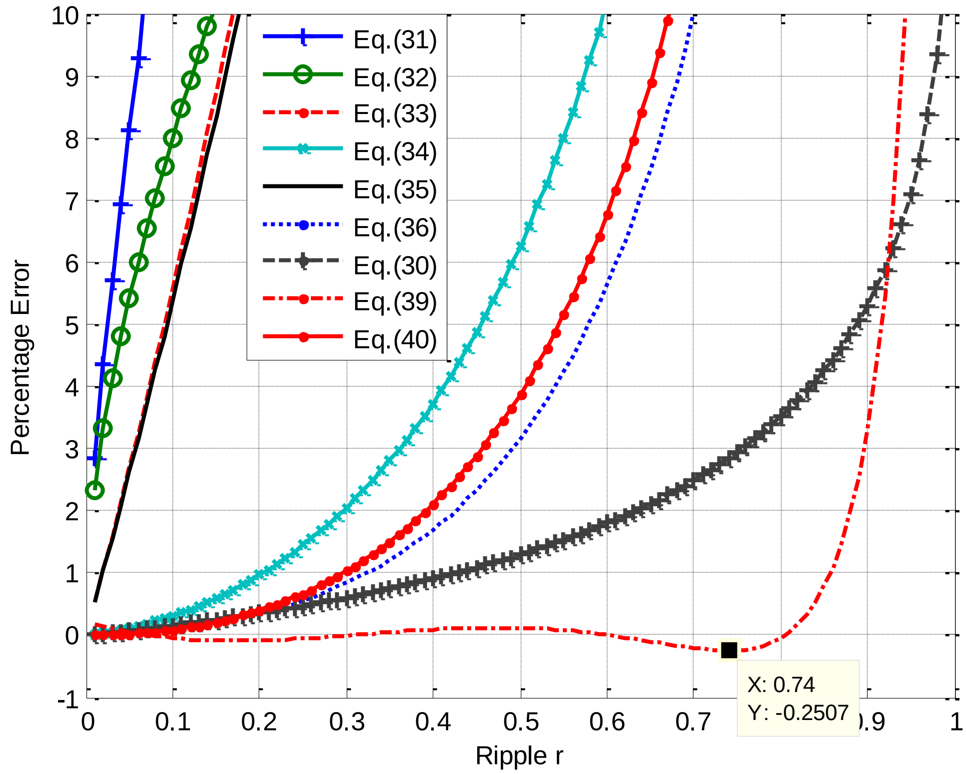

FIGURE 8. Percentage errors of Eq. (30) to Eq. (36), Eq. (39) and Eq. (40), relative to the numerical solution of Eq. (14), for the half-wave rectifier.

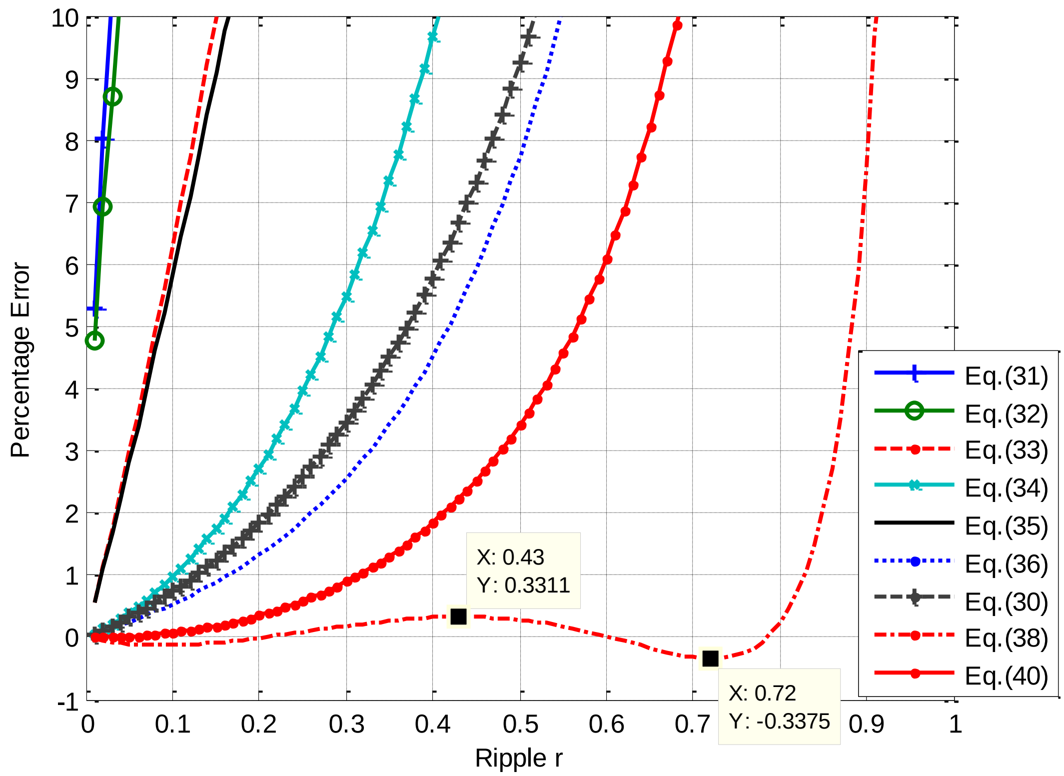

FIGURE 9. Percentage errors of Eq. (30) to Eq. (38) and Eq. (40) relative to the numerical solution of Eq. (22), for the full-wave rectifier.

APPENDIX A

In this appendix, we show that Eq. (30) can be derived from Ludeman's Eq. (5.22) of [2], which states

$$C=-\frac{T_r-\dfrac{\cos^{-1}\left(\dfrac{V_{\min}}{V_p}\right)}{360^o f}}{R\ln\left(\dfrac{V_{\min}}{V_p}\right)}, \quad \text{(A1)}$$

where $T_r=1/f$ for the half-wave rectifier, or $T_r=1/(2f)$ for the full-wave rectifier.

However, from Eq. (1), $\frac{V_{\min}}{V_p}=1-r$. Hence, Eq. (A1) becomes

$$RC=-\frac{T_r-\dfrac{\cos^{-1}(1-r)}{2\pi f}}{\ln(1-r)}, \quad \text{(A2)}$$

or

$$RC=-\frac{\omega T_r-\cos^{-1}(1-r)}{\ln(1-r)} =\frac{-P+\cos^{-1}(1-r)}{\ln(1-r)}. \quad \text{(A3)}$$

APPENDIX B

In this appendix, we show that $\cos^{-1}(1-r)\approx 0$, $\cos^{-1}(1-r)\approx\sqrt{2r}$, or $\cos^{-1}(1-r)\approx\sqrt{\frac{4r}{2-r}}$.

We write $y=\cos^{-1}(1-r)$, or

$$\cos(y)=1-r. \quad \text{(B1)}$$

However, for $r<<1$,

$$\cos(y)\approx 1, \quad \text{(B2)}$$

or $y\approx 0$, *i.e.*, $\cos^{-1}(1-r)\approx 0$.

Also, it is a well-known fact that $\cos y=1-y^2/2$ for small y. Hence, Eq. (B1) becomes

$$1-\frac{y^2}{2}=1-r. \quad \text{(B3)}$$

Solving Eq. (B3) for y gives the desired result, *i.e.*,

$$y=\cos^{-1}(1-r)=\sqrt{2r}. \quad \text{(B4)}$$

However,

$$1-\frac{y^2}{2}\approx\frac{1-y^2/4}{1+y^2/4}. \quad \text{(B5)}$$

The correctness of Eq. (B5) is easily verified by cross-multiplication and ignoring powers greater than 2, because these terms are negligible for small y. hence,

$$\cos(y)=1-r\approx\frac{1-y^2/4}{1+y^2/4}. \quad \text{(B6)}$$

Solving Eq. (B6) for y gives the desired result, *i.e.*, $y=\cos^{-1}(1-r)\approx\sqrt{\frac{4r}{2-r}}$.

APPENDIX C

In this appendix, we show that $\ln(1-r)\approx -r$, or $\ln(1-r)\approx-\frac{2r}{2-r}$, for small r.

$$y=\ln(1-r),$$

or

$$e^y=1-r. \quad \text{(C1)}$$

However, for small y, $e^y\approx 1+y$ [10]. Hence, Eq. (C1) becomes

$$1+y\approx 1-r, \quad \text{(C2)}$$

or $y=-r$, *i.e.*, $y=\ln(1-r)=-r$.

Eq. (C1) can also be expressed as

$$\frac{e^{y/2}}{e^{-y/2}}=1-r, \quad \text{(C3)}$$

or

$$\frac{1+\dfrac{y}{2}}{1-\dfrac{y}{2}}\approx 1-r. \quad \text{(C4)}$$

Solving Eq. (C4) for y gives the desired result, *i.e.*, $\ln(1-r)\approx-\frac{2r}{2-r}$.

APPENDIX D

In this appendix, we derive Eq. (40).
From Eq. (14) and Eq. (22), and using Eq. (7),

$$\cos(\theta_1)\, e^{-[\theta_2-\theta_1]\tan(\theta_1)} - 1 + r = 0. \quad \text{(D1)}$$

where $\theta_2 = 2\pi - \cos^{-1}(1-r)$ for the half-wave rectifier, or $\theta_2 = \pi - \cos^{-1}(1-r)$ for the full-wave rectifier.

For small x, $e^{-x} = \frac{e^{-x/2}}{e^{x/2}} \approx \frac{1-x/2}{1+x/2} = \frac{2-x}{2+x}$. Hence, for small θ_1, *i.e.*, for small ripple, Eq. (D1) can be written as

$$\cos\theta_1 \frac{2-(\theta_2-\theta_1)\tan(\theta_1)}{2+(\theta_2-\theta_1)\tan(\theta_1)} - 1 + r = 0, \quad \text{(D2)}$$

or

$$\frac{2\cos\theta_1 - (\theta_2-\theta_1)\sin(\theta_1)}{2+(\theta_2-\theta_1)\tan(\theta_1)} - 1 + r = 0, \quad \text{(D3)}$$

or, for small θ_1,

$$\frac{2-\theta_1^2-(\theta_2-\theta_1)\theta_1}{2+(\theta_2-\theta_1)\theta_1} - 1 + r = 0, \quad \text{(D4)}$$

or

$$\frac{2-\theta_2\theta_1}{2-\theta_1^2+\theta_2\theta_1} - 1 + r = 0, \quad \text{(D5)}$$

or

$$(1-r)\theta_1^2 - (2-r)\theta_2\theta_1 + 2r = 0. \quad \text{(D6)}$$

Solving Eq. (D6) gives

$$\theta_1 = \frac{(2-r)\theta_2 - \sqrt{(2-r)^2\,\theta_2^2 - 8r(1-r)}}{2(1-r)}. \quad \text{(D7)}$$

Hence, from Eq. (7),

$$\omega RC = \frac{1}{\tan\left(\frac{(2-r)\theta_2 - \sqrt{(2-r)^2\,\theta_2^2 - 8r(1-r)}}{2(1-r)}\right)}. \quad \text{(D8)}$$