The appropiate place of the theme on universal gravitation in teaching programs of introduction to classical mechanics

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Resumen

Se propone un enfoque para la enseñanza de la Mecánica Clásica elemental para los estudiantes de Física, exponiendo la ley de la Gravitación Universal de Newton al principio del curso, justamente después de la introducción de los conceptos básicos de la Cinemática y de la segunda ley de Newton. El objetivo es ayudar al estudiante a comprender mejor esta ley, la cual es fundamental para la Mecánica, y para entender la interpretación dual de la masa como inercial y gravitatoria, conjuntamente con sus consecuencias a lo largo del estudio de los fenómenos de la Mecánica.

Palabras clave: Mecánica clásica elemental, ley de Gravitación Universal, enseñanza de la Física.

Abstract

An approach to teach elementary Classical Mechanics for Physics students is proposed presenting the law of Universal Gravitation of Newton at the beginning of the course, right after the introduction of the basic concepts of Kinematics and the second law of Newton. The aim is to help the student to better understand this law, which is fundamental for Mechanics, and for understanding the dual interpretation of mass as inertial and gravitational, together with its consequences along the study of phenomena of Mechanics.

Keywords: Elementary classical mechanics, law of Universal Gravitation, Physics learning.

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I. INTRODUCTION

Without pretending that the following statement is a definition, it can be said that Physics is the study of forces. Through the analysis of origins and characterization of forces (or potentials), knowledge about natural laws is obtained. From this viewpoint, it seems natural that the first Physics course a student of science takes at the university needs to be mainly concerned with gravitational forces, and specially, with forces of Earth on bodies on its surface. This is actually the case, as can be seen in textbooks of introductory courses of Physics [1, 2, 3, 4]. However in almost all of these books, Newton's law of Universal Gravitation is one of the last chapters of Mechanics. Such a subject distribution arises several questions, such as: how can then these books treat the free fall in the Kinematics chapter? The solution to this problem has been given through the statement that "*all bodies fall with the same acceleration g*", which is presented as an empirical law.

Nevertheless this is later justified once the chapter devoted to gravitation is covered. This approach has two important drawbacks for science students, namely: On one side, the student could be unaware of the unifying character the Gravitation Law has on terrestrial and celestial dynamics of bodies, which is its main achievement. In most cases the student keeps the belief that the goal of the chapter on Gravitation is only to treat dynamics of celestial bodies. On the other side, the student has no opportunity to understand the conceptual difference between gravitational mass and inertial mass and the consequences that this brings about. When the student learns the concept *mass* from the second law of Newton and from the Gravitation Law, he identifies it immediately with the concept of mass of every day's life. Although this is true to a large extent, it has some other aspects the student needs to be aware of when he learns other topics, *e.g.* Special and General Relativity or the *effective mass* of electrons in crystals. Thus under the traditional teaching program, there is a risk even to miss the dependences of g on other parameters such as height and density fluctuations.

The reason behind the traditional textbooks themes organization can be related to commercial aspects. Those books are intended *for Students of Science and Engineering*, and engineering students is a large market. Obviously, engineering students are more interested in practical applications of physics, rather than in such fundamental principles.

II. THE STATEMENT

Here the option to treat the law of Universal Gravitation just after finishing the chapter of Kinematics and the introduction of the second law of Newton is recommended for students of Physics. This needs to be followed by the special case of the attraction between a body and Earth (attraction between a mass point and a sphere of uniform density):

$$F = w = G\frac{M}{R^2}m = gm, \quad (1)$$

where F is the force of the planet Earth on a body of mass m on its surface, w is the weight of the body, G the Universal Gravitation's constant, M is the mass of Earth, R its radius, and $g=GM/R^2$ is the acceleration due to gravity. Accordingly, after the well known explanations of the law of free fall and the conceptual differences of mass, the student will know that every time the weight of a body is defined as mg, the law of Universal Gravitation is applied. Subsequent chapters can be used to establish that both characteristics of mass, namely inertial and gravitational, concur in many phenomena giving as a result that in some cases their effects cancel each other, and that from these cases the law of free fall is only a single example.

Another example of this compensation of both mass characteristics is the stability of the (approximate) circular motion of planets. In this case the attractive gravitational force (gravitational mass) compensates the centripetal force obtained from the centripetal acceleration required by the circular motion, through the application of the second law of Newton (inertial mass).

This applies also in other less known cases. An example is the demonstration that the shortest distance an automobile needs to stop is independent of its mass [1]. Clearly the compensation of both characteristics of mass is fundamental, since the maximum force needed to stop the motion without sliding is the maximum frictional force

$$f_s = \mu_s w = \mu_s m_g\, g, \quad (2)$$

with μ_s the coefficient of static friction and m_g the gravitational mass of the car, is related to the maximum deceleration through the second law of Newton

$$f_s = m_I(-a), \quad (3)$$

where m_i is the inertial mass of the car, and a the acceleration. The shortest distance of stopping is thus obtained by combining (2) and (3) with the condition m_i=m_g, and using the general formula

$$v^2 = v_0^2 + 2ax,$$

where v is the final velocity, v_0 is the initial velocity, and x the displacement. For v=0 at x=x_0 it results

$$x_0 = \frac{v_0^2}{2g\mu_s}.$$

Analysis of the total energy also rveals auch a duality in mass character. Conservation of energy

$$E = T + V, \quad (4)$$

where T represents the kinetic energy, and V the potential energy, comes from the fact that a work performed by an external agent can be expressed by the difference of final and initial kinetic energies ΔT, and also by the difference of final and initial potential energy ΔV. In ΔT the second law of Newton is used, whereas in ΔV, when gravitation is involved, gravitational mass is used. Thus, the mass in T is inertial, and the mass in V is gravitational. From this point of view it is not surprising that the escape speed is also independent of the mass of escaping particles. The escape speed is the smallest initial vertical velocity a projectile needs in order never to return. This problem is solved assuming

$$E_0 = E_\infty = 0,$$

where E_0 is the initial total energy of the projectile, and E_∞ is its total energy at infinity. From (4) the condition is fulfilled if at the start V=-T, or

$$-G\frac{M}{R}m_g = -\frac{1}{2}m_i {v_0}^2,$$

where M is the mass and R the radius of the planet. Again, under the assumption that m_i=m_g, v_0 results

$$v_0 = \sqrt{2gR}.$$

It is also interesting to compare the period of a simple pendulum, which is independent of mass, with the period of a mass attached to a spring, which depends on mass because the potential energy is elastic, although both are harmonic oscillators.

II. CONCLUSION

Teaching a student the law of Universal Gravitation at the beginning of an introductory course, after learning the basic kinematical concepts is valuable. This is one of the most important forces in nature, and makes the student aware of the dual function that mass plays in Mechanics, and its consequences in Dynamics. As a good option, books oriented in such a way, although more devoted to students of Physics, could also be useful for students of Engineering.

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