

Vertical externalities revisited: new results with public inputs and unit taxation

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Abstract

This paper studies the provision of public inputs in a federal system. We use a model with vertical tax and expenditure externalities to analyze the efficiency of equilibria under different settings, particularly Nash and Stackelberg equilibria. Our results discuss some findings from the previous literature. First, both vertical externalities are interrelated each other. Second, the condition for production efficiency in the public sector becomes irrelevant to assess optimality. And third, the replication of the second-best outcome by the federal government behaving as Stackelberg leader crucially depends on the states' reaction function.

Keywords: Fiscal federalism, vertical externality, productive public spending.

JEL Classification: H2, H4, H7

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1 Introduction

The literature on horizontal externalities seems to have taken precedence over the research dealing with vertical externalities. However, policy issues are still dominated by the latter, calling for continuous improvement in the institutional designed of federal or quasi-federal countries. For example, decentralized countries involved in nationwide fiscal reforms and/or facing fiscal consolidation processes (Spain, Italy, US) are experiencing a revival of issues concerning vertical fiscal imbalances and conflicting interactions between different levels of government. The long-term debate on the fiscal relationships between the Member States at different level in the European Union, especially in the Eurozone, may well reflect many of the theoretical and empirical discussions around vertical externalities as well.

In this context, the research on vertical externalities still offers much scope for addressing relevant research and policy questions. For instance, in the presence of vertical expenditure and tax externalities, how general is the standard statement that both types of inefficiencies are independent each other and, consequently, that separate policy measures must be prescribed? Likewise, how sensitive are the usual results on the ability of the federal government to replicate the second-best outcome to different types of public expenditures, taxes or availability of policy instruments?

The previous literature has mainly focused on vertical tax externalities, in which different levels of government share the same tax base. As is well-known, it leads to an over-provision of public goods as long as the deadweight loss of distorting taxation is underestimated by the governments. Flowers (1988) deals with this issue through a Leviathan's approach and shows how the federation may place itself at the downward-sloping part of the Laffer curve. Papers such as Dahlby and Wilson (1994), Boadway and Keen (1996), and Sato (2000) find similar conclusions when a benevolent government is involved¹. Moreover, these contributions propose different systems of vertical transfers that correct tax externalities between governments.

Vertical externalities may also arise, however, in other contexts. Boadway et al. (1998) use a model with heterogeneous and partially-mobile agents to make explicit the trend of the states to be too progressive. In terms of

¹Nevertheless, Keen (1998) claims that the effects of federal taxes on state taxes are not so much straightforward as it might seem: under certain conditions, increases in the federal tax rate may reduce the state tax rates. Empirical evidence is miscellaneous (see, for instance, Esteller-More and Sole-Olle, 2001, and Anderson et al., 2004).

interregional trade, Lucas (2004) has shown how a federal government acting as Stackelberg leader can replicate the unitary nation optimum through matching grants in a federation with vertical and horizontal externalities.

The provision of public inputs in a multi-government environment has been again widely examined in the context of horizontal externalities (see, for instance, Liu and Martinez-Vazquez, 2014, for a recent contribution). By contrast, the literature has not paid much attention upon vertical externalities stemming from the provision of public inputs. This point refers to the positive or negative effects that the productive public spending by one level of government may exert on other levels' revenues. This phenomenon can be found in supranational structures such as the European Union, in which an important share of its budget is devoted to regional policies based on the provision of infrastructures. These types of policies have positive impacts on local, regional and federal budget constraints in many Member States through GDP growth (see, for instance, Becker et al., 2010).

Nevertheless, a few papers have dealt with this issue. Dahlby (1996) describes the effects of expenditure externalities in a federation, and defines a general framework for matching grants in order to eliminate them. Wrede (2000) deals with productivity increasing public services in a federation consisting of Leviathan governments. Dahlby and Wilson (2003) examine a model in which state governments provide a productivity-enhancing public input; they conclude that this externality may have an ambiguous impact on federal revenues, and a matching grant from the federal government to the states may correct it.

This paper aims at providing some theoretical results which, given the specific features of our model, confirm or modify some of the accepted previous results. In particular, we use the Boadway and Keen's (1996) model to discuss the efficiency of equilibria when a public input is provided by the state governments. We consider the positive impact of the public input on wage rate through a higher labor productivity. The federal and state governments set per unit taxes on labor instead of the ad valorem taxes used by Dahlby and Wilson; it allows us to focus on the (likely) positive externality derived from the public input, leaving aside other positive vertical externalities that may arise when ad valorem taxes are involved (Dahlby and Wilson, 2003).

As usual, the behavior of governments has been modeled under different scenarios: as a central government in a unitary country, different governments as Nash competitors, and one level of government (the federal one) acting as Stackelberg leader. Moreover, we investigate how the federal government

is able to achieve the second-best solution. We deal here with restrictions by employing policy instruments: federal government is not allowed to make use of vertical grants aimed at correcting vertical externalities. In this way, the paper tries to reproduce a common feature in real federations, namely, constitutional arrangements may prevent the design of intergovernmental transfers exclusively based on efficiency criteria².

The results show that, as Dahlby and Wilson (2003) and Martinez (2008) have found, the marginal cost of providing the public input may be under or overestimated in a federal system. However, contrary to the previous references, our paper finds that the difference between the unitary and the federal solutions is not independent of the vertical tax externality. The reasoning followed in this paper sharply contrasts to that of Dahlby and Wilson (2003) because we detect that the production efficiency condition does not perform properly as criterion for assessing optimality in federal countries, as they do.

Moreover, since no vertical transfers are available in our model, the ability of federal government behaving as Stackelberg leader to replicate the second-best outcome is not straightforward. This paper demonstrates that when the set of policy instruments is restricted, the effectiveness of the federal tax rate to implement the second-best optimum depends on the state governments' reaction to changes in federal taxes. We also find that the optimal federal tax rate can be positive, unlike Boadway and Keen's (1996) findings but in line with Kotsogiannis and Martinez (2008), who deal with consumption public goods but under an ad valorem tax setting.

The structure of the paper is as follows. Section 2 describes the main features of the model. Section 3 provides the second-best outcome achieved in a unitary country. Next section compares this result to those reached when the federal and state governments play Nash. Section 5 studies whether the federal government behaving as Stackelberg leader is able to replicate the second-best allocation. Finally, section 6 concludes.

2 The model

We assume a country with a federal government and k identical states to avoid unnecessary complexities by dealing with asymmetric allocations and

²This is also related to the literature on optimal taxation and the availability of policy tools (see Stiglitz and Dasgupta (1971) and subsequent papers).

horizontal grants for redistribution aims. Each state is populated by n identical households that are assumed to be completely immobile³. Household's utility function is given by the separable form:

$$u(x, l) + B(G), \quad (1)$$

where x is a private good used as numeraire, l is the labor supplied, and G is a pure public good provided by the federal government. The properties of the function $u(x, l)$ are the standard ones, and $B(G)$ is increasing and concave. The representative household faces the following budget constraint:

$$x = (\omega - \tau)l, \quad (2)$$

where ω is the wage rate and τ the per unit tax on labor. Household's optimization problem consists of maximizing (1) subject to (2), and that yields the labor supply $l(\omega - \tau)$ and the indirect utility function $V(\omega - \tau) + B(G)$. It is assumed that $l' > 0$ ⁴.

Output in the economy is produced using labor services and the public input g according to the following aggregate state production function:

$$F(L, g), \quad (3)$$

where $L = nl$. This function satisfies the usual assumptions: increasing in its arguments and strictly concave. Output can be used costlessly as x, G or g . Labor market is perfectly competitive so that we can write:

$$\omega = F_L[nl(\omega - \tau), g] \quad (4)$$

It allows us to achieve the wage function $\omega(g, \tau, n)$. Some results of comparative statics can be found now; they will be used later:

$$\omega_g = \frac{F_{Lg}}{1 - F_{LL}nl'} > 0 \quad (5)$$

³Relaxing the assumption of complete household immobility would have no effects on the efficiency of the equilibria and governments' behavior, as long as the states are assumed to be symmetric (Proposition 4 in Boadway and Keen, 1996). By contrast, in the presence of (perfect or imperfect) inter-regional population movements and heterogenous states, the second best allocation does not require the equalization of the marginal cost of public funds across regions and layers of government (Sato, 2000).

⁴Hereafter, differentiation is denoted by a prime for functions of a single variable, while a subscript is used for partial derivatives.

$$\omega_\tau = \frac{-F_{LL}nl'}{1 - F_{LL}nl'} > 0 \quad (6)$$

Economic profit (rents) is defined as a residual:

$$\pi(g, \tau, n) = F[nl(\omega(g, \tau, n) - \tau), g] - nl[\omega(g, \tau, n) - \tau]\omega(g, \tau, n) \quad (7)$$

Again, it is useful to obtain some results for later use:

$$\pi_g = F_g - (F_{LL}nl'\omega_g + F_{Lg})nl \leq 0 \quad (8)$$

$$\pi_\tau = (1 - \omega_\tau)F_{LL}n^2l' < 0 \quad (9)$$

Note that the effect of public inputs on rents is ambiguous because g increases output (and hence, the economic profit) but also exerts a positive impact upon the wage rate, reducing rents.

Each level of government sets its own tax rate on labor. Denoting T as the tax rate established by the federal government and t as the corresponding variable at state level, it can be written that $\tau = T + t$. Thus, the revenue raised by the federal government to finance G is:

$$G(T, t, \theta, g, n, S) = knTl(\omega(g, \tau, n) - \tau) + k\theta\pi(g, \tau, n) - kS, \quad (10)$$

where $0 \leq \theta \leq 1$ is the proportional tax rate on profits levied by the federal government, and S is a vertical transfer between both levels of government⁵. Throughout this paper, θ is assumed to be fixed and exogenously determined. The effects of changes in T , t , g and S on the federal budget constraint are given by:

$$G_T = (\omega_\tau - 1)knTl' + knl + k\theta\pi_\tau \quad (11)$$

$$G_t = (\omega_\tau - 1)knTl' + k\theta\pi_\tau = G_T - knl \quad (12)$$

$$G_g = knTl'\omega_g + k\theta\pi_g \quad (13)$$

$$G_S = -k \quad (14)$$

The state revenue constraint is

$$g(t, T, \theta, n, S) = ntl(\omega(g, \tau, n) - \tau) + (1 - \theta)\pi(g, \tau, n) + S \quad (15)$$

⁵ S may have either sign and it is defined as a lump-sum grant in the sense of Boadway and Keen (1996) or Sato (2000).

The state government is in charge of providing the public input. Note that all economic profits are taxed away by both levels of governments because the rents are efficient resources for public sector⁶. For future reference, the impacts of changes in t , T and S are obtained:

$$g_t = (\omega_\tau - 1) ntl' + nl + (1 - \theta) \pi_\tau \quad (16)$$

$$g_T = (\omega_\tau - 1) ntl' + (1 - \theta) \pi_\tau = g_t - nl \quad (17)$$

$$g_S = 1 \quad (18)$$

When one of the equations (12), (13) or (17) is different to zero a vertical externality arises. The equations (12)-(13) show how the federal government's tax revenues are affected by the fiscal decisions taken by the state government on the tax rate and on the provision of the public input, respectively, while the equation (17) is the effect of the federal tax rate upon the state government's revenues.

3 The second-best allocation in a unitary country

The characterization of a vertical externality requires to consider the differences between the optimal solution in a unitary country, and the solution achieved when several levels of government exist. In this section, we obtain the first order conditions for the optimal provision of the national public good G and the public input g in a unitary country.

The central government chooses the values of G , g and τ to maximize the representative household's utility subject to the aggregated budget constraint⁷. Formally,

$$\begin{aligned} \text{Max} \quad & V(\omega - \tau) + B(G) \\ \text{s.t.} \quad & G + kg = kn\tau l(\omega(g, \tau, n) - \tau) + k\pi(g, \tau, n), \end{aligned} \quad (19)$$

⁶We establish here that the country is under-populated in order to avoid that the tax on rents may suffice to finance the first-best level of public good (Wildasin, 1986).

⁷Wildasin (1986) demonstrates that it is relevant to distinguish between to maximize the per capita utility or the total utility. As cited by Mansoorian and Myers (1995), considering the total utility of households as objective function implies that each state authority has a preference for the population size. With symmetric equilibria, this issue is not crucial, but it would prevent from extending the results to an environment in which households mobility is allowed. See footnote 3.

The first order conditions for G , g and τ are, respectively, as follows:

$$B'(G) - \mu = 0 \quad (20)$$

$$V'\omega_g - \mu k + \mu kn\tau l'\omega_g + \mu k\pi_g = 0 \quad (21)$$

$$(\omega_\tau - 1)V' + \mu knl + \mu(\omega_\tau - 1)kn\tau l' + \mu k\pi_\tau = 0, \quad (22)$$

where μ is the Lagrange's multiplier. Combining (20) with (22), using Roy's identity and the expressions (6) and (9), manipulation yields the familiar optimality rule for the provision of national public good G in a second-best environment:

$$\frac{nkB'(G)}{\lambda} = \frac{1}{1 - \frac{\tau l'}{t}}, \quad (23)$$

where λ is the private marginal utility of income. LHS of equation (23) is the sum of marginal rates of substitution between the federal public good G and the private good x . RHS of equation (23) is the marginal cost of public funds (MCPF). As is well-known, this expression is the Samuelson's rule for public good provision corrected by Atkinson and Stern (1974).

After some manipulation with equations (21) and (22), using again Roy's identity and the expressions (6) and (9), the second-best condition for the optimal provision of g can be written as follows:

$$\frac{nV'\omega_g}{\lambda} = \frac{1}{1 - \frac{\tau l'}{t}} \left(1 - n\tau l'\omega_g - \pi_g \right) \quad (24)$$

The LHS of latter condition is the sum of marginal benefits coming from one additional unit of g in terms of the private good x . The RHS of (24) is the marginal cost of providing the public input (MCP), which in turn can be decomposed into the MCPF and the tax revenue effect that arises as long as the public input g may affect positively or negatively the tax bases through labor productivity and economic profits. Whereas in the case of the consumption public good the MCPF and the MCP are identical, this distinction is necessary when the public input is taken into account.

Comparing expressions (23) and (24) a simple result for later use is obtained:

Proposition 1 *In a unitary country with a positive optimal tax rate and $\pi_g \geq 0$, the marginal cost of public funds is higher than the marginal cost of providing the public input g (Sufficient condition).*

If the Roy's identity is used in the LHS of (24), and the expressions (5) and (8) are inserted into (24), manipulation gives:

$$F_g = 1, \quad (25)$$

that is, the production efficiency condition for the provision of public inputs (Diamond and Mirrlees, 1971). It means that the production effects of the public input are equal to its marginal production cost, though distortionary (but optimally set) taxation is used⁸.

4 Vertical externalities when federal and state governments play Nash

The existence of different levels of government may alter the behavior of the agents if they share the same tax base and/or public spending coming from the state governments is able to modify the federal budget constraint. This section deals with the optimal conditions when the state and federal governments behave as Nash competitors, that is, when each government takes as given the tax rates and the level of public expenditure implemented by other governments. Hence, the state's optimization problem consists of choosing the values for g and t in order to maximize the per capita utility of the state, taken its own budget constraint into account. Formally,

$$\begin{aligned} \text{Max} \quad & V(\omega(g, \tau, n) - \tau) + B(G) \\ \text{s.t.} \quad & g = ntl(\omega(g, \tau, n) - \tau) + (1 - \theta)\pi(g, \tau, n) + S \end{aligned} \quad (26)$$

The first order conditions we obtain are:

$$V'\omega_g - \mu + \mu ntl'\omega_g + \mu(1 - \theta)\pi_g = 0 \quad (27)$$

$$(\omega_\tau - 1)V' + \mu nl + \mu(\omega_\tau - 1)ntl' + \mu(1 - \theta)\pi_\tau = 0, \quad (28)$$

The expression that relates the marginal benefits and costs of providing the public input at state level can be derived as before:

$$\frac{nV'\omega_g}{\lambda} = \frac{1}{1 - \frac{tl'}{l} - \theta F_{LL}nl'} \left(1 - ntl'\omega_g - (1 - \theta)\pi_g \right) \quad (29)$$

⁸For further discussion, see Feehan and Matsumoto (2002).

Again, the RHS of equation (29) shows the marginal cost of public input provision when distorting taxes are used and different effects on state tax revenues are involved. A key question arises here concerning the optimality of this result when is compared to the second-best outcome. Our model yields the following proposition:

Proposition 2 *Dealing with the provision of public inputs g , if $T \geq 0$ then the MCPF perceived by state governments that play Nash is smaller than the MCPF in a unitary country. However, the MCP perceived by state governments may be higher, equal or smaller than in a unitary country.*

Proof. Using $\tau = T + t$, an alternative expression of the RHS of equation (29) can be obtained:

$$\frac{1}{1 - \frac{\tau l'}{l} + \frac{T l'}{l} - \theta F_{LL} n l'} \left(1 - n \tau l' \omega_g - \pi_g + n T l' \omega_g + \theta \pi_g \right) \quad (30)$$

The first term is the MCPF. By assumption, $F_{LL} < 0$ so that denominator is bigger than that of expression (24); thus, the MCPF is smaller with state governments. Regarding the marginal cost of provision, nothing can be said about the magnitude of its second term in relation to (24). Note that by (8), π_g may have either sign. ■

The first part of the proposition is a standard result in the literature, without considering whether a consumption public good or a public input are involved. When a vertical tax externality exists, the MCPF for providing both kinds of public expenditures is perceived as lower by state governments. The second part of the proposition pays attention upon the MCP, and claims that the sign of expenditure vertical externality is not determined, so that the state government may under or over-provide the public input.

In this regard, it can be stated that having a positive or negative vertical externality depends firstly on the relative magnitude of the changes in the MCPF and the tax revenue effect, and secondly on the sign of the effect of public inputs on rents. In particular, if it happens to be that $\pi_g \geq 0$ the sign of both combined vertical externalities will depend on the relative magnitudes of both terms in (30) because they change in opposite senses. By contrast, if $\pi_g < 0$ it may occur that both terms in (30) move in the same sense, and consequently an over-provision of public inputs takes place.

In general there exists an indetermination about the sign of vertical externality at play but, under certain circumstances, some remarks on the

magnitude of such externality are provided next. Let $\psi = \frac{MCP_U^g}{MCP_S^g}$ be the ratio between the MCP in a unitary country and the MCP perceived by state governments, both of them referring to g (that is, the RHS of equations (24) and (29), respectively). In terms of the above Proposition 2, such ratio ψ may be lower, equal or higher than 1; in other words, the state government may provide a level of public input below ($\psi > 1$), equal ($\psi = 1$), or higher ($\psi < 1$) than the optimal level, respectively. Next Proposition broadens this comparison (measured by ψ) between the MCP in a unitary versus a federal country.

Proposition 3 *Ceteris paribus,*

- i) ψ is decreasing in the elasticity of wage rate to g if $T > 0$*
- ii) ψ is decreasing in the marginal productivity of g if $0 < \theta < 1$.*
- iii) ψ is increasing in the share of rents levied by the federal government θ when $\pi_g \leq 0$*
- iv) ψ is increasing in the elasticity of the labor supply to the federal tax rate T (in absolute value).*

Proof. i) Using the terms with ω_g in the second term of (30) -and not present in (24)- and the expression (8) for π_g , manipulation yields $nTl'\omega_g + \theta nlF_{LL}l'\omega_g$. Rearranging we can write that $(T - \theta lF_{LL})nl'\omega_g > 0$, given (5), $T > 0$ (by assumption) and $F_{LL} < 0$.

ii) Using the expression (8) and $0 < \theta < 1$, an increase in F_g reduces the second term of (30). But this effect is bigger in the case of numerator of (24), hence ψ decreases.

iii) Differentiating the RHS of (29) with respect to θ yields $\frac{\left(1 - \frac{tl'}{t} - \theta F_{LL}nl'\right)\pi_g - \left(1 - ntl'\omega_g - (1-\theta)\pi_g\right)\left(-F_{LL}nl'\right)}{\left(1 - \frac{tl'}{t} - \theta F_{LL}nl'\right)^2}$. Since both terms of the RHS of (29) are positive, then $F_{LL} < 0$ and $\pi_g \leq 0$ lead to a negative sign in the latter derivative. Thus, MCP_S^g is decreasing in θ , and ψ is increasing in θ .

iv) In the denominator of the MCPF in the expression (30), the term $\frac{tl'}{t}$ is the elasticity of labor supply to the federal tax rate T (in absolute value).

■

In short, the higher the elasticity of wage rate to public inputs and/or the higher the marginal productivity of public inputs, the more likely is to find under-provision of public inputs. By contrast, the higher the federal tax

rate on rents and/or the higher the elasticity of labor supply to the federal tax rate, the more likely is to reach over-provision of public inputs.

Parts i) and ii) of proposition 3 show that the sign of vertical expenditure externality crucially depends on the tax revenue effect produced by the public input provision. In fact, the more productive the public input, the more tax revenues accrue to both governments. In this context, the gap between what the state government perceives and what actually happens in terms of social welfare gets wider, and that obviously leads to under-provision of public inputs.

Part iii) follows an inverse argument. When the public input affects negatively rents, increasing the federal share on economic profits taxes is damaged for the federal government, so that the risk of over-provision of g rises.

Part iv) of proposition 3 reconsiders the canonical statement by Dahlby and Wilson (2003), and later confirmed by Martinez (2008), that the vertical tax externalities do not affect public spending externalities. By contrast, we have found that the extent whereby the MCP_S^g differs from the MCP_U^g (i. e., the sign and magnitude of the externality) depends on the tax rate set by the federal government and/or whether the labor supply is more or less sensitive to the federal tax rate. It therefore means that both externalities are interrelated⁹.

In contrast with that, the reasoning followed by Dahlby and Wilson (2003) is based on the production efficiency condition and concludes that both externalities are independent each other. Nevertheless, recent papers by Blackorby and Brett (2000), Kotsogiannis and Makris (2002) and more recently by Martinez and Sjongren (2014) have proved that considering the production efficiency as criterion for assessing optimality in federal systems may be inappropriate. Our model offers a clear insight about that. Using (5) and (8) in the expression (29), the following is obtained:

$$\left(\frac{n\theta F_{Lg}}{F_g} + (1 - \theta) \right) F_g = 1 \quad (31)$$

⁹In some sense, our vertical expenditure externality holds certain similarities with horizontal externalities. Indeed, assuming a positive impact of state public input on federal tax revenues, it appears a trend towards the under-provision of g that can be seen as state tax rates being too low (as a result of fiscal competition in the case of horizontal externalities). In such a way, Keen and Kotsogiannis (2002) and Madies (2004) have recently shown the interdependence between both externalities.

i. e., the production efficiency does not hold when the governments play Nash. If all the taxes on profits were levied by the state government ($\theta = 0$), the above expression would become $F_g = 1$, that is, the efficiency in production of public inputs would be achieved but the condition for optimality is not still fulfilled (see equation (29) with $\theta = 0$)¹⁰.

5 Federal government plays as Stackelberg leader

The analysis now proceeds by exploring the equilibrium outcome achieved when the federal government behaves as Stackelberg leader, anticipating the effects of its actions on the states' decisions. In this context, the federal government sets its tax rate taking as given the states' reaction function, and in principle is able to replicate the second-best outcome reached by the government in a unitary country. However, the success of this policy is very sensitive to whether the federal government has unrestricted access to vertical transfers or not. As Keen (1998) points out, if vertical transfers are not available for the federal government, to achieve the second-best allocation is not straightforward, even when the states' reaction function is known.

Our aim here is to shed some light about the ability of the federal government to get the second-best outcome when a public input is provided. Vertical transfers are not allowed for the federal government, whose only instrument to affect the behavior of the states is the tax rate T . This approach seeks to show not only how the conclusions of the main branch of literature may be modified when policy instruments are restricted, but also to know under which assumptions a federal system without efficiency-oriented vertical transfers might achieve the second-best allocation. This environment also permits dealing with features of real federations, namely, the intergovernmental grants are not usually designed to correct vertical externalities, and constitutional arrangements sometimes prevent from implementing vertical transfers exclusively based on efficiency criteria.

¹⁰Translating this argument to Dahlby and Wilson's (2003) model, we reach the same conclusion. Using their expressions (6) and (16), an optimal federal tax rate T^* removing both vertical externalities can be achieved (we do something similar in the next section); however, inserting that T^* into their expression (19), the production efficiency is not fulfilled. In other words, the optimality conditions in federal systems and the production efficiency condition do not necessarily coincide.

We should question first whether there exists an optimal federal tax rate that corrects both vertical externalities. Following Boadway and Keen (1996) and given the existing interaction between both types of externalities, we define the marginal vertical externality as follows:

$$\gamma = G_t + G_g, \quad (32)$$

that is, taking into consideration the negative and/or positive effects on the federal revenues generated by the states by means of their own taxes and the provision of public inputs. Since at the optimum $\gamma = 0$, inserting (12) and (13) in (32) and solving for T , the optimal federal tax rate T^* we find is:

$$T^* = \frac{-(\pi_\tau + \pi_g)\theta}{(\omega_\tau + \omega_g - 1)l'n} \leq 0 \quad (33)$$

Since there are no (efficiency-oriented) vertical transfers between levels of government, the federal tax rate T is the unique instrument to offset the two opposite effects that the states' decisions have on the federal revenues. The first effect comes from the fact that the state tax rates exert a negative impact on the federal budget constraint; as pointed out by Boadway and Keen (1996), in that case the federal government should subsidy the (common) tax base that is over-exploited as a result of the tax externality. But secondly, it is also likely that the provision of public inputs increases the federal revenues (positive expenditure externality); thus if it happens to be that t follows T then it may be convenient having a positive federal tax rate to encourage the state taxes. This way, the state resources available for the public input provision will rise. Note that in accordance with the Proposition 3 (iv), the MCP_S^g is decreasing in $\frac{Tl'}{t}$ (ψ is increasing in $\frac{Tl'}{t}$), so T may stimulate the spending in g .

We turn now to the characterization of the state's reaction function with respect to the federal tax rate. So far, each level of government acted independently; under the new framework, by contrast, the federal government knows the effects of its policy on state's behavior. From the state optimization problem (26), it can be readily seen that

$$V'\omega_g g_t + (\omega_\tau - 1)V' = 0. \quad (34)$$

Differentiating this expression with respect to T we obtain:

$$\begin{aligned} (\omega_\tau - 1)(1 + t_T)V''\omega_g g_t + (1 + t_T)V'\omega_{g\tau} g_t + V'\omega_g g_{tt} t_T + V'\omega_g g_{tT} + \\ (\omega_\tau - 1)^2(1 + t_T)V'' + V'\omega_{\tau\tau}(1 + t_T) = 0 \end{aligned}$$

As $g_{tT} = g_{tt} + (\omega_\tau - 1) l' n$, rearranging terms and solving for t_T , the above equation can be rewritten as follows:

$$t_T = \frac{-(\omega_\tau - 1) V' \omega_g n l'}{(\omega_\tau - 1) V'' \omega_g g_t + V' \omega_{g\tau} g_t + V' \omega_g g_{tt} + (\omega_\tau - 1)^2 V'' + V' \omega_{\tau\tau}} - 1 \quad (35)$$

i. e., the state's reaction function. Given the assumptions of our model, the sign of t_T is unclear ($t_T \leq 0$). In other words, the state tax rates may react ambiguously to changes in the federal tax rate.

Even regarding a more general approach, the doubts about the effects of changes in federal taxes on the national tax rate of the federation still remain: the sign of $1 + t_T$ continues being indeterminate¹¹. This ambiguity comes from the unclear net effect of the two vertical externalities when they are jointly considered. Whereas in the case of Boadway and Keen (1996) there exists a remarkable tendency towards over-provision (and the subsequent increase in the tax rates), under-provision of public inputs (or equivalently, state tax rates being too low) can be found when the expenditure externalities are taken into consideration. If this is the case, the national tax rate τ may well go down when the federal government increases its tax rate.

Aimed at assessing how is the response of the state tax rate to changes in the lump-sum transfer, expression (34) is differentiated with respect to S to write:

$$(\omega_\tau - 1) V'' \omega_g g_{tt} t_S + V' \omega_g g_{tt} t_S + (\omega_\tau - 1)^2 V'' t_S = 0, \quad (36)$$

that leads to $t_S = 0$, that is, the tax rate is unaffected by the transfer¹². Contrary to Boadway and Keen (1996), where this situation is caused by a linear utility function in G , our model does not recognize any ability of the vertical transfer for influencing t , regardless of the properties of the utility function. It means that the income effects go entirely to the provision of the state public input. Moreover, this is consistent with the null role played by the vertical transfers as policy instruments in our model.

At this point, the federal's optimization problem we have to solve is the following:

$$\begin{aligned} & \text{Max} \quad V(\omega(g(t, T, \theta, S), \tau, n) - \tau) + B(G(T, t, \theta, S, g(t, T, \theta, S))) \\ & \text{s.t.} : t = t(T, \theta, S) \end{aligned} \quad (37)$$

¹¹Note that $1 + t_T = \frac{d\tau}{dT}$.

¹²This result is based on the assumptions of the model after some manipulation in (36). Details are available upon request.

As can be seen, both the objective function and the federal constraint take into consideration the behavior of the states and the influence of federal decisions on them. In this regard, the federal government chooses T regarding the first order conditions obtained for the state government. Formally:

$$[(g_t t_T + g_T) \omega_g + (\omega_\tau - 1) (1 + t_T)] V' + B' [G_T + G_t t_T + G_g g_T] = 0 \quad (38)$$

Using expression (34) and rearranging terms, one obtains:

$$\frac{knB'}{\lambda} = \frac{nV'\omega_g}{\lambda} \left(\frac{1}{1 + \left(\frac{g_t}{l} - n\right) Tl'\omega_g + \left(\frac{g_t}{nl} - 1\right) \theta\pi_g + \frac{(1+t_T)G_t}{knl}} \right), \quad (39)$$

where (11) and (16) have been used. Expression (39) relates the MCP of G at federal level (MCP_F^G) to the MCP of g at state level (MCP_S^g) when the former government behaves as Stackelberg leader and the latter as follower. Note that if the tax bases are not shared and the provision of public inputs corresponds to the central government exclusively, i. e., $t = g_t = G_t = 0$ and $\theta = 1$, the expression (39) trivially becomes

$$\frac{knB'}{\lambda} = \frac{nV'\omega_g}{\lambda} \left(\frac{1}{1 - nTl'\omega_g - \pi_g} \right), \quad (40)$$

that is, the relation between the MCP of G and the MCP of g at second-best optimum in a unitary country.

Given these two alternative relationships between the MCP under different scenarios, a discussion can be initiated about whether or not the federal government is able to replicate the second-best solution. Let $\eta = \frac{MCP_F^G}{MCP_S^g}$ be the variable that relates both MCP assuming the Stackelberg approach. The relevant issue here is to know to what extent this variable differs from 1; we will accordingly know whether the federal structure of the country leads to an under or over-provision of the public input, using the unitary solution as benchmark.

Proposition 4 *If the federal government plays as Stackelberg leader (with $T^* > 0$) and $\pi_g \geq 0$, then $\eta \leq 1$. Hence, the MCP_F^G may be higher, equal or lower than the MCP_S^g , and the replication of the second-best outcome is not guaranteed.*

Proof. Using (16) and rearranging terms, the expression in parenthesis in equation (39), i. e. the ratio η , can be rewritten as follows:

$$\frac{1}{1 + \frac{g_T}{knl} [G_g + (1 + t_T) G_t]} \quad (41)$$

By (6) and (9), $g_T < 0$; if $\pi_g \geq 0$, then $G_g > 0$ when $T^* > 0$, and $G_t < 0$ by (9), $\forall T^* > 0$. As $1 + t_T \leq 0$, we are not sure if the denominator of (41) is higher, equal or lower than 1. So $\eta \leq 1$. ■

Proposition 4 questions the ability of the federal government to achieve the second-best optimum with no vertical grants as policy instrument. Notice that in a unitary country, also with $\tau > 0$ and $\pi_g \geq 0$, the MCP of G is unambiguously higher than the MCP of g (Proposition 1). From Proposition 4 a necessary condition to ensure the second-best optimum must be established:

Corollary to Proposition 4 *The federal government that plays as Stackelberg can achieve the second-best outcome if, and only if, $1 + t_T > 1$, or what is the same, $t_T > 0$.*

Proof. Given that the necessary condition for achieving the second-best result is that (41) is higher than 1, and since $g_T < 0$, $G_g > 0$, and $G_t < 0$, we then need to have $G_g + (1 + t_T) G_t > 0$. Inserting the formulas (5), (6), (8), (9), and the optimal federal tax rate T^* (33) into this expression, it can be seen that $1 + t_T > 1$ is required in order to obtain that the expression (41) is higher than one. The number of households has been normalized to 1 for making easier the proof. ■

Therefore, the central point to internalize the vertical externalities lies in the states' reaction function. Particularly, the necessary condition is that the state governments increase their taxes after the federal government rise its tax rates, and vice versa; only this way the federal policy-makers acting as Stackelberg can correct both vertical externalities. One of the main implications stemming from it is that the effectiveness of federal policy crucially depends on an empirical issue because the sign of t_T is theoretically ambiguous.

6 Concluding remarks

Sharing tax instruments between the federal and subnational governments is a common feature in federations. It allows that different levels of government

get involved in financing their own public expenditures. However, the concurrency of different tax powers on the same tax base causes that vertical tax externalities come out, and a deviation of the results from the second-best allocation arises.

The vertical externalities also come into being when the public spending provided by one level of government affects other government's decisions. This is the case, for instance, of public inputs such as public investment, education and so on, that may exert different impacts on the tax revenues accruing to other governments. This second vertical externality has received less attention in the literature, though several real examples can be found in countries such as United States, Australia and Spain, or in supranational structures as the European regional policies.

This paper presents a model in which the federal and the state governments set per unit taxes on labor to finance two types of public expenditures. The federal government provides a consumption public good, while the state governments supply a productivity-enhancing public input. The second-best allocation is reached in a unitary country and then used as benchmark for subsequent comparisons. When the Nash behavior is to be assumed for governments, a vertical externality arises from the provision of public inputs and from the tax externality as well. While the former exerts an ambiguous effect on the federal tax revenues, the latter presents a clear negative influence. In this model, the sign and extent of the expenditure externality depend on the tax externality, amongst other things. Here, it has been proved that using the production efficiency condition as optimality criterion in federal systems leads to incorrect conclusions. Moreover, our results drive to distinguish between the cost of public funds and the provision cost of the public input, which includes the former and the tax revenue effect as well.

The ability of federal government to achieve the second-best outcome has been also studied. Our approach restricts the policy instruments available for the federal government, particularly vertical transfers for efficiency purposes. In this context, we cannot ensure that the federal government behaving as Stackelberg leader replicates the second-best equilibrium. We only have some guarantees of that when the states' reaction function indicates that an increase in the federal tax rate is followed by an increment in the state tax rate, and vice versa. Other result we find is that the optimum federal tax rate has not to be necessary negative in order to correct both vertical externalities.

Further research can be initiated on the basis of this paper. One interesting point would come from introducing households mobility across heteroge-

neous regions. It would affect the efficiency of the equilibria, which would have to be restricted in order to avoid multiple solutions. Moreover, horizontal externalities would arise and the set of policy instruments probably should be enlarged to take into consideration transfers between governments; otherwise, the replication of the second-best outcome may well become impossible.

Second, given the critical role of the states' reaction function on the effectiveness of federal policies, empirical researches could focus on how the state governments modify their behaviors when facing federal decisions. To the best of our knowledge, there is a stimulating lack of empirical papers on this issue. Papers such as Besley and Rosen (1998), Esteller-More and Sole-Olle (2001) or Anderson et al. (2004) could be enlarged to deal explicitly with issues related to the interplays between the expenditure and tax externalities and the MCP. The empirical analyses should consider here not only the MCPF (which in fact is not a very usual aspect in this kind of approaches), but also the tax revenue effect arising when there exist complementarities between public spending and tax revenues.

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