The atom: fragments of a networked history

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Abstract

This work presents some historical and disciplinary fragments associated to the development of the atomic model, according to the perception that a diversity of scientific and epistemological questions to be investigated, just as interesting as the models themselves, lies behind each fragment. From the meticulous works developed by Thomson and Rutherford to the brilliant contributions from mathematicians and physicists like Cauchy and Schrödinger, the consistency of the models which they worked on could be identified in such a way that it was possible to further expand the body of knowledge related to the atomic model. This is the starting point for an investigation intending to provide a renewed assessment of the interdisciplinary contributions from current science.

Keywords: Atomic model, interdisciplinary relationships, historic milestones.

Resumen

En este trabajo se presentan algunos fragmentos histórico-disciplinares relacionados con la construcción del modelo atómico, bajo la percepción de que tras cada uno de ellos hay una diversidad de cuestiones científicas y epistemológicas para indagar, tan interesantes como los modelos mismos. Desde los minuciosos trabajos de Thomson y Rutherford hasta las inteligentes contribuciones de matemáticos y físicos como las de Cauchy y Schrödinger, es posible identificar la consistencia de los modelos con que trabajaron, de manera tal que permitieron seguir ampliando el volumen de conocimientos en torno al modelo atómico. Este es el inicio de una exploración que pretende aportar a una valoración renovada de las contribuciones interdisciplinarias de la ciencia actual.

Palabras clave: Modelo atómico, relaciones interdisciplinarias, hitos históricos.

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I. INTRODUCTION

The possession of vast expertise about the atomic model is one the most valuable tools in order to acquire significant knowledge about physics from its most modern perspective.

By “significant”, we mean the possibility of establishing relationships among concepts belonging to a field of knowledge whose borders are receptive to contributions essentially derived from mathematics and chemistry. The above said does not set aside the huge information flow currently transmitted through the massive media focused on the general public, like the television and the internet, though the latter implies a higher level of personal interaction between the user and the communication medium.

The first antecedent can be traced back to the early 19th century, when Dalton, Gay-Lussac and Avogadro started to develop *‘a logical foundation for the existence of the atom’* [1].

Which is the difference between the greek atomic philosophy, posed four hundred years before Christ, and the 19th century stance? That the former was a purely speculative approach, mainly rooted in faith. However, it certainly had the merit of an amazingly profound reflection capacity.

The fact that man has been able to go into the privacy of the atomic-nuclear sphere not only allowed providing answers from another perspective but also opened spaces for new questions. This mechanism, along with the development (sometimes occurring simultaneously and sometimes not) of mathematics and chemistry, has led physics to its present stage, when the conjunction of the classical foundations and new fields of knowledge allow us to envision and project worlds which were unsuspected until just a few decades ago.

The research on electricity and magnetism carried out by Gauss, called the "Prince of mathematics", is one of the most representative examples of such alliances, not only due to its relevance in this particular subject but also because of its impact on the thinking of physicists and chemists, Thomson among them, who conducted nearly all his studies during the troubled period experienced by physics in the late 19th and early 20th centuries.

Another interesting case is that of Ernest Rutherford, whose solid theorical-practical training enabled him to present a revolutionary atomic model based on an extensive background of experiments with radioactive elements.

Furthermore, we cannot fail to mention Planck's daring idea of energy discretization, which encouraged physicist Niels Bohr to consider an atomic model which, though current for about a decade, was very important for considering the atom from the new perspective offered by quantum physics.

It is relevant to bring up the intervention of Schrödinger with his partial derivatives equation, which was used in complex vibrant systems and presents the discontinuous succession of *"proper values"* corresponding to the succession of energy levels found in the atom [2].

II. BETWEEN WAVES AND PARTICLES

When attempting a rough reconstruction of the development of particle physics, it is inevitable to go back to the initial and extraordinary work made by first-class physicists like Thomson, Crookes, Helmholtz, Goldstein, Plücker, Millikan and Rutherford, among others[i].

Part of Thomson's work will be used as an initial reference, since it is possible to collect a significant amount of contributions from his predecessors and contemporaries, based on it.

Immediately after graduating from Cambridge in 1876, Thomson began his investigations from a theorical-mathematical perspective within the field of moving electric charges. He started from Maxwell's work in electromagnetism, first published in 1873, which did not yet clearly reflect the existence of electric "charges". A few years later, in 1881, Helmholtz carried out an exhaustive interpretation of Faraday's writings about his experiences with different electrolytes[ii].

At the same time, there were plenty of studies conducted with discharge tubes aimed at reaching some agreement about the nature of cathode rays. It is worth noting the prevailing atmosphere in the scientific area[iii].

In this sense, Williams Crookes described two properties which gave Thomson clues to think that cathode radiation/radiant matter was of corpuscular nature: that *"matter flows"* were deflected by a magnet and that shadows were cast when opaque objects interposed.

Upon presenting his work in 1897, and true to his strict reasoning, Thomson unfolded two verification procedures for the mass-to-charge ratio obtained.

Mass-to-charge ratios of several ions had been obtained based on electrolysis experiments, and Thomson recognized that calculating the mass-to-charge ratio of the cathode ray particle would help identify such particle, either as an ion or any other charged fragment. Consequently, he determined the mass-to-charge ratio (e/m) using two different methods [1].

A. First method

In the first determination, Thomson bombarded an electrode with cathode rays and measured the current delivered to the electrode and the resulting temperature rise. Based on the temperature rise and electrode capacity, he calculated the energy (*E*) delivered by the cathode ray particles and concluded that it was equivalent to the kinetic energy of such particles:

$$E = \frac{N \cdot mv^2}{2}. \tag{1}$$

Where N: number of particles, m: mass of each particle.

The total charge (*Q*) collected by the electrode during the experiment depends on the number of particles (*N*) and the charge each contains (*e*):

$$Q = Ne. \tag{2}$$

The Q/E quotient between both equations results as follows:

$$\frac{Q}{E} = \frac{2}{v^2}\left(\frac{e}{m}\right). \tag{3}$$

Thomson measured *Q* and *E* and, in order to calculate the e/m ratio, all he needed was to measure the speed of the particles. To this end, he measured the deflection of particles through a magnetic field *H*, whose force is known. When applying such field, the particles move at a speed v following a circular path of radius r:

$$v = \frac{erH}{m}. \tag{4}$$

It follows that:

$$\frac{e}{m} = \frac{v}{rH}. \tag{5}$$

And based on the Equation 2, the speed is equal to:

$$v = \sqrt{\frac{2E}{Q}\left(\frac{e}{m}\right)}.$$

Now the speed value is substituted in (5):

$$\frac{e}{m}=\frac{\sqrt{\frac{2E}{Q}\left(\frac{e}{m}\right)}}{rH},$$

by squaring both members:

$$\frac{e^2}{m^2}=\frac{\frac{2E}{Q}\left(\frac{e}{m}\right)}{r^2H^2}.$$

Simplifying:

$$\frac{e}{m}=\frac{2E}{Qr^2H^2}. \quad (6)$$

The merit of this equation is to allow the measurement of the magnitudes of the second member, thus obtaining the mass-to-charge ratio in a fairly simple way.

B. Second method

The idea is to use a device like that shown below:

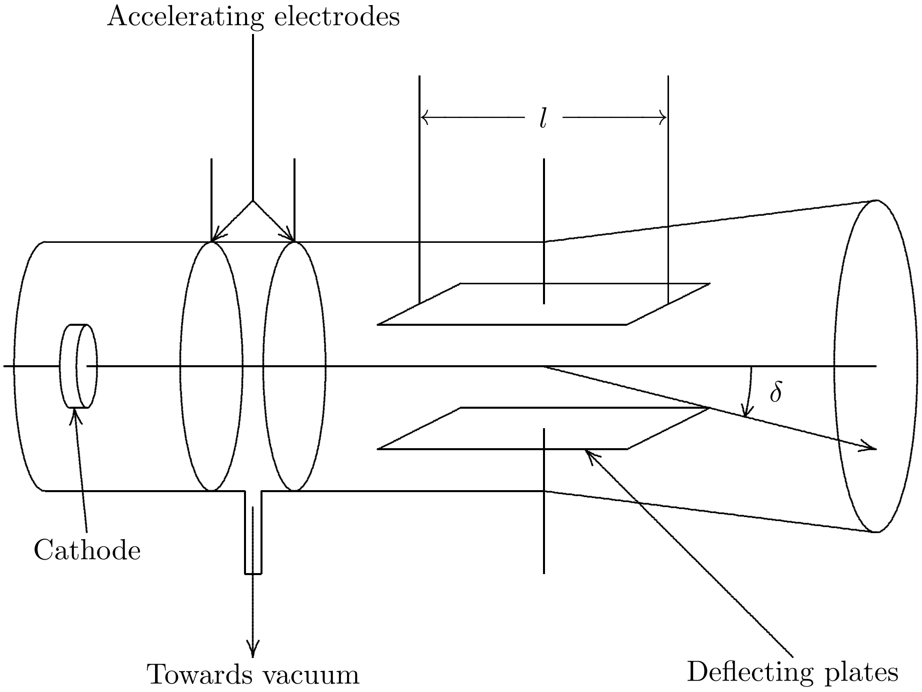

FIGURE 1. Schematic drawing of Thomson's device for measuring the mass-to-charge ratio [1].

A beam of cathode ray particles goes through an area where it can be subject to electric and magnetic fields. Any of those fields, applied in isolation, can deflect the ray from its horizontal path; however, the direction of the magnetic deflection is opposite to that generated by the electric field.

Therefore, if the electric field is applied and kept constant, the magnitude of the magnetic field may be regulated in order to make the beam go back to its original horizontal path.

Under such conditions, the force derived from the magnetic field $(F_m = Hev)$ exerted on particles is equal to that from the electric field $(F_e = eE)$:

$$Hev = eE. \quad (7)$$

Where:

$$v=\frac{E}{H}.$$

The magnetic field is then eliminated, and the ray deflection caused by the electric| field is measured. As particles move through the space between the plates, the electric force eE causes a deflection δ which can be calculated using the similar triangles method, based on the displacement of the spot observed at the end of the tube. The electric force is related to the Newton's second law:

$$eE = F_e = mass \times acceleration,$$

$$eE = ma,$$

$$a=\frac{eE}{m}. \quad (8)$$

Furthermore, the deflection δ can be calculated using the classical movement:

$$\delta=\frac{1}{2}at^2. \quad (9)$$

Then, time can be calculated by relating the length of the plates and the speed:

$$t=\frac{l}{v}. \quad (10)$$

Substituting a and t in the Equation (9):

$$\delta = \frac{1}{2}\frac{E_e}{m}\left(\frac{l}{v}\right)^2 ,$$

$$\frac{e}{m} = \frac{2\delta v^2}{El^2} . \qquad (11)$$

And taking into account:

$$v = \frac{E}{H} .$$

$$\frac{e}{m} = \frac{2\delta}{l^2}\frac{E}{H^2} . \qquad (12)$$

As with the first method, the key point about the expression (12) is that the magnitudes of the second member can be experimentally measured.

The importance of the e/m ratio of the cathode rays became evident when its value was compared to the mass-charge quotient of ions, which had been obtained through electrolysis experiments. The mass-charge ratio of cathode rays is over 1,000 times higher than that of any ion. Besides, while the mass-charge quotient of several ions differed, those of cathode rays were consistent, regardless of the gas used in the discharge tube.

These facts led Thomson to deduce that cathode rays were not electrified atoms, but corpuscular fragments of atoms, *i.e.*, electrons, expressed in current terms [1].

III. CONTRIBUTION OF THOMSON TO THE CORPUSCULAR MODEL AND SOME ANOMALIES

In the early 20th century, and in view of the promising results from several experiences —coupled with the advances in investigations related to spectroscopy and magnetic, electric and optical properties of substances in their various states of aggregation—, favorable conditions were created for scientists to be able to present ideas about how the so much discussed atoms could be like[iv].

Based on this model, it was possible to explain atomic neutrality, the huge difference between atomic mass and electron mass, and also the "mobility" of electrons.

Furthermore, an early, though insufficient, explanation of the emission of electromagnetic radiation was provided: if the atoms were at a minimum energy state, the electrons remained in their equilibrium positions; instead, if the atoms were at an excited state for some reason, the electrons vibrated around such positions. According to the classical electromagnetic theory, these charged particles, when vibrating, released electromagnetic radiation.

The problem was that the experimentally recorded electromagnetic emissions did not match the calculations based on this model.

What calculations are we talking about?

Let us take an example given by [3]:

I) Assuming that an electron charge *–e* within a spherical region carrying uniform positive charge density ρ (hydrogen atom of Thomson). Demonstrate that, if that electron has kinetic energy, its motion will be that of a simple harmonic oscillator whose equilibrium point is the center of the sphere.

Firstly, it should be noted that the charge density (ρ) of an evenly distributed positively charged sphere is the following:

$$\rho = \frac{q}{V} .$$

The volume of a sphere with a radius *r* equals $4\pi r^3 / 3$.

Therefore, the sphere charge can be calculated by means of the following expression:

$$q = V\rho, \qquad i.e. \qquad q = \tfrac{4}{3}\pi r^3 \rho .$$

Furthermore, the law of Coulomb for two point charges states that:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2} .$$

Where q being the sphere charge, q_0 (referred to as e) being the electron charge, and a being the distance the electron moves from the equilibrium point, where *a* is narrower that the sphere radius:

$$F = -\frac{1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi a^3 \rho\right)\frac{e}{a^2} .$$

This results in:

$$F = -\frac{\rho e a}{3\varepsilon_0} .$$

By referring to the general equation to calculate F in a simple harmonic movement (F = -k a), it can be observed that:

$$k = \frac{\rho e}{3\varepsilon_0} .$$

If the electron is released at the a point without initial speed, this force will create a simple harmonic movement along the diameter of the sphere, since it is always directed towards the center and its intensity is proportional to the displacement from the center.

II) It must be noted that the total positive charge has the same magnitude of an electronic charge (so the atom has no net charge) and is distributed on a sphere with a radius $r' = 1.0 \times 10^{-10}$ m. The constant *k* of force and the frequency of movement of electrons must be determined.

As we know, density is the charge-volume ratio:

$$\rho = \frac{e}{\frac{4}{3}\pi r'^3} .$$

Therefore:

$$k = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r'^3} = 10^{-7} c^2 \frac{e^2}{r'^3} ,$$

$$k = 9.0 \times 10^9 \frac{Nm^2}{C^2}\frac{(1.6\times 10^{-19} C)^2}{(10^{-10} m)^3} ,$$

$$k = 2.3 \times 10^2 N/m .$$

Then the frequency of the simple harmonic movement is:

$$f = \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{230 N/m}{9.11\times 10^{-31} kg}} ,$$

$$f = \nu = 2.5 \times 10^{15}\, s^{-1}.$$

Since the radiation emitted by the atom (in analogy with the radiation emitted by the electron oscillating in an antenna) will have the same frequency corresponding to the wavelength:

$$\lambda = \frac{c}{\nu} = \frac{2.9979{\times}10^{8}\, m/s}{2.5{\times}10^{15}/s},$$

$$\lambda = 1.2 \times 10^{-7}\, m = 1200\mathring{A}.$$

This wavelength is found in the electromagnetic spectrum region corresponding to the far-ultraviolet. An electron moving in a stable circular orbit, whose radius is narrower than the radius of the atom of Thomson, makes revolutions at the same frequency and therefore, will also emit radiation at the same frequency. Assuming a radius different from the positively charged sphere would actually result in a different frequency. However, the fact that a hydrogen atom has only one characteristic emission frequency is at odds with the huge number of different frequencies observed in the hydrogen spectrum [3].

Thomson is a true representative of his age. He was fortunate enough to live, in a committed way, in a crucial period for scientific development: the last decades of the 19th century and the first decades of the 20th century. It was then when physics framework was dismantled and reborn with a completely renewed vision.

Being heir to solid mechanics of Newton and Gauss remarkable studies about the behavior of electric fields through closed surfaces, Thomson had enough impetus to decidedly incorporate the *"corpuscle"* (now electron) concept to his investigations, thus introducing the revolutionary idea of the divisibility of atoms. He did it despite strong criticism from the scientific conservative wing at the time [4, 5, 6, 7].

IV. EARLY STAGES OF THE NUCLEAR MODEL: RUTHERFORD

When Ernest Rutherford tested model of Thomson in 1911, he paved the way for nuclear atom.

Let us observe the following figure:

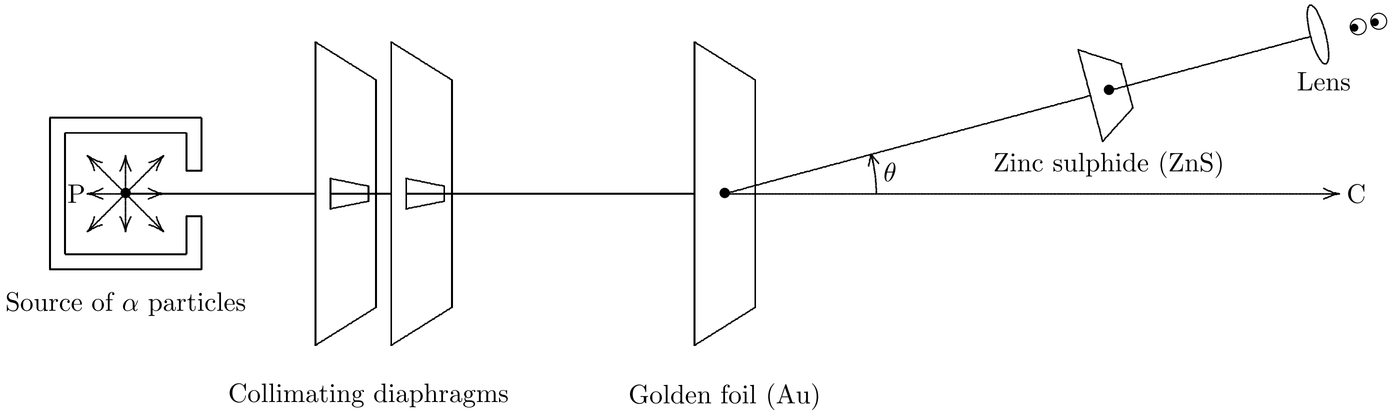

FIGURE 2. Schematic drawing of Rutherford's experiment using alpha particles [1].

The purpose was to study the scattering of α particles (nuclei of atomic mass of helium equal to 4) going through think layers of various substances.

The alpha (α) particles released by the radioactive source go through two collimating diaphragms which direct the beam towards a thin metallic plate. Upon going through it, each α particle is deflected according to its corresponding path, thus deflecting the outgoing beam. Calculating the number of α particles which are scattered into each angle interval between Φ y dΦ, a divergence measurement can be obtained.

The particles impact on a zinc sulphide (*ZnS* crystal) surface, which has the property of emitting one flash after each impact. A microscope is also available to help count the number of impacts per time unit, based on the angular-position detector.

The speed values of the alpha (α) particles can be calculated using the magnetic deflection method (see Equation 4).

Rutherford was aware of the fact that the kinetic energy of α particles was quite strong; therefore, in order to deflect that energy, the atom should support a huge amount of electric force exerted by a massive body. An electron was supposed to be carried by an alpha (α) particle. In this sense, he made a relevant inference: the atom could not be a sphere featuring uniform mass and charge density (as posed in model of Thomson), on the contrary, the atom is highly non-uniform.

While the electrons may take up the volume calculated for the atom (10^{-8} cm), the positive electricity should be concentrated on a very tiny though dense "nucleus".

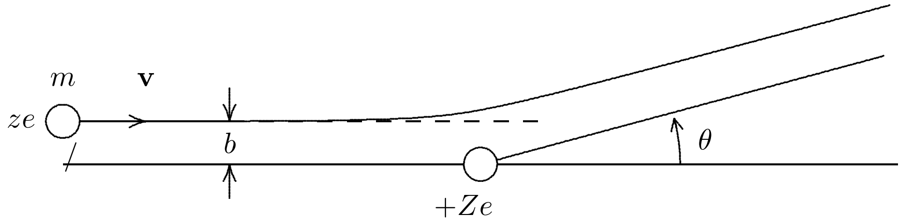

FIGURE 3. Geometrical detail of the deflection an alpha particle may experiment. [1]

Assuming that the forces acting between the nucleus and the alpha (α) particle responded to Coulomb's Law, Rutherford demonstrated that the path followed by such particle, deflected by an atom, must be a hyperbola. Figure 4 shows that the deflection angle Θ (external angle between the asymptotes of the hyperbola) depends of the impact parameter b. It can be mathematically proved that:

$$\tan\left(\frac{\theta}{2}\right)=\frac{Zze^2}{mv^2b}. \qquad (13)$$

z and Z are the atomic numbers of the α particle and the nucleus.
e: magnitude of the electronic charge.
m: mass of α particle.
v: speed of α particle.

If $b = 0$, Θ = 180º, what can be expected from a head-on collision.

When conducting a specific scattering experiment:

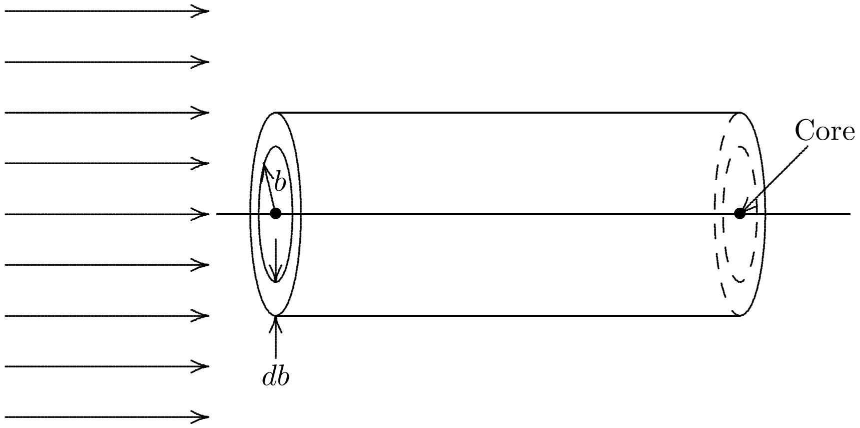

FIGURE 4. Schematic drawing which represents the probability that a particle goes through the ring formed by *b* and *b*+*db* [1].

It can be stated that: z, Z, m and v are constant values and, as a relatively wide α particle beam is used, all values from parameter b are present and the scattering pattern can be observed in all angles.

If we work it out using the ring formed by b and db, it can be observed that the probability of the impact parameter b falling in this area is proportional to the surface of such parameter:

$$\acute{a}rea = 2\pi\cdot b\cdot db. \qquad (14)$$

The area expands along with b; therefore, the highest b values are more likely than low values. This means that there are a few particles which deflect with large angles.

Rutherford deduced that the fraction $f(\Theta)$ of the initial α particles which were scattered through a Θ angle derives from:

$$f(\theta)=2\pi t\rho\left(\frac{Zze^2}{2mv^2}\right)\frac{\sin(\theta)}{\sin^4\left(\frac{\theta}{2}\right)}, \qquad (15)$$

t: plate thickness.
ρ: density in atom/cm^3.

The atomic number of the scattering nucleus (Z) could be calculated based on this expression.

As an added value, the size of the nucleus can also be approximately calculated. Let us consider the following scenario:

An α particle is deflected at 180º and a head-on collision occurs between that particle and a nucleus. Upon that collision, the alpha (α) particle approaches the nucleus until the Coulombic potential repulsion energy, zZe^2/r^2, equals its initial kinetic energy ½ mv^2. Symbolically represented:

$$\frac{1}{2}mv^2=\frac{Zze^2}{r_{\min}}. \qquad (16)$$

$r_{\min}$: distance of closest approach.

It can be calculated based on such equivalence.

For example, in the case of α particles derived from the nucleus breakup and including a scattering copper nucleus:

$$r_{\min} = \frac{2Zze^2}{mv^2} = \frac{2(2.29)(4.8\times10^{-10})^2}{(6.68\times10^{-24})(1.6\times10^9)^2}\,cm,$$

$$r_{\min} = 1.6\times10^{-12}\,cm.$$

According to law of Coulomb, the particles may approach the nucleus up to nearly 10^{-12} *cm* and even be scattered. The nucleus must be smaller than 10^{-12} cm. The law of Coulomb proved inapplicable in the case of lighter nuclei and faster *α* particles, which get the nucleus closer than 0.8 x 10^{-12} *cm*. This means that the positive charge of the nucleus spreads out in a sphere whose approximate radius is 10^{-12} *cm*.

Based on such clever experiments, it can be inferred that not only a qualitative indication of the existence of the nucleus was obtained but also a quantitative measurement.

V. BLACK BODY RADIATION AND A DIFFERENT EXPLANATION BY THE STANDARDS OF THE TIME

One of the most evident signs of the interrelationships underlying the diverse and apparently isolated information which scientists deal with was presented by Max Planck, when he was compelled to provide an explanation for black-body radiation.

By the end of 1870, Planck was pursuing studies in Berlin. His teachers included Helmholtz and Kirchoff, among others. But his natural curiosity led him to explore the work of Rudolph Clausius. In this way, he started his thermodynamics studies. In turn, this led him to work on irreversible radiation processes. The results from such works were published in 1900, immediately followed by strong criticism from Boltzmann. At this point, it must be noted that Planck was aligned with those scientists who rejected energy discretization, which contradicted ideas of Boltzmann. He agreed with Thomson about the corpuscular hypothesis.

Supported by this vast experience in thermodynamics, Planck used arguments derived from that field of knowledge to respond.

The issue Planck tried to figure out was the amount of radiation given off by a heated body. Until then, it had been concluded that the intensity of radiation, caused by vibrating atoms, increased to a maximum value along with wavelengths and then decreased. What Planck intended to find out was an equation which described the amount of radiation emitted for all imaginable wavelengths [8].

A black body may be an enclosed chamber subject to various temperature rises, but also an animal subject to infrared radiation.

The key issue about the black-body concept is the capacity to absorb and emit, almost perfectly, a certain level of electromagnetic radiation.

The varying energy radiated by the temperature of an object was a well established law in the late 20th century, referred to Stefan-Boltzman Law:

$$R_{\mathrm{T}} = \sigma T^4, \qquad (17)$$

where:

R_{T}: Radiance.

σ : Boltzmann constant, (5.67 x 10^{-8} W/ m^2 K^4).

This law states that a 2-fold increase in the temperature of an object causes a 16-fold increase in the radiation rate, but does not indicate how the energy related to the radiation frequency is distributed.

In the early 20th century, Rayleigh and Jeans calculated the value of the radiated energy at an *f* frequency with the black body at a fixed temperature T, applying classical electrodynamics:

$$\rho_{\mathrm{T}}(f) = \frac{8\pi f^2 k_{\mathrm{B}} T}{c^3}. \qquad (18)$$

$\rho_{\mathrm{T}}(f)$: Energy radiated at a given frequency.

k_{B} : Boltzmann constant = 1,381. 10^{-23} J/K.

For example, at 10000 K:

$$\rho_{\mathrm{T}}(f) = \frac{8\pi 10^5(1.381\times10^{-23})}{27\times10^{24}} f^2 \frac{J}{Hz\,m^3},$$

$$= 1.28\times10^{-42} f^2 \frac{J}{Hz\,m^3}.$$

Then, given a frequency of 10^{14} Hz, the electromagnetic energy radiated by the time unit will be equal to:

$$\rho_{\mathrm{T}}(f) = 1.28\times10^{-14}\frac{J}{Hz\,m^3}.$$

Thus it can be observed that, for a given temperature, the energy increases as per the square of the frequency. As the total energy equals the sum of all frequencies from zero to infinity, this formula considers that the total energy radiated will be infinite as well. Due to the amazing discrepancy found when comparing experimental data with the theoretical indication, this fact is known in physics as the *"ultraviolet catastrophe"*.

Faced up to such contradiction, Planck moves away from the classical physics and postulates that the energy is radiated in small separate units to which he called "quanta", a Latin term whose singular form is "quantum" (an amount of something).

Planck integrated the second thermodynamic principle into mechanics. In his studies about the black body, he assumed that the cavity was full of oscillators with very weak buffers and different periods of their own, and supposed that they interchanged energy due to the radiation emitted among them, leading the system to the stationary state of normal distribution of energy.

$$E = \Delta E,\ 2\Delta E, 3\Delta E, \ldots$$

In order to adjust his theory to experimental data, he assumed that a minimum amount of energy, or quantum energy ΔE was proportional to the f frequency of radiation:

$$\Delta E = hf,$$

where h is Planck's constant: $6.626 \times 10^{-34} J\,s$.

Based on his hypothesis, Planck developed the following formula of energy distribution in black body radiation, which reproduces the experimental observations [9]:

$$\rho_T(f) = \frac{8\pi hf^3}{c^2}\frac{1}{\exp[\beta hf]-1}, \quad \beta \equiv \frac{1}{k_B T}. \tag{19}$$

Graphically represented in figure 5.

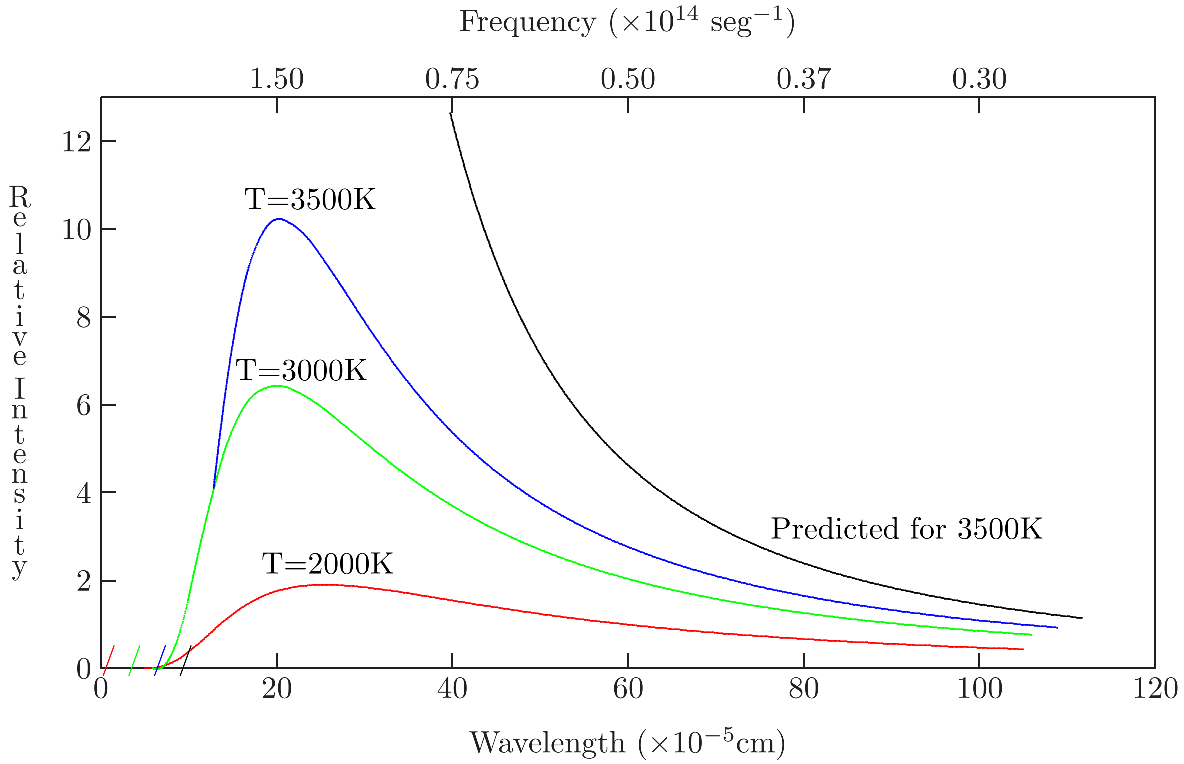

FIGURE 5. The dashed line represents the theoretical variation of energy, calculated through Equation 3, previous to correction of Planck.

According to what was previously stated, the question then is: Is the science developed following a methodological line which gradually articulates logical relationships? During the long scientific processes, there can be small or big leaps (associated to intuition or daring decisions rather than logical inferences) likely to shift investigations towards unpredicted targets. Sometimes they may result in dead end roads, some other times in successful programs, and as a third option, in small contributions to more important modifications. It is worth noting that they may be key unpredicted moments throughout the investigation process [10, 11].

It was within this working and scientific context that Planck established an assumption exclusively intended to justify an algebraic reasoning whose empirical results could be sufficiently consistent. At the time, he never suspected that his assumption would result in the foundation of a new Physics.

It was Danish physicist Niels Bohr who did envisage the potential of this hypothesis and expressed the idea into a model, now called "transition model", which still is a remarkable linking factor in the teaching of science despite the modifications inherent to scientific progress.

VI. THE ELECTRON IN THE HYDROGEN ATOM. AN EXAMPLE OF CENTRAL POTENTIAL APPLICATION

One of the most productive qualities of Mathematics is the possibility of providing tools to explain, with varying degrees of accuracy, how dynamic systems operate. This is the case of the movement of the electrons in the atom.

An initial approach can be made based on the so called Schrödinger equation for a central potential. To begin with, its more general and cryptic format will be considered:

$$H\varphi = E\varphi. \tag{20}$$

E: energy.
φ: wave function.
H: Hamiltonian.

$$H=\frac{p^2}{2m}+V(r), \tag{21}$$

m: electron mass.
p: linear momentum operator.
$V(r)$: central energy operator depending on the radius r.

By introducing algebraically expressed parameters which represent the physical conditions, the following differential equation is obtained:

$$\frac{\hbar^2}{2m}\frac{d^2}{dr^2}P(x)+\left[E-V(r)-\frac{\hbar^2 l(l+1)}{2mr^2}\right]P(x)=0. \tag{22}$$

Just as an introduction and in order to appreciate the relevance of representations, the analysis carried out by [12] based on the Equation 2, starting from the solution of a radial equation, is presented below:

$$\frac{dV(r)}{dr}>0, \quad F=-\Delta V(r)<0.$$

In this case, the value of the potential increases along with the radius.

Working with the Equation (19), the following is obtained:

$$\frac{1}{2}P''(r)+\left[E-\left(V(r)+\frac{l(l+1)}{2r^2}\right)\right]P(r)=0. \tag{23}$$

This can also be expressed as:

$$\frac{1}{2}P''+TP=0. \tag{24}$$

If the kinetic energy is:

$$T=E-U(r),$$

the new function U (r), defined by:

$$U(r)=V(r)+\frac{l(l+1)}{2r^2}, \tag{25}$$

is the sum of the attractive potential and the centripetal potential; the former will prevail for big radiuses and the latter, for small ones.

Figure 7 shows the same information represented in a format which allows a visual reading, thus contributing to a more comprehensive interpretation.

The top section of the schematic drawing outlines the function $U(r)$ in ordinate axes, and the radius in abscissa axes. As the U value is assumed as the source of the energy, when such value is infinite, $V(r)$ is negative.

If the total energy of the moving particle in the central potential is positive, i.e. $E > 0$, the situation is represented by the dotted line parallel to the abscissa axis going through point A. From this point up to infinity, the kinetic energy is always positive and the solution of the Schrödinger equation is similar to that shown below (24):

$$\frac{1}{2}P''+TP=0.$$

This is $\exp(\pm ir\sqrt{2T})$. It is an oscillatory solution which extends to infinity and thus represents a free particle. The solutions of the radial Equation 2 with positive energy corresponds to free particles, and the solution of the differential equation does not require any condition as regards possible energy values, which means that the positive energy spectrum of the differential equation is continuous.

Then, the energy being negative, this situation may be represented by means of the dashed line parallel to the abscissa axis going through B and C. The kinetic energy is positive within the BC interval, and negative outside of it.

Between the points B and C, the solution of the differential equation is also oscillatory, similar to $\exp(\pm ir\sqrt{2T})$; however outside the AB interval, the kinetic energy is negative, $T < 0$, and the solutions are increasing or decreasing in the $\exp(\pm ir\sqrt{2T})$ form. At points A and B, the internal solution of the interval must be smoothly connected with the external solutions, so the function of the internal and external waves, as well as their derivatives, must be equivalent. Figure 1 (bottom) shows a qualitative representation of this type of solution, which also satisfies the conditions: $P(0)=P(\infty)=0$. Such conditions are only satisfied for certain E values, which are distinct values; a discrete spectrum of distinct values for negative energies is obtained. Furthermore, the same figure shows that the radial function $P(r)$ only presents a significant value within a limited radial interval, indicating that only in that region there is a high probability of finding the particle. Therefore, the solutions of the radial Equation (25) with negative energy correspond to bound particles. The radial wave function of a bound state presents nodes, excluding the node of $r = 0$ and including that of $r=\infty$; the number of nodes is designated by $n-1$ (n being, by definition, the main quantum number). This means that the main quantum number can be defined as the sum of the quantum number of the orbital angle momentum plus the number of nodes of the radial function, excluding the origin number but including the infinite number. According to this definition, the result is $n \geq l+1$.

While the interpretation of this type of analysis requires the support of mathematical tools which may or may not be available to secondary-level students, it is possible to introduce some conceptual elements enabling them to connect the study of (electric and/or magnetic) fields with such useful constructs for the development of physics (and chemistry) as electrons. Borderline cases can be used, such as the assumption that the electron is included in a certain volume or it is virtually free due its distance from the nucleus. It is here that different boundary conditions, frequently omitted in the problems presented to students, may appear. The study of differential equations in partial derivatives and the search for solutions, developed by mathematician Cauchy, is one of the contributions which supported physicians and chemists in their efforts to understand the dynamics of the microscopic world systems, in which complexities increase exponentially (from treating one-electron to many-electron systems) due to the multiplying interactions.

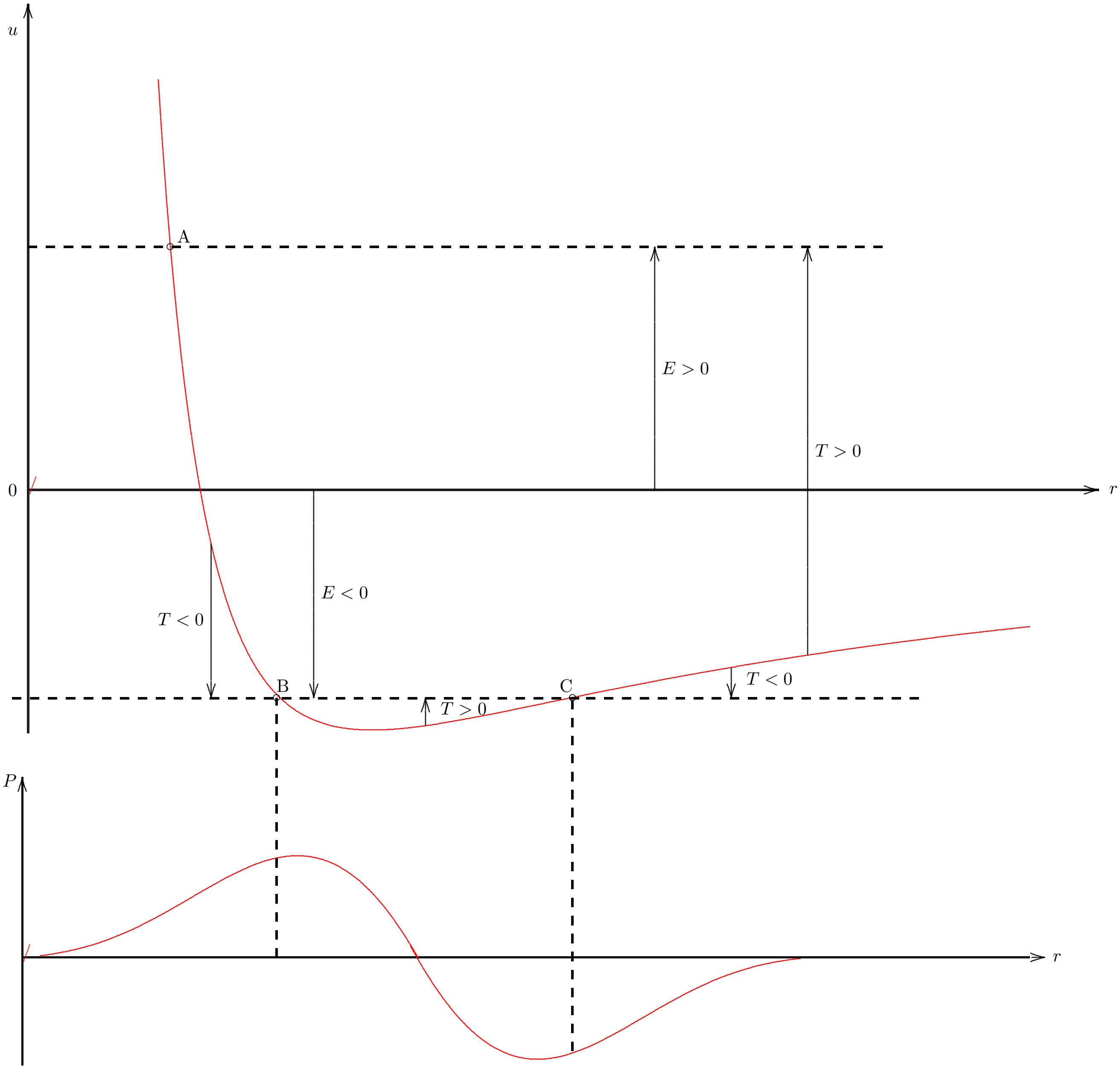

FIGURE 6. Schematic drawing of the potential energy based on the distance from the point of origin. Bottom: Wave function corresponding to a confined state of negative energy [12].

The convergence of analytical and experimental methods led to the current atomic model, which is one of the pillars of modern science.

VII. TO KEEP THINKING ABOUT…

The development of each atomic model involves a complex net of knowledge, personal track records and ideological confrontations which are worth analyzing, as much as the models themselves [13].

Their partial or limited study does not allow us to fully appreciate the huge human undertaking necessary to develop such powerful theories as the quantum theory [9], [14] available today. Based on their solid mathematical foundations, it was possible to provide satisfactory explanations to varied issues such as superconductivity, orientation mechanisms in some birds, etc. Likewise, it was possible to project remarkable applications like quantum computation or the optimization of electronic equipment aimed at laboratory and everyday use.

It is not easy to imagine the present historical moment or what the next challenges to be faced by mankind (which will surely arise). In this sense, we share De la Torre's thinking:

I think that attempting a justification of basic science is a false problem, given that science cannot but exist, as it derives from the intrinsically curious human nature. A justification implies giving the reasons for which the decision of creating or producing what is intended to be justified has been made. Science cannot be justified because it does not derive from a volitional act by which it was decided to create it, but from the ineluctable social manifestation of an individual human characteristic [10].

Although this work presents only some nodes of a complex net, the remaining issue is to continue an extensive and deep investigation of the subject, in order to continue disseminating what constitutes a fundamental axis of physics and chemistry: the atomic model.

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APPENDIX

Los siguientes 4 apartados son de la referencia [4]:

[i] (…) scientific discoveries do not occur separately from technology and culture. Scientists live and work in specific places and periods of time. Their work and discoveries are deeply affected by certain circumstances, such as: a) technical resources available; b) scientific atmosphere (*i.e.,* what can basically be called a fashionable trend about acceptable and interesting research topics); c) global perspective about culture, either explicit or implicit; and d) economic and social scenario which allows time, human energy and money to support scientific investigation. [4].

[ii] Thus established, Faraday's law states that equivalent electric and chemical motion can always be observed through each section of a conductor. The same definite quantity of positive or negative electricity always moves together with each univalent ion or each unit of affinity of a multivalent ion, and accompanies it during all its motions through the interior of the electrolytic fluid. Such quantity can be called electrical charge of the atom. This may be the most remarkable result of Faraday's law. If we accept the hypothesis that elementary substances are composed of atoms, we cannot but conclude that electricity, either positive or negative, is divided into definite elementary portions which behaves like atoms of electricity. [15].

[iii] (…) physicists had two models available which, though contradictory, adjusted to the facts fairly well. Crookes and the British physicists generally advocated for the particle model; however, they recognized that the model specifically developed by Crookes (that of negatively charged molecules) could require some changes. Goldstein, Hertz and other German physicists advocated for the wave model, though the magnetic deflection of rays posed a problem. It is not irrelevant to state that each group must have not disclosed their judgments. However, driven by human nature, every group supported a point of view, at least as a rather solid working hypothesis. It is difficult to drop a point of view, once it has been adopted. In fact, the

controversy turned out to be fruitful, as many of the brilliant experiments carried out during the following two decades were developed to shatter the opposite argument, in one way or another. The relationships among some participants cooled down, though perhaps it was not a high price to pay for what could be called the onset of modern physics [4].

[iv] During the first decade of this century, Thomson himself attempted an explanation of some of the optical and chemical properties of atoms using a model which conceived the atom as a positively charged spherical cloud with embedded electrons. Through mathematical analysis, Thomson found out that certain configurations of those electrons would be mechanically stable. He considered that several configurations might correspond to different chemical elements. The movement of electrons within the cloud involves oscillations whose frequency is roughly adequate to explain light emission and absorption. It is assumed that, in some cases, the outer electrons got detached easily. This accounted for electric conductivity, as well as cathode ray, photoelectrons and beta-rays emission. This model seemed to explain, at least from a qualitative perspective, many of the known atom properties.

Unfortunately, it was proved difficult to perform, based on this model, many measurements which were accurate and experimentally verifiable [4].