INEQUALITY CHANGES OVER TIME, PRO-POOR GROWTH AND RERANKING: LOCAL FOCUS AND GLOBAL ASSESSMENT.

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Abstract

When measuring the change in inequality the conclusion based on an overall inequality measure can hide some important changes at different parts of the distribution. In this paper we propose the decomposition of the inequality change measured through a family of inequality measures proposed by Imedio et al. (2013) into measures of pro-poor income growth and reranking in the income pecking order. Moreover, making use of the result of Imedio et al. (2013) that shows that the Bonferroni index can be seen as the sum of inequality measures that give more weight to inequality in different parts of the income distribution (inequality measures with local focus) we propose a decomposition that reveal the extent to which each part of the income distribution contributes to the global measure of inequality, reranking and pro-poor growth. These results make the analysis of changes in inequality more insightful, as they help to evaluate the homogeneity of the change in inequality, reranking and pro-poor growth along the income distribution and enable to estimate the contribution of the "local" measures to the global one. We use this framework to analyze the effect of the crisis on employee income and self-employment income in 11 European countries.

JEL codes: D31; I32
Key words: inequality, reranking, pro-poor growth, Bonferroni index

1. Introduction

Silber (1995) showed how the Gini index could be the basis for measuring distributional change. He proposed two indices of distributional change, a population- and an income-weighted index, that were derived from the Gini index and whose properties were quite similar to those of the entropy related index of distributional change proposed by Cowell (1985). These indices of distributional change may in fact be considered as indices measuring the degree of income mobility (see, Fields and Ok, 1999, for a thorough review of measures of income mobility). Jenkins and Van kerm (2006) showed that when income inequality is measured using any member of the generalized Gini class of indices, the change in inequality between two points in time can be additively decomposed into two components, one summarizing mobility in the form of reranking, and one summarizing progressivity in income growth. Later, Silber and Weber (2008) proposed a generalization of Gini related mobility indices that allows us to decide either how much weight to give to individuals who were originally poor (for the population weighted mobility indices) or how much weight to give to those individuals who experienced a low rate of growth of income (for the income weighted mobility indices).

In this paper we propose the decomposition of a wider family of inequality measures (Imedio et al. 2013) that contains as particular cases the generalized Gini class of indices, the indices of Aaberge (2000) and the Bonferroni index, into two components, one summarizing reranking and one summarizing progressivity in income growth. We then make use of the decomposition of the Bonferroni index proposed by Imedio et al. (2013) as arithmetic mean of inequality measures that give more weight to inequality in different parts of the income distribution, that is inequality measures with "local" focus. This result allows us to analyze the change in inequality taking into account the different situations that take place along the income distribution, and to reveal the extent to which each part of the income distribution contributes to the global measure of inequality, reranking and pro-poor growth.

These results make the analysis of changes in inequality more insightful, as they help to evaluate the homogeneity of the change in inequality, reranking and pro-poor growth along the income distribution. We show that the crisis has not had an homogeneous effect on the distribution of employee income and self-employment income in the 11 European countries analyzed, and depict where in the income distribution the change in inequality, the reranking and the pro-poor growth is greater.

The paper is structured as follows. Next section summarizes the main results of Imedio et al. (2013) used in this paper. The decomposition of inequality change into

reranking and pro-poor income growth is derived in Section 3. This same section addresses how the inequality change, the pro-poor growth and the reranking can be written as integrated weighted "local" measures. The empirical illustration that reveals the non-homogeneity of the changes in inequality, of reranking and of pro-poor income growth is contained in Section 4. Section 5 provides a summary and concludes.

2. Measuring inequality: the β class of inequality measures

We measure inequality using members of the β family of inequality measures (Imedio et al. 2013). Let us assume that the income distribution of a population is represented by the random variable X, whose domain is the semi-straight positive real, $R_0^+ = [0, \infty)$, where F is its distribution function[1], and $\mu = E(X) = \int_0^\infty x dF(x) < \infty$ its mean income.

Bonferroni (1930) defines his index through a simple transformation of the Lorenz curve, L(p), the Bonferroni curve[2]:

$$B: [0,1] \to [0,1], \quad B(p) = \begin{cases} \frac{L(p)}{p}, 0 < p \leq 1 \\ 0, \qquad p = 0 \end{cases}$$

It satisfies B(p)≤1, 0≤p≤1. For an egalitarian distribution the curve is B(p)=1, 0<p≤1, whereas if the concentration is maximum, the curve is B(p)=0, 0≤p<1 and B(1)=1.

Although from a formal standpoint the Bonferroni curve represents inequality in an equivalent manner to the Lorenz curve and both curves are determined mutually, the information they yield is different. The values of L(p) are fractions of the total income, while the values of B(p) refer to relative income levels.

From curve B(p), the Bonferroni index[3], B, is defined as

$$B = 1 - \int_0^1 B(p)dp = \int_0^1 (1 - B(p))dp = \int_0^1 (\frac{p - L(p)}{p})dp \qquad [1]$$

Its value coincides with the area between the Bonferroni curve of the existing distribution and the curve corresponding to the case of perfect equality. It is evident that B∈ [0, 1].

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[1] Sometimes F is assumed to be continuous in order to obtain theoretical results in a simpler manner. In such a case, f(x)=F´(x) is the density function of the distribution.

[2] In the following equality, if the minimum income is x_0>0, then $B(0) = \lim_{p \to 0^-} \left(\frac{L(p)}{p}\right) = L'(0^+) = x_0/\mu$

[3] Very little attention has been given to this index in the literature on economics up to relatively recent years. Nygard and Sandström (1981), Tarsitano (1990), Giorgi and Crescenzi (2001) and Giorgi and Nadarajah (2010) refer to the properties of this index. In Chakravarty (2007) and Bárcena and Imedio (2008), the B index is interpreted as a deprivation measure.

Let us assume $D: [0,1] \to R$ is a function such that for each $p\epsilon[0,1]$, $D(p)$ measures inequality accumulated up to percentile p and $\omega: [0,1] \to R$ is a non-negative weight function such that $\int_0^\infty \omega(p)dp = \int_0^\infty \omega\big(F(x)\big)dF(x) = 1$. It is clear that the real number

$$I_{D,\omega} = \int_0^1 D(p)\omega(p)dp$$

measures inequality in the distribution F. Its value depends on the functions D and ω, which respectively introduce a way to evaluate cumulative local inequality and a criterion to weight this inequality along the income distribution. This procedure to generate inequality indexes underlies (sometimes in an implicit manner) the papers of Amato (1948), Giacardi (1950a, 1950b), Mehran (1976), Benedetti (1980), Yitzhaki (1983) and Piccolo (1991), among others.

If we consider the Bonferroni curve, the function

$$D_B(p) = 1 - B(p) = \frac{\mu - E(X/X \leq x)}{\mu} \quad 0 \leq p = F(x) \leq 1 \qquad [2]$$

measures the relative difference between the mean income of the population and the mean income of individuals whose income is lower than or equal to x.

The beta class of inequality measures, introduced by Imedio et al. (2013), is a class of indexes that generalizes and comprises different well-known families of inequality measures as particular cases. In this class of inequality measures, local inequality is measured by means of the function $D_B(p)$ defined in [2], and the density functions of the beta distribution in [0, 1] are used as weights. That is:

$$\omega_{(s,t)}: [0,1] \to R_0^+, \ \ \omega_{(s,t)}(p) = (b(s,t))^{-1} p^{s-1}(1-p)^{t-1}, s > 0, t > 0, \qquad [3]$$

where b(s,t) is the Euler beta function.

The above is set out in the following definitions.

Definition 1. (Imedio et al. 2013) For each $(s,t)\epsilon R^+ \times R^+$, the index I(s,t) is given by:

$$I(s,t) = \int_0^1 D_B(p)\omega_{(s,t)}(p)dp = (b(s,t))^{-1} \int_0^1 (1 - B(p))p^{s-1}(1-p)^{t-1}\, dp. \qquad [4]$$

The biparametric set $\boldsymbol{\beta} = \{I(s,t)\}_{s,t>0}$ is the beta class of inequality measures.

It is immediate that I(s,t) is a relative measure of inequality, with I(s,t)=0 in case of equidistribution and I(s,t)=1 in case of maximum concentration. The elements of **β** are consistent with the ordering of the distribution induced by the Bonferroni curve, and for

s≥2 with those induced by the Lorenz[4] curve. Therefore, the elements of **β** satisfy the Pigou-Dalton Transfer Principle: progressive transfers decrease income inequality.

The **β** class adds a broad set of judgements relative to the weight that the social evaluator attaches to the local inequality accumulated in different parts of the distribution. These judgements are derived from the shape of the function ω(s,t)(·).

The **β** class can also be expressed in terms of the relative income as

$$I(s,t) = 1 - \int_0^{\infty} \frac{x}{\mu} \lambda_{(s,t)}(F(x)) dF(x)$$

where

$$\lambda'_{(s,t)}(p) = -\frac{\omega_{(s,t)}(p)}{p},\ 0 < p < 1,\quad \int_0^1 \lambda_{(s,t)}(p)dp = 1,\quad o \quad \lambda_{(s,t)}(1) = 0$$

$\lambda_{(s,t)}(p)$ is a strictly decreasing function in the interval [0,1]. We can observe that each individual's contribution to aggregate inequality is determined by both, the relative income and the income rank.

Among the elements of I(s,t) we find classic indexes such as Bonferroni (s=1 and t=1) and Gini (s=2 and t=1), generalized Gini coefficients (s=2) and indexes of the family of Aaberge (t=1), and uncommon indexes that are interesting by themselves.

The following proposition gives an alternative interpretation of the Bonferroni index in terms of inequality measures that give more weight to inequality in different parts of the distribution.

Proposition 1. (Imedio et al. 2013)

For each $n \in N, n \geq 2$, B is the arithmetic mean of the indexes $\{I(s,t)\}_{s+t=n}$: I(1,n−1), I(2,n−2), ..., I(n−1,1) . That is, B is:

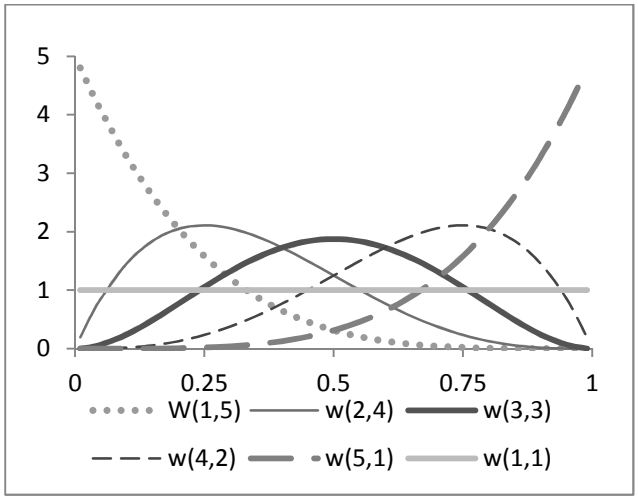
$$B = \frac{1}{(n-1)} \sum_{s=1}^{n-1} I(s, n-s).$$

Therefore, Bonferroni index can be seen as the weighted sum of inequality measures with "local" focus, covering the whole distribution, from the bottom to the top. It considers a broad set of judgements relative to the weight that the social evaluator attaches to the local inequality accumulated in different parts of the distribution. These judgments are derived from the shape of the function $\omega_{(s,t)}$. If 0<s<1, 0<t<1 (respectively if s>1, t>1) less (more) weight is attached to local inequality in the middle incomes and more (less) weight to the tails. These weights are more focused in the middle incomes as s and t are greater and closer to each other. In the rest of the cases, except for s = t = 1, greater

[4] If X and Y are two income distribution functions ($B_X(p) \geq B_Y(p)$, $0 \leq p \leq 1$) $\Rightarrow$ ($I_X(s,t) \leq I_Y(s,t)$). Analogously, the Lorenz consistency: ($L_X(p) \geq L_Y(p)$) $\Rightarrow$ ($I_X(s,t) \leq I_Y(s,t)$).

weight is attached to the local inequality in one of the tails of the distribution. Figure 1 shows the functions ω(s,t), where s+t=6, and s=t=1.

Figure 1. Weighting functions $\omega_{(s,t)}(p)$



This proposition offers new insights that are unavailable from inspecting the Bonferroni index by itself. This expression can be used in the analytical task of assessing the extent to which each part of the income distribution contributes to global inequality as we explain in section 3.

3. Inequality change

In this section we, first, apply the result of the proposition 1 to address how inequality change can be written as integrated weighted "local" measures of inequality change; second, provide an interpretation of the decomposition of the change in inequality in terms of progressivity and reranking components and, finally, obtain the global reranking and progressivity measures as integrated weighted measures with "local" focus.

Consider now the change in I(s,t) between some base year (0) and final year (1) for a fixed population of individuals. Letting $B_j(p)$ denote the Bonferroni curve for year j, the change in I(s,t) can be written as

$$\Delta I(s,t) = I_1(s,t) - I_0(s,t) =$$

$$\int_0^1 \big(1 - B_1(p)\big)\,\omega_{(s,t)}(p)dp - \int_0^1 \big(1 - B_0(p)\big)\,\omega_{(s,t)}(p)dp = \int_0^1 \left(\frac{L_0(p) - L_1(p)}{p}\right)\omega_{(s,t)}(p)dp =$$

$$(b(s,t))^{-1} \int_0^1 (B_0(p) - B_1(p)) p^{s-1}(1-p)^{t-1}\,dp \qquad [6]$$

Or equivalently

$$\Delta I(s,t) = \int_0^\infty \frac{x}{\mu_0}\lambda_{0(s,t)}\big(F_0(x)\big)dF_0(x) - \int_0^\infty \frac{x}{\mu_1}\lambda_{1(s,t)}\big(F_1(x)\big)dF_1(x)$$

Inequality changes are associated with both, changes in individuals' relative incomes and changes in their social weights (which depend on their ranks in the income distribution). As Van Kerm and Jenkins (2006) stated, these two types of changes may not be independent since a large increase in relative income will often be associated with an increase in rank. However, income changes and rank changes are not perfectly correlated.

Moreover, the change in the inequality measure in itself is not informative about local inequality changes. Each index shows more sensitivity to inequality in certain parts of the income distribution and the conclusions based on summary indices may then differ depending on the degree of inequality aversion.

We proceed to address this problem by making use of proposition 1 and suggesting a decomposition of the change in inequality into a weighted sum of the changes in inequality measures more sensitive to certain parts of the income distribution, it is, they show "local" focus.

Corollary 1. From proposition 1 we get that Bonferroni is the arithmetic mean of I(s,n-s) for a given n and s=1, …, n-1. Then we can express the change in inequality measured through the Bonferroni index as the arithmetic mean of changes in inequality with "local"focus.

$$\Delta B = \Delta I(1,1) = B_1 - B_0 = \frac{\sum_{s=1}^{n-1} I_1(s,n-s)}{n-1} - \frac{\sum_{s=1}^{n-1} I_0(s,n-s)}{n-1} \qquad [7]$$

$$= \frac{\sum_{s=1}^{n-1}(I_1(s,n-s) - I_0(s,n-s))}{n-1}$$

As each index I(s, n-s) weights inequality placing more weight in different parts of the distribution, from the bottom to the top part of the distribution, this decomposition enables us to roughly estimate the contribution of each "local" measure to the global change in inequality.

Next we provide an interpretation of the decomposition of the change in inequality in the progressivity and reranking components. We can think of two steps by means of which the inequality may be introduced in stages. Starting from the distribution of income in the initial year, year 0, the individuals keep the rank they had in year 0 but they are now given their income in year 1. Let $C_1^0(p)$ be the concentration curve for income with respect to this "lexicographic income parade". The argument p refers to the individuals' rank in an income parade where individuals are ordered by increasing initial incomes. It is easy to observe that for each p, $C_1^0(p)$ corresponds in fact to the ratio of the mean income of the

Np first people in the present stage of the income parade and the mean income of the population in year 1.

Now we take into account the reranking between year 0 and year 1, final year, by rearranging the individuals from the poorest to the richest in the distribution of year 1. This gives us the true income parade. Call $B_1(p)$ the Bonferroni curve in final year. Clearly $B_1(p)$ gives for each p the ratio of the mean income of the p% (true ranking) poorest people in the year 1 and the mean income in year 1.

The transformation $B_0(p) \rightarrow C_1^0(p) \rightarrow B_1(p)$ tracks the two steps procedure, recording first the progressivity of income growth across the base year income distribution, and then the reranking.

In short we may define P(s,t) and R(s,t) as:

$$P(s,t) = \int_0^1 (C_1^0(p) - B_o(p))\omega_{(s.t)}dp = I(s,t)_0 - I(s,t)_1^0 \quad [8]$$

$$R(s,t) = \int_0^1 (C_1^0(p) - B_1(p))\omega_{(s.t)}dp = I(s,t)_1 - I(s,t)_1^0 \quad [9]$$

where $C_1^0(p)$ is the concentration concentration curve and

$$I(s,t)_1^0 = \int_0^1 \left(1 - C_1^0(p)\right)\omega_{(s.t)}(p)dp$$

an index of concentration.

P(s,t) measures the relative difference between the year 1 mean income of those individuals whose income was lower than or equal to x in year 0 and the year 0 mean income of individuals whose income was lower than or equal to x in year 0, these differences are weighted along the income distribution through $\omega_{(s,t)}$. It summarizes the progressivity or pro-poor income growth across the base year income distribution. When everyone experiences equi-proportionate income growth, relative incomes remain constant, and P(s,t)= 0. When $\mu_1 > \mu_o$ and there is no equi-proportionate income growth, but income growth is more concentrated at the bottom of the distribution P(s,t) >0. Then we could talk about pro-poor income growth (progressive). By contrast, if income gains are more than proportionally concentrated among richer individuals than poorer ones P(s,t) <0. This would be the case of non pro-poor income growth (regressive). The opposite takes place when $\mu_1 < \mu_o$.

R(s,t) measures the relative difference between the year 1 mean income of those individuals whose income was lower than or equal to x in year 0 and the year 1 mean income of individuals whose income is lower than or equal to x in year 1, these differences

are weighted along the income distribution through $\omega_{(s,t)}$. It summarizes the reranking from the initial to the final year.

This leads to the following proposition.

Proposition 2. The change in inequality measured through inequality measure belonging to the β family can be decomposed in two terms, progressivity and reranking, (for s=2, it is the Jenkins and Van kerm, 2006, expression):

$$\Delta I(s,t) = R(s,t) - P(s,t). \qquad [10]$$

R(s,t) and P(s,t) can be interpreted further by rewriting them in terms of the joint distribution of incomes in years 0 and 1. Letting h denote the joint probability density function of incomes in years 0 and 1, we can also write:

$$P(s,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \lambda_{0(s,t)}\big(F_0(x)\big)\left(\frac{x_1}{\mu_1} - \frac{x_0}{\mu_0}\right) h(x_0, x_1)\,dx_0\,dx_1$$

$$R(s,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\lambda_{0(s,t)}\big(F_0(x)\big) - \lambda_{1(s,t)}\big(F_1(x)\big)\right]\left(\frac{x_1}{\mu_1}\right) h(x_0, x_1)\,dx_0\,dx_1$$

These expressions show that P(s,t) is a social-weighted average of the changes in relative incomes between years 0 and 1 with weights determined by year 0 ranks. To interpret the measure further, following Jenkins and Van kerm (2006), P(s,t) can be rewritten as

$$P(s,t) = \frac{\varphi}{1+\varphi} K(s,t)$$

where $\varphi = (\mu_1 - \mu_0)/\mu_0$ is the proportionate change in the average income of the population as a whole, and K(s,t) is a generalized Kakwani (1977)-type index of progressivity summarizing the proportionality of individual income growth,

$$K(s,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \lambda_{0(s,t)}\big(F_0(x)\big)\left(\frac{x_1 - x_0}{\mu_1 - \mu_0} - \frac{x_0}{\mu_0}\right) h(x_0, x_1)\,dx_0\,dx_1$$

Proportionality refers here to the proportionality of individual income changes between years 0 and 1 with respect to the reference point of year 0 incomes. The results are the same as before, when $\mu_1 > \mu_0$, $P(s,t) > 0$ if income growth is concentrated more among poorer individuals in the bottom of the distribution, and $P(s,t) < 0$ when income gains over time are more than proportionally concentrated among richer individuals than poorer ones. The opposite takes place when $\mu_1 < \mu_0$.

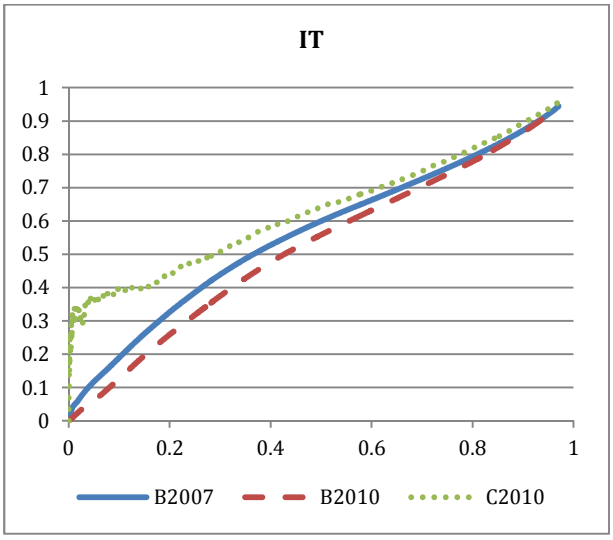
Giving greater weight in the evaluation to initially-poorer individuals is consistent with, but not exactly the same as, a social preference for greater equality in final-year

incomes than in base-year incomes. Greater equality in final-year incomes is guaranteed only if the pattern of income growth does not lead to re-ranking of individuals between the two years that is sufficiently large to offset the progressive income growth (Jenkins and Van Kerm, 2006)

Reranking index R(s,t) is a relative-income-weighted average of changes in social weights. Clearly, when there is no reranking and individual social weights are therefore left unchanged, R(s,t) =0, and R(s,t) >0 otherwise. When s=2 and t=1 then $R(2,1)/I_1(2,1,)$ is the M_{10} asymmetric Gini mobility index', a mobility index in its own right, whose desirable properties are discussed at length by Wodon (2001) and Yitzhaki and Wodon (2004). And this, in turn, has the same form as the Atkinson (1980)-Plotnick (1981) measure of horizontal inequity in the income tax literature.

The decomposition set out in [8], [9] and [10] is based on Bonferroni curves and concentration curves, and can therefore be represented graphically. Figure 2 shows the curves for employee income for Italy taking 2007 as year 0 and 2010 as year 1. The increase or decrease in inequality over this period is represented by the outward shift in the Bonferroni curve. Twice the area between the Bonferroni curves for 2007 and 2010 is the change in the Bonferroni index. With different values for s and t, the differences between the curves are aggregated differently along the abscissa to yield ΔI(s,t).

Figure 2. Bonferroni curves and concentration curves for employee income in Italy.

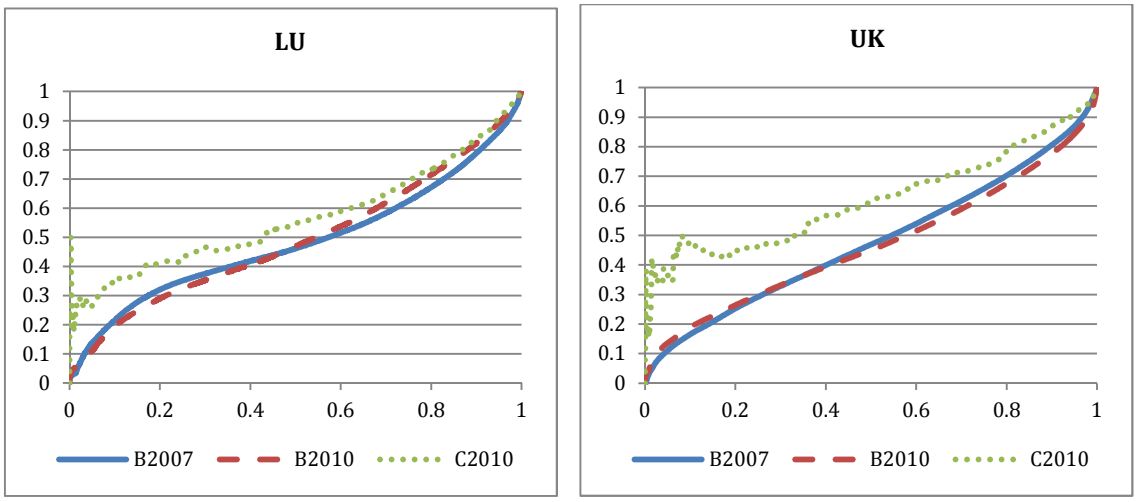


Source: EUSILC 2011

The difference between the Bonferroni curves can be broken down into two parts. One is the difference between the Bonferroni curve for 2007 incomes (B_{2007}) and the concentration curve for 2010 incomes constructed using 2007 income ranks (C_{2010}^{2007}). This summarizes the progressivity of income change: –P(1,1) is twice the area between these

two curves. The second component is the difference between the concentration curve (C_{2010}^{2007}) and the Bonferroni curve for 2010 (B_{2010}), which summarizes the extent of reranking. Note that, by construction, the former lies nowhere below the latter. R(1,1) is twice the area between these two curves. In this case, income decrement is clearly pro-poor, as the concentration curve lies everywhere above the Bonferroni curve for 2007, but this inequality-reducing effect was more than offset by the effect of reranking. In general, the concentration curve may lie wholly below the base year Bonferroni curve, in which case income change is unambiguously regressive. Alternatively, it may have sections above and below the base year Bonferroni curve, in which case it is not clear whether income growth is pro-poor or not. Conclusions based on summary indices may then differ depending on the value of the inequality aversion parameters, s and t. Of course, if the Bonferroni curves intersect the sign (and not only the magnitude) of ΔI(s,t) may depend on s and t, and hence the decomposition as well. This situation arises in our empirical application for Luxembourg and UK (Figure 3).

Figure 3. Bonferroni curves and concentration curves for Luxembourg and United Kingdom, years 2007 and 2010



Source: EU-SILC 2011

As the conclusions might depend on the sensitivity to income inequality in different parts of the distribution, that is, on the values of the inequality aversion parameters, and a global measure is not informative about how income changes are aggregated into the single index number, we go one step further and use the results of the corollary 1 to propose a decomposition that entitle us to disentangle the different situations masked in a summary index of inequality change.

Corollary 1 shows that the change in the Bonferroni index over time is the arithmetic mean of the change in I(s,n-s) for a given n and s=1, …, n-1, that are inequality

measures, more or less sensitive to the bottom, middle or upper part of the distribution, depending on the values of the parameters. This result together with proposition 2 enables to express the change in inequality as a sum of the reranking and progressivity with special focus on certain groups of the income distribution:

Proposition 3. The change in the Bonferroni index can be decomposed into two terms:

$$\Delta B = \Delta I(1,1) = B_1 - B_0 = \frac{\sum_{s=1}^{n-1}(I_1(s,n-s)-I_0(s,n-s))}{n-1} = \quad [11]$$

$$= R(1,1) - P(1,1) = \frac{\sum_{s=1}^{n-1} R(s,n-s)}{n-1} - \frac{\sum_{s=1}^{n-1} P(s,n-s)}{n-1}.$$

The first one is the arithmetic mean of the effect of the reranking when the weights give more importance to different parts of the distribution, from the bottom to the top of the distribution, and the second is the arithmetic mean of the progressivity of income growth when the weights give more importance to different parts of the distribution, from the bottom to the top of the distribution. Therefore we can identify the part of the distribution in which inequality change has more effect and devise if the main cause was the reranking or the progressivity income growth. We can even estimate the contribution to the global measure of progressivity or reranking of the measures that focus on certain groups of the income distribution, from the bottom to the top.

We make use of this result in the next section to analyze the effect of the crisis in terms of inequality in employee income and in self-employment income in 11 European countries and to study how the crisis impacted unevenly in the different parts of the distribution in terms of change in inequality. Moreover, the income growth and reranking components also had a different behavior along the income distribution.

4. Empirical illustration: how crisis affected employee and self-employment income in different ways along the income distribution.

In this section we illustrate the use of the decomposition of the change in inequality applying it to employee and self-employment income measured through the Bonferroni index. We compare the patterns of distributional change in 11 European countries[5] in the period 2007-2010, determine which parts of the distribution contributed more to the change in inequality, to the re-ranking or to the progressivity and thus explain the global result.

[5] AT: Austria; BE: Belgium; DK: Denmark, ES: Spain; FI: Finland; IT: Italy; LU: Luxembourg; NL: The Netherlands; NO: Norway; PT: Portugal; UK: United Kingdom.

We use data of the European Union Statistics on Income and Living Conditions (EU-SILC) for the years 2008-11 (EU-SILC UDB longitudinal file 2011). EU-SILC is an international database that consists of comparable, country-specific data. We select 11 European countries, all of them belonging to EU-15 and Norway. This choice is partly driven by data limitations and partly by the intention to cover the effect of the crisis on a diverse set of countries. We have restricted to the EU-15 to achieve a balance between homogeneity of economic and social development and diversity of institutions during the Recession. Our purpose is mainly illustrative and the analysis is not mean to be an exhaustive examination of all the income sources. We consider two income sources, employee income and self-employment income to analyze the different distributive impact of the crisis on them despite the conclusion we can derive from global measures of inequality change. Employee income is the monetary component of the compensation of employees in cash payable by an employer to an employee (wages and salaries, enhanced rates of pay for overtime, commissions, tips and gratuities, among others). Self-employment income, that is the income received, during the income reference period, by individuals, for themselves or in respect of their family members, as a result of their current or former involvement in self-employment jobs. Those incomes are expressed in constant prices of 2005. The income reference period is the year prior to the interview. The unit of our analysis is the person, and therefore, when we analyze mobility we refer to personal income mobility. To assess the distributive effect of the crisis we study the changes in 2010, the most recent income information, relative to 2007, considered as the baseline.

To reduce the potential influence of outliers, analysis throughout is based on the sample from which incomes in the top 1‰ and the bottom 1‰ have been excluded.[6] We dropped observations with zero and negative incomes as Jenkins and VanKerm (2006). The analysis is carried out over a sample of 13,737 observations for the employee income and 2,278 observations for self-employment income. Our sample is weighted using the EU-SILC longitudinal individual weights (four year duration) corresponding to the 2011 wave. Table 1 shows mean incomes by country.

[6] In this type of analysis each income is measured using a longitudinal average to reduce the potential impact on the estimates of measurement error and transitory income fluctuations, but as our longitudinal dataset spans only four years and we are interested in the income fluctuations in the beginning of the crisis, we have not calculated the longitudinal averages.

Table 1. Mean incomes by country.

	Employee income		Self-employment income	
Country	Mean 2007	Mean 2010	Mean 2007	Mean 2010
AT	265.52	273.65	262.21	270.37
BE	287.18	304.53	224.80	184.43
DK	333.90	350.71	135.84	85.72
ES	198.38	195.93	145.63	155.66
FI	257.26	270.65	128.37	125.04
IT	218.37	217.77	267.10	264.74
LU	393.79	398.26	289.51	367.29
NL	281.09	304.12	225.15	253.42
NO	377.99	402.37	291.30	269.08
PT	122.98	131.92	102.22	91.01
UK	272.85	256.45	283.62	235.46

Our analysis is descriptive in nature, aiming to shed light on the effect of the crisis on inequality in the first years of the crisis, paying special attention to the distribution of progressivity, and reranking over the European countries and trying to depict different effects depending on the part of the distribution where the focus is placed. We are aware of the limitation of the study due to the short period studied, but the data source does not allow following the same individual more than four years.

Table 2 shows the estimates of the inequality change decompositions of employee income for the 11 European countries. Inequality measures for a given year in the cross-sectional database may differ from the estimates of the same year when taking the panel database, since estimation samples are different, in the latter case the sample consists of all individuals present in the database both years.

Table 2. Changes in income inequality of employee incomes measured through the Bonferroni index. Reranking and Progressivity.

Country	B_{2007}	B_{2010}	B_{2010}- B_{2007}	Reranking	Progresivity
AT	545	502	-43	158	201
BE	415	395	-19	87	107
DK	512	500	-12	123	135
ES	453	462	8	142	134
FI	538	529	-9	130	139
IT	435	473	38	108	70
LU	515	509	-6	75	81
NL	566	543	-24	82	106
NO	523	520	-3	130	132
PT	471	496	25	124	99
UK	523	532	9	153	144

We observe that there is not a general trend in the change of employee income inequality. More than half of the countries experienced a reduction in inequality of employee income measured through the Bonferroni index, Austria, Belgium and Netherland being the ones with higher reductions while Italy and Portugal are the ones with greater increments in inequality during this period.

Income changes over the period were progressive or pro-poor in all countries. In those countries with income growth, it was proportionately greater for the relatively poor. In Spain, Italy and United Kingdom, where there was a reduction in the mean employee income, income losses were more concentrated among richer individuals than among poorer ones. But this result about progressivity contrast with the finding that income inequality reduced only for some countries. This is due to the substantial reshuffling of positions in the income distribution.

The re-ranking term accounts for changings in income group membership over time when considering a fixed income group membership (defined by initial income position). In Table 2 we observe that progressivity was relatively high in Austria, and United Kingdom, and at the same time they show high reranking, but reranking is not high enough to offset progressivity and inequality reduces, while in United Kingdom reranking offset progressivity and inequality increases. Luxembourg and Netherlands show low levels of reranking. In both countries progressivity offset reranking and inequality reduces.

Table 3 shows the corresponding estimates for self-employment income. In this type of income we also observe that there is not a general trend in the inequality change. Austria and Netherland are the ones with higher reductions while Luxembourg and Belgium display the greatest increments in inequality during this period. Changes in self-employment incomes are progressive for all countries as in the case of employee income, but in this case mean self-employment income reduces for all countries, except for Austria, Spain, Luxembourg and the Netherlands, and this reduction is more intense for higher incomes. But, again, the considerable reranking in the income distribution offset progressivity in some countries.

Table 3. Changes in income inequality of self-empoyment incomes measured through the Bonferroni index. Reranking and Progressivity.

Country	B_{2007}	B_{2010}	B_{2010}- B_{2007}	Reranking	Progresivity
AT	658	584	-74	197	271
BE	409	544	135	325	191
DK	905	891	-14	42	56
ES	482	502	20	313	293
FI	763	758	-5	127	132
IT	584	586	2	220	218
LU	629	701	72	140	68
NL	749	699	-50	115	165
NO	688	683	-4	104	108
PT	597	590	-7	258	265
UK	667	649	-18	513	531

In Table 3 we observe that progressivity and reranking in self-employment income in relation to the Bonferroni index in the initial year are more disperse than in employee incomes. On the one hand we observe high levels of progressivity in Spain, Portugal and United Kingdom at the same time than high levels of reranking. On the other hand, Denmark displays considerably low levels of progressivity and reranking.

This analysis differs from the one we could have overcome with the decomposition proposed by Silber (1995) and Jenkins and Van Kerm (2006) in the degree of inequality aversion of the index used as the Bonferroni index impose a higher weight on the lower incomes. But we can go deeper and analyze the distribution of the change in inequality, and the distribution of progressivity and re-ranking along the income distribution. In this respect we are decomposing the change in the Bonferroni index into five terms, corresponding to the change in inequality of five measures of inequality that focus on inequality in the low (I(1,5)), middle-low (I(2,4)), middle (I(3,3)), middle-high (I(4,2)) and high (I(5,1)) part of the distribution. With this analysis we would like to study if inequality has been perceived evenly along the distribution. Table 4 shows displays the results.

Table 4. Change in inequality in employee income along the income distribution.

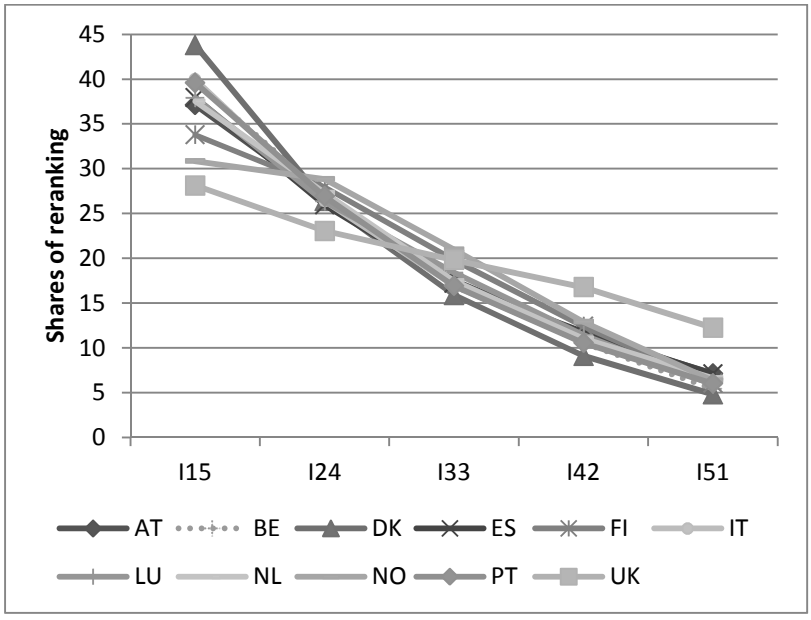
Country		Bonferroni	I15	I24	I33	I42	I51
AT	Change inequality	-43	-45	-53	-47	-39	-33
	Progresivity	201	338	263	186	131	89
	Reranking	158	293	210	139	91	56
BE	Change inequality	-19	-52	-32	-13	-1	3
	Progresivity	107	226	151	88	47	21
	Reranking	87	174	118	75	46	24
DK	Change inequality	-12	-14	-18	-16	-10	-4
	Progresivity	135	284	180	113	66	33
	Reranking	123	270	162	98	56	30
ES	Change inequality	8	18	13	7	4	0
	Progresivity	134	252	172	115	80	50
	Reranking	142	270	185	122	84	50
FI	Change inequality	-9	-5	-8	-11	-11	-10
	Progresivity	139	224	190	139	92	49
	Reranking	130	219	181	128	81	39
IT	Change inequality	38	56	54	41	26	12
	Progresivity	70	160	88	50	32	21
	Reranking	108	216	143	91	58	33
LU	Change inequality	-6	18	12	-6	-24	-30
	Progresivity	81	125	87	75	67	52
	Reranking	75	142	99	69	43	22
NL	Change inequality	-24	-22	-35	-31	-21	-11
	Progresivity	106	176	147	103	66	38
	Reranking	82	154	112	72	45	27
NO	Change inequality	-3	10	2	-6	-10	-9
	Progresivity	132	191	185	143	94	51
	Reranking	130	200	187	136	84	41
PT	Change inequality	25	38	26	21	22	18
	Progresivity	99	207	140	83	44	19
	Reranking	124	245	167	105	65	37
UK	Change inequality	9	-15	0	13	23	26
	Progresivity	144	230	177	138	106	68
	Reranking	153	215	176	152	128	94

We observe that, in general, the change in inequality is not the same if we pay attention to different parts of the distribution. In general, the change in inequality was mostly concentrated in the lower part of the distribution, with the exception of Finland, Luxembourg, Norway and United Kingdom, where the change was more intense in the upper part of the distribution. In particular, Norway and Luxembourg globally decrease inequality, but this decrement is placed mainly in the upper part of the distribution, while in the lower part inequality increases. The opposite takes place in United Kingdom. Therefore, when evaluating changes in income inequality it is important to study the

homogeneity of the change along the distribution. In our illustration , changes in inequality in employee income in the European countries analyzed are more intense for the poorer, with some exceptions.

We now examine the homogeneity in the reranking and progressivity along the income distribution. We observe that in relative terms both are more intense in the lower part of the distribution. Therefore, the main changes in the income pecking order take place in the lower part of the distribution, as Figure 4 shows. Denmark, followed by Portugal are the countries with relatively more reranking in the lower part of the distribution, while United Kingdom is the country with relatively more reranking in the upper part of the distribution.

Figure 4. Reranking in empooyee income along the distribution.



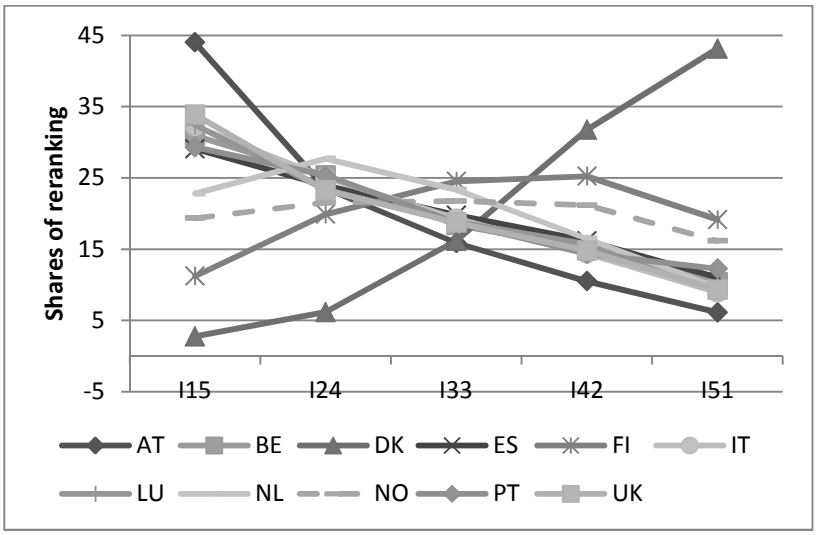
Next we analyze the change in inequality in self-employment income along the distribution. Table 5 presents the results.

Table 5. Change in inequality in self-employemnt income along the income distribution.

Country		Bonferroni	I15	I24	I33	I42	I51
AT	Change inequality	-74	-63	-89	-88	-75	-56
	Progresivity	271	496	320	244	179	116
	Reranking	197	432	231	156	103	60
BE	Change inequality	135	136	171	156	127	84
	Progresivity	191	367	241	143	118	83
	Reranking	325	503	412	299	245	168
DK	Change inequality	-14	0	-1	-4	-17	-46
	Progresivity	56	6	14	38	83	137
	Reranking	42	6	13	34	67	91
ES	Change inequality	20	-7	15	25	32	36
	Progresivity	293	462	360	284	221	136
	Reranking	313	455	375	309	253	173
FI	Change inequality	-5	-2	-3	-2	-5	-13
	Progresivity	132	74	129	157	165	134
	Reranking	127	71	126	155	160	121
IT	Change inequality	2	10	6	0	-4	-2
	Progresivity	218	344	269	212	162	100
	Reranking	220	355	275	212	158	99
LU	Change inequality	72	76	76	78	77	53
	Progresivity	68	151	87	54	34	15
	Reranking	140	228	164	132	110	68
NL	Change inequality	-50	-22	-45	-63	-67	-53
	Progresivity	165	153	205	197	162	108
	Reranking	115	131	159	134	95	56
NO	Change inequality	-4	2	-6	-8	-6	-4
	Progresivity	108	99	117	120	116	88
	Reranking	104	100	112	113	110	84
PT	Change inequality	-7	9	1	-12	-21	-15
	Progresivity	265	369	326	254	206	173
	Reranking	258	378	327	242	185	158
UK	Change inequality	-18	-32	-45	-36	-3	28
	Progresivity	531	901	643	517	383	212
	Reranking	513	870	598	481	379	239

In this case we observe that changes in inequality are more intense in the upper part of the distribution, with the exception of Italy, Luxembourg and United Kingdom. With respect to progressivity and reranking, both are more intense in the lower part of the distribution, but there are clear exceptions. Denmark shows more reranking and progressivity in the upper part of the distribution, and in Finland, Netherlands and Norway reranking is more intense in the middle of the distribution. Figure 5 shows how reranking is shared along the income distribution.

Figure 5. Reranking in self-employemtn income along the distribution.



We observe that decomposition of the change in the Bonferroni index can be used to analyze how inequality change, reranking and progressivity allocate along the income distribution. This decomposition allows the identification of parts of the distribution that has suffered the changes in income inequality with more intensity and we can depict the contribution of reranking and progressivity to this change.

5. Conclusions

Countries with similar inequality changes do not necessarily have similar achievement in pro-poor economic changes and re-ranking nor a similar behavior in the different parts of the distribution. The decomposition of income inequality measures that we propose in this paper allows an interpretation of the change in inequality focusing on the change in inequality in different parts of the distribution and also enables us to analyze the intensity of progressivity, and reranking along the distribution. Interpreting the change in inequality as a weighted measure of inequality change indexes that focus on different parts of the distribution, also offers the prospect of constructing new reranking and pro-poor growth measures that focus on the changes in different parts of the distribution. Therefore we can pinpoint the contribution of each "local" index to the global measure of inequality change, progressivity and reranking..

Making use of this decomposition we analyze the effect of the economic crisis in 11 European countries and compare the patterns of distributional change in employee income and self-employment income, determining which parts of the distribution contributed more to the reranking or to the changes in the progressivity and thus unmasking the different situations that the global result encompasses.

As far as the impact of the crisis on income (employee or self-employment) inequality is concerned, there is no uniform pattern among the EU countries. The vehemence with which the crisis affects a country is linked to its labour market regulation. In general the incomes of poorer households were less affected by the recession than those of richer individuals as the change in incomes is progressive and in all countries employee and self-employment income changes are pro-poor, but reranking of individuals in some countries offset progressivity and inequality increases.

The analysis of the changes along the distribution shows that changes in inequality are not homogeneous along the income distribution and confirm the asymmetric effect of the crisis on employee income and self-employment incomes. Inequality changes in the distribution of the former were concentrated on the lower part of the distribution with more reranking in lower incomes, while changes in self-employment income inequality is more intense in the upper or middle part of the distribution for most of the countries, and reranking is more concentrated on the upper part of the distribution but for the Nordic countries and the Netherlands.

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